电路理论基础

——电路理论（高级篇）

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第14章 正弦稳态电路的频率响应

● 重 点

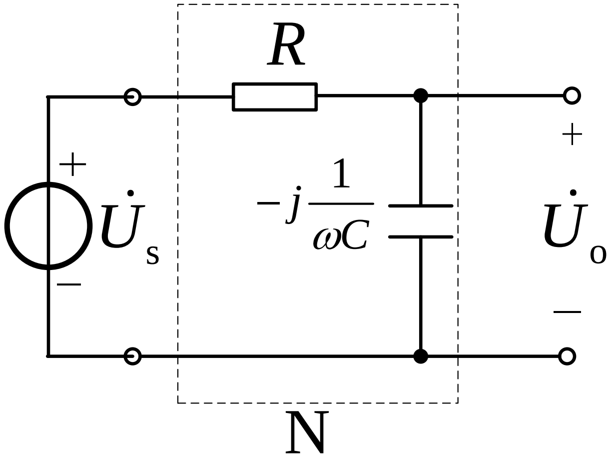
1. 熟练掌握传递函数与频率响应

2. 熟练掌握谐振电路，明确谐振电路的特点、带通滤波的概念

Q1：下面的正弦稳态电路，参数一定，只改变电源的频率，响应如何变化？

$$\dot{U}_o(\omega)=\frac{-j\dfrac{1}{\omega C}}{R-j\dfrac{1}{\omega C}}\times\dot{U}_s(\omega)$$

$$=\frac{U_s}{\sqrt{1+(\omega CR)^2}}\angle\phi_s-\arctan(\omega CR)$$



Q2：用什么描述响应随频率的变化规律？

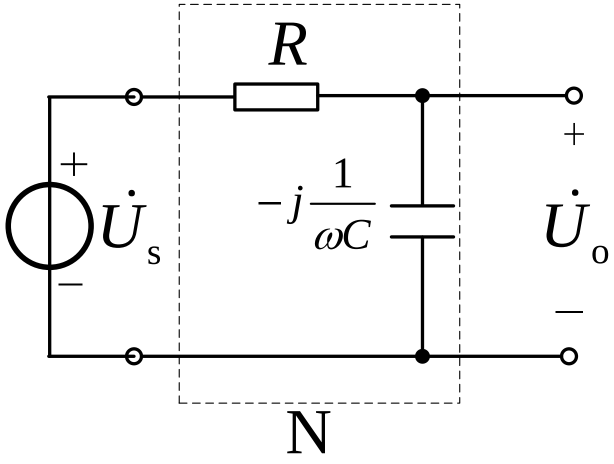
$$\frac{\dot{U}_o(\omega)}{\dot{U}_s(\omega)}=\frac{1}{1+j\omega RC}=\frac{1}{\sqrt{1+(\omega CR)^2}}\angle-\arctan(\omega CR)$$

Q3：研究响应随频率变化的特点有何意义？

$$u_s=\sum_k\sqrt{2}U_k\cos(k\omega t+\phi_k)$$

◆ 传递函数

$$H(\omega)=\frac{\dot{U}_{\mathrm{o}}(\omega)}{\dot{U}_{\mathrm{s}}(\omega)}=\frac{1}{\sqrt{1+(\omega CR)^2}}\angle-\arctan(\omega CR)$$



$$|H(\omega)|=\frac{1}{\sqrt{1+(\omega CR)^2}}$$ 幅频响应

$$\angle H(\omega)=-\arctan(\omega CR)$$ 相频响应

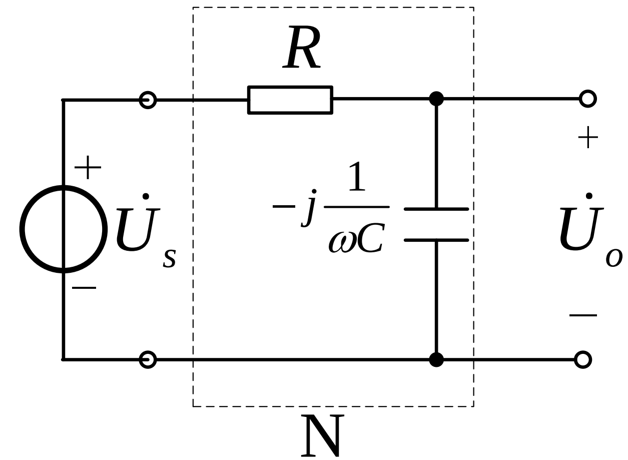
正弦稳态电路频率响应的描述方法

- 用输出相量和输入相量之比（传递函数）来描述
- 比值的模反映输出正弦量幅值随输入量频率的变化规律
- 比值的相位反映输出正弦量初相随输入量频率的变化规律

14.2 传递函数与频率响应

◆ 传递函数

$$H(\omega)=\frac{\text{响应}}{\text{激励}}=\frac{\dot{U}_o(\omega)}{\dot{U}_s(\omega)}$$



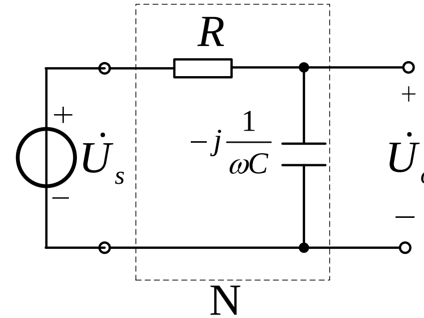
◆ 正弦稳态电路的传递函数

$$H(\omega)=\frac{\dot{Y}(\omega)}{\dot{X}(\omega)}$$

- □ 电压增益（激励为电压源，响应为电压）
- □ 电流增益（激励为电流源，响应为电流）
- □ 转移阻抗（激励为电流源，响应为电压）
- □ 转移电导（激励为电压源，响应为电流）

◆ 频率响应

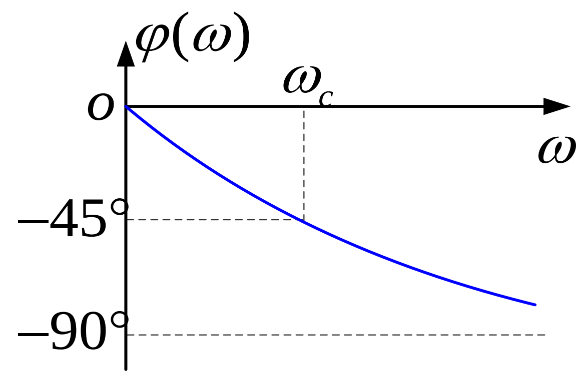
$$H(\omega)=\frac{\dot U_o(\omega)}{\dot U_s(\omega)}=\frac{-j\dfrac{1}{\omega C}}{R-j\dfrac{1}{\omega C}}=\frac{1}{1+j\omega RC}$$



频率响应：正弦稳态电路中，响应随激励频率的变化规律。

$$H(\omega)=|H(\omega)|\angle\varphi(\omega)$$

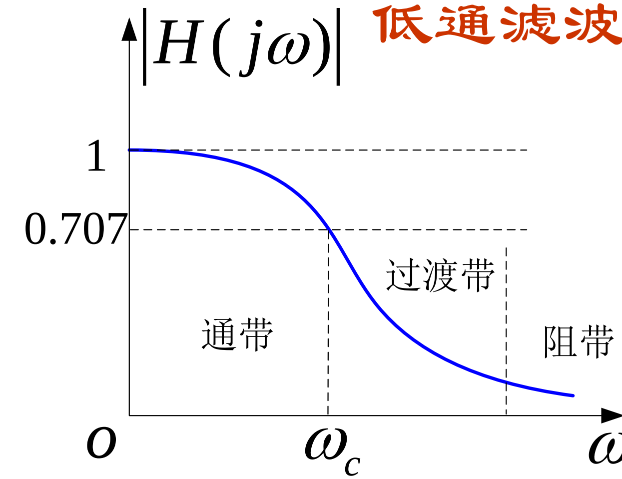
幅频响应　相频响应



$$H(\omega)=\frac{1}{\sqrt{1+(\omega CR)^2}}\angle-\arctan\omega RC$$

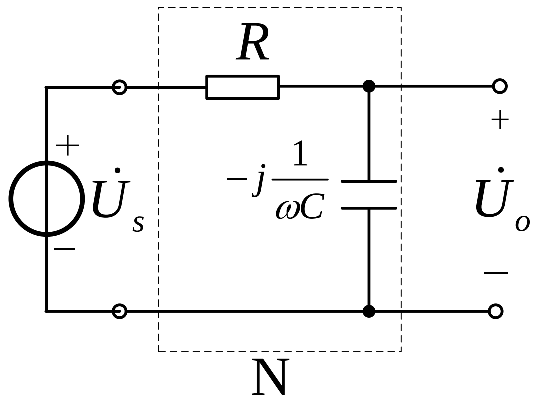
$$H(0)=1\angle 0^\circ \qquad H(\infty)=0\angle-90^\circ$$

低通滤波器



◆ **频率响应**

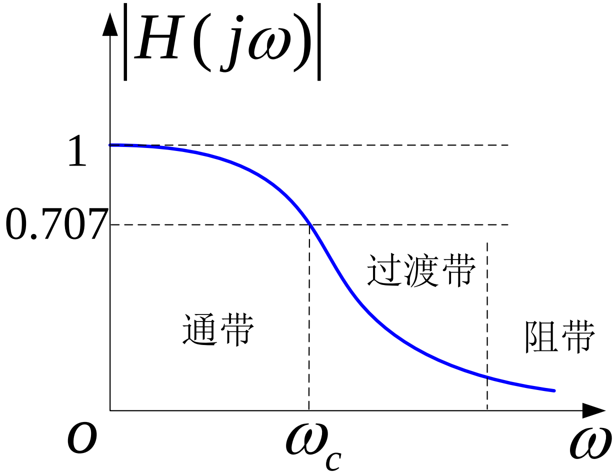
$$H(\omega)=\frac{\dot{U}_{o}(\omega)}{\dot{U}_{s}(\omega)}=\frac{-j\dfrac{1}{\omega C}}{R-j\dfrac{1}{\omega C}}=\frac{1}{1+j\omega RC}$$



频率响应：正弦稳态电路中，响应随激励角频率的变化规律。

$$u_{s}=\sum_{k}\sqrt{2}U_{k}\cos(k\omega t+\phi_{k})$$

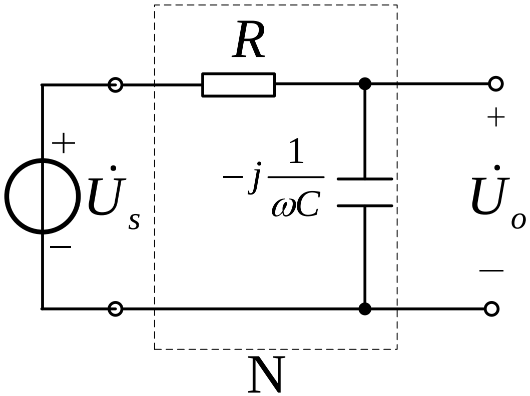
$$\frac{U_{ok}}{U_{sk}}=|H(\omega)|=\frac{1}{\sqrt{1+(k\omega CR)^{2}}}$$



截止频率 $\omega_{c}=1/RC$

例：请参考如图所示电路，设计一种高通滤波器，要求截止频率

$\omega_c = 1000\ \text{rad/s}$。



$$\omega_c = \frac{1}{RC} = 1000$$

$$C = 5uF$$

$$R = \frac{1}{\omega_c C} \quad \rightarrow R = \frac{1}{5*10^{-6}*1000} = 200\Omega$$

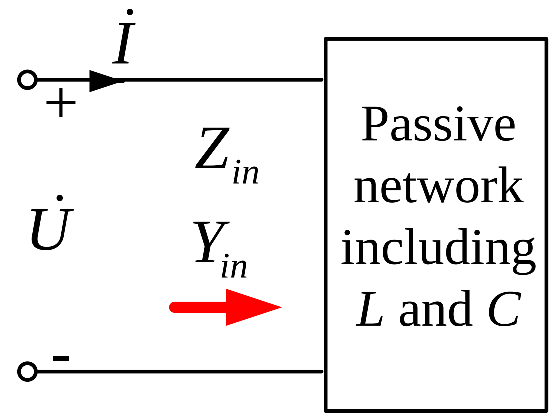
◆ 谐振电路

$$Z_{in} = R(\omega) + jX(\omega) \qquad X(\omega) = 0$$

$$Y_{in} = G(\omega) + jB(\omega) \qquad B(\omega) = 0$$

$\dot{U}$ 与 $\dot{I}$ 同相

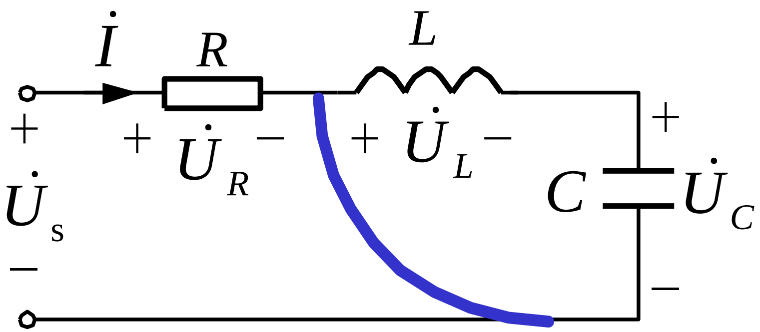
$$P = UI \qquad Q = 0$$



- 通过改变电源频率
- 通过改变电感或电容

◆ RLC串联谐振电路

$$Z = R + \mathrm{j}(\omega L - \frac{1}{\omega C})$$



$$\omega = \frac{1}{\sqrt{LC}} = \omega_0$$ 谐振频率

对外相当于短路！

谐振时的电气特点:

(1) $\dot{U}_S$ 与 $\dot{I}_0$ 同相位

(2) $|Z(\omega_0)| = R = |Z_{\min}(\omega)|$

(3) $|\dot{I}_0| = \left|\frac{\dot{U}_S}{R}\right| = |\dot{I}_{\max}(\omega)|$

(4) $\dot{U}_{R0} = \dot{U}_S$

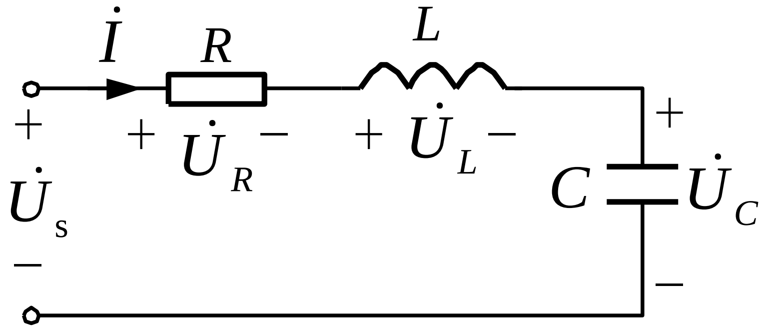
(5) $\dot{U}_{L0} = \mathrm{j}\omega_0 L\dot{I}_0 = \mathrm{j}\frac{\omega_0 L}{R}\dot{U}_S$

$$\dot{U}_{C0} = -\mathrm{j}\frac{1}{\omega_0 C}\dot{I}_0 = -\mathrm{j}\frac{1}{\omega_0 CR}\dot{U}_S$$

$$U_{L0} = U_{C0} = QU_S$$

Q —— 品质因数

◆ RLC串联谐振电路



□ 能量储存

$$w_0 = w_{L0} + w_{C0} = \frac{1}{2}Li_0{}^2 + \frac{1}{2}Cu_{C0}{}^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= L\left(\frac{U_s}{R}\right)^2 (\cos\omega_0 t)^2 + \frac{1}{2}C\left(\frac{\sqrt{2}U_s}{\omega_0 RC}\right)^2 [\cos(\omega_0 t - 90°)]^2 \longrightarrow = L\left(\frac{U_s}{R}\right)^2$$

□ 一个周期内消耗的能量

$$w_{R0} = \int_0^{T_0} p_{R0}\,\mathrm{d}\,t = \int_0^{T_0} i_0{}^2 R\,\mathrm{d}\,t = \int_0^{T_0}\left(\frac{\sqrt{2}U_s\cos\omega_0 t}{R}\right)^2 R\,\mathrm{d}\,t = \frac{U_s{}^2}{R}\frac{2\pi}{\omega_0}$$

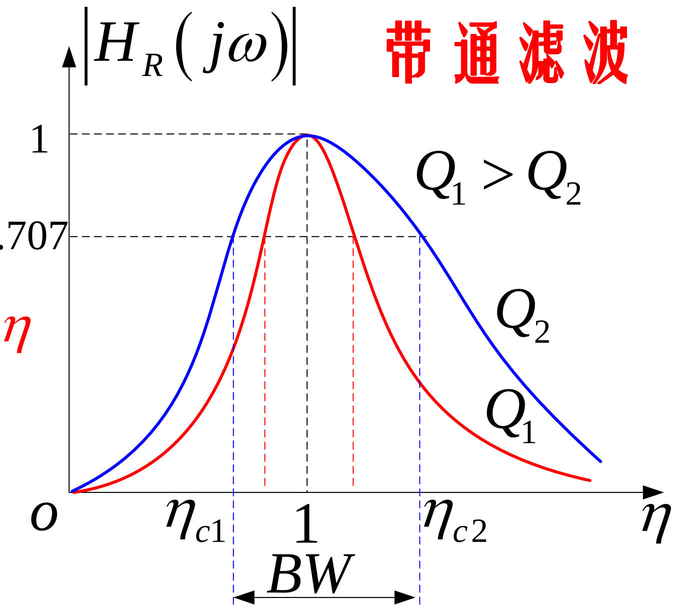
$$\boxed{\frac{w_0}{w_{R0}} = \frac{1}{2\pi}\frac{\omega_0 L}{R} = \frac{Q}{2\pi}}$$

$$|H_R(\omega)| = \left|\frac{\dot{U}_R(\omega)}{\dot{U}_S(\omega)}\right| = \frac{R}{\left|R + j(\omega L - \dfrac{1}{\omega C})\right|}$$

$$= \frac{1}{\sqrt{1 + (\dfrac{\omega L}{R} - \dfrac{1}{R\omega C})^2}} = \frac{1}{\sqrt{1 + Q^2(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega})^2}}$$

$$\frac{\omega}{\omega_0} = \eta$$

带通滤波



□ 半功率频率

$$P(\omega_{c1,c2}) = \frac{1}{2}P(\omega_0) = \frac{1}{2}\frac{U_S^2}{R} = \frac{(U_S/\sqrt{2})^2}{R}$$

$$|H_R(\eta_{c1,c2})| = \frac{1}{\sqrt{2}} = 0.707$$

$$\left.\begin{aligned} \eta_{c1} &= -\frac{1}{2Q} + \sqrt{(\frac{1}{2Q})^2 + 1} \\ \eta_{c2} &= \frac{1}{2Q} + \sqrt{(\frac{1}{2Q})^2 + 1} \end{aligned}\right\}$$

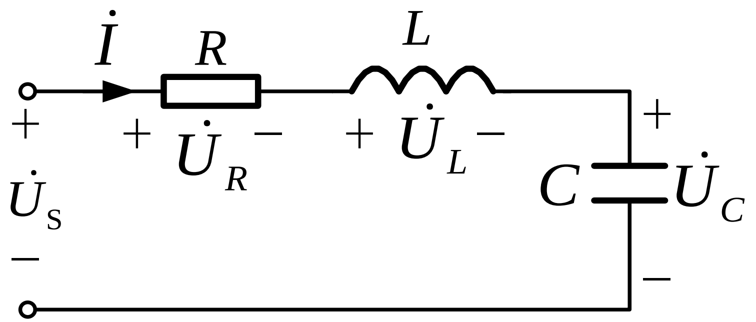
□ 带宽

$$BW = (\eta_{c2} - \eta_{c1})\omega_0 = \frac{\omega_0}{Q}$$

$$\sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\eta_{c1}\eta_{c2}}\,\omega_0 = \omega_0$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{1}{2}BW \quad (\text{for } Q \geq 10)$$

◆ **RLC串联谐振电路**



$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_{c1}\omega_{c2}}$$

$$BW = \omega_{c2} - \omega_{c1} = \frac{R}{L} \qquad BW = \frac{\omega_0}{Q}$$

$$Q = \frac{U_{L0}}{U_S} = \frac{U_{C0}}{U_S} \qquad Q = \frac{X_{L0}}{R} = \frac{X_{C0}}{R} = \frac{\sqrt{L/C}}{R} \qquad Q = 2\pi\frac{w_0}{w_{R0}} \qquad Q = \frac{\omega_0}{BW}$$

$$\omega_{c1,c2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \mp\frac{\omega_0}{2Q} + \omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2}$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{BW}{2} \qquad (\text{For } Q \geq 10)$$

例：已知 ***RLC*** **串联谐振的带宽** BW=20 rad/s，角频率 $\omega_0 = 1000$ rad/s. (1) 品质因数Q. (2) 如果 C=5μF, 求电感 L和电阻 R. (3) 半功率频率.

$$BW = \frac{\omega_0}{Q} \qquad \rightarrow Q = \frac{\omega_0}{BW} = \frac{1000}{20} = 50$$

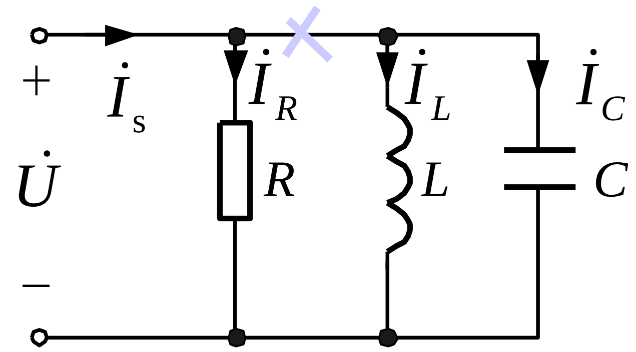
$$\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \qquad L = 200\text{mH}$$

$$Q = \frac{\omega_0 L}{R} \qquad \rightarrow R = \frac{\omega_0 L}{Q} = 4\Omega$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{BW}{2} = 990\text{rad}/\text{s}, 1010\text{rad}/\text{s}$$

◆ RLC并联谐振电路

近似开路



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- $\dot{I}_{R0} = \dot{I}_S$　　电压与电流同相
- $|Y(\omega_0)| = G = |Y_{\min}(\omega)|$
- $|\dot{U}_0| = \left|\dfrac{\dot{I}_S}{G}\right| = |\dot{U}_{\max}(\omega)|$
- $\dot{I}_{L0} = \dfrac{\dot{U}_0}{j\omega_0 L} = -j\dfrac{R}{\omega_0 L}\dot{I}_S$
- $\dot{I}_{C0} = j\omega_0 C\dot{U}_0 = j\omega_0 CR\dot{I}_S$
- $I_{L0} = I_{C0} = QI_S$

$$\omega_{c1,c2} = \mp\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$BW = \frac{1}{RC}$$

串联 $\left\{ \begin{array}{c} R \to G \\ L \to C \\ C \to L \\ \dot{U} \to \dot{I} \\ \dot{I} \to \dot{U} \end{array} \right\}$ 并联

例： ***RLC*** **并联谐振电路，电感** L=100mH，**谐振频率为** $\omega_0 = 100k$ rad/s，**品质因数** Q=*50*. **求带宽，半功率频率，以及电容** C **和电阻** R .

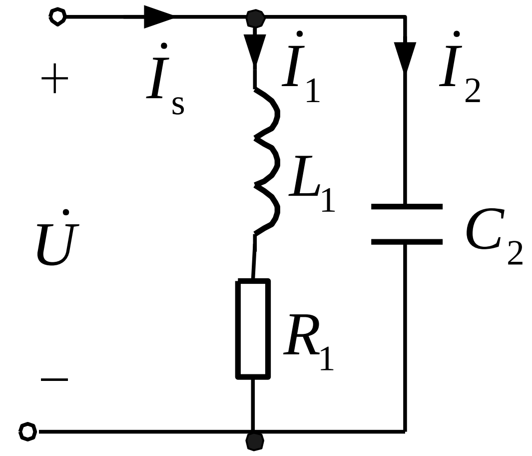
$$BW = \frac{\omega_0}{Q} = \frac{100k}{50} = 2krad/s$$

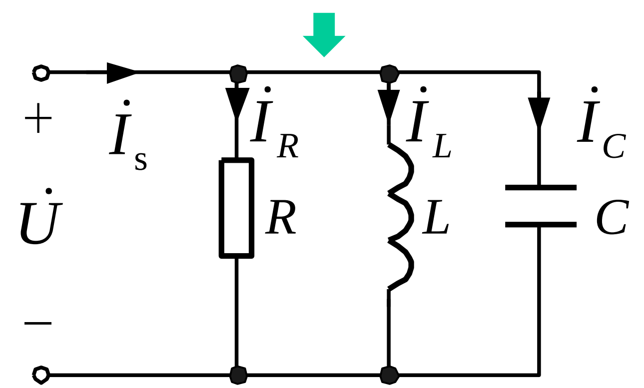
$$\omega_0 = \frac{1}{\sqrt{LC}} = 100k \qquad C = 1nF$$

$$Q = \frac{\omega_0 C}{G} \qquad \rightarrow R = \frac{Q}{\omega_0 C} = 500k\Omega$$

$$\omega_{c1,c2} \approx \omega_0 \mp \frac{BW}{2} = 99\text{krad/s},\ 101\text{krad/s}$$

◆ 其他谐振电路





$$Y=\frac{1}{R_1+\mathrm{j}\omega L_1}+\mathrm{j}\omega C_2$$

$$=\frac{R_1}{R_1^2+(\omega L_1)^2}+\mathrm{j}[\omega C_2-\frac{\omega L_1}{R_1^2+(\omega L_1)^2}]$$

$$=G+\mathrm{j}(\omega C-\frac{1}{\omega L})$$

$$\omega_0 C_2=\frac{\omega_0 L_1}{R_1^2+(\omega_0 L_1)^2}$$

$$\omega_0=\frac{1}{\sqrt{L_1 C_2}}\sqrt{1-\frac{C_2 R_1^2}{L_1}} \qquad (L_1>C_2R_1^2)$$

$$Q=\frac{B_{L0}}{G}=\frac{\omega_0 L_1}{R_1}=\sqrt{\frac{L_1}{C_2 R_1^2}-1}$$

$$I_{20}=QI_s$$

$$I_{10}=\sqrt{I_s^2+(QI_s)^2}$$

$$=I_s\sqrt{1+Q^2}$$

谢 谢!