EECS 545: Machine Learning

Lecture 22. Reinforcement Learning 2

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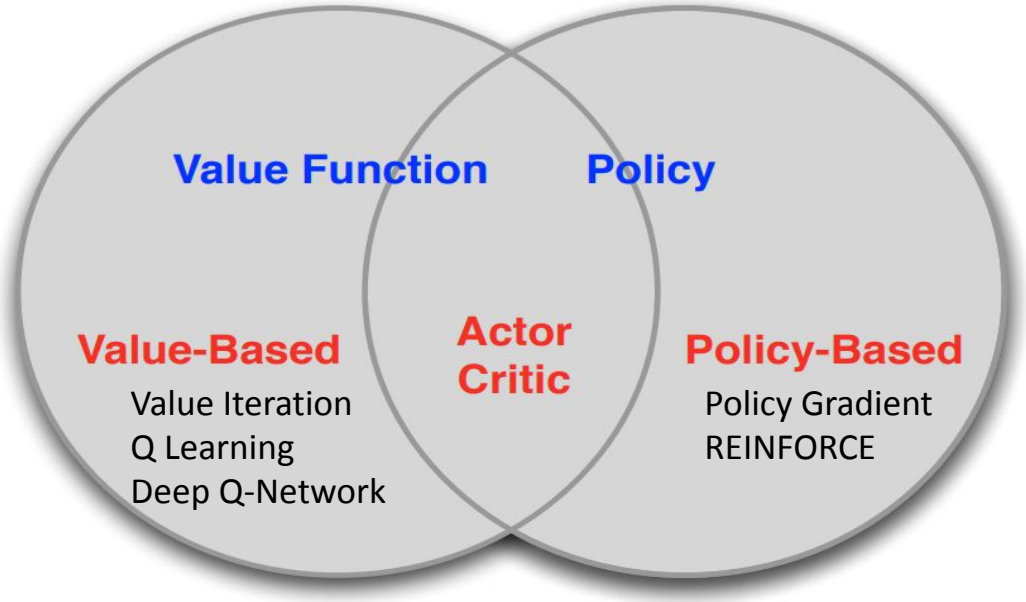
03/30/2022

Outline

- Q-Learning
- Deep Q-Network and its variants
- Policy-based methods
- REINFORCE



Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \ldots$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

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How good is a state?

$$R^a_{ss'} = R(s)$$

Notation: reward is a function of current state s.

The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

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How good is a state-action pair?

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s, a) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

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Optimal policy and value function

The *optimal policy* is the policy that achieves the highest value for every state

$$\pi^\star = \underset{\pi}{\operatorname{argmax}}\ V^\pi(s)$$

And its value function is written $V^\star = V^{\pi^\star}$ (but there are an exponential number of policies, so this formulation is not very useful)

Slide credit: J. Zico Kolter

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And its value function is written $V^\star = V^{\pi^\star}$ (but there are an exponential number of policies, so this formulation is not very useful)

Instead, we can directly define the optimal value function using the ***Bellman optimality equation***

$$V^\star(s) = R(s) + \gamma \max_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} P(s'|s,a)\, V^\star(s')$$

Notation: reward is a function of current state s.

$$R^a_{ss'} = R(s)$$

and optimal policy is simply the action that attains this max

$$\pi^\star(s) = \operatorname*{argmax}_a \sum_{s' \in \mathcal{S}} P(s'|s,a)\, V^\star(s')$$

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Computing the optimal policy

How do we compute the optimal policy? (or equivalently, the optimal value function?

(A solution:) **Value iteration**: repeatedly update an estimate of the optimal value function according to Bellman optimality equation

1. Initialize an estimate for the value function arbitrarily

$$\hat{V}(s) \leftarrow 0, \quad \forall s \in \mathcal{S}$$

2. Repeat update

$$\hat{V}(s) \leftarrow R(s) + \gamma \max_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} P(s'|s,a)\, \hat{V}(s'), \quad \forall s \in \mathcal{S}$$

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Bellman equation for Q-value function

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

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Q* satisfies the following **Bellman equation**:

$$Q^*(s,a) = \mathbb{E}_{s'\sim\mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s, a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s',a') are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s',a')$

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The optimal policy **π*** corresponds to taking the best action in any state as specified by Q*

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Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s,a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s',a') | s,a\right]$$

Q_i will converge to Q* as i -> infinity

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What's the problem with this?

Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

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Solution: use a function approximator to estimate Q(s,a). E.g. a neural network!

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Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

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Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

If the function approximator is a deep neural network => **deep q-learning**!

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Deep Q-Network (DQN)

Solving for the optimal policy: Q-learning

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$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning**!

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Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

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Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(y_i - Q(s,a;\theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s, a \right]$

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Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i)) \nabla_{\theta_i} Q(s,a;\theta_i) \right]$$

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Case Study: Playing Atari Games

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

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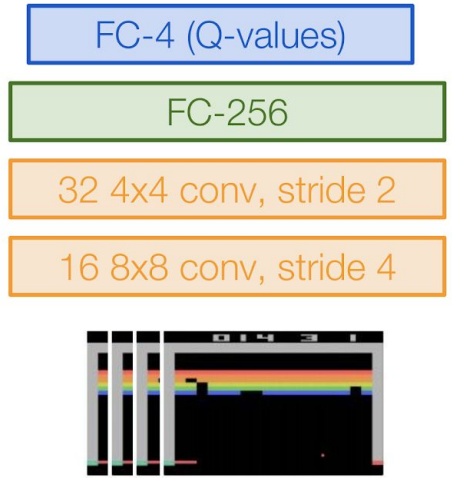
Q-network Architecture

[Mnih et al. NIPS Workshop 2013; Nature 2015]

$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

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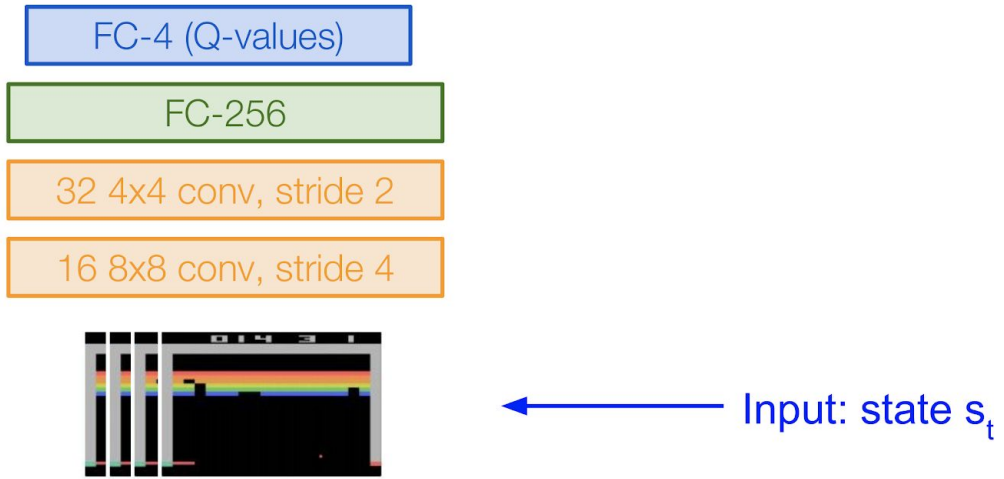

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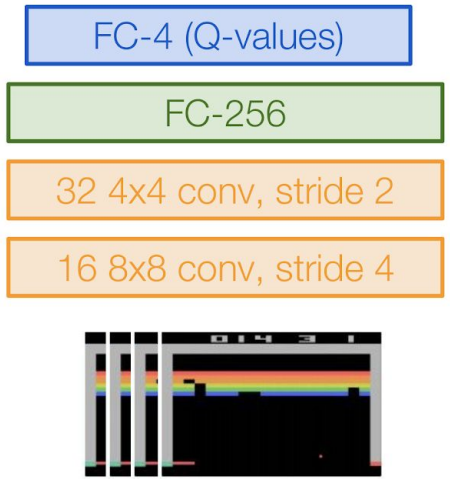

Familiar conv layers,
FC layer

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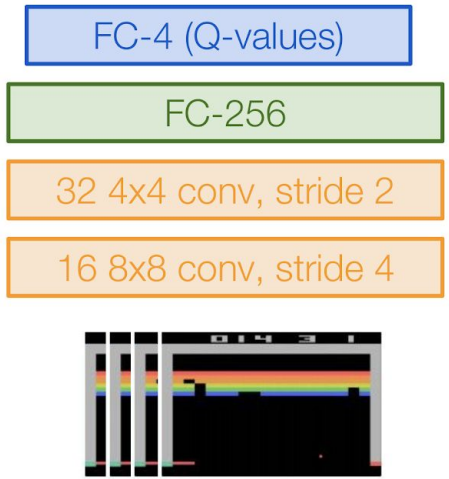

Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

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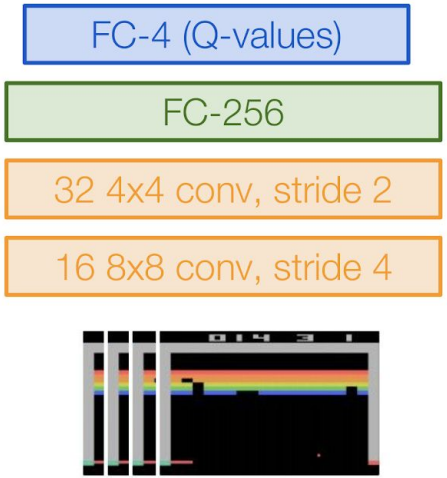
A single feedforward pass to compute Q-values for all actions from the current state => efficient!



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Number of actions between 4-18 depending on Atari game

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Training Q-network: Loss function (from before)

[Mnih et al. NIPS Workshop 2013; Nature 2015]

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Training Q-network: Experience Replay

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

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Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

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Each transition can also contribute to multiple weight updates
=> greater data efficiency

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Putting it together: Deep Q-learning with Experience Replay

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

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Putting it together: Deep Q-learning with Experience Replay

[Mnih et al. NIPS Workshop 2013; Nature 2015]

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for episode $= 1, M$ **do** ← Play M episodes (full games)
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Initialize state (starting game screen pixels) at the beginning of each episode

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For each timestep t of the game

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Putting it together: Deep Q-learning with Experience Replay

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
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    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
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  end for
end for

With small probability select a random action (explore), otherwise select greedy action from policy

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Take action a_t, and observe reward r_t and next state s_{t+1}

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

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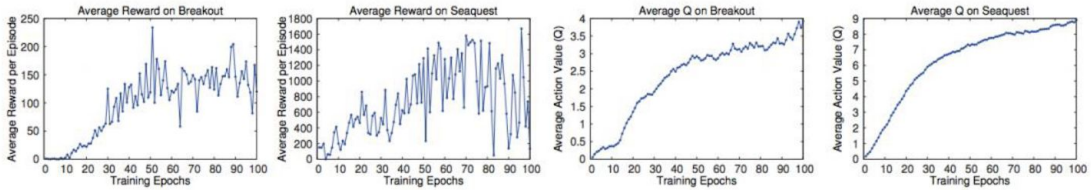
Experience replay: Sample a random minibatch of transitions from replay memory and perform gradient descent step

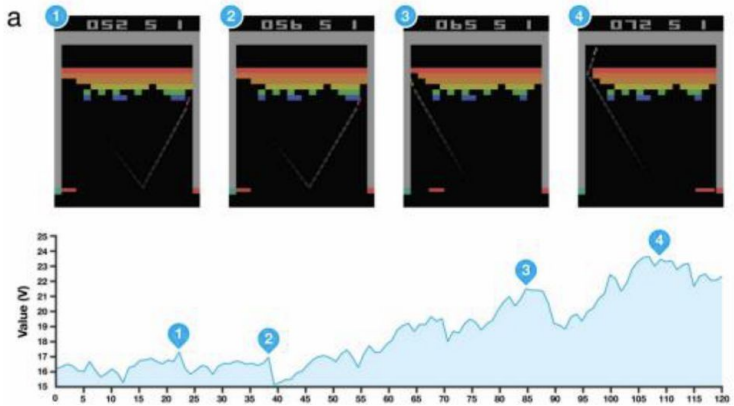
Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

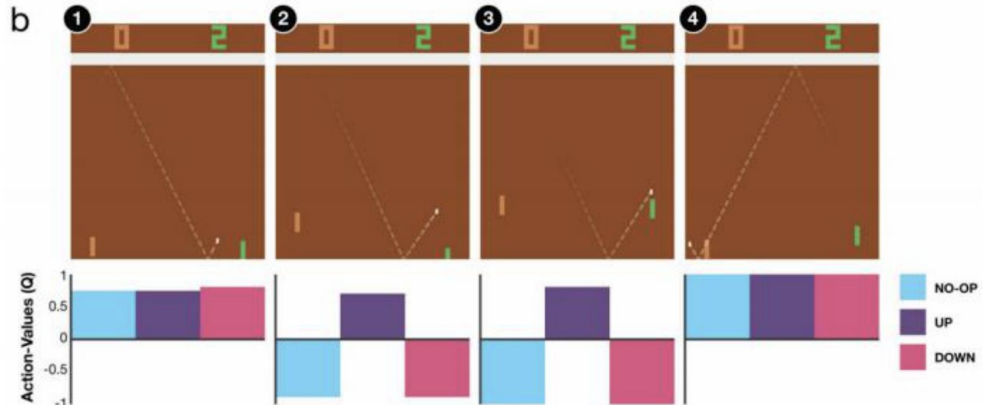
Video from: https://www.youtube.com/watch?v=V1eYniJ0Rnk

Are the Q-values accurate?



As predicted Q increases, so does the return





Slide credit: Sergey Levine

Double DQN

Overestimation in Q-learning

$$\text{target value } y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$$

this last term is the problem

Slide credit: Sergey Levine

Overestimation in Q-learning

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imagine we have two random variables: X_1 and X_2

$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$

Slide credit: Sergey Levine

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hence $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ *overestimates* the next value!

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value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

Slide credit: Sergey Levine

Double Q-learning

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if the noise in these is decorrelated, the problem goes away!

idea: don't use the same network to choose the action and evaluate value!

"double" Q-learning: use two networks:

$$Q_{\phi_A}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_B}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi_A}(\mathbf{s}'))$$

$$Q_{\phi_B}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_A}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi_B}(\mathbf{s}'))$$

if the two Q's are noisy in *different* ways, there is no problem

Slide credit: Sergey Levine

Double Q-learning in practice

where to get two Q-functions?

just use the current and target networks!

standard Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}'))$

double Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi}(\mathbf{s}', \mathbf{a}'))$

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double Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg\max_{\mathbf{a}'} Q_{\phi}(\mathbf{s}', \mathbf{a}'))$

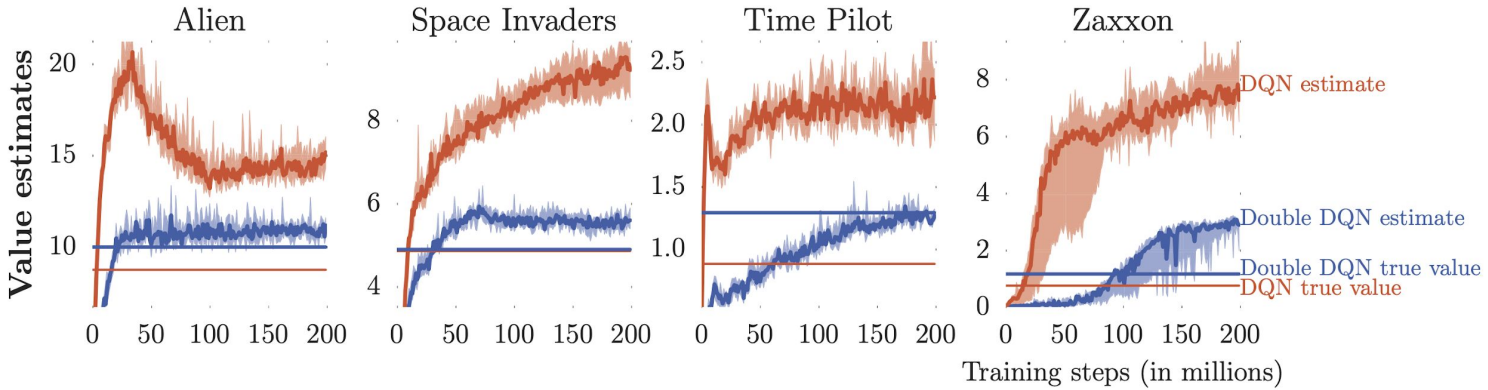
just use current network (not target network) to evaluate action

still use target network to evaluate value!

Slide credit: Sergey Levine

Q-learning vs Double Q-learning

- Q-Network estimated via Double Q-Learning is closer to the true return

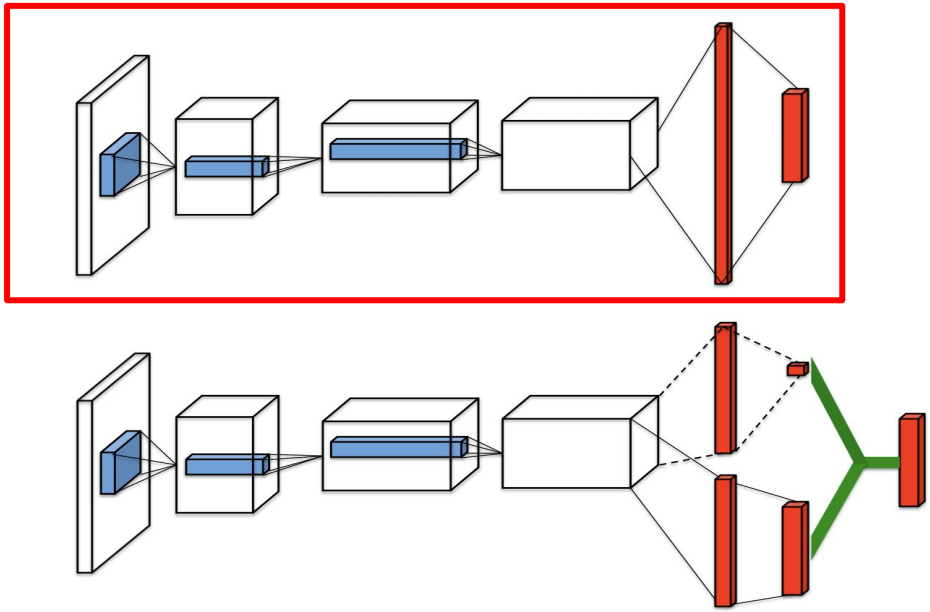


The straight horizontal orange (for DQN) and blue (for Double DQN) lines in the top row are computed by running the corresponding agents after learning, and averaging the actual discounted return obtained from each visited state. These straight lines would match the learning curves at the right side of the plots if there is no bias.

Figure credit: Hado van Hasselt et al, 2015

Dueling Networks

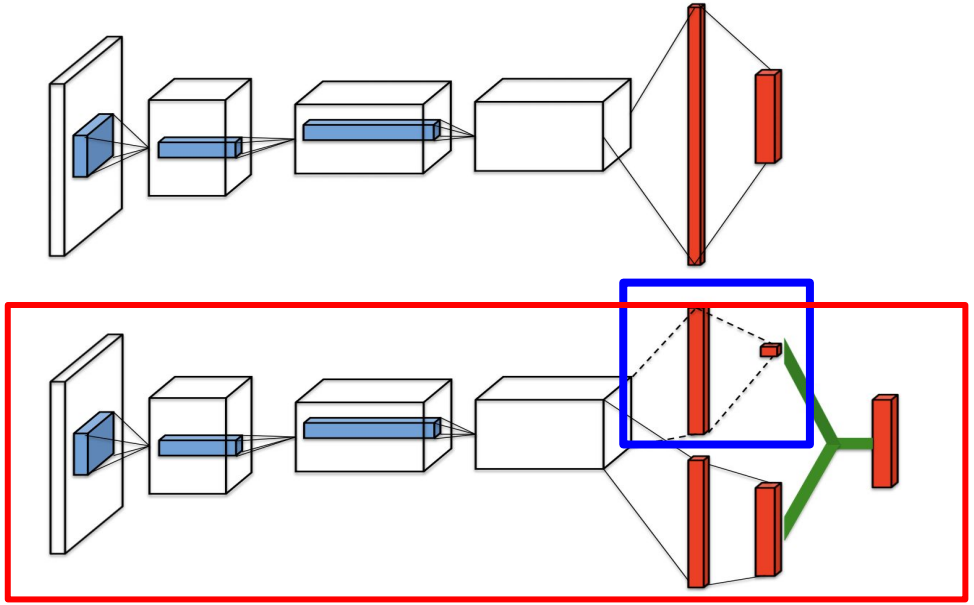
- **Top:** Standard Q-network



Dueling Networks

- **Top:** Standard Q-network
- **Bottom:** Dueling network estimates a **state** value,

$$V(s;\theta,\beta)$$



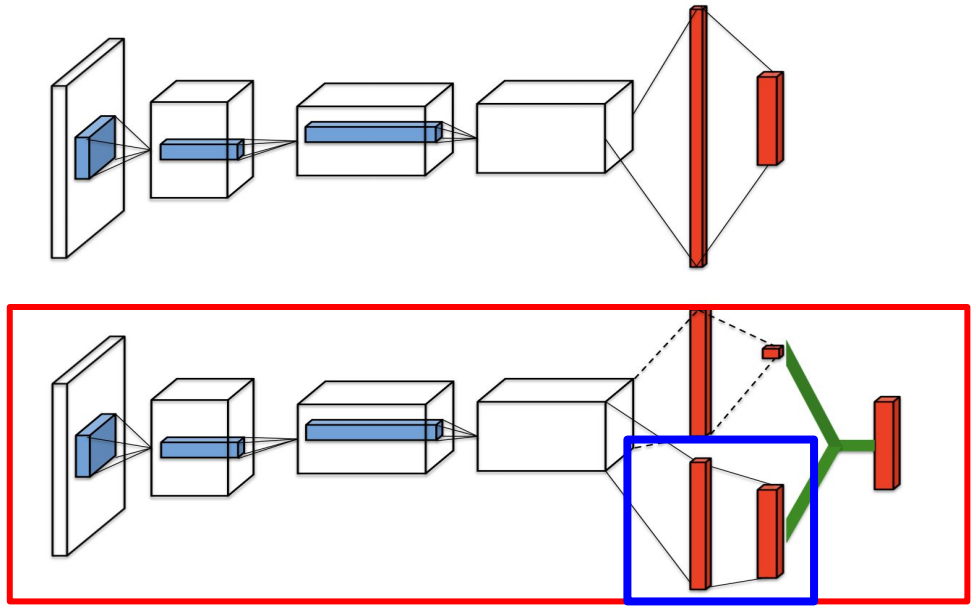
Dueling Networks

- **Top:** Standard Q-network
- **Bottom:** Dueling network estimates a state value,

$$V(s;\theta,\beta)$$

and the **advantage** for each action

$$A(s,a;\theta,\alpha)$$



Dueling Networks

- **Top:** Standard Q-network
- **Bottom:** Dueling network estimates a state value,

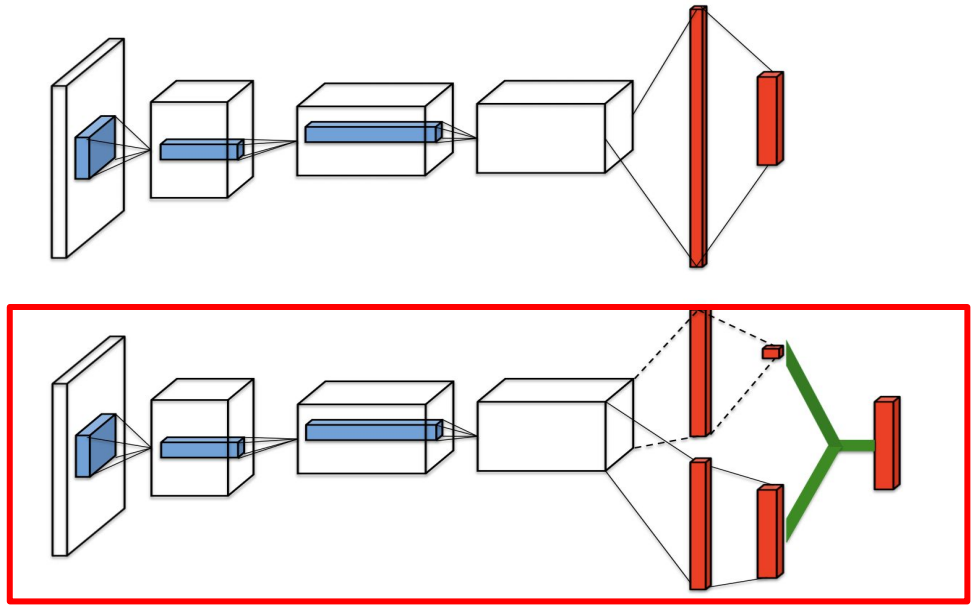
$$V(s;\theta,\beta)$$

and the advantage for each action

$$A(s,a;\theta,\alpha)$$

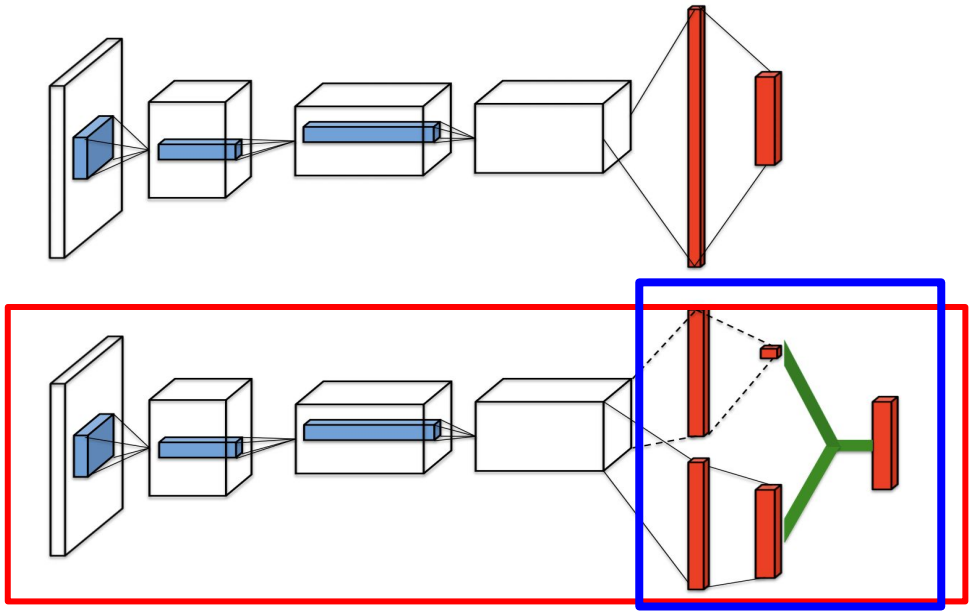
such that the sum of the two estimates the **Q-value** function

$$Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + A(s,a;\theta,\alpha)$$



Dueling Networks

- **Top:** Standard Q-network
- **Bottom:** Dueling network estimates a state value, and advantage for each action.



To address the issue of identifiability, the dueling networks compute Q by

$$Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + \left(A(s,a;\theta,\alpha) - \max_{a' \in |\mathcal{A}|} A(s,a';\theta,\alpha) \right)$$

Figure credit: Wang et al 2016

Dueling Networks

- **Top:** Standard Q-network
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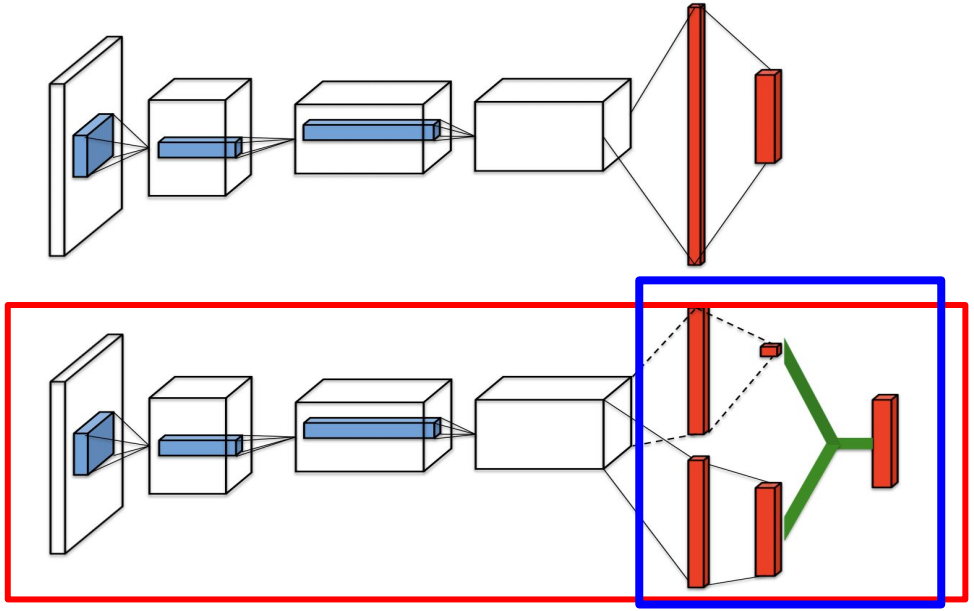
  Implementation detail: For numerical stability, Wang et al. used the following form

$$Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + \left(A(s,a;\theta,\alpha) - \frac{1}{|\mathcal{A}|}\sum_{a'} A(s,a';\theta,\alpha) \right)$$



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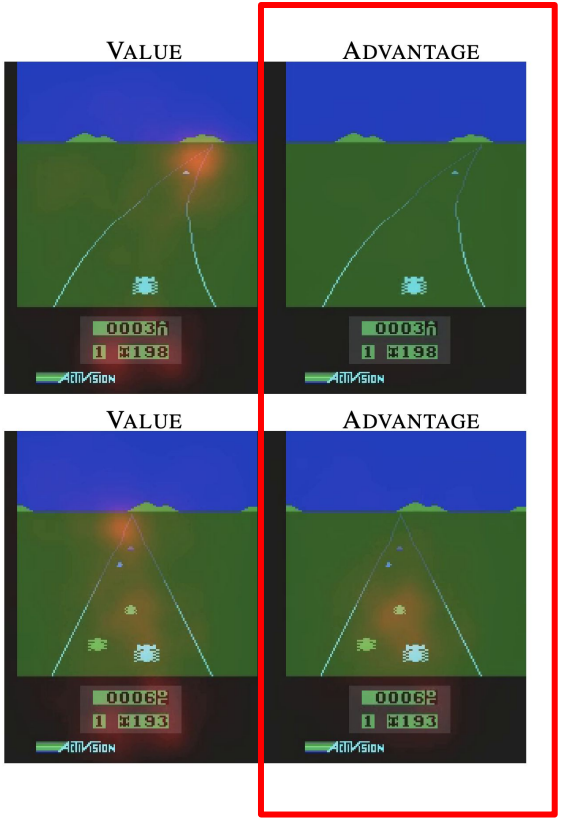
Dueling Networks

- Dueling Networks can choose to focus on what affects the action choice A(s, a)



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Dueling Networks

- Dueling Networks can choose to focus on what affects the action choice A(s, a) or simply what affects the current state expected return

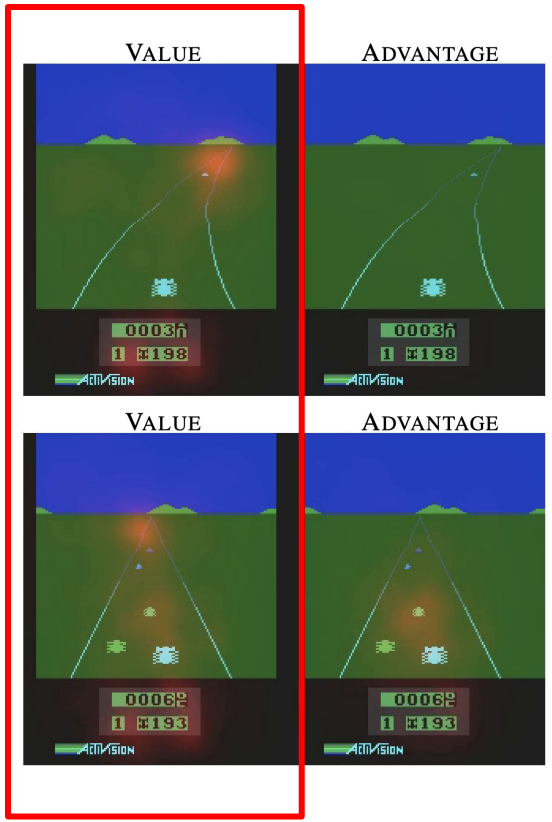
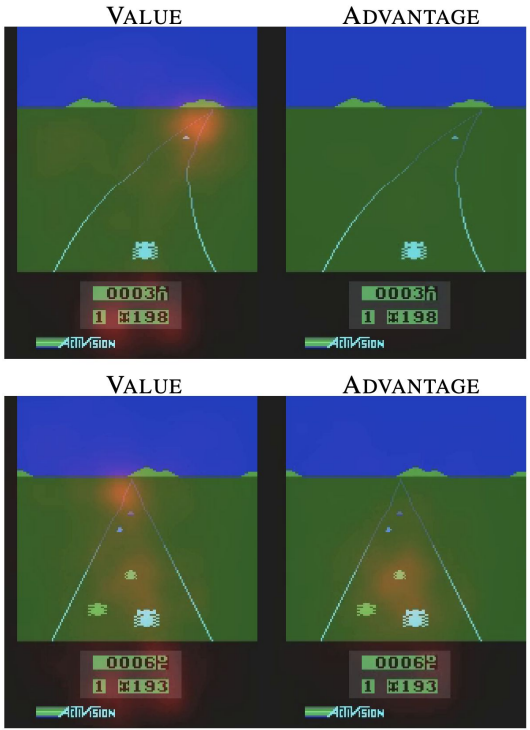


Figure credit: Wang et al 2016

Dueling Networks

- Dueling Networks can choose to focus on what affects the action choice A(s, a) or simply what affects the current state expected return
- In this scenario the Q-network can intuitively focus on relevant parts of the state when making a decision



Video on Double DQN and Dueling Networks:
https://www.youtube.com/watch?v=XjsY8-P4WHM&ab_channel=AndrewMelnik

Figure credit: Wang et al 2016

Dueling Networks

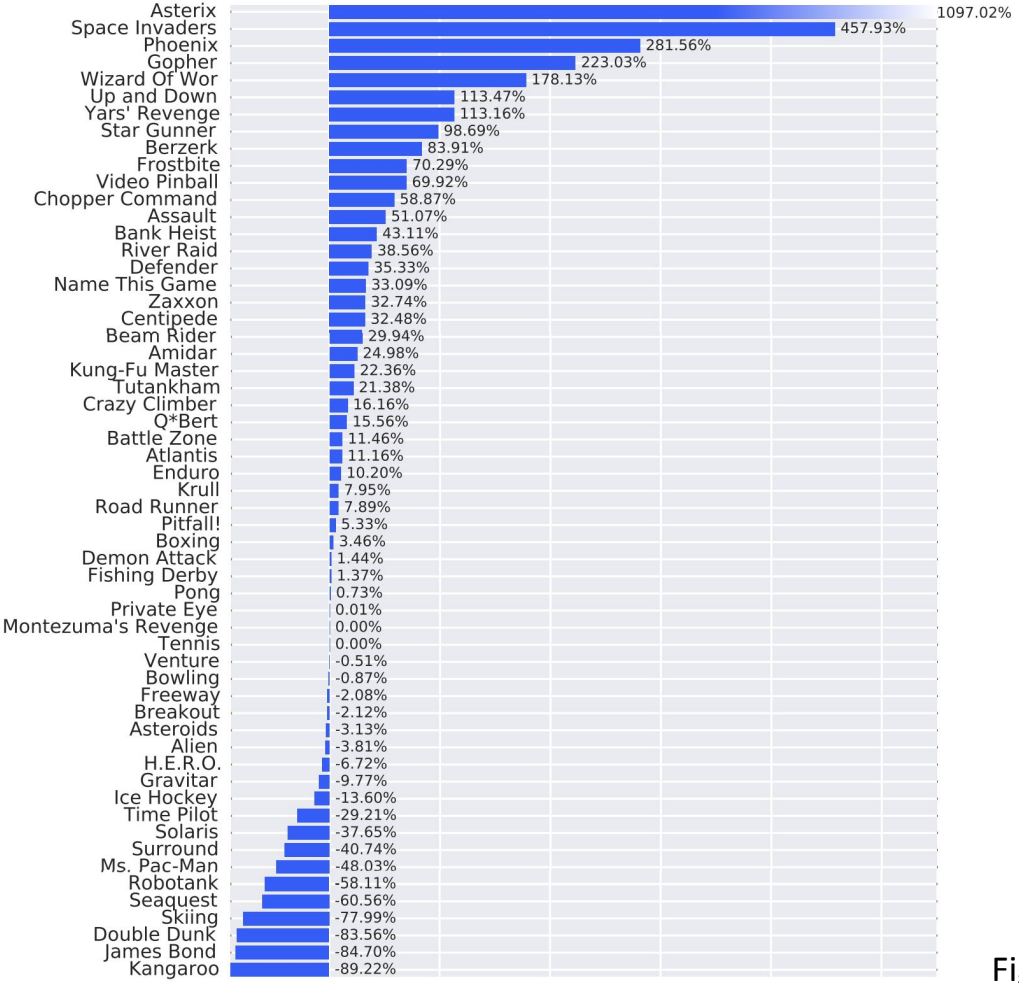


Figure credit: Wang et al 2016

Simple practical tips for Q-learning

- Q-learning takes some care to stabilize
  - Test on easy, reliable tasks first, make sure your implementation is correct

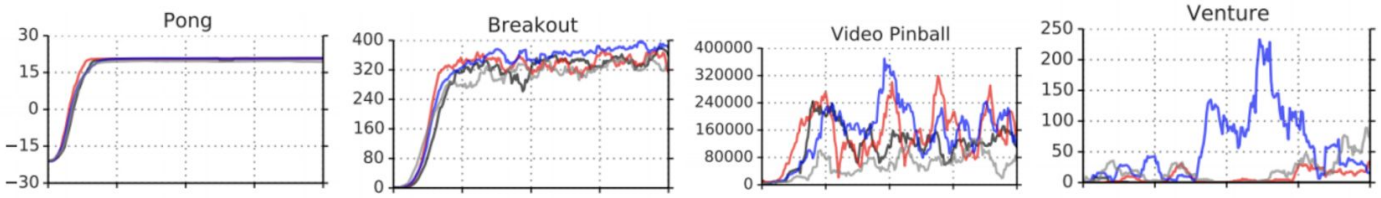


Figure: From T. Schaul, J. Quan, I. Antonoglou, and D. Silver. "Prioritized experience replay". *arXiv preprint arXiv:1511.05952* (2015), Figure 7

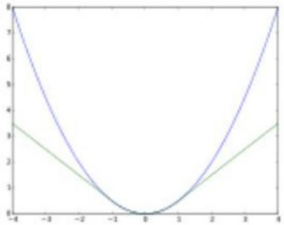
- Large replay buffers help improve stability
- It takes time, be patient – might be no better than random for a while
- Start with high exploration (epsilon) and gradually reduce

Slide credit: John Schulman, Sergey Levine

Advanced tips for Q-learning

- Bellman error gradients can be big; clip gradients or user Huber loss

$$L(x) = \begin{cases} x^2/2 & \text{if } |x| \leq \delta \\ \delta|x| - \delta^2/2 & \text{otherwise} \end{cases}$$



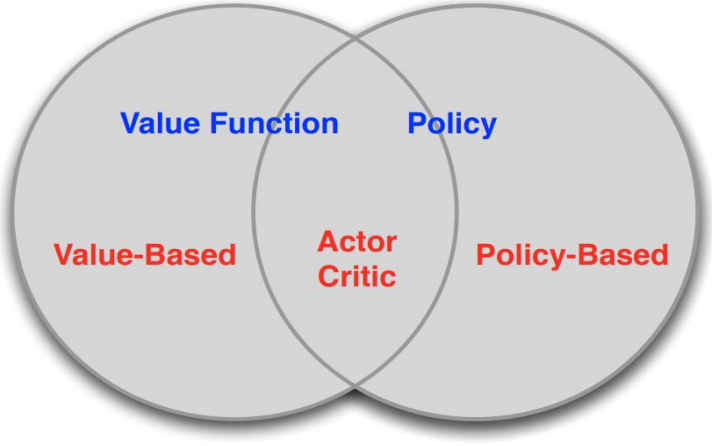
- Double Q-learning helps *a lot* in practice, simple and no downsides
- N-step returns also help a lot, but have some downsides
- Schedule exploration (high to low) and learning rates (high to low), Adam optimizer can help too
- Run multiple random seeds, it's very inconsistent between runs

Slide credit: Sergey Levine

RL methods taxonomy:
value-based, policy-based, actor-critic

RL methods taxonomy

- Value Based
  - Learned Value Function
  - Implicit policy
  - Eg: Q-learning, DQN
- Policy Based
  - No Value Function
  - Learned Policy
  - Eg: REINFORCE, TRPO, PPO
- Actor-Critic
  - Learned Value Function
  - Learned Policy
  - Eg: A2C, A3C



Slide credit: David Silver

Policy-Based RL

Advantages

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

Disadvantages

- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Slide credit: David Silver

Policy gradients

What is a problem with Q-learning?
The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Policy gradients

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But the policy can be much simpler: just close your hand
Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Policy gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta\right]$$

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We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Policy gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta\right]$$

We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

Gradient ascent on policy parameters!

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Aside: Score Function Gradient Estimator

Consider an expectation $E_{x \sim p(x \mid \theta)}[f(x)]$. Want to compute gradient wrt θ

$$\nabla_\theta E_x[f(x)] = \nabla_\theta \int \mathrm{d}x \; p(x \mid \theta) f(x)$$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

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$$
\begin{aligned}
\nabla_\theta E_x[f(x)] &= \nabla_\theta \int \mathrm{d}x \; p(x \mid \theta) f(x) \\
&= \int \mathrm{d}x \; \nabla_\theta p(x \mid \theta) f(x) \\
&= \int \mathrm{d}x \; p(x \mid \theta) \frac{\nabla_\theta p(x \mid \theta)}{p(x \mid \theta)} f(x) \\
&= \int \mathrm{d}x \; p(x \mid \theta) \nabla_\theta \log p(x \mid \theta) f(x) \\
&= E_x[f(x) \nabla_\theta \log p(x \mid \theta)].
\end{aligned}
$$

$$
\boxed{\nabla_\theta E_x[f(x)] = E_x[f(x) \nabla_\theta \log p(x \mid \theta)]}
$$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Where r(τ) is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE algorithm

Expected reward:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} \left[r(\tau) \right]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$

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Now let's differentiate this: $\nabla_\theta J(\theta) = \int_{\tau} r(\tau) \nabla_\theta p(\tau;\theta) \mathrm{d}\tau$

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Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$

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Intractable! Gradient of an expectation is problematic when p depends on θ

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However, we can use a nice trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_\theta \log p(\tau;\theta)$

If we inject this back:

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE algorithm

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If we inject this back:

$$\nabla_\theta J(\theta) = \int_{\tau} \left(r(\tau) \nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta) \mathrm{d}\tau$$

$$= \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau) \nabla_\theta \log p(\tau;\theta)\right]$$

Can estimate with Monte Carlo sampling

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau;\theta) = \prod_{t \geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE algorithm

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We have: $p(\tau;\theta) = \prod_{t \geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

Thus: $\log p(\tau;\theta) = \sum_{t \geq 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)$

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And when differentiating: $\nabla_\theta \log p(\tau;\theta) = \sum_{t \geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$

Doesn't depend on transition probabilities!

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Doesn't depend on transition probabilities!

Therefore when sampling a trajectory τ, we can estimate J(θ) with

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

Intuition

Gradient estimator: $\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t | s_t)$

Interpretation:

- If r(τ) is high, push up the probabilities of the actions seen
- If r(τ) is low, push down the probabilities of the actions seen

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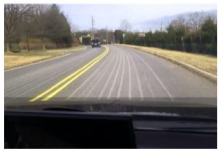
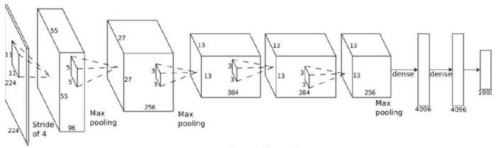
- Quite similar to supervised learning



$\mathbf{s}_t$



$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

$\mathbf{a}_t$

maximum likelihood: $\nabla_\theta J_{\mathrm{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_t^i, s_t^i) \right)$

$\mathbf{s}_t$
$\mathbf{a}_t$

$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$

policy gradient: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_t^i, s_t^i) \right) r(\tau^i)$

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

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Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

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However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?

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Variance reduction

Gradient estimator: $\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t | s_t)$

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Variance reduction

Gradient estimator: $\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t|s_t)$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

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$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

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Variance reduction: Baseline

Problem: The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

What is important then? Whether a reward is better or worse than what you expect to get

Idea: Introduce a baseline function dependent on the state.
Concretely, estimator is now:

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

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Variance reduction: Baseline

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

But.. are we allowed to do that ?

Subtracting a baseline is unbiased in expectation

In expectation, the gradient estimator remains unchanged

How to choose the baseline?

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

A simple baseline: constant moving average of rewards experienced so far from all trajectories

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A simple baseline: constant moving average of rewards experienced so far from all trajectories

Variance reduction techniques seen so far are typically used in “Vanilla REINFORCE”

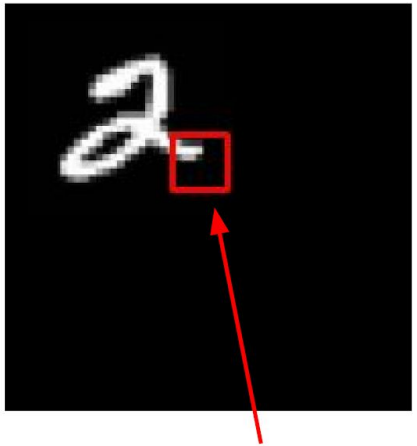
Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

REINFORCE in action: Recurrent Attention Model (RAM)

Objective: Image Classification

Take a sequence of "glimpses" selectively focusing on regions of image, to predict class

- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image



State: Glimpses seen so far
Action: (x,y) coordinates (center of glimpse) of where to look next in image
Reward: 1 at the final timestep if image correctly classified, 0 otherwise

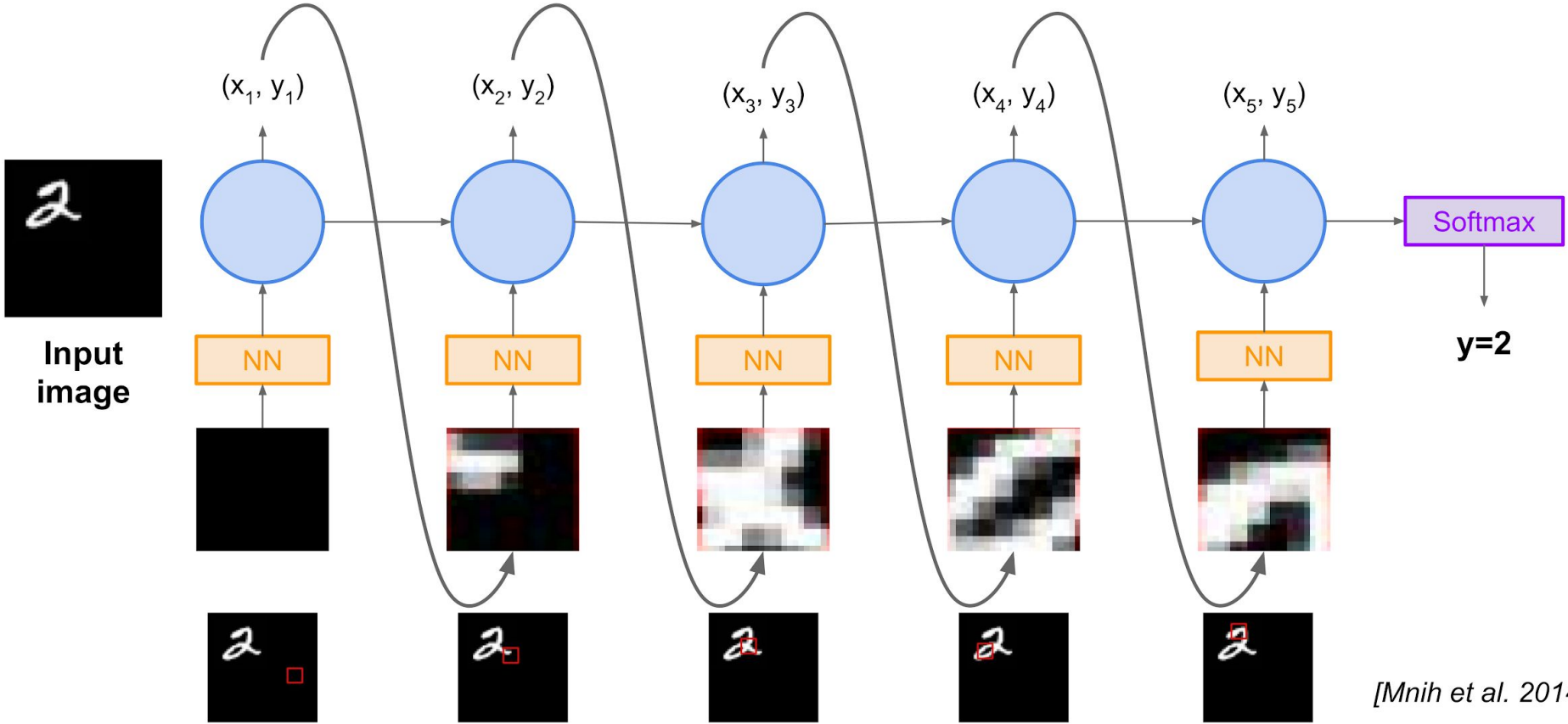
Glimpsing is a non-differentiable operation =>
learn policy for how to take glimpse actions using REINFORCE

Given state of glimpses seen so far,
use RNN to model the state and output next action

[Mnih et al. 2014]

Slide credit: Fei fei Li & Justin Johnson & Serena Yeung

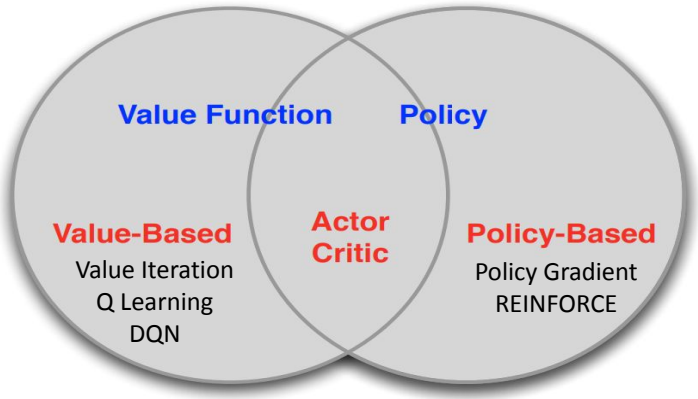
REINFORCE in action: Recurrent Attention Model (RAM)



[Mnih et al. 2014]

Summary

- Value Functions and Q-Learning
  - Value functions contain the expected future reward from state s_t.
  - Use Bellman optimality equation find optimal value functions
- DQN and its variants
  - Use deep neural networks to estimate Q-functions
  - Variants: Double DQN, Dueling DQN, Recurrent DQN, …
- Policy-based RL Introduction
- Policy gradient methods:
  - REINFORCE



Quiz

https://forms.gle/8WrCqh8aNQ3S9iJL7