概率论与数理统计

华中科技大学 概率统计系

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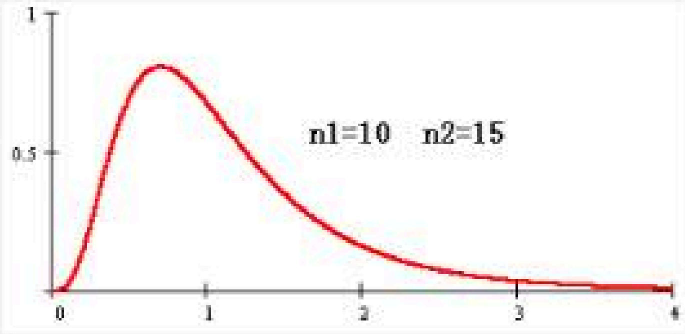
F 分布

设$X \sim \chi^2(n_1)$与$Y \sim \chi^2(n_2)$相互独立，则称$F = \dfrac{X/n_1}{Y/n_2}$

服从自由度为(n_1,n_2)的F分布，记为$F \sim F(n_1,n_2)$.

$$f(x) = \begin{cases} \dfrac{\Gamma(\frac{n_1+n_2}{2})}{\Gamma(\frac{n_1}{2})\Gamma(\frac{n_2}{2})} \dfrac{\left(\frac{n_1}{n_2}\right)\left(\frac{n_1}{n_2}x\right)^{\frac{n_1}{2}-1}}{\left(1+\frac{n_1}{n_2}x\right)^{\frac{n_1+n_2}{2}}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



上侧分位数

$$P(F > F_\alpha(n_1,n_2)) = \alpha$$

F 分布

$$\frac{1}{F} = \frac{Y/n_2}{X/n_1} \sim F(n_2, n_1)$$

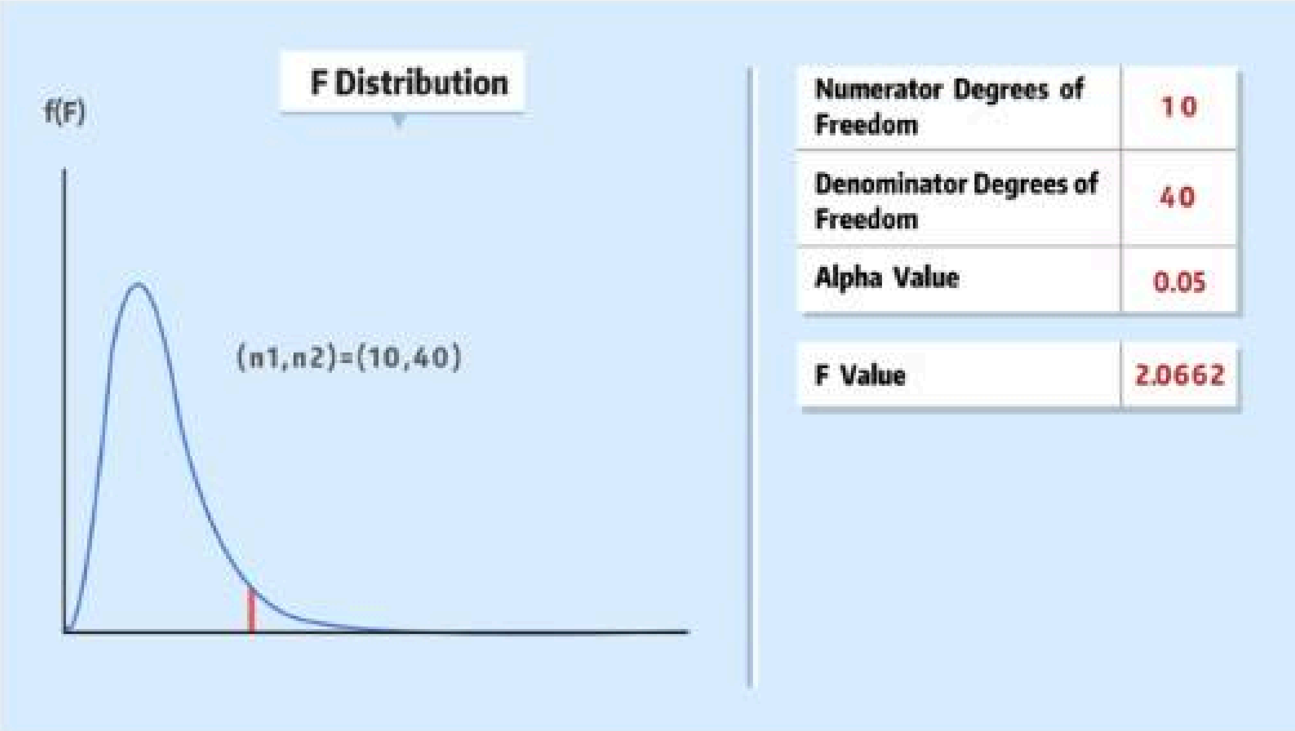
查表 $F_{0.975}(15,10) = \dfrac{1}{F_{0.025}(10,15)} = \dfrac{1}{3.06} = 0.33$

$$= P\left(\frac{1}{F} > \frac{1}{F_\alpha(n_1,n_2)}\right) = 1-\alpha$$

$\alpha = 0.025$

$n_2 \backslash n_1$	8	9	10	12	15	20
13	3.39	3.31	3.25	3.15	3.05	2.95
14	3.29	3.21	3.15	3.05	2.95	2.84
15	3.20	3.12	3.06	2.96	2.86	2.76
16	3.12	3.05	2.99	2.89	2.79	2.68
17	3.06	2.98	2.92	2.82	2.72	2.62
18	3.01	2.93	2.87	2.77	2.67	2.56

F 分布


F Distribution
f(F)
(n1,n2)=(10,40)
Numerator Degrees of Freedom
10
Denominator Degrees of Freedom
40
Alpha Value
0.05
F Value
2.0662

F 分布

例 $X \sim N(0,\sigma^2), X_1, X_2, X_3, X_4 \ i.i.d.$

$Y = (X_1 + X_2)^2 / (X_3 - X_4)^2 \sim ?$

解 $X_1 + X_2,\ X_3 - X_4 \ \ i.i.d.\ \ N(0, 2\sigma^2)$

$$\chi_1^2 = [(X_1 + X_2)/\sqrt{2}\sigma]^2 \sim \chi^2(1)$$

$$\chi_2^2 = [(X_3 - X_4)/\sqrt{2}\sigma]^2 \sim \chi^2(1)$$

$$\frac{\chi_1^2/1}{\chi_2^2/1} = \frac{(X_1 + X_2)^2}{(X_3 - X_4)^2} = Y \sim F(1,1)$$

单正态总体的抽样分布

抽样定理 设$X_1, X_2, \ldots, X_n$为来自正态总体$N(m, s^2)$的一个样本，则

$$(1)\ \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right);$$

证明(1)

$$\bar{X} = \sum_{i=1}^{n} \frac{1}{n} X_i \sim N\left(\sum_{i=1}^{n} \frac{1}{n}\mu_i, \sum_{i=1}^{n} \frac{1}{n^2}\sigma_i^2\right)$$

$$\sim N\left(\mu, \frac{\sigma^2}{n}\right)$$