Project 3 Due Date: 11:59pm, Sunday April 16

In this project, we implement the steepest descent method (SD) and the BFGS method for finding a local minimizer of a multivariate function. We consider a two dimensional quadratic function

$$f(\mathbf{x}) = 100(x_1 - x_2)^2 + (x_2 - 1)^2, \tag{1}$$

which has a unique minimizer at $\mathbf{x}^\star = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

1. Given $\mathbf{x^k}$ and the search direction $\mathbf{p^k}$, the ideal step size α_k is determined by minimizing the 1-dimensional function $\varphi(\alpha) = f(\mathbf{x}^k + \alpha_k \mathbf{p^k})$. For the quadratic function f defined in (1), find the explicit formula for α_k.

2. In general, there is no analytical formula for the best α_k, nor is it necessary. Implement the Backtracking algorithm (in note Lec23) for finding an acceptable stepsize (and update $\mathbf{x^{k+1}}$), using $\rho = 0.001$ and $\sigma = 0.9$.

3. For the quadratic function f given in (1), implement the SD algorithm with backtracking for α_k for finding the local minimizer. Take the error tolerance to be 10^{-10} and $\mathbf{x}^0 = \mathbf{0}$, output the error $||\mathbf{x}^k - \mathbf{x}^\star||$ and the iteration number k into a data file. Plot the error vs iteration number.

4. Likewise, implement the BFGS algorithm (note Lec25). Repeat the same test, and plot error vs iteration number.

Please attach your code to the report in the PDF format.