Assignment 5

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Question: A man is know to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution: Let the event that the man reports the result to be six be E.

The event that six actually occurs on the die be X and $\overline{X}$ be the event that six does not occur.

Then,

$$\text{Probability that die rolls six} = P(X) = \frac{1}{6} \tag{1}$$

$$\text{Probability that die does not roll six} = P(\overline{X}) = \frac{5}{6} \tag{2}$$

Probabilities when the man reports the result to be six,

$$\text{Probability that six actually occurs} = P(E|X) = \frac{3}{4} \tag{3}$$

$$\text{Probability that it is not a six} = P(E|\overline{X}) = \frac{1}{4} \tag{4}$$

Now by Bayes's Formula we get,

Probability that the man reports the result to be six is actually a six,

$$P(X|E) = \frac{P(X) \times P(E|X)}{P(X) \times P(E|X) + P(\overline{X}) \times P(E|\overline{X})} \tag{5}$$

Now using the values from (1), (2), (3) and (4),

$$P(X|E) = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} \tag{6}$$

$$= \frac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}} \tag{7}$$

$$= \frac{\frac{3}{24}}{\frac{8}{24}} \tag{8}$$

$$= \frac{3}{8} \tag{9}$$