Assignment-6

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Outline

Question

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution

Let the event that the man reports the actual six be $E = 1$ and lies as six be $E = 0$.

The event that six actually occurs on the die be $X = 1$ and $X = 0$ be the event that six does not occur.

Description of event	Events
Die rolls the number six	X=1
Die does not roll the no. six	X=0
Man speaks truth	E=1
Man does not speaks truth	E=0

Then,

$$\text{Probability that die rolls six} = P(X = 1) = \frac{1}{6} \qquad (1)$$

$$\text{Probability that die does not roll six} = P(X = 0) = \frac{5}{6} \qquad (2)$$

Probabilities when the man reports the result to be six,

$$\text{Probability that six actually occurs} = P(E = 1 | X = 1) = \frac{3}{4} \qquad (3)$$

$$\text{Probability that it is not a six} = P(E = 0 | X = 0) = \frac{1}{4} \qquad (4)$$

Now by Bayes's Formula we get,
Probability that the man reports the result to be six is actually a six,

$$P(X = 1|E = 1) = \frac{P(X = 1) \times P(E = 1|X = 1)}{P(X = 1) \times P(E = 1|X = 1) + ..}(\text{cont.}) \quad (5)$$

$$= \frac{1}{... + P(X = 0) \times P(E = 0|X = 0)} \quad (6)$$

Now using the values from (1), (2), (3) and (4),

$$P(X = 1|E = 1) = \frac{\dfrac{1}{6} \times \dfrac{3}{4}}{\dfrac{1}{6} \times \dfrac{3}{4} + \dfrac{5}{6} \times \dfrac{1}{4}} \quad (7)$$

$$= \frac{\dfrac{3}{24}}{\dfrac{3}{24} + \dfrac{5}{24}} \tag{8}$$

$$= \frac{\dfrac{3}{24}}{\dfrac{8}{24}} \tag{9}$$

$$= \frac{3}{8} \tag{10}$$