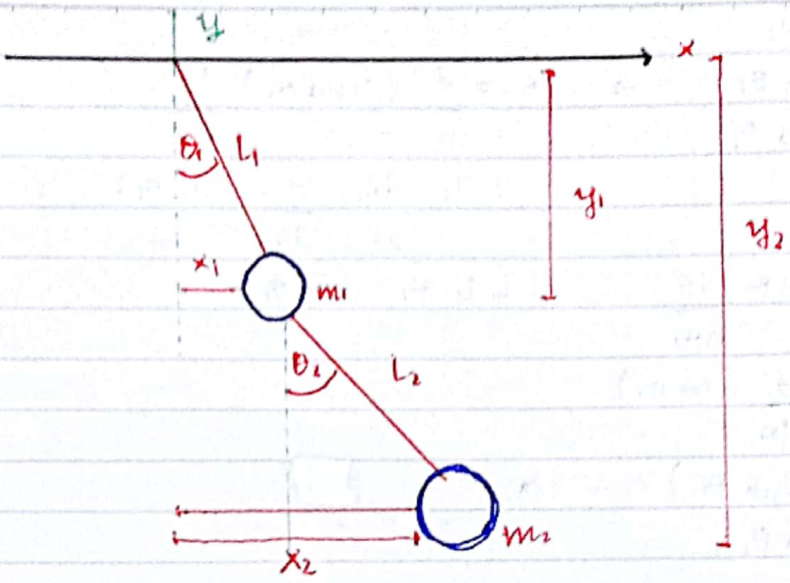
Inayah Mutmainah ; kemaluBin hardiansyah ; Nuril camila B.

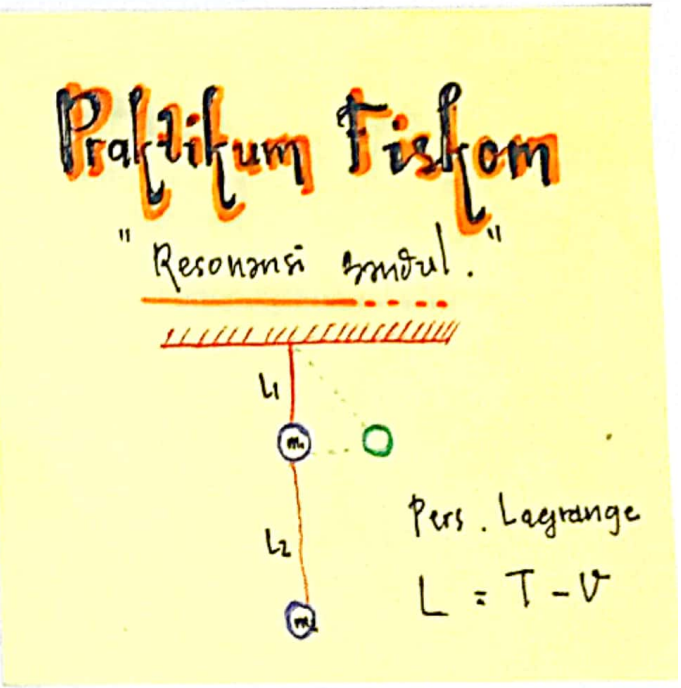


Tinjau m_2

$$x_2 = x_1 + L_2 \sin\theta_2$$
$$= L_1 \sin\theta_1 + L_2 \sin\theta_2$$
$$\dot{x}_2 = \frac{d}{dt}(L_1 \sin\theta_1 + L_2 \sin\theta_2)$$
$$= L_1 \dot{\theta}_1 \cos\theta_1 + L_2 \dot{\theta}_2 \cos\theta_2$$

$$y_2 = y_1 - L_2 \cos\theta_2$$
$$= -L_1 \cos\theta_1 - L_2 \cos\theta_2$$
$$\dot{y}_2 = \frac{d}{dt}(-L_1 \cos\theta_1 - L_2 \cos\theta_2)$$
$$= L_1 \dot{\theta}_1 \sin\theta_1 + L_2 \dot{\theta}_2 \cdot \sin\theta_2$$

$$V = m_1 \cdot g \cdot y_1 + m_2 \cdot g \cdot y_2$$
$$= m_1 \cdot g(-L_1 \cos\theta_1) + m_2 \cdot g(-L_1 \cos\theta_1 - L_2 \cdot \cos\theta_2)$$
$$= -m \cdot g\, L_1 \cos\theta_1 - m_1 g\, L_1 \cos\theta_1 - m_2 g \cdot L_2 \cos\theta_2$$
$$= -(m_1 + m_2) g L_1 \cos\theta_1 - m_2 g L_2 \cos\theta_2$$



Tinjauan m_1

$x_1 = l_1 \sin\theta_1 \rightarrow \dot{x} = \frac{d}{dt}(l_1 \sin\theta_1)\frac{d\theta}{d\theta_1}$

$y_1 = -l_1 \cos\theta_1 \quad = l_1 \frac{d\theta_1}{dt}\frac{d}{d\theta}(\sin\theta_1)$

$\downarrow \quad = l_1 \dot{\theta}_1 \cos\theta_1$

$\dot{y} = \frac{d}{dt}(-l_1 \cos\theta_1)\frac{d\theta}{d\theta_1}$

$= -l_1 \frac{d\theta_1}{dt}\frac{d}{d\theta_1}(\cos\theta_1)$

$= -l_1 \dot{\theta}_1 (-\sin\theta_1)$

$= l_1 \dot{\theta}_1 \sin\theta_1$

sumbu x

$\dot{x}_1 = l_1 \dot{\theta}_1 \cos\theta_1$

$\dot{x}_2 = l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2$

sumbu y

$\dot{y}_2 = l_1 \dot{\theta}_1 \sin\theta_1$

$\dot{y}_2 = l_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2$

$V = -(m_1+m_2)\, g\, l_1 \cos\theta_1 - m g l_2 \cos\theta_2$ → potensial

$\sin^2\theta + \cos^2\theta - 1$

- Analisis Energi kinetik (T) sistem

$T = 1/2\, m v_1^2 + 1/2\, m v_2^2$

$= 1/2\, m(\dot{x}_1, \dot{y}_1)^2 + 1/2\, m(\dot{x}_2 + \dot{y}_2)^2$

$= 1/2\, m[(l_1 \dot{\theta}_1 \cos\theta_1)^2 + (l_1 \dot{\theta}_1 \sin\theta_1)^2] + 1/2\, m[(l_1\dot{\theta}_1\cos\theta_1 + l_2\dot{\theta}_2\cos\theta_2)^2 + (l_1\dot{\theta}_1\sin\theta_1 + l_2\dot{\theta}_2\sin\theta_2)^2]$

$= 1/2\, m[l_1^2\dot{\theta}_1^2\cos^2\theta_1 + l_1^2\dot{\theta}_1^2\sin\theta_1 + l_1^2\dot{\theta}_1^2\cos^2\theta + 2(l_1 l_2 \dot{\theta}_1\dot{\theta}_2\cos\theta_1.\cos\theta_2) + l_2^2\dot{\theta}_2^2\cos\theta_2 + l_1^2\dot{\theta}_1^2\sin^2\theta_1 + 2(l_1 l_2 \dot{\theta}_1\dot{\theta}_2\sin\theta_1\sin\theta_2) + l_2^2\dot{\theta}_2^2\sin^2\theta_2]$

$= 1/2\, m[[l_1^2\dot{\theta}_1^2(\sin^2\theta_1+\cos^2\theta_1)] + [l_1^2\dot{\theta}_1^2(\sin^2\theta_1+\cos^2\theta_1) + l_2^2\dot{\theta}_2^2(\sin^2\theta_2 + \cos^2\theta_2) + 2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2(\cos\theta_1.\cos\theta_2 + \sin\theta_1.\sin\theta_2)]]$

$T = 1/2\, m_1[l_1^2\dot{\theta}_1^2] + 1/2\, m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2))$

$L = T - V$ → Lagrange

$= 1/2\, m_1(l_1^2\dot{\theta}_1^2) + 1/2\, m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)) + ((m_1+m_2) g l_1 \cos\theta_1 - m g l_2 \cos\theta_2)$

Turunkan fungsi terhadap $(\dot{\theta}_1, \dot{\theta}_2, \theta_1, \theta_2)$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1 + m_2 l_1^2 \dot{\theta}_1 + 2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \theta_1} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_2 \sin(\theta_1 - \theta_2).(\dot{\theta}_1 - \dot{\theta}_2)$$

$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_2 \sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2)$
$m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1 = 0$
$(m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1 = 0$
$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g \sin\theta_1) = 0$
$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g \sin\theta_1 = 0$
$2m l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 - (2m) g \theta_1 = 0$
$2m l \ddot{\theta}_1 + m l \ddot{\theta}_2 - 2m g \theta_1 = 0$

$$L = 1/2 m_1 l_1^2 \dot{\theta}_1^2 + 1/2 m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1 + m_2) g l_1 \sin\theta_1 + m_2 g l_2 \sin\theta_2$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 \sin(\theta_1 - \theta_2).(\dot{\theta}_1 - \dot{\theta}_2)$$

$m_2 l_1^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) + m_2 l_1 l_2 \ddot{\theta}_2 \dot{\theta}_2 \sin(\theta_1-\theta_2) - m_2 g l_2 \sin\theta_2 = 0$

$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + m_2 l_1 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) + m_2 l_1 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_1) - m_2 g \sin\theta_2 = 0$

$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) - m_2 g \sin\theta_2 = 0$

$m l \ddot{\theta}_2 + m l \ddot{\theta}_1 - m g \theta_2 = 0$

$m(l\ddot{\theta}_2 + l\ddot{\theta}_1 - g\theta) = 0$