0.1 建立决策矩阵 Establishing a decision matrix

依据题目描述：随着资源环境的变化，雄性占比可大致认为在 0.5-0.8 之间。我们不妨以 0.05 为步长，采样 7 组数据，建立决策矩阵
Based on the description of the topic: as the resource environment changes, the male share can be roughly considered to be between 0.5 and 0.8. We may wish to sample 7 sets of data in steps of 0.05 and build a decision matrix$(A_{i\times j})_{m\times n}$

0.2 原始决策矩阵正向化 Raw Decision Matrix Normalization

构建决策矩阵时我们需要将原始矩阵正向化处理。在四个评价指标中，我们认为寄生虫传播为极小型指标，使用公式转化为极大型指标
When constructing the decision matrix, we need to normalize the original matrix. Among four evaluation criteria, we consider parasite transmission as a minimization criterion, which we transform into a maximization criterion using a formula.

$$\widetilde{x_i} = \max x_i - x_i$$

同时竞争与合作关系为中间型指标.
Meanwhile, competitive and cooperative interactions are considered as an intermediate criterion, which we also transform into a maximization criterion using a formula.

$$\widetilde{x_i} = 1 - \frac{|x_i - x_{best}|}{\max(|x_i - x_{best}|)}$$

食物链的影响与生态位的改变为区间型指标，使用公式转化为极大型指标
Food web dynamics and ecological niche modification are regarded as range criteria, which we convert into maximization criteria using a formula.

$$M = \max(a - \min x_i, \max x_i - b)$$

Where,

$$\widetilde{x_i} = \begin{cases} 1 - \frac{a - x_i}{M} & , x_i < a \\ 1 & , a \leq x_i \leq b \\ 1 - \frac{x_i - b}{M} & , x_i > b \end{cases}$$

0.3 正向决策矩阵标准化 Normalization of forward decision matrices

为了消除不同指标量纲的影响我们进行标准化上述操作后正向化的矩阵如下

In order to eliminate the effect of different metrics scales we can normalize the matrix. The normalized matrix after the above operation is as follows

$$\widetilde{X} = \begin{pmatrix} \widetilde{x}_{11} & \cdots & \widetilde{x}_{in} \\ \vdots & \ddots & \vdots \\ \widetilde{x}_{n1} & \cdots & \widetilde{x}_{nn} \end{pmatrix}$$

Let the matrix to which it is normalized be Z, and each element in Z is given by

$$z_{ij} = \frac{\widetilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \widetilde{x}_{ij}^2}}$$

0.4 熵权法计算各项指标权重 Entropy weighting method to calculate the weights of indicators

为了确保权重的客观性，我们采用熵权法构造权重指标。在标准化处理后的非负矩阵。我们计算比重矩阵 P 其中 P 的每一个元素 p_{ij} 计算公式如下

In order to ensure the objectivity of the weights, we use the entropy weight method to construct the weight indicators. In the non-negative matrix after normalization, we calculate the weight matrix P where each element p_{ij} of

P is calculated as follows

$$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}}$$

根据信息论中信息熵的定义，引入信息熵

According to the definition of information entropy in information theory, we introduce the information entropy

$$E_j = -\frac{\sum_{i=1}^{n} p_{ij} \log p_{ij}}{\log n}$$

if $p_{ij} = 0$ then let $E_j = 0$

By calculating the information utility value $D_j = 1 - E_j$ We Calculate the indicator weights

$$w_j = \frac{D_j}{\sum_{j=1}^{n} D_j}$$

then derive the weight vector W

0.5 计算得分和排序 Calculating scores and ranking

For an already normalized forward matrix Z

Define the maximum value

$$\begin{aligned} Z^+ &= (Z_1^+, Z_2^+, \cdots, Z_m^+) \\ &= (\max(z_{11}, z_{21}, \cdots, z_{n1}), \max(z_{12}, z_{22}, \cdots, z_{n2}), \max(z_{1m}, z_{2m}, \cdots, z_{nm})) \end{aligned}$$

Similarly, define the minimum value

$$\begin{aligned} Z^- &= (Z_1^-, Z_2^-, \cdots, Z_m^-) \\ &= (\min(z_{11}, z_{21}), \cdots, z_{n1}), \min(z_{12}, z_{22}, \cdots, z_{n2}), \min(z_{1m}, z_{2m}, \cdots, z_{nm})) \end{aligned}$$

Under the introduction of the entropy weight method of weights Define the distance between the ith evaluation object and the maximum value

$$D_i^+ = \sqrt{\sum_{j=1}^{m} w_j (Z_j^+ - z_{ij})^2}$$

Define the distance of the ith evaluated object from the minimum value

$$D_i^- = \sqrt{\sum_{j=1}^{m} w_j (Z_j^- - z_{ij})^2}$$

Then the score of the ith evaluation object can be calculated as

$$S_i = \frac{D_i^-}{D_i^- + D_i^+}$$