ASSIGNMENT-2

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Question: Solve the equation for x:

$sin^{-1}x + sin^{-1}(1-x) = cos^{-1}x, x \neq 0$

Solution: Taking the equation:

$$sin^{-1}x + sin^{-1}(1-x) = cos^{-1}x \tag{1}$$

Also, we know that

$$cos^{-1}x + sin^{-1}x = \frac{\pi}{2} \tag{2}$$

Using (1) and (2) we get,

$$\implies sin^{-1}x + sin^{-1}(1-x) = \frac{\pi}{2} - sin^{-1}x \tag{3}$$

$$\implies sin^{-1}(1-x) = \frac{\pi}{2} - 2sin^{-1}x \tag{4}$$

$$\implies sin\left(\frac{\pi}{2} - 2sin^{-1}x\right) = (1-x) \tag{5}$$

$$\implies cos(2sin^{-1}x) = (1-x) \tag{6}$$

We know that,

$$cos(2y) = 1 - 2sin^2y \tag{7}$$

From equation (6) and (7) we get,

$$\implies 1 - 2sin^2(sin^{-1}x) = (1-x) \tag{8}$$

Also,

$$sin(sin^{-1}x) = x \tag{9}$$

Now taking (8) and (9)

$$\implies 1 - 2x^2 = (1-x) \tag{10}$$

$$\implies 2x^2 - x = 0 \tag{11}$$

$$\implies x(2x-1) = 0 \tag{12}$$

$$\implies x = 0, \frac{1}{2} \tag{13}$$

As already mentioned that $x \neq 0$. Therefore $x = \frac{1}{2}$