Task 1: Analyzing the Free Energy Function

Image 1

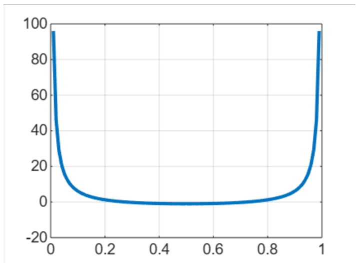


Image 2

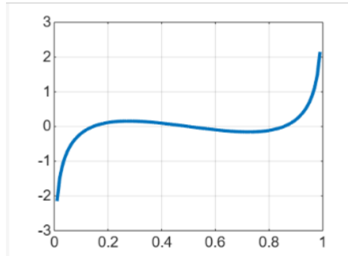
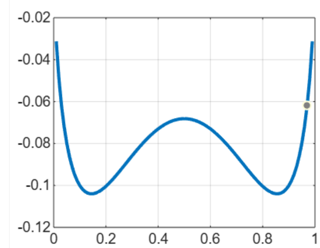


Image 3



Analysis of the Three Plots:

1. **Plot of Free Energy Function(image 3)**:

    - This shows the double-well potential characteristic of phase separation systems
    - The curve has two minima (at approximately $\phi \approx 0.1$ and $\phi \approx 0.9$)
    - There's a local maximum at around $\phi \approx 0.5$

2. **Plot of Chemical Potential (image 2)**:

    - This is the first derivative of the free energy
    - It crosses zero at three points: these correspond to the two minima and one maximum in the free energy curve
    - Zero-crossings appear to be at approximately $\phi \approx 0.1$, $\phi \approx 0.5$, and $\phi \approx 0.9$

3. **Plot of Second Derivative (image 1)**:

    - Shows where the curvature of the free energy function changes
    - Negative values indicate the spinodal region (approximately $0.3 < \phi < 0.7$)
    - Very high positive values near $\phi = 0$ and $\phi = 1$

Determining the Key Regions:

Spinodal Region:

- This is where the second derivative is negative (ddF < 0)
- From your Figure 3, this occurs approximately between $\phi \approx 0.3$ and $\phi \approx 0.7$
- In this region, any small fluctuation will lead to spontaneous phase separation

Miscibility Gap:

- This corresponds to the range between the minima of the free energy function
- From your Figure 6, these minima are at approximately $\phi \approx 0.1$ and $\phi \approx 0.9$
- This represents the range of compositions where phase separation occurs, resulting in two equilibrium phases with compositions at the boundaries of this gap

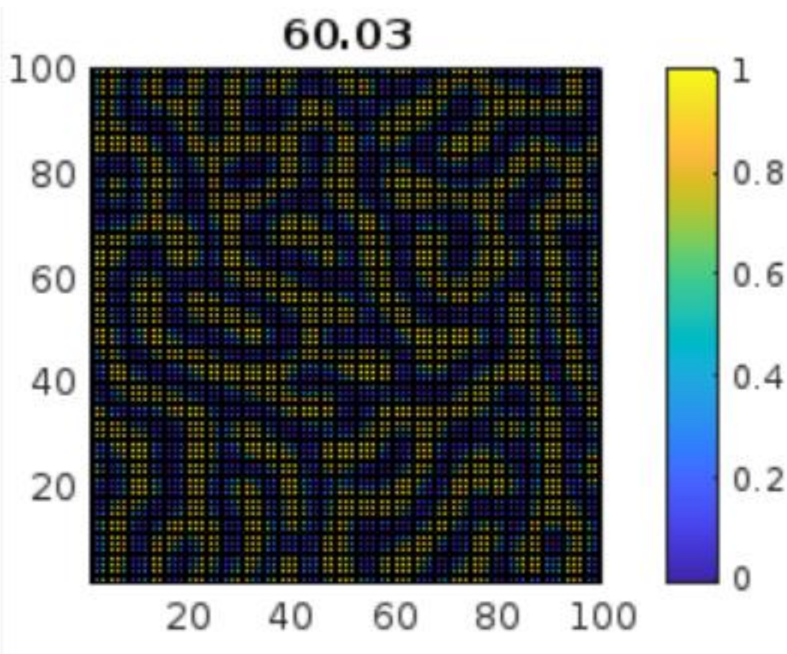
Physical Interpretation:

- For any overall composition within the miscibility gap ($0.1 < \phi < 0.9$), the system will separate into two phases with compositions $\phi \approx 0.1$ and $\phi \approx 0.9$
- If the composition is within the spinodal region ($0.3 < \phi < 0.7$), this separation happens spontaneously (spinodal decomposition)
- If the composition is in the metastable region ($0.1 < \phi < 0.3$ or $0.7 < \phi < 0.9$), separation requires nucleation and growth

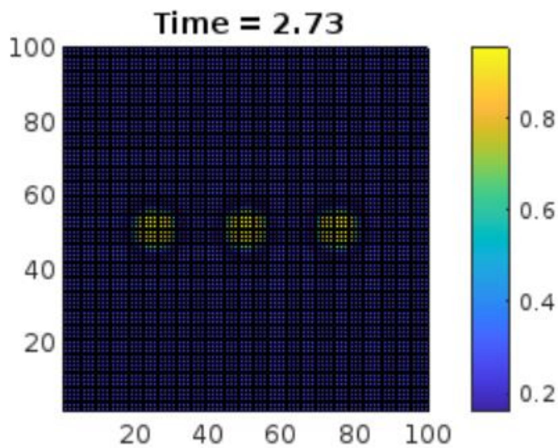
Summary for Task 1:

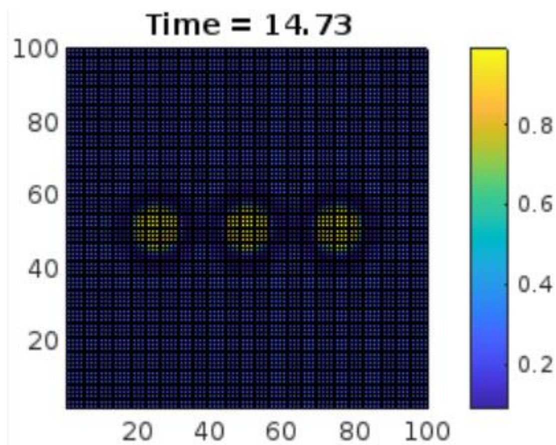
- Miscibility gap: $\phi \approx 0.1$ to $\phi \approx 0.9$
- Spinodal region: $\phi \approx 0.3$ to $\phi \approx 0.7$
- Metastable regions: $0.1 < \phi < 0.3$ and $0.7 < \phi < 0.9$

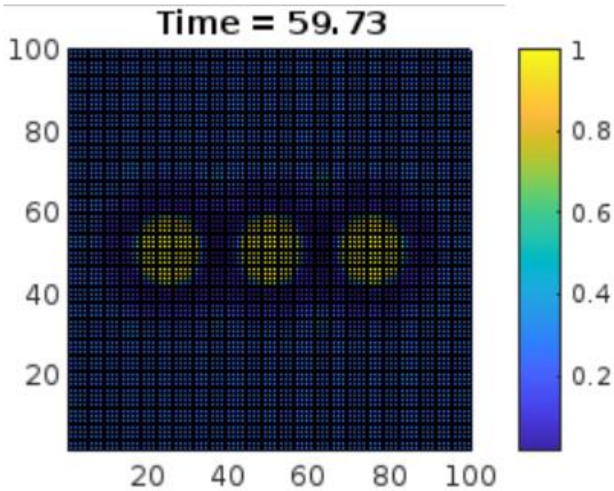
Task 2: Completing the Cahn-Hilliard01.m Code



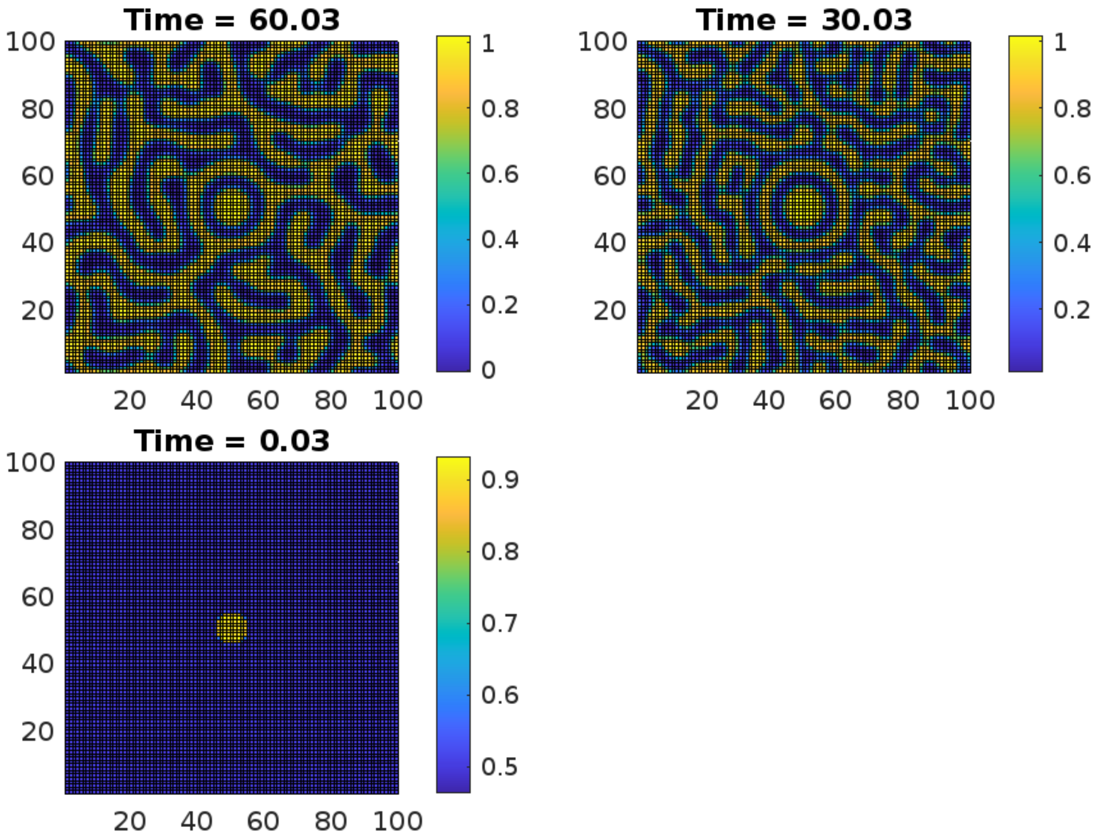
Task 3: Modifying Initial Condition for Metastable Region







Task 4: Modifying Initial Condition for Nucleation-Growth Dynamics.



Discussion of Observations

Task 3 - Metastable Region:

- The system remains largely uniform for many iterations.
- Fluctuations alone are not enough to cause phase separation quickly.
- If phase separation begins, it occurs through isolated regions—indicating the need for nucleation.

Interpretation:

This behavior confirms the region $\phi \approx 0.25$ lies in the **metastable range**. Without a large enough fluctuation (nucleus), the system resists decomposition.

Task 4 - Nucleation and Growth:

- A nucleus at $\phi = 0.9$ embedded in a $\phi = 0.25$ background begins to grow.
- Over time, the nucleus expands as the system lowers its free energy.
- Eventually, the domain separates into distinct phases.

Interpretation:
This shows the **nucleation-growth mechanism**. A critical-sized nucleus grows in a metastable environment, initiating phase separation—matching what is expected for compositions outside the spinodal range.