11) Given an array of [4,-2,5,3,10,-5,2,8,-3,6,7,-4,1,9,-1,0,-6,-8,11,-9] integers, find the maximum and minimum product that can be obtained by multiplying two integers from the array.

Sol) Given array is [4,-2,5,3,10,-5,2,8,-3,6,7,-4,1,9,-1,0,-6,-8,11,-9]

we need to consider the largest and smallest products that can be formed by selecting two numbers from the array.

1. Sort the array.

Sorted array is [-9,-8,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11]

2. Identify possible candidates for maximum product

3. Identify possible candidates for minimum product.

Calculating maximum product :-

-> the two largest positive numbers are 10 and 11

$10 \times 11 = 110$

-> the two smallest negative numbers are -9 and -8.

$-9 \times -8 = 72$

the maximum product is 110.

Calculating minimum products

-> the largest postive and negative number is 11 and -9

$11 \times -9 = -99$

-> the smaller negative numbers are

$-9 \times -8 = 72$

-99 is smaller than -72 so

maximum product = 110

and minimum product = -99

12) Demonstrate the Binary Search method to Search for the Key = 23 from the array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}.

Sol: Given Key = 23 and array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}.

1. Initialize pointers

low = 0 and high = 9

Calculate mid = $\left[\frac{low + high}{2}\right] = \left[\frac{6+9}{2}\right] = 4$

Compare arr(mid) with Key :

arr(4) = 16

Since 16 < 23 update low = mid + 1 = 5

Calculate mid = $\left[\frac{low + high}{2}\right] = \left[\frac{5+9}{2}\right] = 7$

Compare arr(mid) with Key :

arr(7) = 56

Since 56 > 23 update high = mid - 1 = 6

mid = $\left(\frac{5+6}{2}\right) = 5$

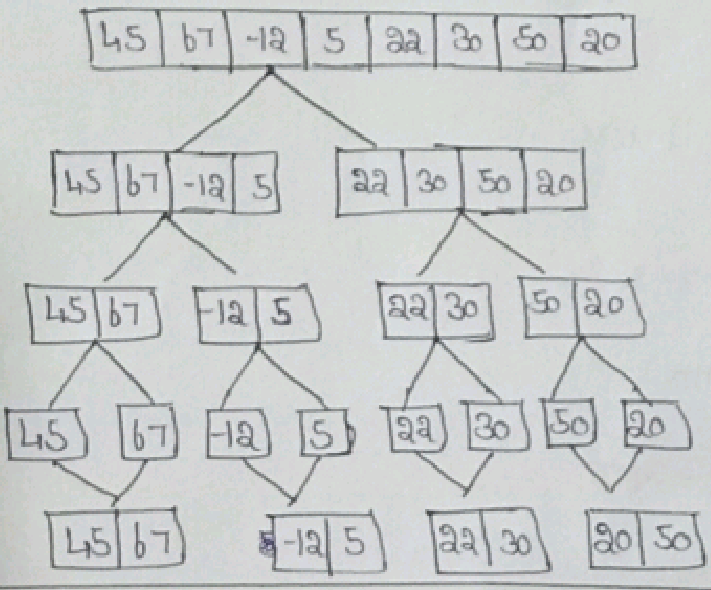
arr(mid) = arr(5) = 23

23 == 23 the Key is found

∴ the Key = 23 is found at index 5.

13) Apply merge sort and other list of 8 elements, Data d = (45, 67, -12, 5, 22, 30, 50, 20) Set up Recurrence Relation for the number of Key Comparisions made by merge Sort.

Sol:



-12	5	45	67

20	22	30	50

-12	5	20	22	30	45	50	67

$\therefore$ The sorted list = [-12, 5, 20, 22, 30, 45, 50, 67]

Recurrence Relation for comparisons:

$$T(n) = 2 + (n/2) + O(n)$$

If $n = 1$, $T(1) = 0$ Base case

-> At each level of Recursion we make at most $n-1$ comparisions to merge two halves of size $n/2$ so it becomes

$$T(n) = 2T + (n/2) + (n-1)$$

Solving Recurrence Relation we get

$$T(n) = n \log 2(n) - n + 1$$

$$T(n) = O(n \log n)$$

$\therefore$ The Recurrence Relation is $T(n) = 2T(n/2) + O(n)$

or more precisely

$$T(n) = n \log_2(n) - n + 1$$

14) Find the no. of times to perform solving swapping for selection sort also estimates the time complexity for the order of notation sets (12, 7, 5, -2, 18, 6, 13, 4).

Sol: The selection sort algorithm always makes exactly $n-1$ swaps in the worst case, where n is the no. of elements in the list

Given $S = \{12, 7, 5, -2, 18, 6, 13, 4\}$:

No of elements, $n = 8$

No. of swaps $= n-1 = 8-1 = 7$

Time complexity: The time complexity of selection sort in Big O notation is $O(n^2)$.

So, the number of swaps is 7, and the time complexity is $O(n^2)$.

15) Find the index of the target value 10 using binary search from the following list of elements [2, 4, 6, 8, 10, 12, 14, 16, 18, 20].

Sol: Given list = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} and value = 10

low = 0 and high = 9

$mid = \frac{low + high}{2} = \frac{0+9}{2} = 4$

List (4) = mid = 10 mid == value

Since 10 == 10 the target is found at index

∴ the target value = 10 is found at index.