

Assignment 5

Nwjwr Khungur Brahma (AI20BTECH11016)

Question: A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution: Let the event that the man reports the actual six be $E = 1$ and lies as six be $E = 0$.

The event that six actually occurs on the die be $X = 1$ and $X = 0$ be the event that six does not occur. Then,

$$\text{Probability that die rolls six} = P(X = 1) = \frac{1}{6} \quad (1)$$

$$\text{Probability that die does not roll six} = P(X = 0) = \frac{5}{6} \quad (2)$$

Probabilities when the man reports the result to be six,

$$\text{Probability that six actually occurs} = P(E = 1|X = 1) = \frac{3}{4} \quad (3)$$

$$\text{Probability that it is not a six} = P(E = 0|X = 0) = \frac{1}{4} \quad (4)$$

Now by Bayes's Formula we get,

Probability that the man reports the result to be six is actually a six,

$$P(X = 1|E) = \frac{P(X = 1) \times P(E = 1|X = 1)}{P(X = 1) \times P(E = 1|X = 1) + P(X = 0) \times P(E = 0|X = 0)} \quad (5)$$

Now using the values from (1), (2), (3) and (4),

$$P(X = 1|E) = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} \quad (6)$$

$$= \frac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}} \quad (7)$$

$$= \frac{\frac{3}{24}}{\frac{8}{24}} \quad (8)$$

$$= \frac{3}{8} \quad (9)$$