CHAPTER 15 : RANDOM VARIABLES

1. The discrete random variable X has the probability distribution as follows,

x	-60	20	24	60
$P(X = x)$	p	0.10	q	0.25

It is given that $P(X \le 20) = 0.25$.

(a) Determine the values of p and q. [2]

(b) Find $E(X)$ and $E\{[X - E(X)]^2\}$. [5]

(a) $P(X \le 20) = 0.25$:	$p + 0.1 = 0.25$	$\Rightarrow$	$p = 0.15$
$\sum P(X = x) = 1$:	$(0.15) + 0.1 + q + 0.25 = 1$	$\Rightarrow$	$q = 0.5$

(b) $E(X) = (-60)(0.15) + 20(0.1) + 24(0.5) + 60(0.25) = 20$

$E\{[X - E(X)]^2\} = E\{[X - 20]^2\} = (-80)^2(0.15) + 0 + (4)^2(0.5) + (40)^2(0.25) = 1368$

Alternative

$E(X^2) = (-60)^2(0.15) + (20)^2(0.1) + (24)^2(0.5) + (60)^2(0.25) = 1768$

$E\{[X - E(X)]^2\} = E\{[X - 20]^2\} = E(X^2) - 40\,E(X) + E(400)$
$= 1768 - 40(20) + 400 = 1368$

2. A discrete random variable X which takes the values 0, 1, 2 and 3, and its probability distribution is shown in the table below.

x	0	1	2	3
$P(X = x)$	a	b	$\frac{2}{3}a$	0.15

If $E(X) = 1.20$, find the values of a and b. [4]

Hence, show that X is not a binomial random variable. [2]

$\Sigma P(X = x) = 1$: $a + b + \frac{2}{3}a + 0.15 = 1 \Rightarrow b + \frac{5}{3}a = 0.85$

$E(X) = 1.20$: $0 + b + 2(\frac{2}{3}a) + 3(0.15) = 1.20 \Rightarrow b + \frac{4}{3}a = 0.75$

$\therefore\ a = 0.3,\ b = 0.35$

Assume $X \sim B(3, p)$, since given that $E(X) = 1.20$: $np = 3p = 1.2 \Rightarrow p = 0.4$

Since $P(X = 0) = 0.6^3 = 0.216$ ($\neq a$) $\Rightarrow$ X is **not** a binomial random variable

Allternative :

Since $P(X = 3) = 0.4^3 = 0.064$ ($\neq 0.15$) $\Rightarrow$ X is **not** a binomial random variable

3. The number of car accidents at a particular junction is not more than four accidents in a day. The probability that there is no car accident in a day is 0.4 and the probabilities that there are at most one, two and three car accidents in a day are 0.7, 0.85 and 0.95 respectively.

(a) Construct a probability distribution table for the number of car accidents at the junction in a day. [3]

(b) Calculate the mean of the number of car accidents at the junction in a day. [2]

(a) X = **no.** of car **accidents** at the junction **in a day**

$P(X = 0) = 0.4$, $P(X \le 1) = 0.7$, $P(X \le 2) = 0.85$ and $P(X \le 3) = 0.95$

x	0	1	2	3	4
$P(X = x)$	0.40	0.30	0.15	0.10	0.05

(b) Mean $= E(X) = 0(0.4) + 1(0.3) + 2(0.15) + 3(0.1) + 4(0.05) = 1.1$

4. On average, 48 cars enter a car park of a shopping complex per hour. The number of cars which enter the car park has a Poisson distribution.

(a) Find the probability that at most two cars enter the car park in a five-minute interval. [3]

(b) Determine time interval, in minutes, if the probability that at least one car enters the car park in this interval is 0.85. [4]

(a) X = no. of cars entering in 5 min, $X \sim P_o\left(\frac{5}{60} \times 48\right) \Rightarrow X \sim P_o(4)$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = e^{-4}\left(1 + 4 + \frac{4^2}{2!}\right) = 0.23810$

(b) T = no. of cars entering in t min. interval, $T \sim P_o\left(\frac{t}{60} \times 48\right) \Rightarrow T \sim P_o\left(\frac{4}{5}t\right)$

$P(T \ge 1) = 0.85 \Rightarrow 1 - P(T = 0) = 0.85$

$0.85 = 1 - e^{-\frac{4}{5}t} \Rightarrow -\frac{4}{5}t = \ln(0.15)$ $\therefore\ t = 2.3714$

5. A laboratory equipment shop gets 45% of the electronic weighing scale supplies from country Q. Five electronic weighing scales are selected at random.

(a) Determine the mean and standard deviation for the number of the electronic weighing scales supplied by country Q. [4]

(b) Find the probability that less than three electronic weighing scales are supplied by country Q. [4]

X = no. of electronic weighing scales supplied by country Q, $X \sim B(5, 0.45)$
(a) Mean = 5(0.45) = 2.25 , Standard deviation = $\sqrt{5(0.45)(0.55)}$ = 1.1124
(b) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$
$= {}^5C_0 \cdot (0.45)^0(0.55)^5 + {}^5C_1 \cdot (0.45)^1(0.55)^4 + {}^5C_2 \cdot (0.45)^2(0.55)^3 = 0.59313$

6. Studies conducted in the past show that the mean number of serious accidents in a certain factory is one per year. The factory wishes to find the probability of the number of serious accidents occurring on a yearly basis for the purpose of revising the insurance premium of its employees. The factory is required to increase the insurance premium if the probability of having less than two serious accident-free years out of five years is more than 0.35.

(a) Define the relevant random variable, and identify its distribution. [3]

(b) Find the probability that

(i) there is no serious accident occurring in the next one year, [2]

(ii) there are at least three serious accidents occurring in the next two years, [4]

(iii) there are less than two accident-free years occurring out of five years. [5]

(c) Based on the result in (b)(iii), state whether the factory is required to increase the insurance premium or not. [1]

(a) X = number of serious accidents in a certain factory in **a year**, $X \sim P_o(1)$

(b) (i) $P(X = 0) = e^{-1} = 0.3679$
(ii) Y = number of serious accidents in a certain factory in **2 year**, $Y \sim P_o(2)$
$P(Y \geq 3) = 1 - P(Y = 0, 1, 2) = 1 - e^{-2}(1 + 2 + \frac{2^2}{2!}) = 0.3233$
(iii) W = number of accident-free years, $W \sim B(5, 0.3679)$
$P(W < 2) = P(W = 0, 1) = {}^5C_0(0.3679)^0(0.6321)^5 + {}^5C_1(0.3679)(0.6321)^4 = 0.3946$

(c) Since $P(W < 2) > 0.35$, the factory is required to increase the insurance premium.

7. The continuous random variable X has the probability density function

$$f(x) = \begin{cases} k & ,0 \leq x < 1 \\ \frac{k}{x^2} & ,1 \leq x < 4 \\ 0 & ,\text{otherwise} \end{cases}$$

(a) Show that $k = \frac{4}{7}$. [3]

(b) Calculate the mean and Var(X). [7]

(a) $P(-\infty < X < \infty) = 1$: $\int_0^1 k\,dx + \int_1^4 \frac{k}{x^2}\,dx = 1 \Rightarrow [kx]_0^1 + [-\frac{k}{x}]_1^4 = 1$
$[k - 0] - [\frac{k}{4} - k] = 1 \Rightarrow k = \frac{4}{7}$

(b) $E(X) = \int_0^1 kx\,dx + \int_1^4 \frac{k}{x}\,dx = [\frac{2}{7}x^2]_0^1 + [\frac{4}{7}\ln x]_1^4$
$= \frac{2}{7}[1 - 0] + \frac{4}{7}[\ln 4 - 0] = \frac{2}{7} + \frac{4}{7}\ln 4$
$E(X^2) = \int_0^1 kx^2\,dx + \int_1^4 k\,dx = [\frac{4}{21}x^3]_0^1 + [\frac{4}{7}x]_1^4$
$= \frac{4}{21}[1 - 0] + \frac{4}{7}[4 - 0] = \frac{40}{21}$
$\therefore \text{Var}(X) = \frac{40}{21} - (\frac{2}{7} + \frac{4}{7}\ln 4)^2 = \frac{268}{147} - \frac{16}{49}\ln 4 - \frac{16}{49}(\ln 4)^2$ **or** 0.7429

8. The cumulative distribution function for the continuous random variable X is given by

$$F(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{3}x^2 & , 0 \le x < 1 \\ x - \frac{1}{6}x^2 - \frac{1}{2} & , 1 \le x < 3 \\ 1 & , x \ge 3 \end{cases}$$

(a) Calculate the median. [4]

(b) Find $P\left(|X-1| < \frac{1}{2}\right)$. [2]

(c) Determine the probability density function of X. [2]

(a) $F(m) = 0.5 \Rightarrow 1 \le m < 3$
$m - \frac{1}{6}m^2 - \frac{1}{2} = 0.5 \Rightarrow m^2 - 6m + 6 = 0$ $m = 3 - \sqrt{3}, 3 + \sqrt{3} \Rightarrow$ (since $1 \le m < 3$) : $m = 3 - \sqrt{3}$
(b) $P\left(\lvert X-1\rvert < \frac{1}{2}\right) = P(0.5 < X < 1.5) = \left[(1.5) - \frac{1}{6}(1.5)^2 - \frac{1}{2}\right] - \frac{1}{3}(0.5)^2 = \frac{13}{24}$
(c) $f(x) = \frac{d}{dx}[F(x)] = \begin{cases} \frac{2}{3}x & , 0 \le x < 1 \\ 1 - \frac{1}{3}x & , 1 \le x < 3 \\ 0 & , \text{otherwise} \end{cases}$

9. The continuous random variable X has probability density function

$$f(x) = \begin{cases} x-1 & , 1 < x \le 2 \\ k & , 2 < x \le 3 \\ 0 & , \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{1}{2}$. [2]

(b) Find the cumulative distribution function. [4]

(c) Sketch the graph of the cumulative distribution function. [2]

(d) Find $P\left(\frac{3}{2} \le X \le \frac{5}{2}\right)$. [2]

(a) $\frac{1}{2}(1)(0+1) + k(1) = 1 \Rightarrow k = \frac{1}{2}$

Alternative (a)

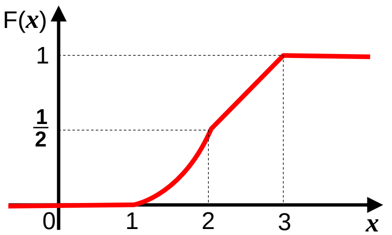
$\int_1^2 (x-1)\,dx + \int_2^3 k\,dx = 1 \Rightarrow \frac{1}{2}\left[(x-1)^2\right]_1^2 + [kx]_2^3 = 1$ $\therefore \frac{1}{2}(1-0) + k(3-2) = 1 \Rightarrow k = \frac{1}{2}$

(b) $F(x) = \begin{cases} 0 & , x \le 1 \\ \frac{1}{2}x^2 - x + c_1 & , 1 < x \le 2 \\ \frac{1}{2}x + c_2 & , 2 < x \le 3 \\ 1 & , 3 < x \end{cases}$
$\underline{x=1}$: $0 = \frac{1}{2} - 1 + c_1 \Rightarrow c_1 = \frac{1}{2}$ $\qquad$ $\underline{x=3}$: $\frac{1}{2}(3) + c_2 = 1 \Rightarrow c_2 = -\frac{1}{2}$
$\therefore F(x) = \begin{cases} 0 & , x \le 1 \\ \frac{1}{2}(x^2 - 2x + 1) & , 1 < x \le 2 \\ \frac{1}{2}(x-1) & , 2 < x \le 3 \\ 1 & , 3 < x \end{cases}$

Alternative (b)

(b) $1 < x \le 2$: $F(x) = \int_1^x (x-1)\,dx = \left[\frac{1}{2}x^2 - x\right]_1^x = \left[\frac{1}{2}x^2 - x\right] - \left[\frac{1}{2} - 1\right] = \frac{1}{2}(x^2 - 2x + 1)$
$2 < x \le 3$: $F(x) = \int_1^2 (x-1)\,dx + \int_2^x \frac{1}{2}\,dx = \frac{1}{2}(4 - 4 + 1) + \left[\frac{1}{2}x\right]_2^x$
$= \frac{1}{2} + \frac{1}{2}[x - 2] = \frac{1}{2}(x-1)$
$\therefore F(x) = \begin{cases} 0 & , x \le 1 \\ \frac{1}{2}(x^2 - 2x + 1) & , 1 < x \le 2 \\ \frac{1}{2}(x-1) & , 2 < x \le 3 \\ 1 & , 3 < x \end{cases}$

(c)



(d) $P\left(\frac{3}{2} \le X \le \frac{5}{2}\right) = P\left(X \le \frac{5}{2}\right) - P\left(X < \frac{3}{2}\right) = \frac{1}{2}\left[\frac{5}{2} - 1\right] - \frac{1}{2}\left[\left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right) + 1\right] = \frac{5}{8}$

10. A continuous random variable X has probability density function

$$f(x) = \begin{cases} k & ,0 \le x \le 2 \\ \dfrac{m}{x^2} & ,2 < x \le 5 \\ 0 & ,5 < x \end{cases}$$

It is given that $3\,P(X \le 2) = 2\,P(X > 2)$.

(a) Show that $k = \frac{1}{5}$ and find the value of m. [5]

(b) Find the cumulative distribution function of X. [5]

(c) Determine $P(1 \le X \le 3)$. [2]

(d) Calculate the median value of X. [3]

(a) $3P(X \le 2) = 2\,P(X > 2) = 2[1 - P(X \le 2)] \Rightarrow P(X \le 2) = \frac{2}{5}$
$2k = \frac{2}{5} \Rightarrow k = \frac{1}{5}$
$P(-\infty < X < \infty) = 1:\ 2 \times \frac{1}{5} + \int_2^5 mx^{-2}\,dx = 1 \Rightarrow \frac{2}{5} + \left[-\frac{m}{x}\right]_2^5 = 1$
$-\frac{m}{5} + \frac{m}{2} = \frac{3}{5} \Rightarrow m = 2$

Alternative (a)

$\int_0^2 k\,dx + \int_2^5 \frac{m}{x^2}\,dx = 1$,	$3\int_0^2 k\,dx = 2\int_2^5 \frac{m}{x^2}\,dx$
$[kx]_0^2 + \left[-\frac{m}{x}\right]_2^5 = 1$	$3[kx]_0^2 = 2\left[-\frac{m}{x}\right]_2^5$
$(2k - 0) + (-\frac{m}{5} + \frac{m}{2}) = 1$	$3(2k - 0) = 2(-\frac{m}{5} + \frac{m}{2})$
$\therefore\ k = \frac{1}{5},\ m = 2$	

(b) $F(x) = \begin{cases} 0 & ,x < 0 \\ \frac{x}{5} + c_1 & ,0 \le x \le 2 \\ -\frac{2}{x} + c_2 & ,2 < x \le 5 \\ 1 & ,x > 5 \end{cases}$
$\because F(x)$ is continuous, $0 = \frac{1}{5}(0) + c_1 \Rightarrow c_1 = 0$
$\frac{1}{5}(2) + 0 = -\frac{2}{2} + c_2 \Rightarrow c_2 = \frac{7}{5}$
$F(x) = \begin{cases} 0 & ,x < 0 \\ \frac{x}{5} & ,0 \le x \le 2 \\ \frac{7}{5} - \frac{2}{x} & ,2 < x \le 5 \\ 1 & ,x > 5 \end{cases}$

Alternative (b)

$P(X \le x) = \int_0^x \frac{1}{5}\,dx = \left[\frac{x}{5}\right]_0^x = \frac{1}{5}x$
$P(X \le x) = \frac{2}{5} + \int_2^x \frac{2}{x^2}\,dx = \frac{2}{5} + \left[-\frac{2}{x}\right]_2^x = \frac{7}{5} - \frac{2}{x}$
$F(x) = \begin{cases} 0 & ,x < 0 \\ \frac{x}{5} & ,0 \le x \le 2 \\ \frac{7}{5} - \frac{2}{x} & ,2 < x \le 5 \\ 1 & ,x > 5 \end{cases}$

(c) $P(1 \le X \le 3) = P(X \le 3) - P(X < 1) = (\frac{7}{5} - \frac{2}{3}) - \frac{1}{5} = \frac{8}{15}$

(d) Since $P(X \le m) = \frac{1}{2}$, $2 < m \le 5$
$\frac{7}{5} - \frac{2}{m} = \frac{1}{2} \Rightarrow m = \frac{20}{9}$

11. A continuous random variable X has a probability density function,

$$f(x) = \begin{cases} \dfrac{2(k-x)}{k^2} & ,0 \le x \le k \\ 0 & ,\text{otherwise} \end{cases}$$

where k is a positive constant

(a) Find the mean of X in terms of k. [3]

(b) Show that the standard deviation of X is equal to $\dfrac{k}{3\sqrt{2}}$. [4]

(c) Find $P(X < \mu + \sigma\sqrt{2})$, where μ and σ are the mean and standard deviation of X respectively. [5]

(a) Mean $= \displaystyle\int_0^k x.\frac{2(k-x)}{k^2}\,dx = \frac{1}{k^2}\left[kx^2 - \frac{2x^3}{3}\right]_0^k = \frac{1}{k^2}\left[k^3 - \frac{2k^3}{3}\right] - 0 = \frac{1}{3}k$

(b) $E(X^2) = \displaystyle\int_0^k x^2.\frac{2(k-x)}{k^2}\,dx = \frac{2}{k^2}\left[\frac{kx^3}{3} - \frac{x^4}{4}\right]_0^k = \frac{2}{k^2}\left[\frac{k^4}{3} - \frac{k^4}{4}\right] - 0 = \frac{1}{6}k^2$

Variance of $X = \dfrac{1}{6}k^2 - \left(\dfrac{1}{3}k\right)^2 \quad \Rightarrow \quad$ Standard deviation of $X = \dfrac{k}{3\sqrt{2}}$

(c) $P(X < \mu + \sigma\sqrt{2}) = P(X < \frac{2}{3}k) = \displaystyle\int_0^{\frac{2}{3}k} \frac{2(k-x)}{k^2}\,dx = \frac{1}{k^2}\left[2kx - x^2\right]_0^{\frac{2}{3}k}$

$= \dfrac{1}{k^2}\left[\dfrac{4k^2}{3} - \dfrac{4k^2}{9}\right] - 0 = \dfrac{8}{9}$

12. (a) Describe briefly the standard normal random variable. [2]

(b) The life span of a certain light bulb is a normal random variable with a mean of 950 hours and a standard deviation of 50 hours.

(i) Find the probability that a randomly chosen light bulb has a life span of more than 1000 hours. [3]

(ii) Determine the value of h such that 99% of the light bulbs have life span between $(950 - h)$ hours and $(950 + h)$ hours. [5]

(iii) Find the probability that at most one out of eight independently selected light bulbs has a life span of more than 1000 hours. [5]

(a) Standard normal random variable is a **continuous** random variable **and** with a mean **zero** and variance **1**.

(b) (i) Let X be the life span of a certain light bulb, $X \sim N(950, 50^2)$

$P(X > 1000) = P(Z > \frac{1000-950}{50}) = P(Z > 1) = 0.1587$

(ii) $P(950 - h < X < 950 + h) = 0.99 \Rightarrow P(-\frac{h}{50} < Z < \frac{h}{50}) = 0.99$

$\therefore P(Z < \frac{h}{50}) = 0.995$ **or** $P(Z > \frac{h}{50}) = 0.005 \Rightarrow \frac{h}{50} = 2.576 \qquad \therefore h = 128.8$

Note : **Do not accept** confidence interval method, cause **population mean** is known.

(iii) Let Y = no. of light bulbs that $(X > 1000)$, $Y \sim B(8, 0.1587)$

$P(Y \le 1) = P(Y = 0) + P(Y = 1) = (0.8413)^8 + 8(0.1587)(0.8413)^7 = 0.6299$

13. The masses of guavas in a farm are normally distributed with mean, μ, and standard deviation, σ. The mass percentages of guava that less than 400 g is 15.87% and more than 500 g is 6.68%.

(a) Determine the values of μ, and σ. [7]

(b) Find the probability that a guava has a mass of at most 500 g given that it is more than 400 g. [3]

An agriculture officer thinks that the number of guavas, Y, on a branch of guava trees in the farm may be modelled by a Poisson distribution.

(c) If $P(Y = 0) = 0.0954$, find the parameter of the Poisson distribution, [3]

(d) Give **two** reasons why Poisson distribution cannot be an exact model for Y. [2]

(a) X = mass of guava :
$P(X < 400) = 0.1587$: $\dfrac{400-\mu}{\sigma} = -1 \Rightarrow 400 - \mu = -\sigma$ ---- (1)
$P(X > 500) = 0.0668$: $\dfrac{500-\mu}{\sigma} = 1.5 \Rightarrow 500 - \mu = 1.5\sigma$ ---- (2)
$\therefore \quad \mu = 440 \;,\; \sigma = 40$

(b) $P(X \le 500 \mid X > 400) = \dfrac{P(400 < X \le 500)}{P(400 < X)} = \dfrac{(1-0.0668)-0.1587}{1-0.1587} = 0.92060$

(c) $Y \sim P_o(\lambda)$, $P(Y = 0) = 0.0954$: $e^{-\lambda} = 0.0954$
$\therefore \lambda = -\ln 0.0954 = 2.350$

(d) R_1 : The **space** (a branch of tree) defined in Y is **not a constant /consistent**(not specific)
R_2 : The **size** of guavas taken into account is **not specified**.

14. It is known that 2% of printers sold by a supplier malfunction within one year. The supplier offers a warranty period of one year.

(a) Find the probability that, out of 20 printers, at least three printers malfunction within the warranty period. [4]

(b) Determine the least number of printers, such that the probability that at least one printer malfunctions within the warranty period is more than 0.9995. [5]

(c) Using a suitable approximation, find the probability that, out of 1000 printers, there are between 10 to 18 printers (inclusive) malfunction within the warranty period. [6]

(a) Let X = number of printers malfunction within the warranty period, $X \sim B(20, 0.02)$
$P(X \ge 3) = 1 - P(X = 0, 1, 2) = 1 - \sum_{x=0}^{2} {}^{20}C_x\,(0.02)^x((0.98)^{20-x} = 0.0070687$

(b) Let Y = number of printers malfunction within the warranty period, $Y \sim B(\boldsymbol{n}, 0.02)$
$P(Y \ge 1) > 0.9995$: $P(Y = 0) < 0.0005 \Rightarrow 0.98^{n} < 0.0005$ $n \log 0.98 < \log 0.0005 \Rightarrow n > 376.23$ $\therefore$ Least number of printers required = 377

(c) $T \sim B(1000, 0.02) \xrightarrow{np=20,\, nq=980} T \sim N(20, 19.6)$		
$P(10 \le T \le 18) \longrightarrow P(9.5 \le T \le 18.5)$		
$= P\left(\dfrac{9.5-20}{\sqrt{19.6}} \le Z \le \dfrac{18.5-20}{\sqrt{19.6}}\right) = P(-2.372 \le Z \le -0.339)$		
$= P(Z \le 2.372) - P(Z \le 0.339)$ $= 0.9912 - 0.6327$ $= 0.3585$	$= P(-0.339) - P(-2.372)$ $= 0.3673 - 0.0088$ $= 0.3585$	$= R(-2.372) - R(-0.339)$ $= 0.9912 - 0.6327$ $= 0.3585$

15. In a given population, 25% of the population choose jogging as their leisure activity.

(a) Find the probability that, of the 20 people selected at random,

(i) exactly five people choose jogging as their leisure activity, [3]

(ii) at most 17 people do not choose jogging as their leisure activity. [4]

(b) Use a suitable approximation to find the probability that, out of 60 selected at random, 12, 13, 14 or 15 choose jogging as their leisure activity. Justify the choice of your approximation. [8]

(a) (i) X = No. of people who choose jogging as their leisure activity

$X \sim B(20\,,\,0.25)$

$P(X=5) = {}^{20}C_5 \,.\, (0.25)^5\,(0.75)^{15} = 0.20233$

(ii) $P(X \ge 3)$ $= 1 - [P(X=0)+P(X=1)+P(X=2)]$ $= 1 - \sum_{x=0}^{2} {}^{20}C_x \,.\, (0.25)^x\,(0.75)^{20-x}$ $= 0.90874$	$P(X \ge 3) = 1 - P(X<3)$ $= P_3 + P_4 + P_5 + \ldots + P_{20}$ $= \sum_{x=3}^{20} {}^{20}C_x \,.\, (0.25)^x\,(0.75)^{20-x}$ $= 0.90874$

Alternative

(ii) Y = No. of people who <u>do not</u> choose jogging as their leisure activity

$Y \sim B(20\,,\,0.75)$

$P(Y \le 17)$ $= P_0 + P_1 + P_2 + \ldots + P_{17}$ $= \sum_{y=0}^{17} {}^{20}C_y \,.\, (0.75)^y\,(0.25)^{20-y}$ $= 0.90874$	$P(Y \le 17) = 1 - P(Y>17)$ $= 1 - [P(Y=18)+P(Y=19)+P(Y=20)]$ $= 1 - \sum_{y=18}^{20} {}^{20}C_y \,.\, (0.75)^y\,(0.25)^{20-y}$ $= 0.90874$

(b) Use normal approximation to binomial

$\mu = 60(0.25) = 15 \qquad \sigma = \sqrt{60(0.25)(0.75)} = 3.3451 \text{ or } \sigma^2 = 11.25$

$P(12 \le X \le 15) \longrightarrow P(11.5 \le X \le 15.5)$

$$= P\left(\frac{11.5-15}{3.3451} \le Z \le \frac{15.5-15}{3.3451}\right) = P(-1.044 \le Z \le 0.149)$$

$= P(Z \le 0.149) - P(Z \le -1.044)$ $= 0.5592 - (1 - 0.8518)$ $= 0.4110$	$= P(0.149) - P(-1.044)$ $= 0.5592 - 0.1482$ $= 0.4110$	$= R(-1.044) - R(0.149)$ $= 0.8518 - 0.4408$ $= 0.4110$

Justification : $np = 15\ (>5)$ and $nq = 45\ (>5)$

16. The discrete random variable X has probability function

$$P(X=x) = \begin{cases} k(4-x)^2 & , x = 1,2,3 \\ 0 & , \text{otherwise} \end{cases}, \quad \text{where } k \text{ is a constant.}$$

(a) Determine the value of k and tabulate the probability distribution of X. [3]

(b) Find $E(7X-1)$ and $Var(7X-1)$. [7]

(a) $\sum P(X=x) = 1 : \quad k(3)^2 + k(2)^2 + k(1)^2 = 1 \quad \Rightarrow \quad k = \frac{1}{14}$

x	1	2	3
$P(X=x)$	$\frac{9}{14}$	$\frac{4}{14}$	$\frac{1}{14}$

(b) $E(7X-1) = \sum (7x-1).P(X=x) = 6\left(\frac{9}{14}\right) + 13\left(\frac{4}{14}\right) + 20\left(\frac{1}{14}\right) = 9$

$Var(7X-1) = E[(7X-1)^2] - [E(7X-1)]^2$

$$= \left[(6)^2.\left(\frac{9}{14}\right) + (13)^2.\left(\frac{4}{14}\right) + (20)^2.\left(\frac{1}{14}\right)\right] - (9)^2 = 19$$