参考答案

备注：目前参考答案写的基本上是英文版本，仅供参考。

第一题

The rank vector r would be a 1/n vector (n=3). If the transition matrix is M, then the surfer’s distribution after one step is Mr.

$$r=\begin{bmatrix}1/3\\1/3\\1/3\end{bmatrix}$$

$$M=\begin{bmatrix}1/3 & 1/2 & 0\\1/3 & 0 & 1/2\\1/3 & 1/2 & 1/2\end{bmatrix}$$

$$Mr=\begin{bmatrix}1/3 & 1/2 & 0\\1/3 & 0 & 1/2\\1/3 & 1/2 & 1/2\end{bmatrix}\begin{bmatrix}1/3\\1/3\\1/3\end{bmatrix}=\begin{bmatrix}5/18\\5/18\\8/18\end{bmatrix}$$

Then, after two steps it is $M(Mr)=M^2r$, and recursively and so on.

第二题

The rank vector r would be a 1/n vector (n=3). The google Matrix is $A=\beta M+(1-\beta)\left[\frac{1}{N}\right]_{N\times N}$

$$r=\begin{bmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\end{bmatrix}$$

$$A=0.8*\begin{bmatrix}\frac{1}{3} & \frac{1}{2} & 0\\\frac{1}{3} & 0 & \frac{1}{2}\\\frac{1}{3} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}+0.2\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & \frac{7}{15} & \frac{1}{15}\\\frac{1}{3} & \frac{1}{15} & \frac{7}{15}\\\frac{1}{3} & \frac{7}{15} & \frac{7}{15}\end{bmatrix}$$

Then the surfer’s distribution after one step is Ar

$$Ar = \begin{bmatrix} \frac{1}{3} & \frac{7}{15} & \frac{1}{15} \\ \frac{1}{3} & \frac{1}{15} & \frac{7}{15} \\ \frac{1}{3} & \frac{7}{15} & \frac{7}{15} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 13/45 \\ 13/45 \\ 19/45 \end{bmatrix}$$

Then, after two steps it is $A(Ar) = A^2 r$, and recursively and so on.

第三题

To determine the Topic-Sensitive PageRank, firstly consider figure as shown below.

$$\beta M = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix}$$

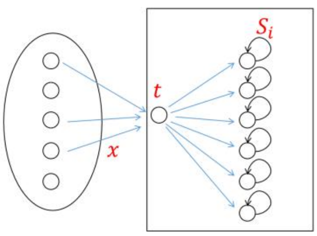
$$V' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} V + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix}$$

Then solve the equation and the result is given below.

$$V = \left[\frac{27}{70}, \frac{6}{35}, \frac{19}{70}, \frac{6}{35} \right]^T$$

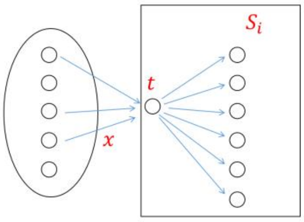
第四题

(a)Each supporting page links to itself instead of to the target page:



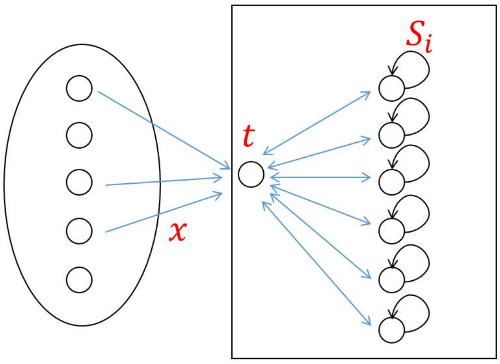
$$y_t = x$$
$$S_i = \beta y/m + (1-\beta)/n + \beta S_i$$
$$S_i = \frac{\beta}{1-\beta} \cdot \frac{x}{m} + \cancel{\frac{1-\beta}{n}}$$

(b) Each supporting page links nowhere.



$$
\begin{aligned}
y_t &= x + \cancel{(1-\beta)/n} \\
S_i &= \beta y/m + (1-\beta)/n \\
&= \beta \cdot \frac{x}{m} + \cancel{\frac{1-\beta}{n}}
\end{aligned}
$$

(c) Each supporting page links both to itself and to the target page.



$$
\begin{aligned}
y_t &= x + \beta m S_i/2 \\
S_i &= \beta y/m + \frac{1-\beta}{n} + \beta S_i/2 \\
S_i &= \frac{2\beta}{2m - \beta m - \beta^2 m} \cdot x \\
y_t &= \frac{m(2-\beta)}{2m - \beta m - \beta^2 m} \cdot x
\end{aligned}
$$

第五题

由图 5-1 可知，链接矩阵 A= $\begin{bmatrix} 0111 \\ 1001 \\ 1000 \\ 0110 \end{bmatrix}$，A 的转置矩阵 A^T = $\begin{bmatrix} 0110 \\ 1001 \\ 1001 \\ 1100 \end{bmatrix}$

初始化导航度 h=$\begin{bmatrix}0.25\\0.25\\0.25\\0.25\end{bmatrix}$，权威度 a=$\begin{bmatrix}0.25\\0.25\\0.25\\0.25\end{bmatrix}$

那么，接下来循环迭代执行:

h=Aa=$\begin{bmatrix}1.5\\1\\0.5\\1\end{bmatrix}$，进行归一化处理可得 h=$\begin{bmatrix}0.71\\0.47\\0.24\\0.47\end{bmatrix}$

a=$A^T h = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$,进行归一化处理可得 a=$\begin{bmatrix}0.5\\0.5\\0.5\\0.5\end{bmatrix}$

然后重复执行 h=Aa,a=$A^T h$获得结果进行归一化处理,直到结果和上一轮循环的向量之间的差异足够小停止计算，所获得的结果为每个节点的导航度和权威度。