电路理论基础

—电路理论（基础篇）

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第11章 正弦稳态电路的功率

11.1 概述

11.2 瞬时功率

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11.9 拓展与应用

●重 点

1. 熟练掌握瞬时功率，有功功率与无功功率，视在功率及功率因数

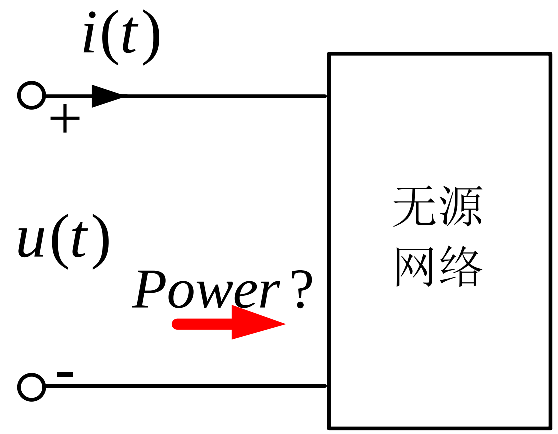
2. 复功率及功率守恒，功率因数校正

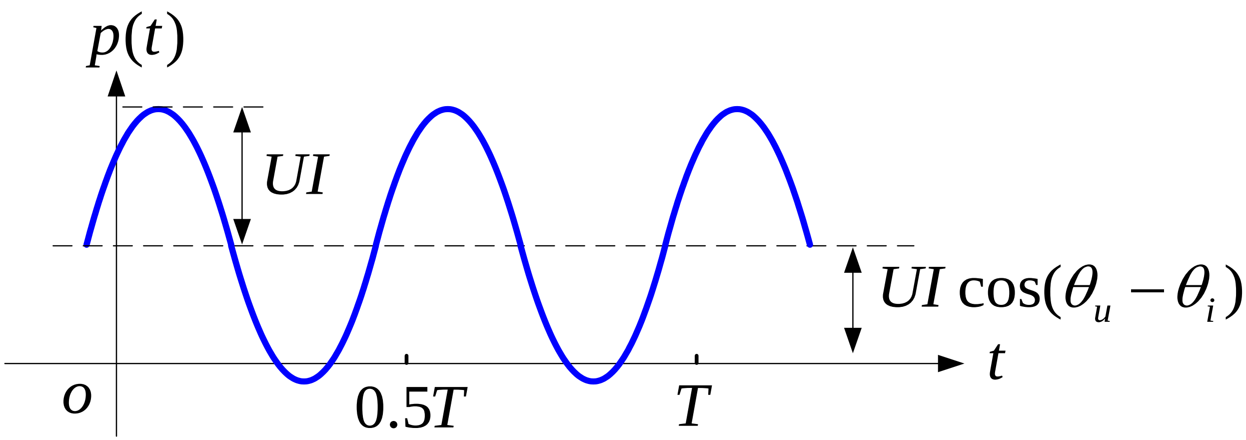
3. 最大有功功率传输

◆ 无源网络的瞬时功率

$$p(t)=u(t)i(t)=2UI\cos(\omega t+\theta_u)\cos(\omega t+\theta_i)$$

$$=UI\cos(\theta_u-\theta_i)+UI\cos(2\omega t+\theta_u+\theta_i)$$





R $\quad p(t)=UI[1+\cos 2(\omega t+\theta_i)]\geq 0$

L $\quad p(t)=UI\cos(90°)+UI\cos[2(\omega t+\theta_i)+90°]$ $\boxed{=-UI\sin 2(\omega t+\theta_i)}$

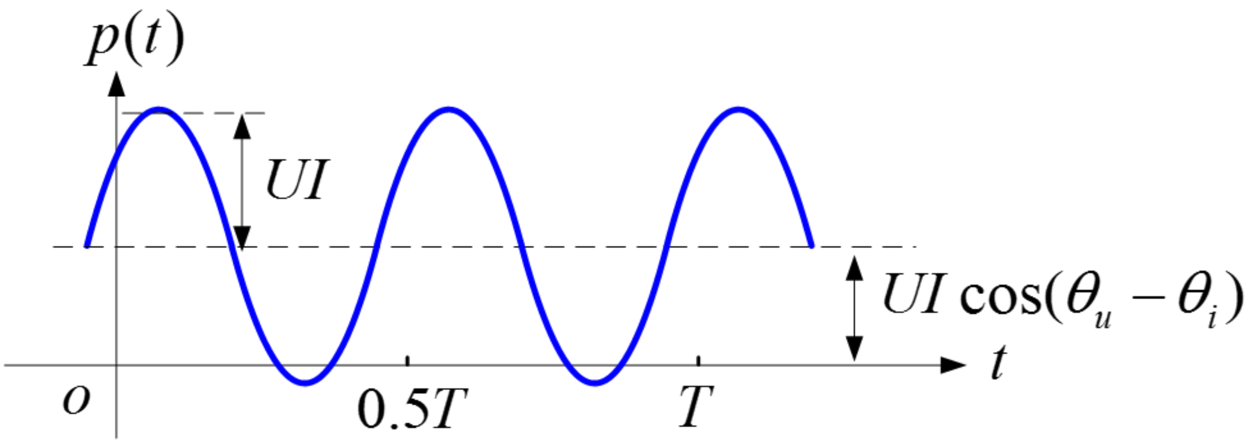
C $\quad p(t)=UI\cos(-90°)+UI\cos[2(\omega t+\theta_i)-90°]$ $\boxed{=UI\sin 2(\omega t+\theta_i)}$

◆ 元件的瞬时功率

R $p(t)=UI[1+\cos 2(\omega t+\theta_i)]\geq 0$ ➢ 电阻恒吸收功率

L $p(t)=UI\cos(90°)+UI\cos[2(\omega t+\theta_i)+90°]$ $\boxed{=-UI\sin 2(\omega t+\theta_i)}$

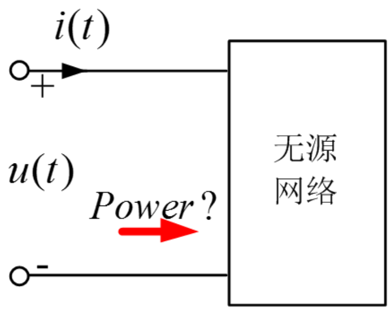
C $p(t)=UI\cos(-90°)+UI\cos[2(\omega t+\theta_i)-90°]$ $\boxed{=UI\sin 2(\omega t+\theta_i)}$



➢ 电感、电容的平均功率为零

➢ 电感、电容的功率有互补性

◆ RLC支路的瞬时功率



$$p(t)=u(t)i(t)=2UI\cos(\omega t+\theta_u)\cos(\omega t+\theta_i)$$

$$=UI\cos(\theta_u-\theta_i)+UI\cos(2\omega t+\theta_u+\theta_i)$$

$$=UI\cos(\theta_u-\theta_i)[1+\cos 2(\omega t+\theta_i)]-UI\sin(\theta_u-\theta_i)\sin 2(\omega t+\theta_i)$$

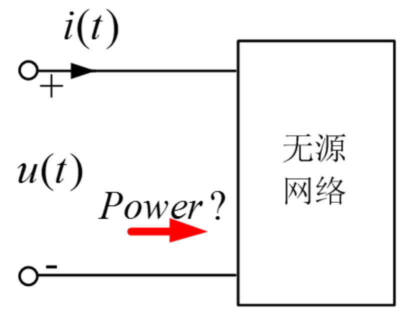
$$2\omega t+\theta_u+\theta_i=[2(\omega t+\theta_i)+(\theta_u-\theta_i)]$$

$UI\cos(\theta_u-\theta_i)[1+\cos 2(\omega t+\theta_i)]$ 表示什么?

$-UI\sin(\theta_u-\theta_i)\sin 2(\omega t+\theta_i)$ 表示什么?

11.2 瞬时功率

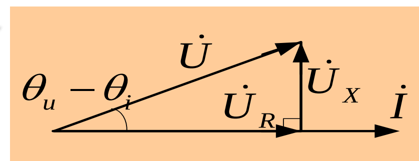
◆RLC支路的瞬时功率



$$p(t)=u(t)i(t)=2UI\cos(\omega t+\theta_u)\cos(\omega t+\theta_i)$$

$$=UI\cos(\theta_u-\theta_i)+UI\cos(2\omega t+\theta_u+\theta_i)$$

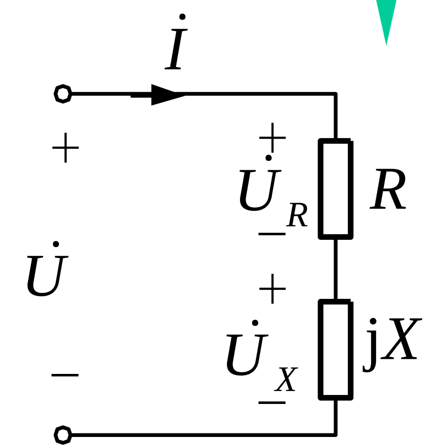
$$=\underbrace{UI\cos(\theta_u-\theta_i)[1+\cos 2(\omega t+\theta_i)]}_{\text{吸收功率}}\underbrace{-UI\sin(\theta_u-\theta_i)\sin 2(\omega t+\theta_i)}_{\text{交换功率}}$$



$$=p_R(t)+p_X(t)$$

$$p_R(t)=u_R(t)i(t)=\sqrt{2}U\cos(\theta_u-\theta_i)\cos(\omega t+\theta_i)\sqrt{2}I\cos(\omega t+\theta_i)$$

$$=UI\cos(\theta_u-\theta_i)[1+\cos 2(\omega t+\theta_i)]$$

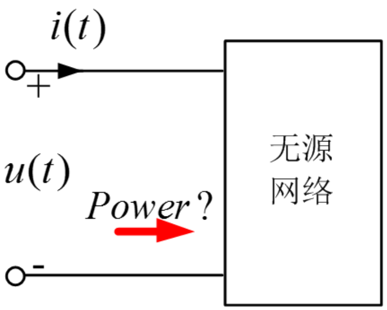


$$p_X(t)=u_X(t)i(t)$$

$$=\sqrt{2}U\left|\sin(\theta_u-\theta_i)\right|\cos(\omega t+\theta_i\pm 90^\circ)\sqrt{2}I\cos(\omega t+\theta_i)$$

$$=-UI\sin(\theta_u-\theta_i)\sin 2(\omega t+\theta_i)$$

11.3 有功功率与无功功率



$$p(t)=u(t)i(t)=2UI\cos(\omega t+\theta_u)\cos(\omega t+\theta_i)$$

$$=UI\cos(\theta_u-\theta_i)+UI\cos(2\omega t+\theta_u+\theta_i)$$

$$=\underbrace{UI\cos(\theta_u-\theta_i)[1+\cos 2(\omega t+\theta_i)]}_{\text{吸收功率}}-\underbrace{UI\sin(\theta_u-\theta_i)\sin 2(\omega t+\theta_i)}_{\text{交换功率}}$$

$$=p_R(t)+p_X(t)$$

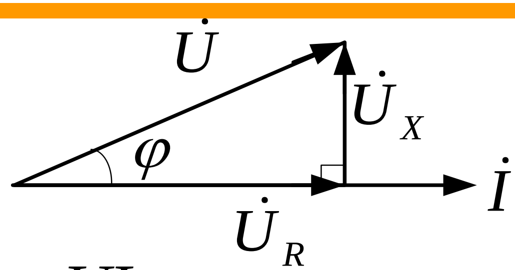
有功功率　　瞬时功率的平均值，也称平均功率

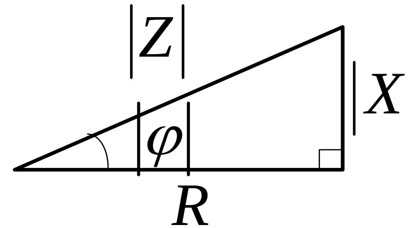
$$P=\frac{1}{T}\int_0^T p(t)dt=UI\cos(\theta_u-\theta_i)=UI\cos\varphi$$ W 瓦　为吸收功率！

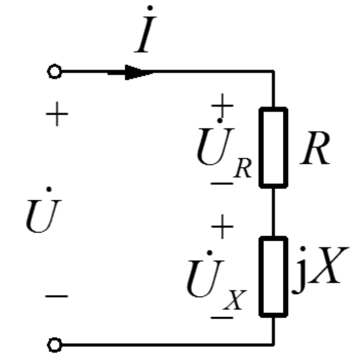
无功功率　　网络与电源往复交换功率的幅值

$$Q=UI\sin(\theta_u-\theta_i)=UI\sin\varphi$$ var 乏　吸收还是发出？

11.3 有功功率与无功功率

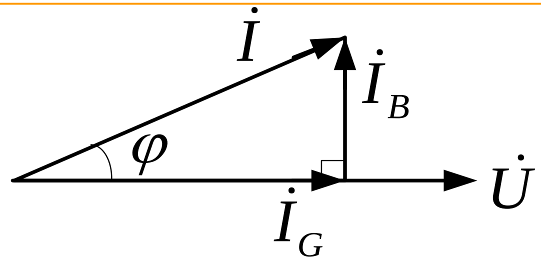


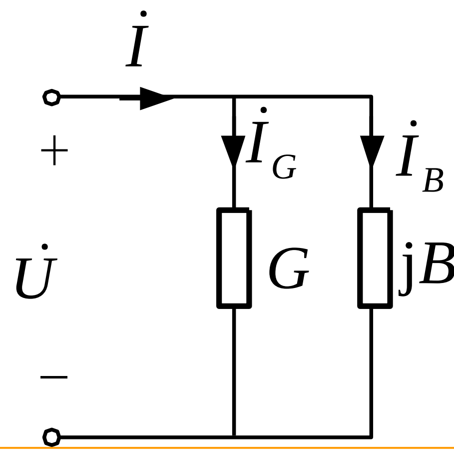




$$P = UI\cos\varphi = U_R I \;= RI^2$$

$$Q = UI\sin\varphi = \pm U_X I = XI^2$$

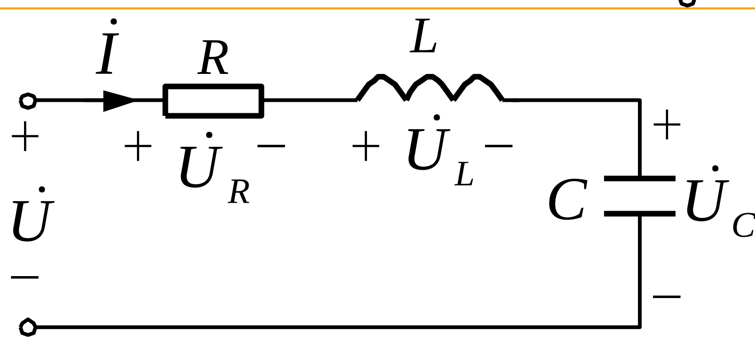




$$P = UI\cos\varphi = UI_G = GU^2$$

$$Q = UI\sin\varphi = \pm UI_B = -BU^2$$

R	$P_R = UI$	$Q_R = 0$
L	$P_L = 0$	$Q_L = UI$
C	$P_C = 0$	$Q_C = -UI$

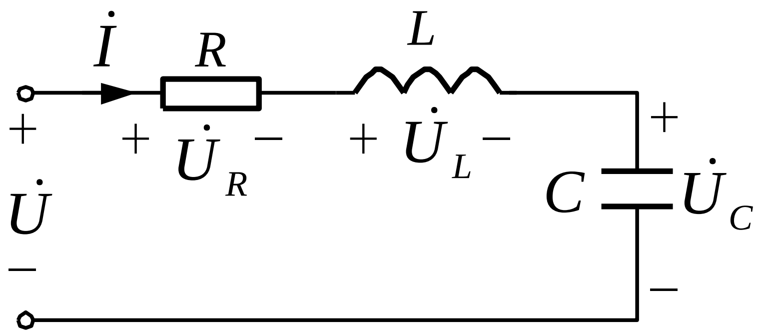


$$P = UI\cos\varphi = RI^2$$

$$Q = UI\sin\varphi = (\omega L - \frac{1}{\omega C})I^2$$

已知：R=100欧, L=0.2亨, C=10微法,　$u=100\sqrt{2}\cos(1000t+60^\circ)$

计算RLC支路的有功功率与无功功率。



$$I=\frac{1}{\sqrt{2}}A$$

$$P=UI\cos\varphi=100*\frac{1}{\sqrt{2}}*\frac{\sqrt{2}}{2}=50w$$

$$Q=UI\sin\varphi=(\omega L-\frac{1}{\omega C})I^2=50\,\text{var}$$

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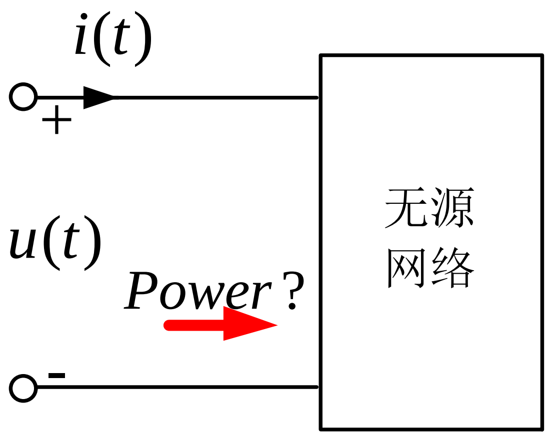
11.4 视在功率及功率因数

视在功率：负载消耗或电源提供的有功功率的上限。

$$S = UI = \sqrt{P^2 + Q^2}$$

单位：VA(伏安)



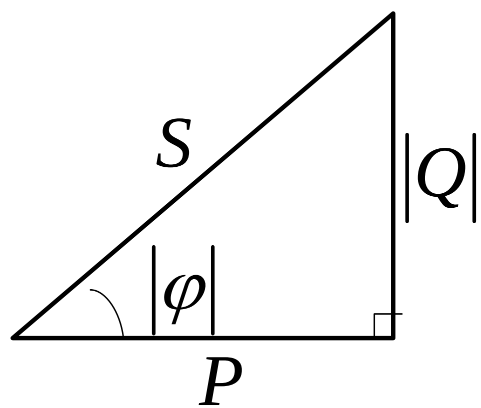
功率因数：有功功率与视在功率的比值

$$\lambda = \cos(\theta_u - \theta_i) = \cos\varphi \ = \frac{P}{S}$$

$Q > 0$，感性网络，滞后功率因数

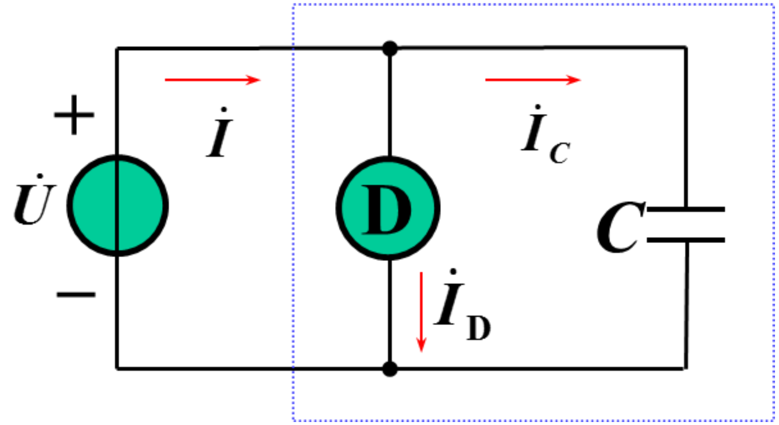
$Q = 0$，阻性网络，单位功率因数

$Q < 0$，容性网络，超前功率因数



功率三角形

已知：电动机 P_D=1000W，U=220V，f=50Hz，C=30uF，$\cos\phi_D = 0.8$(滞后）。求负载电路的功率因数。



解：$I_D = \dfrac{P}{U\cos\varphi_D} = \dfrac{1000}{220\times 0.8} = 5.68\text{A}$

$$\cos\phi_D = 0.8(\text{滞后})\ \phi_D = 36.9^\circ$$

设 $\dot{U} = 220\angle 0^\circ$

$$\dot{I}_D = 5.68\angle -36.9^\circ$$

$$\dot{I}_C = \text{j}\omega C 220\angle 0^\circ = \text{j}2.08$$

$$\dot{I} = \dot{I}_D + \dot{I}_C = 4.54 - \text{j}1.33 = 4.73\angle -16.3^\circ$$

$$\therefore\ \cos\varphi = \cos[0^\circ - (-16.3^\circ)] = 0.96\ \ (\text{滞后})$$

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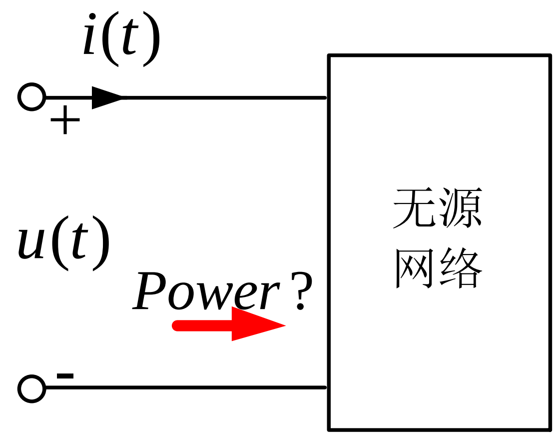
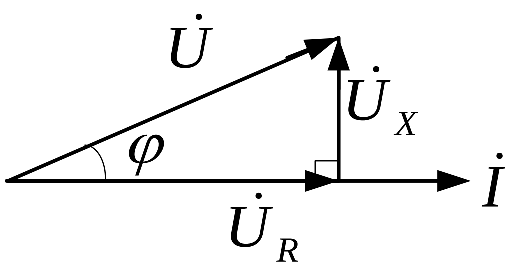
复功率

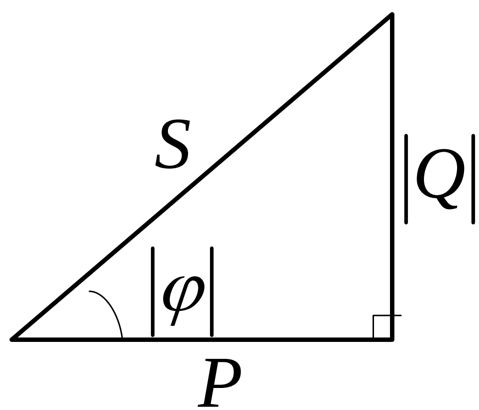
$$\bar{S}=P+jQ=UI\cos\varphi+jUI\sin\varphi=\dot{U}\dot{I}^{*}$$

电压相量和电流相量的共轭复数的乘积

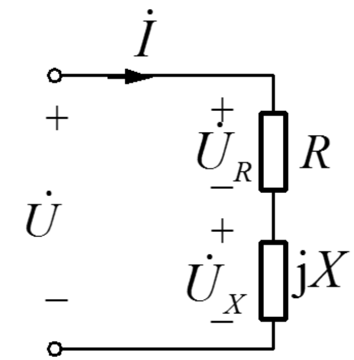


$$\bar{S}=\dot{U}\dot{I}^{*}=(Z\dot{I})\dot{I}^{*}=I^{2}Z$$





功率三角形

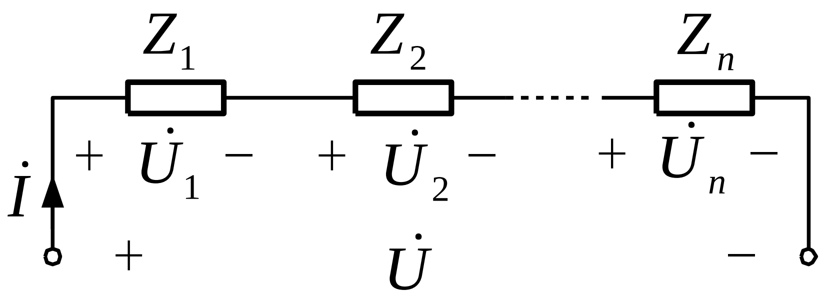


复功率守恒

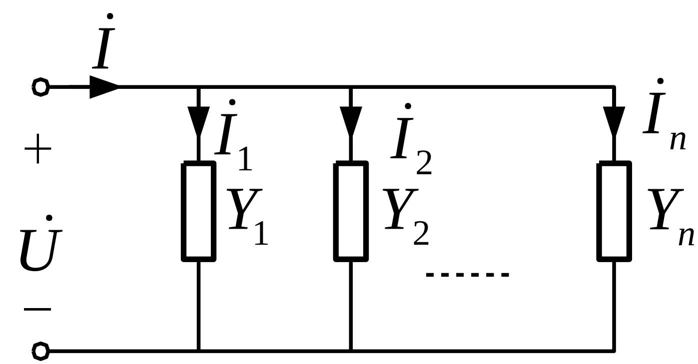
$$\bar{S}=\dot{U}\dot{I}^{*}=\left(\sum_{k=1}^{n}\dot{U}_k\right)\dot{I}^{*}$$

$$=\sum_{k=1}^{n}(\dot{U}_k\dot{I}^{*})$$

$$=\sum\bar{S}_k$$



$$\boxed{\bar{S}=\sum\bar{S}_k=\sum P_k+j\sum Q_k}$$



$$\bar{S}=\dot{U}\dot{I}^{*}=\dot{U}\left(\sum_{k=1}^{n}\dot{I}_k^{*}\right)=\left(\sum_{k=1}^{n}\dot{U}\dot{I}_k^{*}\right)=\left(\sum_{k=1}^{n}\bar{S}_k\right)$$

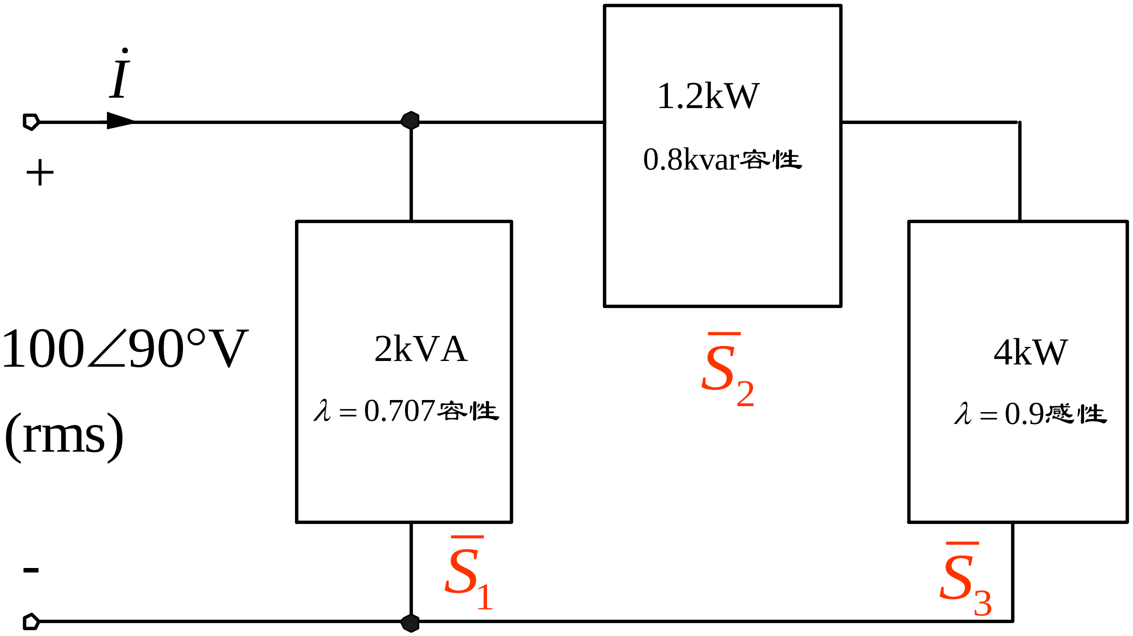
例 确定电源提供的复功率及电流

$$\bar{S}_1 = 2\times(0.707 - j0.707) = 1.414 - j1.414$$

$$\bar{S}_2 = 1.2 - j0.8$$

$$\bar{S}_3 = 4 + j4\times\tan(\arccos 0.9) = 4 + j1.937$$

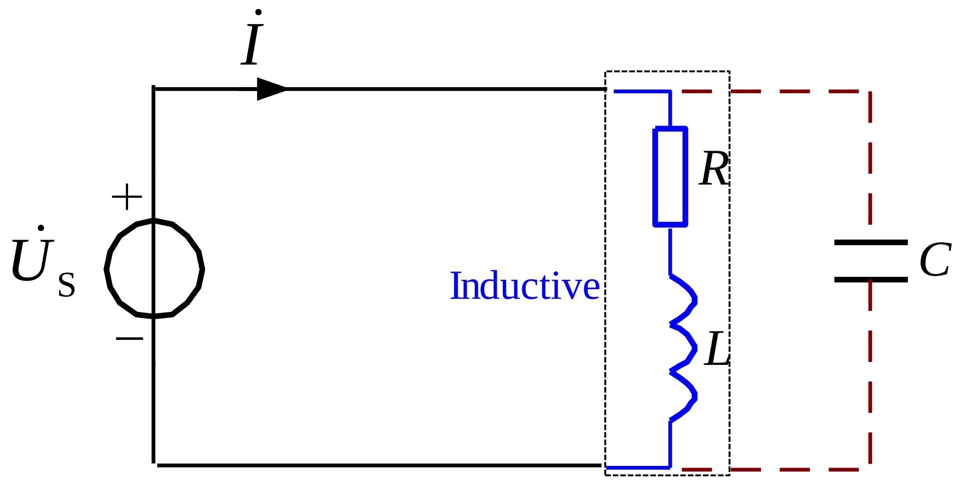
$\dot{U} = 100\angle 90°\text{V}$ (rms)



$$\bar{S} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 6.614 - j0.277 = 6.62\angle -2.4°\text{kVA}$$

$$\dot{I} = \left(\frac{\bar{S}}{\dot{U}}\right)^* = \left(\frac{6620\angle -2.4°}{100j}\right)^* = 66.2\angle 92.4°A$$

无功补偿



容量为2500VA, 电压为220V (rms), 频率为50Hz 的正弦电源，通过输电线路给额定电压为220V，功率为 1210W、功率因数为0.5的感性负载供电，欲使功率因数提高到0.9，确定并联电容值。

方法1：利用功率守恒计算并联电容

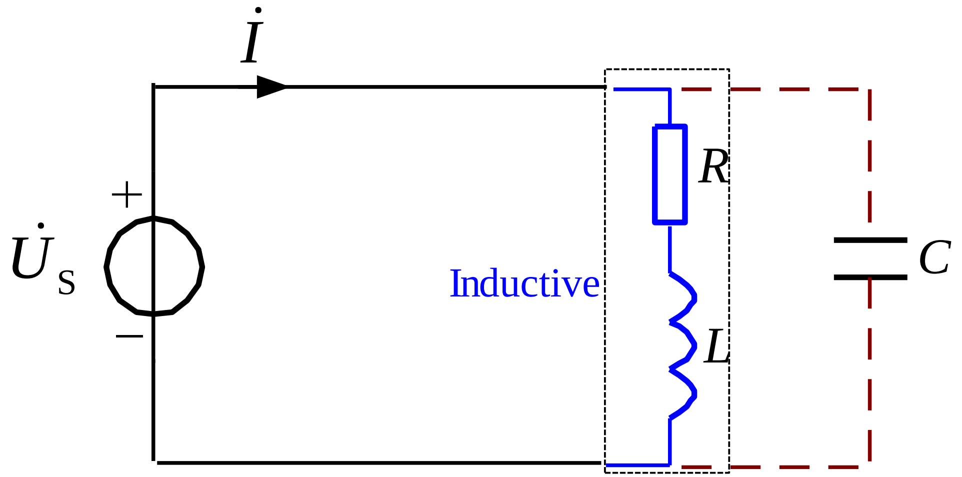
方法2：通过计算线路电流来计算并联电容

电源:

2500VA, 220V (rms), 50Hz

感性负载:

1210W ,功率因数0.5 ⟶ 0.9



方法1

$$Q_1 = 1210\tan(\arccos 0.5) = 2096\text{Var}$$

$$Q_2 = 1210\tan(\arccos 0.9) = 586\text{Var}$$

$$Q_C = Q_1 - Q_2 = \omega C U_s^2 = 2\pi \times 50 \times 220^2 C$$

$$\boxed{C = 99.3\mu\text{F}}$$

$$S_1 = \sqrt{1210^2 + (1210\sqrt{3})^2} = 1210/0.5 = 2420\text{VA}$$

$$S_2 = \sqrt{1210^2 + (586)^2} = 1210/0.9 = 1344.5\text{VA}$$

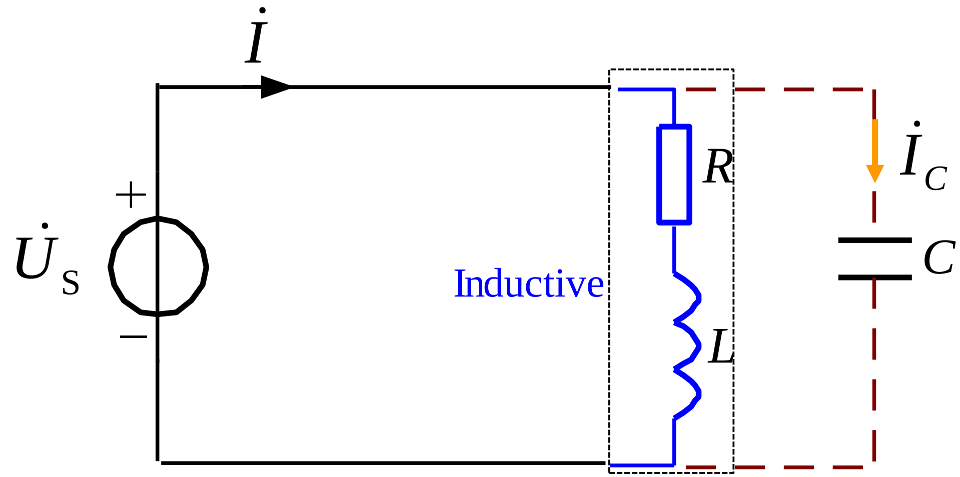
提高电源容量利用率

电源:

2500VA, 220V (rms), 50Hz

感性负载:

1210W ,功率因数0.5 → 0.9



方法2 $\dot{U}_s = 220\angle 0°$

$$\dot{I}' = \frac{P}{U_s \cos\varphi_1}\angle -\varphi_1 = \frac{1210}{220\times 0.5}\angle -60° = 11\angle -60°$$

$$\dot{I}'' = \frac{P}{U_s \cos\phi_2}\angle -\phi_2 = \frac{1210}{220\times 0.9}\angle -25.84° = 6.11\angle -25.84°$$

$$\dot{I}_C = \frac{\dot{U}_s}{-\mathrm{j}X_C} = \frac{220}{-\mathrm{j}X_C} \qquad \dot{I}'' = \dot{I}_C + \dot{I}' \Rightarrow\ 6.11\angle -25.84° = 11\angle -60° + \frac{220}{-\mathrm{j}X_C}$$

$$C = 99.3\mu F$$

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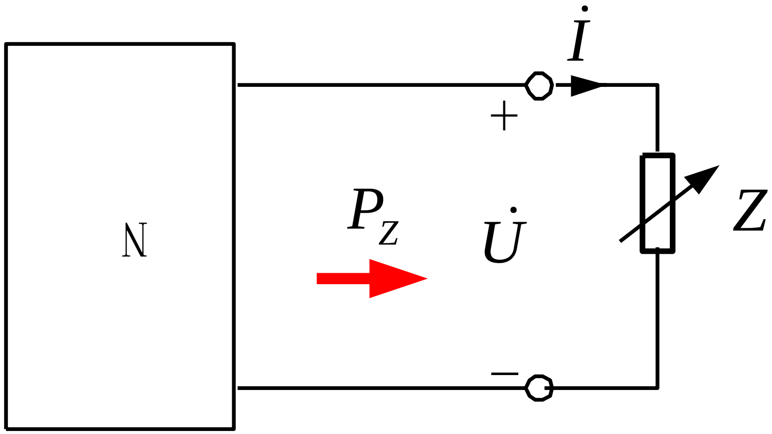
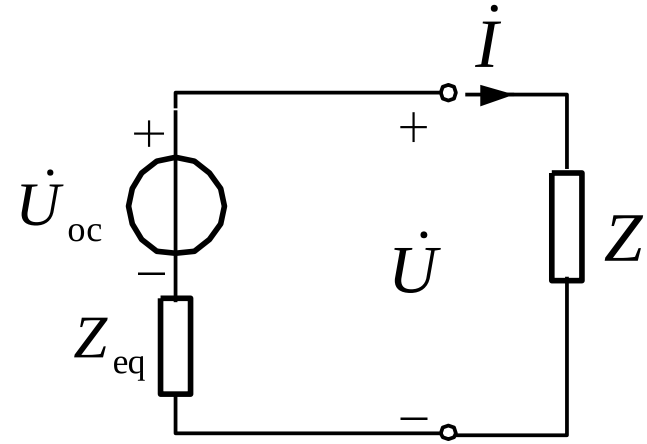
11.7 最大有功功率传输





$$Z = ? \Rightarrow P_Z = \max = ?$$

$$Z_{eq} = R_{eq} + jX_{eq} \qquad Z = R + jX$$

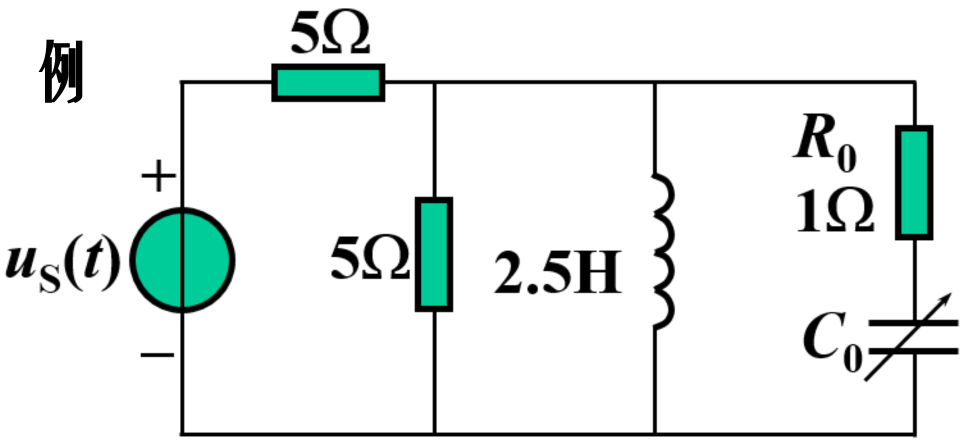
$$P_Z = I^2 R = \frac{U_{oc}^2 R}{\left|Z_{eq} + Z\right|^2} = \frac{U_{oc}^2 R}{(R + R_{eq})^2 + (X + X_{eq})^2}$$

$$\begin{cases} \dfrac{\partial P_Z}{\partial X} = 0 \\ \dfrac{\partial P_Z}{\partial R} = 0 \end{cases} \Rightarrow \begin{cases} X + X_{eq} = 0 \\ R = R_{eq} \end{cases}$$

$$\boxed{Z = Z_{eq}^* \qquad P_{Z\max} = \frac{U_{oc}^2}{4R_{eq}}}$$

最大有功功率传输条件：共轭匹配

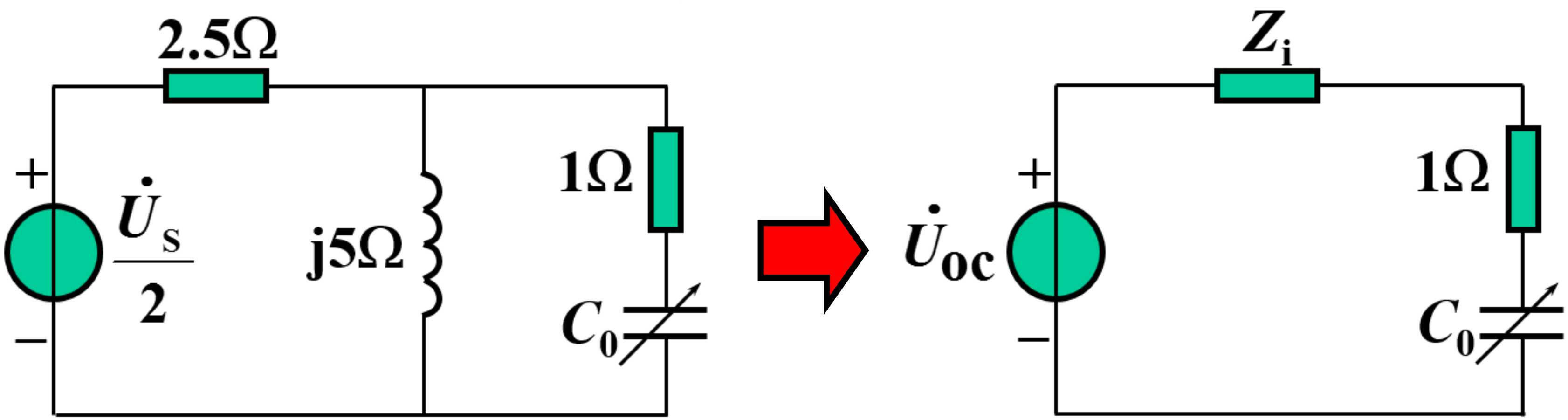
例



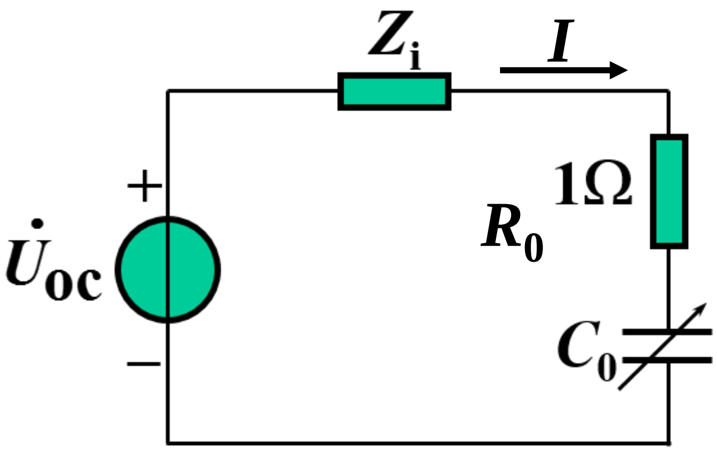
已知：$u_S(t)=\sqrt{2}\cos(2t-45^\circ)$ V

要使R_0上获最大有功功率，则C_0为何值？

解：用戴维南等效电路：$\dot{U}_S = 1\angle -45^\circ$ V



$$\dot{U}_{oc} = \frac{0.5\angle -45^\circ}{2.5+\text{j}5}\cdot \text{j}5 = 0.447\angle -18.4^\circ \text{ V} \quad Z_i = \frac{2.5\times \text{j}5}{2.5+\text{j}5} = 2+\text{j}1\,\Omega$$



$$P_0 = I^2 R$$

要使R_0上功率最大，只需使电流I 最大即可。

$$\dot{I} = \frac{\dot{U}_{oc}}{Z_i + R_0 - j1/(\omega C_0)}, \quad I = \frac{U_{oc}}{|Z_i + R_0 - j1/(\omega C_0)|}$$

$$\begin{aligned} Z_i + R_0 - j1/(\omega C_0) &= 2 + j1 + 1 - j1/(\omega C_0) \\ &= 3 + j[1 - 1/(\omega C_0)]\ \Omega \end{aligned}$$

$$\frac{1}{\omega C_0} = 1, \quad C_0 = \frac{1}{\omega} = \frac{1}{2} = 0.5\,\mathrm{F}$$

第11章 正弦稳态电路的功率

11.1 概述

11.2 瞬时功率

11.3 有功功率与无功功率

11.4 视在功率及功率因数

11.5 复功率及功率守恒

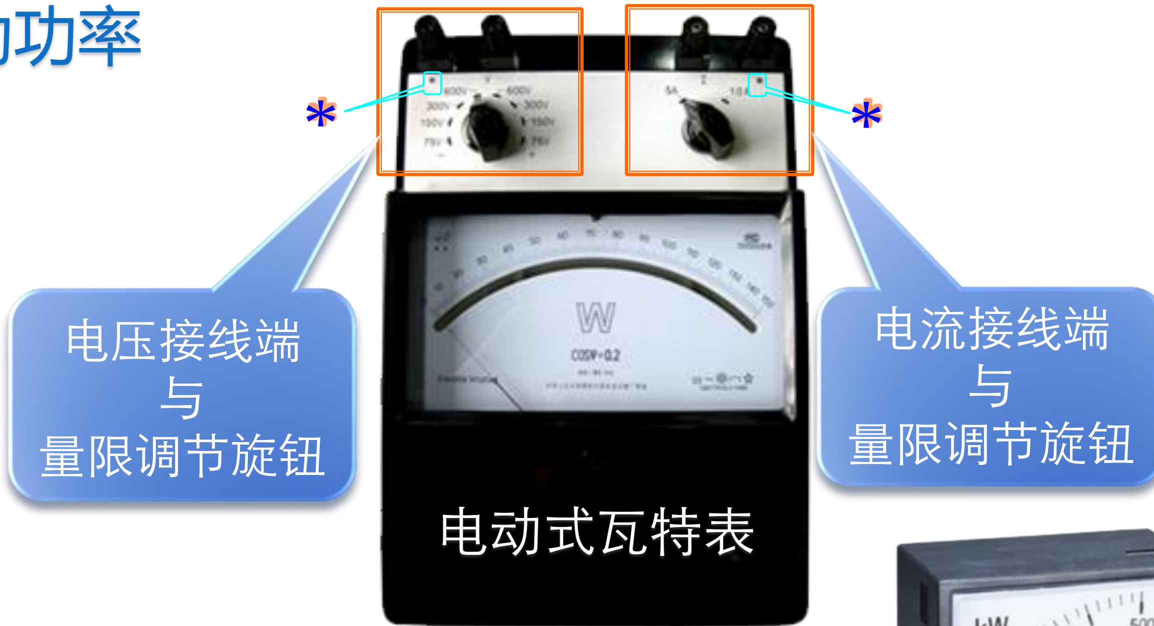
11.6 功率因数校正

11.7 最大有功功率传输

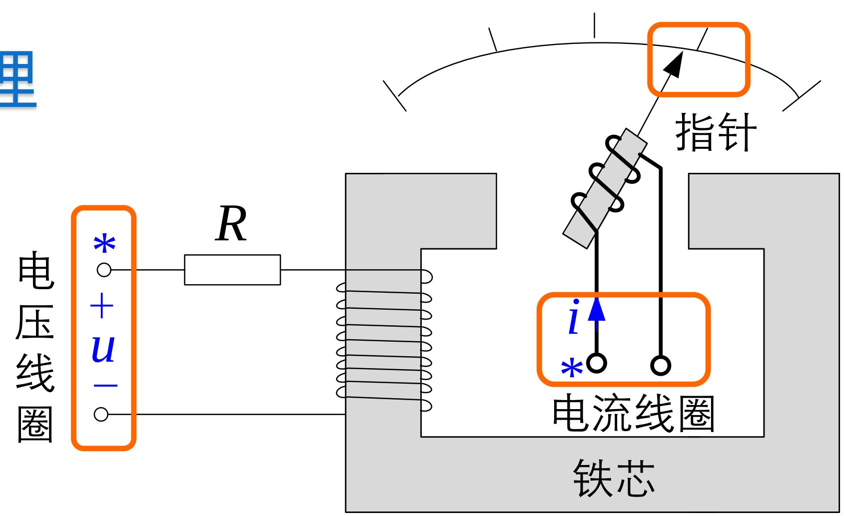
11.8 有功功率测量

11.9 拓展与应用

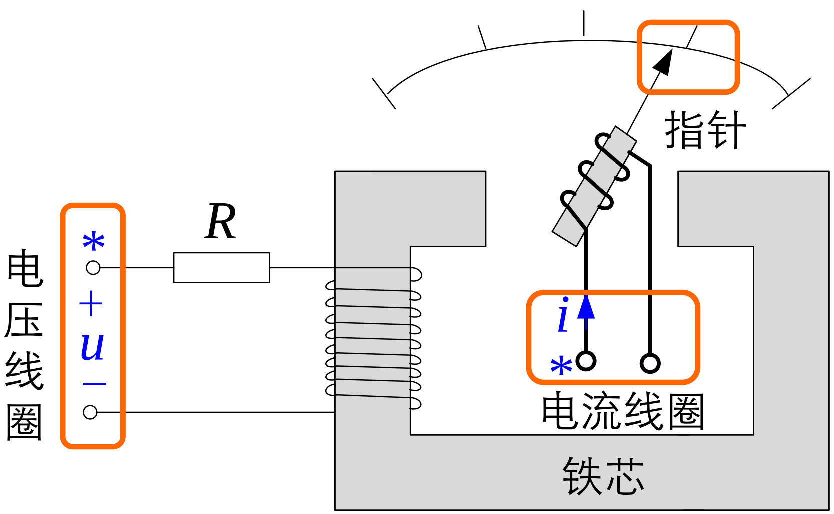
☞ 瓦特表测量有功功率



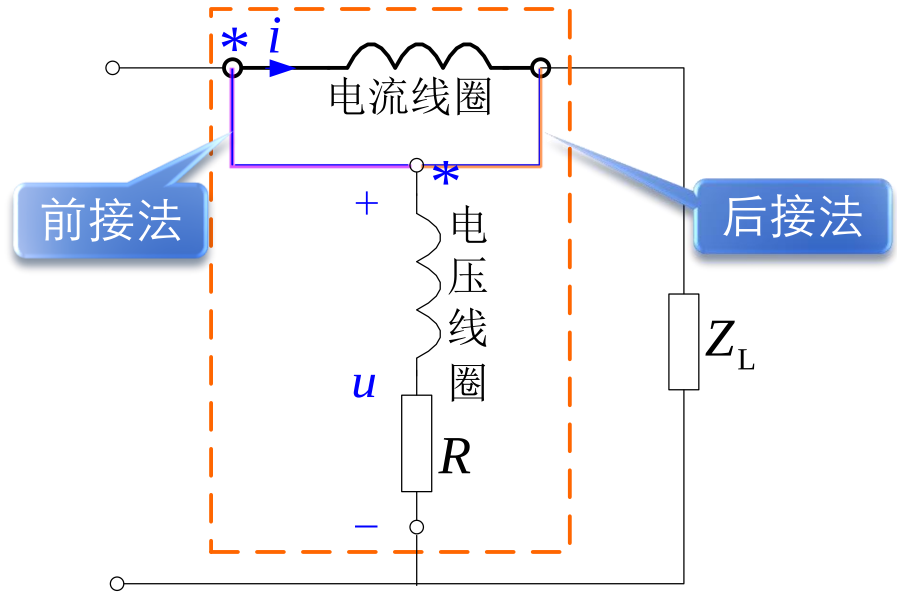
☞ 电动式瓦特表原理



11.8 有功功率测量



☞瓦特表的接线方式



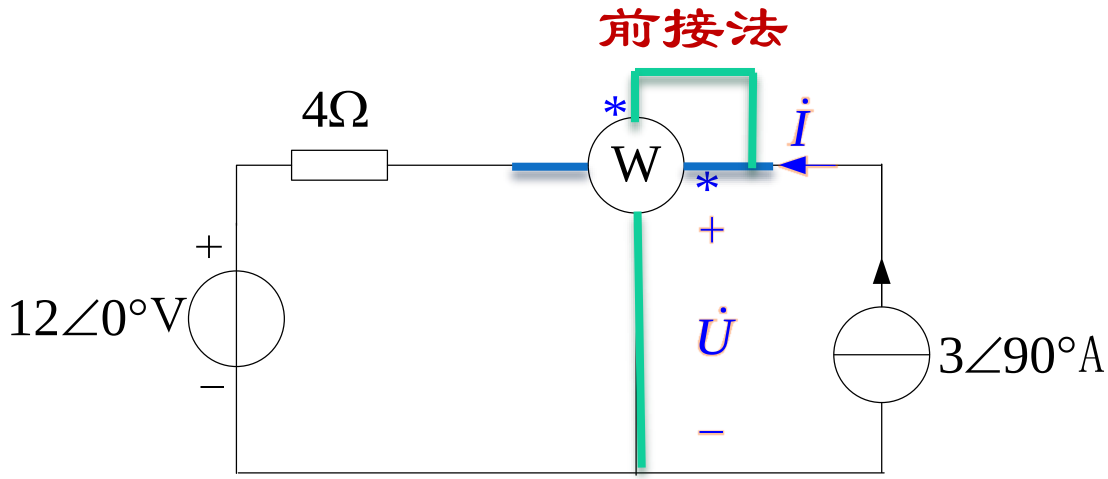
电流线圈与负载串联

电压线圈与负载并联

☞瓦特表的读数

$$\boxed{P=}\frac{1}{T}\int_0^T uidt \;=\boxed{\mathrm{Re}[\dot{U}\times\dot{I}^*]}$$

例　确定瓦特表的读数，及读数的物理含义。



瓦特表的读数 $P = \mathrm{Re}[\dot{U} \times \dot{I}^*]$

$$\dot{U} = 4\times 3\angle 90° + 12\angle 0° = 12\sqrt{2}\angle 45°\ \mathrm{V}$$

$$P = \mathrm{Re}[12\sqrt{2}\angle 45° \times 3\angle -90°] = 36\mathrm{W}$$

是电流源发出的有功功率，

也是电压源和电阻吸收的有功功率之和。

谢 谢!