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CSE (AI)

1) Solve the following Recurrence Relation.

a) $x(n) = x(n-1) + 5$ for $n > 1$ with $x(1) = 0$

1) write down the first two terms to identify the pattern.

$x(1) = 0$

$x(2) = x(1) + 5 = 5$

$x(3) = x(2) + 5 = 10$

$x(4) = x(3) + 5 = 15$

2) Identify the pattern (or) the general term.

-> the first term $x(1) = 0$

The common difference $d = 5$

the general formula for the nth term of an AP is.

$x(n) = x(1) + (n-1) \cdot d$

Substituting the given values

$x(n) = 0 + (n-1) \cdot 5 = 5(n-1)$

The solution is $x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ for $n > 1$ with $x(1) = 4$

1) write down the first two terms to identify the pattern.

$x(1) = 4$

$x(2) = 3x(1) = 3 \times 4 = 12$

$x(3) = 3x(2) = 36$

$x(4) = 3x(3) = 108$.

2) Identify the general term.

-> the first term $x(1) = 4$

-> the common Ratio $r = 3$

The general formula for the nth term of a GP is $x(n) = x(1) \cdot r^{n-1}$

Substituting the given values

$x(n) = 4 \cdot 3^{n-1}$

The solution is $x(n) = 4 \cdot 3^{n-1}$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^k$).

For $n = 2^k$, we can write Recurrence in terms of k.

1) Substitute $n = 2^k$ in the Recurrence

$$x(2^k) = x(2^{k-1}) + 2^k$$

2) write down the first few terms to identify the pattern.

$x(1) = 1$

$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$

$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$

$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$

3) Identify the general term by finding the pattern:

we observe that :-

$$x(2^k) = x(2^{k-1}) + 2^k$$

we sum the series :

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \dots$$

Since $x(1) = 1$!

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \dots$$

the geometric series with the term $a = 2$ and the last term 2^k except for the additional $+1$ term.

the sum of a geometric series S with Ratio $r = 2$ is given by.

$$S = \frac{ar^n - 1}{r - 1}$$

Here $a = 2$, $r = 2$ and $\cancel{k} n = k$:

$$S = 2\,\frac{2^k - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1}$$

Adding the $+1$ term

$$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

Solution is $x(2^k) = 2^{k+1} - 1$

d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (Solve for $n = 3^k$).

For $n = 3^k$, we can write the Recurrence in terms of k.

1) Substitute $n = 3^k$ in the Recurrence

$$x(3^k) = x(3^{k-1}) + 1$$

2) write down the first few terms to identify the pattern.

$$x(1) = 1$$
$$x(3) = x(3^1) = x(1) + 1 = 1 + 1 = 2$$
$$x(9) = x(3^2) = x(3) + 1 = 2 + 1 = 3$$
$$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4$$

3) Identify the general term.

we observe that:-

$$x(3^k) = x(3^{k-1}) + 1$$

Summing up the series

$$x(3^k) = 1 + 1 + 1 + \cdots + 1$$
$$x(3^k) = k + 1$$

the Solution is $x(3^k) = k + 1$

2) Evaluate the following Recurrences complexity.

i) $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$.

the Recurrence Relation can be solved using iteration method.

1) Substitute $n = 2^k$ in the Recurrence

2) Iterate the Recurrence

for $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1) + 1$

$k=2$: $T(2^2) = T(8) = T(n) + 1 = (T(1) + 2) + 1 = T(1) + 2$

$k=3$: $T(2^3) = T(8) = T(n) + 1 = (T(1) + 2) + 1 = T(1) + 3$

3) Generalize the pattern

$$T(2^k) = T(1) + k$$

Since $n = 2^k$, $k = \log_2 n$.

$$T(n) = T(2^k) = T(1) + \log_2 n$$

4) Assume $T(1)$ is a constant c.

$$T(n) = c + \log_2 n$$

∴ The solution is $T(n) = O(\log n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$ where c is constant and n is input size).

The recurrence can be solved using the master's theorem. for divide-and-conquer recurrence of the form.

$$T(n) = aT(n/b) + f(n)$$

where $a = 2$, $b = 3$ and $f(n) = cn$

Lets determine the value of $\log_b a$:

$$\log_b a = \log_3 2$$

using the properties of logarithms

$$\log_3 2 = \frac{\log 2}{\log 3}$$

now we compare $f(n) = cn$ with $n^{\log_3 2}$:

$$f(n) = O(n)$$

$$n = n^1$$

Since $\log_3 2$ we are in the third case of the master's theorem.

$$f(n) = O(n^c) \text{ with } c > \log_b a$$

the solution is :

$$T(n) = O(f(n)) = O(cn) = O(n)$$

3) Consider the following Recurrence algorithm ?

min [A (0... n-2)]

if $n=1$ Return A[0]

Else temp = min (1A1 0.... n-2)

if temp <= A (n-1) Return temp

else

Return A (n-1)

a) what does this Algorithm compute ?

The given algorithm, min [A (0,.... n-1)] computes the minimum value in the array 'A'. from index '0' for 'n-1' if does this by Recursively finding the minimum value in the sub array A (0.... n-2) and then comparing it with the last element A (n-1) to determine the overall maximum value.

b) set up a Recurrence Relation for the algorithm basic operation count and solve it.

the solution is $T(n) = n$

This means the algorithm performs n basic operations for an input array of size n.

4) Analyze the order of growth.

1) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation.

To analyze the order of growth and use the Ω notation, we need to compare the given function $f(n)$ and $g(n)$.

Given functions:

$F(n) = 2n^2+5$

$g(n) = 7n$

order of growth using Ω g(n) notation :

the notation Ω g(n) describes a lower bound on the growth rate that for sufficiently large n, f(n), grows at least as for as g(n)

$$f(n) = c \cdot g(n)$$

lets analyze $f(n) = 2n^2+5$ with Respect to $g(n)=7n$

1) Identify dominant terms :

-> the dominant terms in f(n) is $2n^2$ since it grows faster than the constant terms as n increases.

-> the dominant term in g(n) is 7n.

2) Establish the inequality :

-> we want to find constants c and n_0 such that :

$$2n^2+5 \geq c \cdot 7n \text{ for all } n \geq n_0$$

3) Simplify the inequality :

-> Ignore the lower order term 5 for larger.

$$2n^2 \geq 7cn$$

-> Divide both sides by n.

$$2n \geq 7c$$

-> solve for n:

$$n \geq 7/2$$

4) choose constants

Let c = 1

$$n \geq \frac{7 \cdot 1}{2} = 3.5$$

$\therefore$ for $n \geq n$, the inequality holds:

$$2n^2+5 \geq 7n. \text{ for all } n \geq n$$

we have shown that there exist contants $c=1$ and $n_0=n$ such that for all $n \geq n_0$:

$$2n^2+5 \geq 7n$$

Thus, we can conclude that :-

$$f(n) = 2n^2+5 = \Omega(7n)$$

In Ω notation, the dominant term $2n^2$ in $f(n)$ clearly grows faster than $7n$ hence.

$$f(n) = \Omega(n^2)$$

However for the specific comparision asked $f(n) = \Omega(7n)$ is also correct.

Showing that $f(n)$ grows at least as fast as $7n$.