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6) Big omega notation: prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$.

Sol: $g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants c and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + 2/n + 4/n^2$ as 1

Example $c = 1/2$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$

$1 + 2/n + 4/n^2 \geq 1$ $(1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1/2$ $(n \geq 1, n_0 = 1)$

thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$.

7) Big theta notation: Determine whether $h(n) = 4n^2 + 3n$ is $\theta(n^2)$ or not

Sol: $c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n$

$h(n) \leq c_2 n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

Let's $c_2 = 5$

Divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

Lower bound :-

$h(n) = 4n^2 + 3n$

$h(n) \geq c_1 n^2$

$4n^2 + 3n \geq c_1 n^2$

Let's $c_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

Divide both sides by n^2

$4 + \frac{3}{n} \geq 4$

$h(n) = 4n^2 + 3n$ $(c_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\theta(n^2)$

3) Let $f(n) = n^3 - 2n^2 + n$ and $g(n) - n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify your answer

Sol: $f(n) \geq c \cdot g(n)$

Substuting $f(n)$ and $g(n)$ into this inequality we get

$n^3 - 2n^2 + n \geq c(-n^2)$

find c and n_0 holds $n \geq n_0$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n + cn^2 \geq 0$

$n^3 + \cancel{2}(c-2)n^2 + n \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$ $(n^3 \geq 0)$

$n^3+(1-2)n^2+n = n^3-n^2+n \geq 0 \quad (c=2)$

$f(n) = n^3-2n^2+n$ is $\Omega(g(n)) = \Omega(-n^2)$

therefore the statement $f(n) = \Omega(g(n))$ is True.

2) Determine whether $h(n) = n\log n + n$ is in $\Theta(n\log n)$ prove a Rigorous proof for your conclusion.

Sol:- $C_1 n\log n \leq h(n) \leq C_2 n\log n$

upper bound:

$h(n) \leq C_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \leq C_2 n\log n$

Divide both sides by $n\log n$

$1+\frac{n}{n\log n} \leq 2$

$1+\frac{1}{\log n} \leq C_2$ (simplify)

$1+\frac{1}{\log n} \leq 2 \quad (C_2=2)$

Then $h(n)$ is $O(n\log n) \quad (C_2=2, n_0=2)$

Lower bound:

$h(n) \geq C_1 n\log n$

$h(n) = n\log n + n$

$n\log n + n \geq C_1 n\log n$

Divide both sides by $n\log n$

$1+\frac{n}{n\log n} \geq C_1$

$1+\frac{1}{\log n} \geq C_1$ (simplify)

$1+\frac{1}{\log n} \geq 1 \quad (C_1=1)$

$\frac{1}{\log n} \geq 0$ for all $n>1$

$h(n)$ is $\Omega(n\log n) \quad (C_1=1, n_0=1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

10) Solve the following Recurrence Relations and find the order of growth for Solutions. $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

Sol: $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4$, $b = 2$, $f(n) = n^2$

Applying master theorem

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - \epsilon})$ $\left(\begin{array}{l}\epsilon > 0 \\ T(n) = \Theta(n^{\log_b a})\end{array}\right)$

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a}\log n)$.

$f(n) = \Omega(n^{\log_b a + \epsilon})$, then $T(n) = f(n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$ (comparing $f(n)$ with $n^{\log_b a}$).

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$, (case 2)

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a}\log n) = \Theta(n^2\log n)$

order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2\log n)$.