

ASSIGNMENT-2

Nwjwr Khungur Brahma(AI20BTECH11016)

Question: Solve the equation for x:

$$\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x, x \neq 0$$

Solution: Taking the equation:

$$\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x \quad (1)$$

Also, we know that

$$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2} \quad (2)$$

Using (1) and (2) we get,

$$\Rightarrow \sin^{-1} x + \sin^{-1}(1 - x) = \frac{\pi}{2} - \sin^{-1} x \quad (3)$$

$$\Rightarrow \sin^{-1}(1 - x) = \frac{\pi}{2} - 2\sin^{-1} x \quad (4)$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right) = (1 - x) \quad (5)$$

$$\Rightarrow \cos(2\sin^{-1} x) = (1 - x) \quad (6)$$

We know that,

$$\cos(2y) = 1 - 2\sin^2 y \quad (7)$$

From equation (6) and (7) we get,

$$\Rightarrow 1 - 2\sin^2(\sin^{-1} x) = (1 - x) \quad (8)$$

Also,

$$\sin(\sin^{-1} x) = x \quad (9)$$

Now taking (8) and (9)

$$\Rightarrow 1 - 2x^2 = (1 - x) \quad (10)$$

$$\Rightarrow 2x^2 - x = 0 \quad (11)$$

$$\Rightarrow x(2x - 1) = 0 \quad (12)$$

$$\Rightarrow x = 0, \frac{1}{2} \quad (13)$$

As already mentioned that $x \neq 0$. Therefore $x = \frac{1}{2}$

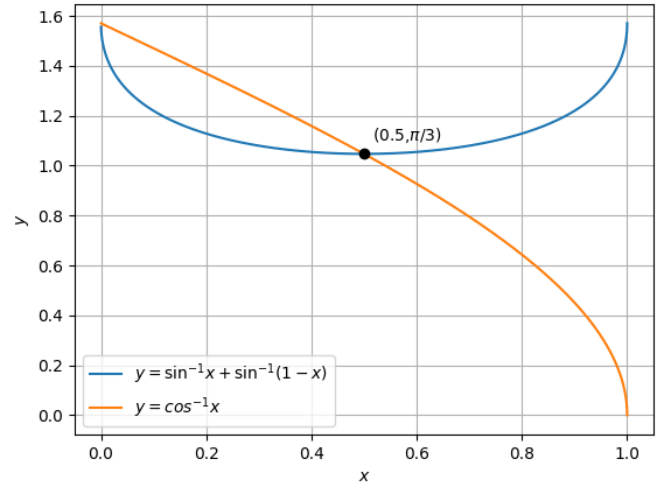


Fig. 1. Graph showing the intersection of $y = \sin^{-1} x + \sin^{-1}(1 - x)$ and $y = \cos^{-1} x$