Assignment-6

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Outline

Question

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution

Let the event that the man reports the actual six be $E = 1$ and lies as six be $E = 0$.
The event that six actually occurs on the die be $X = 1$ and $X = 0$ be the event that six does not occur.

Description of event	Events
Die rolls the number six	X=1
Die does not roll the no. six	X=0
Man speaks truth	E=1
Man does not speaks truth	E=0

Then,

$$\text{Probability that die rolls six} = P(X = 1) = \frac{1}{6} \tag{1}$$

$$\text{Probability that die does not roll six} = P(X = 0) = \frac{5}{6} \tag{2}$$

Probabilities when the man reports the result to be six,

$$\text{Probability that six actually occurs} = P(E = 1|X = 1) = \frac{3}{4} \tag{3}$$

$$\text{Probability that it is not a six} = P(E = 0|X = 0) = \frac{1}{4} \tag{4}$$

Now by Bayes's Formula we get,
Probability that the man reports the result to be six is actually a six,

$$P(X=1|E) = \frac{P(X=1) \times P(E=1|X=1)}{P(X=1) \times P(E=1|X=1) + P(X=0) \times P(E=0|X=0)} \tag{5}$$

Now using the values from (1), (2), (3) and (4),

$$P(X=1|E) = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} \tag{6}$$

$$= \frac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}} \tag{7}$$

$$= \frac{\frac{3}{24}}{\frac{8}{24}} \tag{8}$$

$$= \frac{3}{8} \tag{9}$$