

**МІНІСТЕРСТВО ОСВІТИ І НАУКИ, МОЛОДІ ТА СПОРТУ УКРАЇНИ  
НАВЧАЛЬНО-НАУКОВИЙ КОМПЛЕКС  
«ІНСТИТУТ ПРИКЛАДНОГО СИСТЕМНОГО АНАЛІЗУ»  
НАЦІОНАЛЬНОГО ТЕХНІЧНОГО УНІВЕРСИТЕТУ УКРАЇНИ  
«КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ»  
КАФЕДРА МАТЕМАТИЧНИХ МЕТОДІВ СИСТЕМНОГО АНАЛІЗУ**

**Лабораторна робота №6  
з курсу «Чисельні методи»**

**Тема: Обчислення власних чисел та відповідних власних  
векторів матриць**

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## Варіант – 18

**Мета роботи:** набути вміння та досвід використання методів розв'язання часткової та повної проблеми власних значень. Закріпити та поглибити теоретичні знання методів розв'язання часткової та повної проблеми власних значень.

### Завдання на роботу:

**Завдання 1.** Знайти найбільше та власне число та відповідний власний вектор степеневим методом або методом скалярних добутків з точністю  $\varepsilon=0,0001$  (матриця з таблиці 1).

**Завдання 2.** Розв'язати повну проблему власних значень:

- а) методом Якобі (матриця з таблиці 1);
- б) QR або LR методом (матриця з таблиці 2)

### Матриця з таблиці 1:

$$\begin{pmatrix} 931 & 585 & 351 & -75 & 390 \\ 585 & 867 & -14 & 129 & 306 \\ 351 & -14 & 801 & -308 & -32 \\ -75 & 129 & 308 & 356 & -175 \\ 390 & 306 & -32 & -175 & 839 \end{pmatrix}$$

### Матриця з таблиці 2:

$$\begin{pmatrix} -973 & -378 & -195 & 683 & -487 \\ 905 & -779 & -69 & 676 & 362 \\ 707 & 223 & -868 & 724 & 135 \\ 715 & 630 & 128 & -917 & 408 \\ 308 & 596 & -572 & -204 & -54 \end{pmatrix}$$

## Текст програми

```
import numpy as np
import scipy.linalg

def Power_method(matrix, eps = 1e-4):
    """
    This function finds max eigenvalue and eigenvector by using power method.

    :param matrix: given matrix
    :param eps: accuracy of computation

    :return: cortege of eigenvalue and eigenvector
    """
    k, n = 1, len(matrix)
    x = 1
    Xn = np.random.random(n) #eigenvector
    yn = matrix @ Xn
    l_n = yn / Xn #lambda
    Xn = yn / np.max(yn) #normalization
    dl = np.copy(l_n) #delta lamda: lambda[k + 1] - lambda[k]

    with open('Result_1(6).txt', 'a') as file:

        file.write('-----POWER METHOD-----\n')
        file.write('\nGiven matrix =\n' + str(matrix) + '\n')
        while all(abs(i) > eps for i in dl):

            k+=1
            l_o = np.copy(l_n)
            yn = matrix @ Xn
            l_n = yn / Xn
            Xn = yn / np.max(yn)
            dl = np.array([abs(l_n[i] - l_o[i]) for i in range(n)])

            file.write('\ny[' + str(k) + '] =' + str(yn) + '\n')
            file.write('lambda[' + str(k) + '] =' + str(l_n) + '\n')
            file.write('X[' + str(k) + '] =' + str(Xn) + '\n')

            x = matrix @ Xn.transpose() - np.mean(l_n) * Xn.transpose()

        file.write('\nAnswer:\neigenvalue:' + str(np.mean(l_n)))
        file.write('\neigenvector:\n' + str(Xn))

        file.write('\n\nResidual vector:\n')
        file.write('Ax - lx = ' + str(x) + '\n')

    return np.mean(l_n), Xn
```

```

def Jacobi_method(matrix, eps = 1e-4):
    """
    This function solves full problem of eigenvalues

    :param matrix: given matrix
    :param eps: accuracy of computation

    :return: eigenvalues and eigenvectors
    """

    temp, count = 1, 0
    check = np.zeros((len(matrix), len(matrix[1])))
    Eigenvectors = np.eye(5)
    A = np.copy(matrix)

    with open('Result_1(6).txt', 'a') as file:
        file.write('\n-----Jacobi Method-----\n')
        while temp > eps:
            count += 1

            file.write('\n\nIteration: ' + str(count))
            file.write('\nA =\n' + str(matrix))

            mask = np.ones(matrix.shape, dtype=bool)
            np.fill_diagonal(mask, 0)
            key_element = max(matrix[mask], key=abs) #find the biggest value
            a = np.argwhere(matrix == key_element) #find indices with the biggest
            value

            file.write('\n\nIndices with max absolute value: i = ' + str(a[0, 0])
            + ' j = ' + str(a[0, 1]))

            t = (matrix[a[0, 0], a[0, 0]] - matrix[a[0, 1], a[0, 1]]) / (2 * key_
            element)

            if t == 0:
                c = s = 1 / np.sqrt(2)
            else:
                tau = np.sqrt(1 + t ** 2)
                if t > 0:
                    t = -t + tau
                else:
                    t = -t - tau

                c = 1 / np.sqrt(1 + t ** 2)
                s = c * t

            file.write('\nAngular parameters: c = ' + str(c) + ' s = ' + str(s))
            file.write('\nCheck: c^2 + s^2 == ' + str(c ** 2 + s ** 2))

            Temp = np.array([i ** 2 for i in matrix])

```

```

omega_2 = np.sum(Temp) - np.trace(Temp)
check_o = np.sum(check) - np.trace(check)
check = np.copy(Temp)

file.write('\nDelta = ' + str(np.trace(Temp)))
file.write('\n2 * Omega = ' + str(omega_2))
if count != 1:
    file.write('\nCheck delta = ' + str(np.trace(Temp) - np.trace(check)))
    file.write('\nCheck 2 * Omega = ' + str(omega_2 - check_o))
else:
    file.write('\nCheck delta = 0')
    file.write('\nCheck 2 * Omega = 0')
file.write('\nDelta + 2 * Omega = ' + str(np.sum(Temp)))

T = np.eye(matrix.shape[1])
T[a[0, 0], a[0, 0]] = c
T[a[0, 1], a[0, 1]] = c
T[a[0, 0], a[0, 1]] = -s
T[a[0, 1], a[0, 0]] = s

B = T.transpose() @ (matrix @ T)
matrix = np.copy(B)

Eigenvectors = Eigenvectors.dot(T)

Temp = np.array([i ** 2 for i in matrix])
temp = np.sum(Temp) - np.trace(Temp)

file.write('\n\nAnswer:\nEigenvalues:' + str(np.diag(matrix)))
file.write('\nEigenvectors:\n' + str(Eigenvectors.transpose()))

file.write('\n\nResidual vectors:\n')
for j in range(len(matrix)):
    x = A @ Eigenvectors.transpose()[j] - matrix[j, j] * Eigenvectors.transpose()[j]
    file.write('x[' + str(np.diag(matrix)[j]) + '] = ' + str(x) + '\n')

return np.diag(matrix), Eigenvectors.transpose()

def LU_decomposition(matrix, eps = 1e-4):
    """
    This function solves full problem of eigenvectors and eigenvalues using LU-decomposition

    :param matrix: given matrix
    :param eps: accuracy of computation

    :return: eigenvalues and eigenvectors
    """
    A = np.copy(matrix)

```

```

LL = UU = np.eye(len(matrix))
count = 0
x, y = np.linalg.eig(matrix)

with open('Result_1(6).txt', 'a') as file:
    file.write('\n-----LU-algorithm-----\n')
    file.write('Given matrix = \n' + str(matrix) + '\n')

    while abs(np.linalg.det(matrix) - np.prod(np.diag(matrix))) > eps:
        P, L, U = scipy.linalg.lu(matrix)
        L = P @ L
        matrix = U @ L
        LL = LL @ L
        UU = np.linalg.inv(L) @ UU

        count += 1
        if count == 1 or count % 5 == 0:
            file.write('\nIteration ' + str(count))
            file.write('\nL = \n' + str(L) + '\n')
            file.write('U = \n' + str(U) + '\n')
        matrix = np.sort(np.diag(matrix))
        matrix = matrix[::-1]

    file.write('\n\nAnswer:\nEigenvalues: ' + str(matrix))
    file.write('\nEigenvectors:\n' + str(y.transpose()))

    file.write('\n\nResidual vectors\n')
    for j in range(len(matrix)):
        x = A @ y.transpose()[j] - matrix[j] * y.transpose()[j]
        file.write('x[' + str(matrix[j]) + '] = ' + str(x) + '\n')

    return matrix, y.transpose()

matrix_1 = np.loadtxt('matrix_1.txt', 'f')
matrix_2 = np.loadtxt('matrix_2.txt', 'f')

eigenvalue_p, eigenvector_p = Power_method(matrix_1)
print('-----Power method-----\nEigenvalue:\n', eigenvalue_p)
print('Eigenvector: \n', eigenvector_p)

eigenvalues_j, eigenvectors_j = Jacobi_method(matrix_1)
print('\n-----Jacobi method-----\nEigenvalues: \n', eigenvalues_j)
print('Eigenvectors:\n', eigenvectors_j)

eigenvalues_lu, eigenvectors_lu = LU_decomposition(matrix_2)
print('\n-----LU algorithm-----\nEigenvalues: \n', eigenvalues_lu)
print('Eigenvectors: \n', eigenvectors_lu)

```

**Текстовый файл matrix\_1.txt**

931 585 351 -75 390  
585 867 -14 129 306  
351 -14 801 -308 -32  
-75 129 -308 356 -175  
390 306 -32 -175 839

**Текстовый файл matrix\_2.txt**

-973 -378 -195 683 -487  
905 -779 -69 676 362  
707 223 -868 724 135  
715 630 128 -917 408  
308 596 -572 -204 -54

## Результати роботи програми

-----Power method-----

Eigenvalue

1807.509782582055

Eigenvector

[ 1. 0.81718438 0.35760705 -0.13614489 0.67365449]

-----Jacobi method-----

Eigenvalues

[1807.50900619 195.01967057 1019.25754891 95.57503608 676.63873826]

Eigenvectors

[[ 0.66401316 0.542621 0.23745611 -0.09040216 0.44731534]

[-0.66779667 0.63032902 0.24667846 -0.30783007 0.03351597]

[ 0.1038325 -0.391686 0.79648724 -0.40911244 -0.18448845]

[-0.23892253 -0.08107463 0.46833826 0.76672504 0.35935347]

[-0.21276503 -0.38504126 -0.17038689 -0.37660035 0.79725451]]

-----LU algorithm-----

Eigenvalues

[ -94.78896 -261.58487 -551.79395 -1297.6287 -1385.2035 ]

Eigenvectors

[[ -0.4128942 -0.01850755 -0.1918212 0.03821078 0.88933706]

[ 0.3987725 -0.26899958 -0.22199246 -0.34829584 -0.7733234 ]

[-0.45127386 0.2880226 0.42270043 0.4257158 0.59454626]

[ 0.69283634 -0.43206507 0.06005434 -0.5733523 -0.03095425]

[ 0.70856494 -0.27461526 0.06993608 -0.6370592 -0.10856643]]



## Текстовый файл Result\_1(6).txt

-----POWER METHOD-----

Given matrix =

[[ 931. 585. 351. -75. 390.]

[ 585. 867. -14. 129. 306.]

[ 351. -14. 801. -308. -32.]

[ -75. 129. -308. 356. -175.]

[ 390. 306. -32. -175. 839.]]

y[2]=[1910.15693641 1562.68534304 629.85975181 -277.22385588 1432.91074919]

lambda[2]=[1910.15693641 1925.6241214 1700.09524101 3601.67656166 1516.08622442]

X[2]=[ 1. 0.81809265 0.32974241 -0.14513146 0.75015342]

y[3]=[1828.76847507 1500.49491793 624.36595233 -253.97035662 1284.56131258]

lambda[3]=[1828.76847507 1834.13813171 1893.49605415 1749.93323282 1712.39813983]

X[3]=[ 1. 0.82049474 0.34141334 -0.13887507 0.70241878]

y[4]=[1815.18445908 1488.61441521 633.28128046 -246.67429877 1243.77865452]

lambda[4]=[1815.18445908 1814.28879334 1854.88147972 1776.2316327 1770.70815622]

X[4]=[ 1. 0.82008989 0.34887985 -0.13589489 0.68520786]

y[5]=[1810.63259162 1483.27677603 638.90047661 -244.95335553 1228.45434875]

lambda[5]=[1810.63259162 1808.67585646 1831.29085263 1802.52067757 1792.8199961 ]

X[5]=[ 1. 0.81920362 0.35286036 -0.13528606 0.67846694]

y[6]=[1808.83666676 1480.46847522 642.12946695 -244.89727773 1222.29359783]

lambda[6]=[1808.83666676 1807.2045052 1819.78349283 1810.21807889 1801.552192 ]

X[6]=[ 1. 0.81846443 0.35499583 -0.13538938 0.67573464]

y[7]=[1808.09594185 1478.94829136 643.96957571 -245.20898496 1219.72475638]  
lambda[7]=[1808.09594185 1806.97930582 1814.02013265 1811.13896866 1805.03511317]  
X[7]=[ 1. 0.81795897 0.35615896 -0.13561724 0.67459073]

y[8]=[1807.77946932 1478.11433844 645.0151111 -245.51336913 1218.61299504]  
lambda[8]=[1807.77946932 1807.07638837 1811.03152821 1810.34037743 1806.44788875]  
X[8]=[ 1. 0.81764085 0.35679967 -0.13580936 0.67409383]

y[9]=[1807.63887638 1477.65272624 645.60784025 -245.73318031 1218.11187529]  
lambda[9]=[1807.63887638 1807.21490336 1809.44070727 1809.39799688 1807.03607169]  
X[9]=[ 1. 0.81744907 0.35715532 -0.13594152 0.67386904]

y[10]=[1807.57376557 1477.39564304 645.94330331 -245.87517176 1217.87633792]  
lambda[10]=[1807.57376557 1807.32438192 1808.57814019 1808.68338024 1807.2893465 ]  
X[10]=[ 1. 0.81733629 0.35735377 -0.13602497 0.67376301]

y[11]=[1807.54235223 1477.25187333 646.13293853 -245.96199648 1217.76111943]  
lambda[11]=[1807.54235223 1807.39786526 1808.10443078 1808.21206546 1807.40275812]  
X[11]=[ 1. 0.81727096 0.3574649 -0.13607537 0.67371097]

y[12]=[1807.52662283 1477.17124901 646.24005121 -246.01348672 1217.70273457]  
lambda[12]=[1807.52662283 1807.44370158 1807.84199754 1807.92074724 1807.45569065]  
X[12]=[ 1. 0.81723347 0.35752727 -0.13610504 0.67368454]

y[13]=[1807.51849584 1477.125952 646.30051906 -246.04346933 1217.67227666]  
lambda[13]=[1807.51849584 1807.47119558 1807.69575078 1807.74691302 1807.48141189]  
X[13]=[ 1. 0.81721208 0.35756233 -0.13612224 0.67367071]

y[14]=[1807.51419106 1477.10047142 646.33464214 -246.0607312 1217.65602421]

lambda[14]=[1807.51419106 1807.48731594 1807.61392839 1807.64530739 1807.49437063]

X[14]=[ 1. 0.81719993 0.35758206 -0.13613212 0.67366333]

y[15]=[1807.51186767 1477.08612623 646.35389367 -246.07059797 1217.64720486]

lambda[15]=[1807.51186767 1807.49663699 1807.56802912 1807.58667046 1807.50109943]

X[15]=[ 1. 0.81719304 0.35759317 -0.13613775 0.67365931]

y[16]=[1807.51059647 1477.07804569 646.36475319 -246.07621157 1217.64236092]

lambda[16]=[1807.51059647 1807.50197951 1807.54223604 1807.55310237 1807.50467718]

X[16]=[ 1. 0.81718915 0.35759943 -0.13614095 0.67365711]

y[17]=[1807.50989423 1477.07349233 646.37087822 -246.0793957 1217.63967782]

lambda[17]=[1807.50989423 1807.50502449 1807.52772467 1807.53398494 1807.50661358]

X[17]=[ 1. 0.81718695 0.35760295 -0.13614277 0.67365589]

y[18]=[1807.50950369 1477.0709259 646.37433264 -246.0811982 1217.63818293]

lambda[18]=[1807.50950369 1807.50675366 1807.51955415 1807.52313402 1807.50767516]

X[18]=[ 1. 0.8171857 0.35760494 -0.13614379 0.6736552 ]

y[19]=[1807.5092855 1477.06947915 646.37628078 -246.08221725 1217.63734674]

lambda[19]=[1807.5092855 1807.50773328 1807.51495143 1807.51698873 1807.50826242]

X[19]=[ 1. 0.817185 0.35760606 -0.13614437 0.67365482]

y[20]=[1807.50916324 1477.06866349 646.37737941 -246.08279286 1217.63687776]

lambda[20]=[1807.50916324 1807.50828738 1807.51235768 1807.51351349 1807.50858932]

X[20]=[ 1. 0.81718461 0.3576067 -0.1361447 0.67365461]

y[21]=[1807.50909458 1477.06820361 646.37799896 -246.08311782 1217.63661424]

lambda[21]=[1807.50909458 1807.50860047 1807.51089571 1807.51155008 1807.50877207]

X[21]=[ 1. 0.81718438 0.35760705 -0.13614489 0.67365449]

Answer:

eigenvalue:1807.509782582055

eigenvector:

[ 1. 0.81718438 0.35760705 -0.13614489 0.67365449]

Residual vector:

$Ax - lx = [-7.26616863e-04 \ -8.21534466e-04 \ 1.03336846e-04 \ -8.97119699e-05 \ -6.11717829e-04]$

-----Jacobi Method-----

Iteration: 1

A =

[[ 931. 585. 351. -75. 390.]

[ 585. 867. -14. 129. 306.]

[ 351. -14. 801. -308. -32.]

[ -75. 129. -308. 356. -175.]

[ 390. 306. -32. -175. 839.]]

Indices with max absolute value:  $i = 0 \ j = 1$

Angular parameters:  $c = 0.726160863859085 \ s = 0.6875248357691722$

Check:  $c^2 + s^2 == 0.9999999999999998$

Delta = 3090708.0

$2 * \Omega = 1720274.0$

Check delta = 0

Check  $2 * \Omega = 0$

Delta +  $2 * \Omega = 4810982.0$

Iteration: 2

A =

```
[[ 1.48487456e+03  5.25747894e-14  2.45257116e+02  3.42286390e+01
   4.93585337e+02]
 [ 2.58652807e-14  3.13125440e+02 -2.51487469e+02  1.45239114e+02
  -4.59294616e+01]
 [ 2.45257116e+02 -2.51487469e+02  8.01000000e+02 -3.08000000e+02
  -3.20000000e+01]
 [ 3.42286390e+01  1.45239114e+02 -3.08000000e+02  3.56000000e+02
  -1.75000000e+02]
 [ 4.93585337e+02 -4.59294616e+01 -3.20000000e+01 -1.75000000e+02
   8.39000000e+02]]
```

Indices with max absolute value: i = 0 j = 4

Angular parameters: c = 0.8796297088079835 s = 0.4756590957633232

Check:  $c^2 + s^2 == 1.0$

Delta = 3775157.9999999999

2 \* Omega = 1035824.0

Check delta = 0.0

Check 2 \* Omega = -684450.0

Delta + 2 \* Omega = 4810981.9999999999

Iteration: 3

A =

```
[[ 1.75178046e+03 -2.18467662e+01  2.00514354e+02 -5.31318140e+01
  -6.17753989e-14]
 [-2.18467662e+01  3.13125440e+02 -2.51487469e+02  1.45239114e+02
  -4.04009189e+01]
 [ 2.00514354e+02 -2.51487469e+02  8.01000000e+02 -3.08000000e+02
  -1.44806928e+02]
 [-5.31318140e+01  1.45239114e+02 -3.08000000e+02  3.56000000e+02
```

-1.70216363e+02]

[-4.50347880e-14 -4.04009189e+01 -1.44806928e+02 -1.70216363e+02  
5.72094105e+02]]

Indices with max absolute value:  $i = 2$   $j = 3$

Angular parameters:  $c = 0.8903894208773017$   $s = -0.4551996036792907$

Check:  $c^2 + s^2 == 0.9999999999999999$

$\Delta = 4262410.9691125965$

$2 * \Omega = 548571.0308874035$

Check  $\Delta = 0.0$

Check  $2 * \Omega = -487252.9691125965$

$\Delta + 2 * \Omega = 4810982.0$

Iteration: 4

A =

[[ 1.75178046e+03 -2.18467662e+01 2.02721440e+02 4.39660494e+01  
-6.17753989e-14]  
[-2.18467662e+01 3.13125440e+02 -2.90034569e+02 1.48423743e+01  
-4.04009189e+01]  
[ 2.02721440e+02 -2.90034569e+02 9.58460853e+02 2.27504397e-14  
-5.14521364e+01]  
[ 4.39660494e+01 1.48423743e+01 4.46661154e-14 1.98539147e+02  
-2.17474905e+02]  
[-4.50347880e-14 -4.04009189e+01 -5.14521364e+01 -2.17474905e+02  
5.72094105e+02]]

Indices with max absolute value:  $i = 1$   $j = 2$

Angular parameters:  $c = 0.9337328002507004$   $s = 0.3579707498329797$

Check:  $c^2 + s^2 == 1.0$

$\Delta = 4452138.9691125965$

$$2 * \Omega = 358843.0308874026$$

$$\text{Check } \delta = 0.0$$

$$\text{Check } 2 * \Omega = -189728.00000000093$$

$$\Delta + 2 * \Omega = 4810981.999999999$$

Iteration: 5

A =

$$\begin{bmatrix} 1.75178046e+03 & 5.21693038e+01 & 1.97108161e+02 & 4.39660494e+01 \\ -6.17753989e-14 & 5.21693038e+01 & 2.01933146e+02 & -4.77508944e-14 & 1.38588117e+01 \\ -5.61420230e+01 & 1.97108161e+02 & -5.34755999e-14 & 1.06965315e+03 & -5.31313585e+00 \\ -3.35802002e+01 & 4.39660494e+01 & 1.38588117e+01 & -5.31313585e+00 & 1.98539147e+02 \\ -2.17474905e+02 & -4.50347880e-14 & -5.61420230e+01 & -3.35802002e+01 & -2.17474905e+02 \\ 5.72094105e+02 \end{bmatrix}$$

Indices with max absolute value:  $i = 3$   $j = 4$

$$\text{Angular parameters: } c = 0.9087177598294037 \quad s = 0.4174111078668488$$

$$\text{Check: } c^2 + s^2 == 1.0$$

$$\Delta = 4620379.072084397$$

$$2 * \Omega = 190602.92791560292$$

$$\text{Check } \delta = 0.0$$

$$\text{Check } 2 * \Omega = -168240.10297179967$$

$$\Delta + 2 * \Omega = 4810982.0$$

Iteration: 6

A =

$$\begin{bmatrix} 1.75178046e+03 & 5.21693038e+01 & 1.97108161e+02 & 3.99527299e+01 \end{bmatrix}$$

-1.83519174e+01]  
 [ 5.21693038e+01 2.01933146e+02 -4.77508944e-14 -1.08405557e+01  
 -5.68020753e+01]  
 [ 1.97108161e+02 -5.34755999e-14 1.06965315e+03 -1.88448895e+01  
 -2.82971624e+01]  
 [ 3.99527299e+01 -1.08405557e+01 -1.88448895e+01 9.86440583e+01  
 5.47583827e-14]  
 [-1.83519174e+01 -5.68020753e+01 -2.82971624e+01 2.12821502e-14  
 6.71989193e+02]]

Indices with max absolute value: i = 0 j = 2

Angular parameters: c = 0.9658703351409492 s = 0.2590260521544324

Check: c<sup>2</sup> + s<sup>2</sup> == 1.0000000000000004

Delta = 4714969.740614537

2 \* Omega = 96012.25938546285

Check delta = 0.0

Check 2 \* Omega = -94590.66853014007

Delta + 2 \* Omega = 4810982.0

Iteration: 7

A =

[[ 1.80464071e+03 5.03887830e+01 5.17282795e-14 3.37078393e+01  
 -2.50552749e+01]  
 [ 5.03887830e+01 2.01933146e+02 -1.35132088e+01 -1.08405557e+01  
 -5.68020753e+01]  
 [ 8.88774253e-14 -1.35132088e+01 1.01679290e+03 -2.85505176e+01  
 -2.25777650e+01]  
 [ 3.37078393e+01 -1.08405557e+01 -2.85505176e+01 9.86440583e+01  
 5.47583827e-14]  
 [-2.50552749e+01 -5.68020753e+01 -2.25777650e+01 2.12821502e-14



6.71989193e+02]]

Indices with max absolute value:  $i = 4$   $j = 1$

Angular parameters:  $c = 0.9929791109958173$   $s = -0.11828983526050145$

Check:  $c^2 + s^2 == 1.0000000000000002$

$\Delta = 4792672.99514241$

$2 * \Omega = 18309.004857592285$

Check  $\delta = 0.0$

Check  $2 * \Omega = -77703.25452787057$

$\Delta + 2 * \Omega = 4810982.000000002$

Iteration: 8

A =

[[ 1.80464071e+03 4.70712246e+01 5.17282795e-14 3.37078393e+01

-3.08398454e+01]

[ 4.70712246e+01 1.95166530e+02 -1.60890542e+01 -1.07644454e+01

-6.98905944e-14]

[ 8.88774253e-14 -1.60890542e+01 1.01679290e+03 -2.85505176e+01

-2.08207737e+01]

[ 3.37078393e+01 -1.07644454e+01 -2.85505176e+01 9.86440583e+01

1.28232755e+00]

[-3.08398454e+01 -7.63744201e-14 -2.08207737e+01 1.28232755e+00

6.78755809e+02]]

Indices with max absolute value:  $i = 0$   $j = 1$

Angular parameters:  $c = 0.9995733291660824$   $s = 0.029208896244720253$

Check:  $c^2 + s^2 == 1.0000000000000002$

$\Delta = 4799125.946670248$

$2 * \Omega = 11856.05332975369$

Check  $\delta = 0.0$

$$\text{Check } 2 * \Omega = -6452.951527838595$$

$$\Delta + 2 * \Omega = 4810982.000000002$$

Iteration: 9

A =

$$\begin{bmatrix} 1.80601619\text{e}+03 & -5.59148565\text{e}-13 & -4.69943514\text{e}-01 & 3.33790396\text{e}+01 \\ & & & -3.08266869\text{e}+01 \end{bmatrix}$$

$$\begin{bmatrix} -5.85085350\text{e}-13 & 1.93791045\text{e}+02 & -1.60821894\text{e}+01 & -1.17444213\text{e}+01 \\ & & & 9.00797844\text{e}-01 \end{bmatrix}$$

$$\begin{bmatrix} -4.69943514\text{e}-01 & -1.60821894\text{e}+01 & 1.01679290\text{e}+03 & -2.85505176\text{e}+01 \\ & & & -2.08207737\text{e}+01 \end{bmatrix}$$

$$\begin{bmatrix} 3.33790396\text{e}+01 & -1.17444213\text{e}+01 & -2.85505176\text{e}+01 & 9.86440583\text{e}+01 \\ & & & 1.28232755\text{e}+00 \end{bmatrix}$$

$$\begin{bmatrix} -3.08266869\text{e}+01 & 9.00797844\text{e}-01 & -2.08207737\text{e}+01 & 1.28232755\text{e}+00 \\ & & & 6.78755809\text{e}+02 \end{bmatrix}$$

Indices with max absolute value:  $i = 3$   $j = 0$

Angular parameters:  $c = 0.9998091003460464$   $s = -0.019538752908752465$

$$\text{Check: } c^2 + s^2 == 1.0$$

$$\Delta = 4803557.347034693$$

$$2 * \Omega = 7424.652965311892$$

$$\text{Check } \delta = 0.0$$

$$\text{Check } 2 * \Omega = -4431.400364441797$$

$$\Delta + 2 * \Omega = 4810982.000000005$$

Iteration: 10

A =

$$\begin{bmatrix} 1.80666850\text{e}+03 & -2.29471345\text{e}-01 & -1.02769531\text{e}+00 & 7.38031839\text{e}-14 \\ & & & -3.07957470\text{e}+01 \end{bmatrix}$$

$$\begin{bmatrix} -2.29471345\text{e}-01 & 1.93791045\text{e}+02 & -1.60821894\text{e}+01 & -1.17421793\text{e}+01 \end{bmatrix}$$

9.00797844e-01]  
 [-1.02769531e+00 -1.60821894e+01 1.01679290e+03 -2.85358852e+01  
 -2.08207737e+01]  
 [ 8.33670648e-14 -1.17421793e+01 -2.85358852e+01 9.79917490e+01  
 1.88439777e+00]  
 [-3.07957470e+01 9.00797844e-01 -2.08207737e+01 1.88439777e+00  
 6.78755809e+02]]

Indices with max absolute value:  $i = 0$   $j = 4$

Angular parameters:  $c = 0.9996280270423864$   $s = -0.02727283541082302$

Check:  $c^2 + s^2 == 0.9999999999999999$

$\Delta = 4805785.667604493$

$2 * \Omega = 5196.332395510748$

Check  $\delta = 0.0$

Check  $2 * \Omega = -2228.3205698011443$

$\Delta + 2 * \Omega = 4810982.0000000004$

Iteration: 11

$A =$

[[ 1.80750870e+03 -2.53953300e-01 -4.59471500e-01 -5.13928703e-02  
 -5.08076533e-13]  
 [-2.53953300e-01 1.93791045e+02 -1.60821894e+01 -1.17421793e+01  
 8.94204437e-01]  
 [-4.59471500e-01 -1.60821894e+01 1.01679290e+03 -2.85358852e+01  
 -2.08410571e+01]  
 [-5.13928703e-02 -1.17421793e+01 -2.85358852e+01 9.79917490e+01  
 1.88369683e+00]  
 [-4.96867940e-13 8.94204437e-01 -2.08410571e+01 1.88369683e+00  
 6.77915609e+02]]

Indices with max absolute value:  $i = 2$   $j = 3$

Angular parameters:  $c = 0.9995189839282224$   $s = -0.0310129129088848$

Check:  $c^2 + s^2 == 1.0000000000000002$

$\Delta = 4807682.423675697$

$2 * \Omega = 3299.576324304566$

Check  $\delta = 0.0$

Check  $2 * \Omega = -1896.7560712061822$

$\Delta + 2 * \Omega = 4810982.000000002$

Iteration: 12

A =

```
[[ 1.80750870e+03 -2.53953300e-01 -4.57656645e-01 -6.56176991e-02
   -5.08076533e-13]
 [-2.53953300e-01  1.93791045e+02 -1.57102945e+01 -1.22352866e+01
    8.94204437e-01]
 [-4.57656645e-01 -1.57102945e+01  1.01767830e+03 -1.93397403e-12
   -2.08894512e+01]
 [-6.56176991e-02 -1.22352866e+01 -1.91654726e-12  9.71063422e+01
    1.23644885e+00]
 [-4.96867940e-13  8.94204437e-01 -2.08894512e+01  1.23644885e+00
    6.77915609e+02]]
```

Indices with max absolute value:  $i = 4$   $j = 2$

Angular parameters:  $c = 0.9981293701571575$   $s = 0.061137226218366605$

Check:  $c^2 + s^2 == 0.9999999999999998$

$\Delta = 4809311.01716591$

$2 * \Omega = 1670.9828340923414$

Check  $\delta = 0.0$

Check  $2 * \Omega = -1628.5934902122244$

$\Delta + 2 * \Omega = 4810982.000000003$

Iteration: 13

A =

```
[[ 1.80750870e+03 -2.53953300e-01 -4.56800538e-01 -6.56176991e-02
   -2.79798578e-02]
 [-2.53953300e-01  1.93791045e+02 -1.57355755e+01 -1.22352866e+01
   -6.79521148e-02]
 [-4.56800538e-01 -1.57355755e+01  1.01895782e+03 -7.55930529e-02
   -1.03715615e-13]
 [-6.56176991e-02 -1.22352866e+01 -7.55930529e-02  9.71063422e+01
   1.23413591e+00]
 [-2.79798578e-02 -6.79521148e-02 -1.17943352e-13  1.23413591e+00
   6.76636093e+02]]
```

Indices with max absolute value: i = 2 j = 1

Angular parameters: c = 0.9998183573749538 s = -0.019059177738008307

Check:  $c^2 + s^2 == 0.9999999999999999$

Delta = 4810183.755508027

2 \* Omega = 798.2444919748232

Check delta = 0.0

Check 2 \* Omega = -872.7383421175182

Delta + 2 \* Omega = 4810982.0000000002

Iteration: 14

A =

```
[[ 1.80750870e+03 -2.62613413e-01 -4.51877423e-01 -6.56176991e-02
   -2.79798578e-02]
 [-2.62613413e-01  1.93491083e+02  1.03790833e-12 -1.22345049e+01
   -6.79397718e-02]
 [-4.51877423e-01  1.03091857e-12  1.01925778e+03  1.57615181e-01
```

1.29511143e-03]  
 [-6.56176991e-02 -1.22345049e+01 1.57615181e-01 9.71063422e+01  
 1.23413591e+00]  
 [-2.79798578e-02 -6.79397718e-02 1.29511143e-03 1.23413591e+00  
 6.76636093e+02]]

Indices with max absolute value: i = 1 j = 3

Angular parameters: c = 0.9922837113963048 s = -0.12398804820455472

Check:  $c^2 + s^2 == 1.0$

Delta = 4810678.972180537

2 \* Omega = 303.0278194639832

Check delta = 0.0

Check 2 \* Omega = -495.21667251084

Delta + 2 \* Omega = 4810982.000000001

Iteration: 15

A =

[[ 1.80750870e+03 -2.52451202e-01 -4.51877423e-01 -9.76722986e-02  
 -2.79798578e-02]  
 [-2.52451202e-01 1.95019811e+02 -1.95423986e-02 -5.72912860e-14  
 -2.20433632e-01]  
 [-4.51877423e-01 -1.95423986e-02 1.01925778e+03 1.56398976e-01  
 1.29511143e-03]  
 [-9.76722986e-02 -4.66829332e-14 1.56398976e-01 9.55776137e+01  
 1.21618924e+00]  
 [-2.79798578e-02 -2.20433632e-01 1.29511143e-03 1.21618924e+00  
 6.76636093e+02]]

Indices with max absolute value: i = 3 j = 4

Angular parameters: c = 0.9999978095799773 s = -0.002093044492503851

Check:  $c^2 + s^2 == 1.0000000000000002$

Delta = 4810978.338401915

$2 * \Omega = 3.6615980863571167$

Check delta = 0.0

Check  $2 * \Omega = -299.36622137762606$

Delta +  $2 * \Omega = 4810982.000000001$

Iteration: 16

A =

[[ 1.80750870e+03 -2.52451202e-01 -4.51877423e-01 -9.76135215e-02  
-2.81842290e-02]  
[-2.52451202e-01 1.95019811e+02 -1.95423986e-02 4.61377398e-04  
-2.20433149e-01]  
[-4.51877423e-01 -1.95423986e-02 1.01925778e+03 1.56395923e-01  
1.62245861e-03]  
[-9.76135215e-02 4.61377398e-04 1.56395923e-01 9.55750681e+01  
-4.94008085e-12]  
[-2.81842290e-02 -2.20433149e-01 1.62245861e-03 -4.97530737e-12  
6.76638638e+02]]

Indices with max absolute value: i = 0 j = 2

Angular parameters: c = 0.9999998356832164 s = -0.0005732656804221699

Check:  $c^2 + s^2 == 1.0$

Delta = 4810981.296634454

$2 * \Omega = 0.7033655475825071$

Check delta = 0.0

Check  $2 * \Omega = -2.9582325387746096$

Delta +  $2 * \Omega = 4810982.000000002$

Iteration: 17

A =

```
[[ 1.80750896e+03 -2.52439958e-01 -2.31265161e-11 -9.77031619e-02
   -2.81851545e-02]
 [-2.52439958e-01  1.95019811e+02 -1.96871170e-02  4.61377398e-04
   -2.20433149e-01]
 [-2.30909027e-11 -1.96871170e-02  1.01925752e+03  1.56339939e-01
    1.60630129e-03]
 [-9.77031619e-02  4.61377398e-04  1.56339939e-01  9.55750681e+01
   -4.94008085e-12]
 [-2.81851545e-02 -2.20433149e-01  1.60630129e-03 -4.97530737e-12
    6.76638638e+02]]
```

Indices with max absolute value: i = 1 j = 0

Angular parameters: c = 0.9999999877455853 s = 0.00015655295955241446

Check:  $c^2 + s^2 == 0.9999999999999999$

Delta = 4810981.705020866

$2 * \Omega = 0.29497913643717766$

Check delta = 0.0

Check  $2 * \Omega = -0.4083864111453295$

Delta +  $2 * \Omega = 4810982.000000003$

Iteration: 18

A =

```
[[ 1.80750900e+03  1.70580065e-10  3.08205331e-06 -9.77032329e-02
   -2.81506447e-02]
 [ 1.70553617e-10  1.95019772e+02 -1.96871168e-02  4.46081674e-04
   -2.20437558e-01]
 [ 3.08205334e-06 -1.96871168e-02  1.01925752e+03  1.56339939e-01
    1.60630129e-03]
 [-9.77032329e-02  4.46081674e-04  1.56339939e-01  9.55750681e+01
    0.00000000e+00]]
```



-4.94008085e-12]

[-2.81506447e-02 -2.20437558e-01 1.60630129e-03 -4.97530737e-12  
6.76638638e+02]]

Indices with max absolute value:  $i = 4$   $j = 1$

Angular parameters:  $c = 0.9999998952548418$   $s = -0.0004577011091325725$

Check:  $c^2 + s^2 == 0.9999999999999999$

$\Delta = 4810981.8324727295$

$2 * \Omega = 0.1675272723659873$

Check  $\delta = 0.0$

Check  $2 * \Omega = -0.12745186407119036$

$\Delta + 2 * \Omega = 4810982.0000000002$

Iteration: 19

$A =$

[[ 1.80750900e+03 -1.28844107e-05 3.08205331e-06 -9.77032329e-02  
-2.81506417e-02]  
[-1.28844107e-05 1.95019671e+02 -1.96863795e-02 4.46081627e-04  
3.74427402e-11]  
[ 3.08205334e-06 -1.96863795e-02 1.01925752e+03 1.56339939e-01  
1.61531194e-03]  
[-9.77032329e-02 4.46081627e-04 1.56339939e-01 9.55750681e+01  
-2.04177017e-07]  
[-2.81506417e-02 3.74404212e-11 1.61531194e-03 -2.04177052e-07  
6.76638739e+02]]

Indices with max absolute value:  $i = 3$   $j = 2$

Angular parameters:  $c = 0.9999999856759951$   $s = -0.00016925722851219552$

Check:  $c^2 + s^2 == 0.9999999999999999$

$\Delta = 4810981.929658163$

$$2 * \Omega = 0.07034183759242296$$

$$\text{Check } \delta = 0.0$$

$$\text{Check } 2 * \Omega = -0.09718543477356434$$

$$\Delta + 2 * \Omega = 4810982.000000001$$

Iteration: 20

A =

$$\begin{bmatrix} 1.80750900e+03 & -1.28844107e-05 & -1.34549252e-05 & -9.77032321e-02 \\ & -2.81506417e-02 & & \\ -1.28844107e-05 & 1.95019671e+02 & -1.96863037e-02 & 4.49413682e-04 \\ & 3.74427402e-11 & & \\ -1.34549251e-05 & -1.96863037e-02 & 1.01925755e+03 & -1.21342035e-11 \\ & 1.61531188e-03 & & \\ -9.77032321e-02 & 4.49413682e-04 & -1.21182240e-11 & 9.55750417e+01 \\ & -4.77580236e-07 & & \\ -2.81506417e-02 & 3.74404212e-11 & 1.61531188e-03 & -4.77580271e-07 \\ & 6.76638739e+02 \end{bmatrix}$$

Indices with max absolute value:  $i = 0 \quad j = 3$

$$\text{Angular parameters: } c = 0.9999999983714023 \quad s = -5.7071845638168827e-05$$

$$\text{Check: } c^2 + s^2 == 1.0000000000000002$$

$$\Delta = 4810981.978542517$$

$$2 * \Omega = 0.021457483991980553$$

$$\text{Check } \delta = 0.0$$

$$\text{Check } 2 * \Omega = -0.04888435360044241$$

$$\Delta + 2 * \Omega = 4810982.000000001$$

Iteration: 21

A =

$$\begin{bmatrix} 1.80750901e+03 & -1.29100595e-05 & -1.34549251e-05 & -9.68509789e-10 \\ & & & \end{bmatrix}$$

-2.81506416e-02]  
 [-1.29100596e-05 1.95019671e+02 -1.96863037e-02 4.49412946e-04  
 3.74427402e-11]  
 [-1.34549251e-05 -1.96863037e-02 1.01925755e+03 -7.80031613e-10  
 1.61531188e-03]  
 [-9.68503321e-10 4.49412946e-04 -7.80015636e-10 9.55750361e+01  
 -2.08418931e-06]  
 [-2.81506416e-02 3.74404212e-11 1.61531188e-03 -2.08418935e-06  
 6.76638739e+02]]

Indices with max absolute value:  $i = 0$   $j = 4$

Angular parameters:  $c = 0.9999999996901718$   $s = -2.4892899083642983e-05$

Check:  $c^2 + s^2 == 1.0$

$\Delta = 4810981.997634361$

$2 * \Omega = 0.0023656412959098816$

Check  $\delta = 0.0$

Check  $2 * \Omega = -0.01909184269607067$

$\Delta + 2 * \Omega = 4810982.0000000002$

Iteration: 22

$A =$

[[ 1.80750901e+03 -1.29100595e-05 -1.34951349e-05 -9.16628274e-10  
 -2.18632874e-09]  
 [-1.29100596e-05 1.95019671e+02 -1.96863037e-02 4.49412946e-04  
 -2.83926070e-10]  
 [-1.34951349e-05 -1.96863037e-02 1.01925755e+03 -7.80031613e-10  
 1.61531155e-03]  
 [-9.16621806e-10 4.49412946e-04 -7.80015636e-10 9.55750361e+01  
 -2.08418934e-06]  
 [-2.18631532e-09 -2.83928388e-10 1.61531155e-03 -2.08418937e-06

6.76638738e+02]]

Indices with max absolute value:  $i = 2$   $j = 1$

Angular parameters:  $c = 0.999999997147713$   $s = -2.3884251270382814e-05$

Check:  $c^2 + s^2 == 1.0$

$\Delta = 4810981.999219279$

$2 * \Omega = 0.0007807239890098572$

Check  $\delta = 0.0$

Check  $2 * \Omega = -0.0015849173069000244$

$\Delta + 2 * \Omega = 4810982.000000003$

Answer:

Eigenvalues:[1807.50900619 195.01967057 1019.25754891 95.57503608 676.63873826]

Eigenvectors:

[[ 0.66401316 0.542621 0.23745611 -0.09040216 0.44731534]

[-0.66779667 0.63032902 0.24667846 -0.30783007 0.03351597]

[ 0.1038325 -0.391686 0.79648724 -0.40911244 -0.18448845]

[-0.23892253 -0.08107463 0.46833826 0.76672504 0.35935347]

[-0.21276503 -0.38504126 -0.17038689 -0.37660035 0.79725451]]

Residual vectors:

$x[1807.5090061862575] = [ 7.22099230e-06 -2.85113799e-06 -1.39332273e-05 9.49522584e-06$   
 $2.05486276e-06]$

$x[195.0196705689863] = [-1.15955599e-04 -4.34565211e-05 2.07405799e-04 3.45728505e-04$   
 $1.55753462e-04]$

$x[1019.2575489102006] = [-0.00035264 -0.00062928 -0.00027844 -0.00060712 0.00128177]$

$x[95.57503607804681] = [-2.99674832e-04 2.84084531e-04 1.11206225e-04 -1.37553119e-04$   
 $1.34025976e-05]$

$x[676.6387382565093] = [ 0.00016819 -0.0006325 0.00128561 -0.00066245 -0.00029875]$

-----LU-algorithm-----

Given matrix =

```
[[ -973. -378. -195. 683. -487.]  
 [ 905. -779. -69. 676. 362.]  
 [ 707. 223. -868. 724. 135.]  
 [ 715. 630. 128. -917. 408.]  
 [ 308. 596. -572. -204. -54.]]
```

Iteration 1

L =

```
[[ 1.      0.      0.      0.      0.      ]  
 [-0.9301131  1.      0.      0.      0.      ]  
 [-0.7266187  0.04569491  1.      0.      0.      ]  
 [-0.73484075 -0.31154746  0.09346028  0.3904772  1.      ]  
 [-0.31654677 -0.42132726  0.7405112  1.      0.      ]]
```

U =

```
[[ -973.    -378.    -195.     683.    -487.     ]  
 [  0.    -1130.5828 -250.37206  1311.2672  -90.965065]  
 [  0.      0.    -998.24994  1160.3623  -214.70667 ]  
 [  0.      0.      0.    -294.5872  -87.49166 ]  
 [  0.      0.      0.      0.     76.02267 ]]
```

Iteration 5

L =

```
[[ 1.0000000e+00  0.0000000e+00  0.0000000e+00  0.0000000e+00  
  0.0000000e+00]  
 [ 3.5437319e-01  1.0000000e+00  0.0000000e+00  0.0000000e+00  
  0.0000000e+00]  
 [-2.6155986e-02  2.0161691e-01  1.0000000e+00  0.0000000e+00  
  0.0000000e+00]
```

```
[ 2.4123492e-03 -1.1550991e-02  3.2183249e-02  1.0000000e+00
 0.0000000e+00]
[-5.3129584e-06  3.0160405e-05 -1.5869109e-04 -7.8109181e-03
 1.0000000e+00]]
```

U =

```
[[ -2358.0212  1411.4507 -1016.565  -662.3399  2111.484 ]
 [  0.      -672.41943 -517.4674  -372.40277  316.85306 ]
 [  0.       0.      -638.0721  115.256996  565.59344 ]
 [  0.       0.       0.      -261.4954  -410.40768 ]
 [  0.       0.       0.       0.      -92.95862 ]]
```

Iteration 10

L =

```
[[ 1.0000000e+00  0.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
 [ 2.2401722e-02  1.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
 [-1.5531597e-04  7.7693854e-03  1.0000000e+00  0.0000000e+00
 0.0000000e+00]
 [ 2.5076127e-07 -6.8092913e-06  7.7953783e-04  1.0000000e+00
 0.0000000e+00]
 [-3.3842460e-12  1.1632193e-10 -2.2287054e-08 -4.8129477e-05
 1.0000000e+00]]
```

U =

```
[[ -1526.2444  955.05634 -1057.2661  -687.92914  2111.484 ]
 [  0.      -1175.0046  -240.58696  -181.44923  -281.03314 ]
 [  0.       0.      -553.3139   165.64787   606.2905 ]
 [  0.       0.       0.      -261.50012  -427.96045 ]
 [  0.       0.       0.       0.      -94.777245]]
```

Iteration 15

L =

```
[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
  0.0000000e+00]
 [ 8.0452003e-03 1.0000000e+00 0.0000000e+00 0.0000000e+00
  0.0000000e+00]
 [-1.1357916e-06 1.4353648e-04 1.0000000e+00 0.0000000e+00
  0.0000000e+00]
 [ 4.3627331e-11 -2.8796838e-09 1.8656001e-05 1.0000000e+00
  0.0000000e+00]
 [-3.6803912e-18 3.1311274e-16 -3.3311971e-12 -3.0081961e-07
  1.0000000e+00]]
```

U =

```
[[ -1455.0896   940.27124 -1058.2612  -688.0875   2111.484 ]
 [    0.    -1235.2651  -176.814   -139.79195  -408.79907 ]
 [    0.      0.    -551.8177   166.57736   608.627 ]
 [    0.      0.      0.    -261.5823  -428.37796 ]
 [    0.      0.      0.      0.    -94.788895]]
```

Iteration 20

L =

```
[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
  0.0000000e+00]
 [ 3.9623962e-03 1.0000000e+00 0.0000000e+00 0.0000000e+00
  0.0000000e+00]
 [-9.5085104e-09 2.3443652e-06 1.0000000e+00 0.0000000e+00
  0.0000000e+00]
 [ 8.7438152e-15 -1.1052595e-12 4.4667235e-07 1.0000000e+00
  0.0000000e+00]
 [-4.6086059e-24 7.5746399e-22 -4.9830911e-16 -1.8794806e-09
```

1.0000000e+00]]

U =

[[ -1426.4469 940.0039 -1058.285 -688.08844 2111.484 ]

[ 0. -1260.1093 -148.89879 -121.63837 -464.50494]

[ 0. 0. -551.79425 166.59135 608.6766 ]

[ 0. 0. 0. -261.5848 -428.38797]

[ 0. 0. 0. 0. -94.78896]]

Iteration 25

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00  
0.00000000e+00]

[ 2.25754199e-03 1.00000000e+00 0.00000000e+00 0.00000000e+00  
0.00000000e+00]

[-8.52046489e-11 3.60905439e-08 1.00000000e+00 0.00000000e+00  
0.00000000e+00]

[ 1.87598587e-18 -4.03148428e-16 1.06946745e-08 1.00000000e+00  
0.00000000e+00]

[-6.17766104e-30 1.73479603e-27 -7.45423567e-20 -1.17425826e-11  
1.00000000e+00]]

U =

[[ -1411.5059 939.9996 -1058.2856 -688.08844 2111.484 ]

[ 0. -1273.4484 -133.8819 -111.87444 -494.4667 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Iteration 30

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00



```

0.0000000e+00]
[ 1.3955300e-03  1.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
[-7.9359056e-13  5.3685018e-10  1.0000000e+00  0.0000000e+00
 0.0000000e+00]
[ 4.1835139e-22 -1.4266303e-19  2.5606253e-10  1.0000000e+00
 0.0000000e+00]
[-8.6071922e-36  3.8472740e-33 -1.1150833e-23 -7.3365068e-14
 1.0000000e+00]]

```

U =

```

[[-1402.7131   939.9995 -1058.2856  -688.08844  2111.484 ]
 [  0.    -1281.4308  -124.894844 -106.031136 -512.3976 ]
 [  0.      0.    -551.79395   166.59155   608.6774 ]
 [  0.      0.      0.    -261.58487  -428.38818 ]
 [  0.      0.      0.      0.    -94.78896 ]]

```

Iteration 35

L =

```

[[ 1.0000000e+00  0.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
 [ 9.0690615e-04  1.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
 [-7.5699453e-15  7.8158565e-12  1.0000000e+00  0.0000000e+00
 0.0000000e+00]
 [ 9.5546747e-26 -4.9522898e-23  6.1309035e-12  1.0000000e+00
 0.0000000e+00]
 [-1.2282381e-41  8.3606203e-39 -1.6680595e-27 -4.5836876e-16
 1.0000000e+00]]

```

U =

```

[[-1397.1686   939.9995 -1058.2856  -688.08844  2111.484 ]

```

```
[ 0.   -1286.5161  -119.169556 -102.3086  -523.8206 ]
[ 0.      0.   -551.79395  166.59155  608.6774 ]
[ 0.      0.      0.   -261.58487  -428.38818 ]
[ 0.      0.      0.      0.   -94.78896 ]]
```

Iteration 40

L =

```
[[ 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
 [ 6.08745730e-04  1.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
 [-7.33368471e-17  1.12196656e-13  1.00000000e+00  0.00000000e+00
    0.00000000e+00]
 [ 2.21627577e-29 -1.69737414e-26  1.46792192e-13  1.00000000e+00
    0.00000000e+00]
 [ 0.00000000e+00  1.82168800e-44 -2.49525994e-31 -2.86378717e-18
    1.00000000e+00]]
```

U =

```
[[ -1393.5187   939.9995 -1058.2856  -688.08844  2111.484 ]
 [  0.   -1289.8857  -115.37576  -99.8419  -531.38995]
 [  0.      0.   -551.79395  166.59155  608.6774 ]
 [  0.      0.      0.   -261.58487  -428.38818]
 [  0.      0.      0.      0.   -94.78896]]
```

Iteration 45

L =

```
[[ 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
 [ 4.1755592e-04  1.00000000e+00  0.00000000e+00  0.00000000e+00
    0.00000000e+00]
```

```

[-7.1791264e-19 1.5953299e-15 1.0000000e+00 0.0000000e+00
0.0000000e+00]
[ 5.1945940e-33 -5.7676354e-30 3.5146448e-15 1.0000000e+00
0.0000000e+00]
[ 0.0000000e+00 0.0000000e+00 -3.7326735e-35 -1.7892314e-20
1.0000000e+00]]

```

U =

```

[[-1391.0474 939.9995 -1058.2856 -688.08844 2111.484 ]
[ 0. -1292.1772 -112.79585 -98.164474 -536.53735 ]
[ 0. 0. -551.79395 166.59155 608.6774 ]
[ 0. 0. 0. -261.58487 -428.38818 ]
[ 0. 0. 0. 0. -94.78896 ]]

```

Iteration 50

L =

```

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00]
[ 2.9069203e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00]
[-7.0781829e-21 2.2535891e-17 1.0000000e+00 0.0000000e+00
0.0000000e+00]
[ 1.2262533e-36 -1.9481574e-33 8.4151129e-17 1.0000000e+00
0.0000000e+00]
[ 0.0000000e+00 0.0000000e+00 -5.5837274e-39 -1.1178724e-22
1.0000000e+00]]

```

U =

```

[[-1389.3422 939.9995 -1058.2856 -688.08844 2111.484 ]
[ 0. -1293.7632 -111.01039 -97.00358 -540.09973]
[ 0. 0. -551.79395 166.59155 608.6774 ]
[ 0. 0. 0. -261.58487 -428.38818]

```

[ 0. 0. 0. 0. -94.78896]]

Iteration 55

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00  
0.0000000e+00]  
[ 2.0447001e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00  
0.0000000e+00]  
[-7.0134392e-23 3.1689219e-19 1.0000000e+00 0.0000000e+00  
0.0000000e+00]  
[ 2.9091657e-40 -6.5529088e-37 2.0148298e-18 1.0000000e+00  
0.0000000e+00]  
[ 0.0000000e+00 0.0000000e+00 -8.3517388e-43 -6.9842214e-25  
1.0000000e+00]]

U =

[[ -1388.1501 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1294.8743 -109.75967 -96.19037 -542.59515]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818]  
[ 0. 0. 0. 0. -94.78896]]

Iteration 60

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00  
0.0000000e+00]  
[ 1.4486843e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00  
0.0000000e+00]  
[-6.9736364e-25 4.4416812e-21 1.0000000e+00 0.0000000e+00  
0.0000000e+00]  
[ 6.8663625e-44 -2.1976704e-40 4.8241060e-20 1.0000000e+00

```

0.0000000e+00]
[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -4.3635879e-27
1.0000000e+00]]

```

U =

```

[[-1387.3091  939.9995 -1058.2856 -688.08844 2111.484 ]
[ 0.    -1295.6593 -108.876076 -95.61586 -544.3581 ]
[ 0.      0.    -551.79395 166.59155 608.6774 ]
[ 0.      0.      0.    -261.58487 -428.38818 ]
[ 0.      0.      0.      0.    -94.78896 ]]

```

Iteration 65

L =

```

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00]
[ 1.0316848e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00
0.0000000e+00]
[-6.9512659e-27 6.2113961e-23 1.0000000e+00 0.0000000e+00
0.0000000e+00]
[ 0.0000000e+00 -7.2867520e-44 1.1550354e-21 1.0000000e+00
0.0000000e+00]
[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -2.7262739e-29
1.0000000e+00]]

```

U =

```

[[-1386.712  939.9995 -1058.2856 -688.08844 2111.484 ]
[ 0.    -1296.217 -108.24811 -95.20756 -545.611 ]
[ 0.      0.    -551.79395 166.59155 608.6774 ]
[ 0.      0.      0.    -261.58487 -428.38818]
[ 0.      0.      0.      0.    -94.78896]]

```

Iteration 70

L =

```
[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
    0.0000000e+00]
 [ 7.3740608e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00
    0.0000000e+00]
 [-6.9412062e-29 8.6720653e-25 1.0000000e+00 0.0000000e+00
    0.0000000e+00]
 [ 0.0000000e+00 0.0000000e+00 2.7655005e-23 1.0000000e+00
    0.0000000e+00]
 [ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -1.7033159e-31
    1.0000000e+00]]
```

U =

```
[[-1386.2864  939.9995 -1058.2856 -688.08844 2111.484 ]
 [  0.    -1296.6151 -107.79991 -94.916145 -546.50525 ]
 [  0.      0.    -551.79395 166.59155  608.6774 ]
 [  0.      0.      0.    -261.58487 -428.38818 ]
 [  0.      0.      0.      0.    -94.78896 ]]
```

Iteration 75

L =

```
[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
    0.00000000e+00]
 [ 5.28445235e-05 1.00000000e+00 0.00000000e+00 0.00000000e+00
    0.00000000e+00]
 [-6.93991082e-31 1.20934195e-26 1.00000000e+00 0.00000000e+00
    0.00000000e+00]
 [ 0.00000000e+00 0.00000000e+00 6.62143664e-25 1.00000000e+00
    0.00000000e+00]
 [ 0.00000000e+00 0.00000000e+00 0.00000000e+00 -1.06419432e-33
    1.00000000e+00]]
```

U =

```
[[ -1385.9817  939.9995 -1058.2856 -688.08844 2111.484 ]
 [   0.    -1296.9001 -107.47906 -94.70753 -547.1454 ]
 [   0.      0.    -551.79395 166.59155 608.6774 ]
 [   0.      0.      0.    -261.58487 -428.38818]
 [   0.      0.      0.      0.    -94.78896]]
```

Iteration 80

L =

```
[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00]
 [ 3.7940736e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00]
 [-6.9448934e-33 1.6850476e-28 1.0000000e+00 0.0000000e+00
 0.0000000e+00]
 [ 0.0000000e+00 0.0000000e+00 1.5853704e-26 1.0000000e+00
 0.0000000e+00]
 [ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -6.6488519e-36
 1.0000000e+00]]
```

U =

```
[[ -1385.7632  939.9995 -1058.2856 -688.08844 2111.484 ]
 [   0.    -1297.1046 -107.24886 -94.55785 -547.6047 ]
 [   0.      0.    -551.79395 166.59155 608.6774 ]
 [   0.      0.      0.    -261.58487 -428.38818]
 [   0.      0.      0.      0.    -94.78896]]
```

Iteration 85

L =

```
[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
 0.0000000e+00]
```

```
[ 2.7276859e-05  1.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
[-6.9543910e-35  2.3464667e-30  1.0000000e+00  0.0000000e+00
 0.0000000e+00]
[ 0.0000000e+00  0.0000000e+00  3.7958510e-28  1.0000000e+00
 0.0000000e+00]
[ 0.0000000e+00  0.0000000e+00  0.0000000e+00 -4.1540568e-38
 1.0000000e+00]]
```

U =

```
[[ -1385.6063   939.9995 -1058.2856  -688.08844  2111.484 ]
 [   0.    -1297.2515  -107.083466  -94.45031  -547.93475 ]
 [   0.         0.    -551.79395   166.59155   608.6774 ]
 [   0.         0.         0.    -261.58487  -428.38818 ]
 [   0.         0.         0.         0.    -94.78896 ]]
```

Iteration 90

L =

```
[[ 1.0000000e+00  0.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
[ 1.9629177e-05  1.0000000e+00  0.0000000e+00  0.0000000e+00
 0.0000000e+00]
[-6.9671506e-37  3.2660964e-32  1.0000000e+00  0.0000000e+00
 0.0000000e+00]
[ 0.0000000e+00  0.0000000e+00  9.0884040e-30  1.0000000e+00
 0.0000000e+00]
[ 0.0000000e+00  0.0000000e+00  0.0000000e+00 -2.5953589e-40
 1.0000000e+00]]
```

U =

```
[[ -1385.4935   939.9995 -1058.2856  -688.08844  2111.484 ]
 [   0.    -1297.357   -106.964485  -94.372955  -548.1721 ]]
```



[ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 95

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00  
 0.0000000e+00]  
 [ 1.4135506e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00  
 0.0000000e+00]  
 [-6.9822793e-39 4.5447339e-34 1.0000000e+00 0.0000000e+00  
 0.0000000e+00]  
 [ 0.0000000e+00 0.0000000e+00 2.1760359e-31 1.0000000e+00  
 0.0000000e+00]  
 [ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -1.6213023e-42  
 1.0000000e+00]]

U =

[[ -1385.4122 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.4332 -106.878815 -94.31726 -548.343 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 100

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00  
 0.00000000e+00]  
 [ 1.01844535e-05 1.00000000e+00 0.00000000e+00 0.00000000e+00  
 0.00000000e+00]  
 [-6.99920557e-41 6.32252737e-36 1.00000000e+00 0.00000000e+00

```

0.00000000e+00]
[ 0.00000000e+00 0.00000000e+00 5.21008116e-33 1.00000000e+00
0.00000000e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 -9.80908925e-45
1.00000000e+00]]

```

U =

```

[[-1385.3538  939.9995 -1058.2856 -688.08844 2111.484 ]
[ 0.    -1297.4879 -106.81711 -94.27714 -548.4661 ]
[ 0.      0.    -551.79395 166.59155 608.6774 ]
[ 0.      0.      0.    -261.58487 -428.38818]
[ 0.      0.      0.      0.    -94.78896]]

```

Iteration 105

L =

```

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00]
[ 7.3404071e-06 1.00000000e+00 0.00000000e+00 0.00000000e+00
0.00000000e+00]
[-7.0205053e-43 8.7943373e-38 1.00000000e+00 0.00000000e+00
0.00000000e+00]
[ 0.00000000e+00 0.00000000e+00 1.2474495e-34 1.00000000e+00
0.00000000e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
1.00000000e+00]]

```

U =

```

[[-1385.3118  939.9995 -1058.2856 -688.08844 2111.484 ]
[ 0.    -1297.5273 -106.772644 -94.24822 -548.5548 ]
[ 0.      0.    -551.79395 166.59155 608.6774 ]
[ 0.      0.      0.    -261.58487 -428.38818 ]
[ 0.      0.      0.      0.    -94.78896 ]]

```

Iteration 110

L =

```
[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
    0.0000000e+00]
 [ 5.2919463e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00
    0.0000000e+00]
 [-7.0064923e-45 1.2231080e-39 1.0000000e+00 0.0000000e+00
    0.0000000e+00]
 [ 0.0000000e+00 0.0000000e+00 2.9867675e-36 1.0000000e+00
    0.0000000e+00]
 [ 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00
    1.0000000e+00]]
```

U =

```
[[ -1385.2812   939.9995 -1058.2856  -688.08844  2111.484 ]
 [    0.    -1297.5559  -106.740585  -94.22738  -548.6188 ]
 [    0.     0.    -551.79395   166.59155   608.6774 ]
 [    0.     0.     0.    -261.58487  -428.38818 ]
 [    0.     0.     0.     0.    -94.78896 ]]
```

Iteration 115

L =

```
[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]
 [3.8158578e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]
 [0.0000000e+00 1.7008961e-41 1.0000000e+00 0.0000000e+00 0.0000000e+00]
 [0.0000000e+00 0.0000000e+00 7.1512156e-38 1.0000000e+00 0.0000000e+00]
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]
```

U =

```
[[ -1385.2594   939.9995 -1058.2856  -688.08844  2111.484 ]
 [    0.    -1297.5763  -106.717476  -94.21236  -548.66486 ]]
```

[ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 120

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [2.7518668e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 2.3681944e-43 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 1.7122144e-39 1.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2435 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.5912 -106.700806 -94.20152 -548.6981 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 125

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [1.9847466e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 4.2038954e-45 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 4.0994987e-41 1.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2322 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6018 -106.68878 -94.19371 -548.7221 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]]

[ 0. 0. 0. 0. -94.78896]]

Iteration 130

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[1.4315709e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 9.8231022e-43 1.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.224 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1297.6095 -106.68011 -94.18807 -548.7394 ]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818]  
[ 0. 0. 0. 0. -94.78896]]

Iteration 135

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[1.0326249e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 2.3822074e-44 1.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2181 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1297.615 -106.67386 -94.18401 -548.75183]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818]  
[ 0. 0. 0. 0. -94.78896]]

Iteration 140

L =

```
[[1.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00]
 [7.44883e-07 1.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00]
 [0.00000e+00 0.00000e+00 1.00000e+00 0.00000e+00 0.00000e+00]
 [0.00000e+00 0.00000e+00 0.00000e+00 1.00000e+00 0.00000e+00]
 [0.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00 1.00000e+00]]
```

U =

```
[[ -1385.2139   939.9995 -1058.2856  -688.08844  2111.484  ]
 [   0.    -1297.619   -106.66935  -94.181076  -548.7608  ]
 [   0.       0.    -551.79395   166.59155   608.6774  ]
 [   0.       0.       0.    -261.58487  -428.38818  ]
 [   0.       0.       0.       0.    -94.78896  ]]
```

Iteration 145

L =

```
[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]
 [5.373348e-07 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]
 [0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]
 [0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]
 [0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]
```

U =

```
[[ -1385.2108   939.9995 -1058.2856  -688.08844  2111.484  ]
 [   0.    -1297.6218  -106.666084  -94.17896  -548.76733  ]
 [   0.       0.    -551.79395   166.59155   608.6774  ]
 [   0.       0.       0.    -261.58487  -428.38818  ]
 [   0.       0.       0.       0.    -94.78896  ]]
```

Iteration 150

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [3.8762343e-07 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2086 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6239 -106.66374 -94.17744 -548.7719 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818]  
 [ 0. 0. 0. 0. -94.78896]]

Iteration 155

L =

[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]  
 [2.796282e-07 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[ -1385.207 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6254 -106.662056 -94.17633 -548.7753 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 160

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [2.0172335e-07 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2058 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6266 -106.660835 -94.17554 -548.7777 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 165

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [1.4552394e-07 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2051 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6272 -106.65995 -94.174965 -548.7795 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 170

L =

[[1.000000000e+00 0.000000000e+00 0.000000000e+00 0.000000000e+00  
 0.000000000e+00]  
 [1.04981964e-07 1.000000000e+00 0.000000000e+00 0.000000000e+00  
 0.000000000e+00]



[0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00  
0.00000000e+00]  
[0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00  
0.00000000e+00]  
[0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00  
1.00000000e+00]]

U =

[[ -1385.2045 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1297.6278 -106.6593 -94.17455 -548.78076]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818]  
[ 0. 0. 0. 0. -94.78896]]

Iteration 175

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[7.5735066e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2039 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1297.6284 -106.65884 -94.174255 -548.7817 ]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818 ]  
[ 0. 0. 0. 0. -94.78896 ]]

Iteration 180

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[5.463626e-08 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[ -1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6287 -106.6585 -94.17404 -548.7823 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818]  
 [ 0. 0. 0. 0. -94.78896]]

Iteration 185

L =

[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]  
 [3.941533e-08 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]  
 [0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[ -1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]  
 [ 0. -1297.6287 -106.658264 -94.17388 -548.7828 ]  
 [ 0. 0. -551.79395 166.59155 608.6774 ]  
 [ 0. 0. 0. -261.58487 -428.38818 ]  
 [ 0. 0. 0. 0. -94.78896 ]]

Iteration 190

L =

[[1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]  
 [2.8434743e-08 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]  
 [0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00 0.00000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1297.6287 -106.6581 -94.17377 -548.7831 ]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818]  
[ 0. 0. 0. 0. -94.78896]]

Iteration 195

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[2.0513202e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[ -1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]  
[ 0. -1297.6287 -106.657974 -94.17369 -548.7834 ]  
[ 0. 0. -551.79395 166.59155 608.6774 ]  
[ 0. 0. 0. -261.58487 -428.38818 ]  
[ 0. 0. 0. 0. -94.78896 ]]

Iteration 200

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[1.4798499e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]  
[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

```
[[-1385.2035  939.9995 -1058.2856 -688.08844 2111.484 ]
 [  0.    -1297.6287 -106.65789 -94.17363 -548.7837 ]
 [  0.      0.    -551.79395 166.59155 608.6774 ]
 [  0.      0.      0.    -261.58487 -428.38818]
 [  0.      0.      0.      0.    -94.78896]]
```

Iteration 205

L =

```
[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]
 [1.0675836e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]
 [0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]
 [0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]
```

U =

```
[[-1385.2035  939.9995 -1058.2856 -688.08844 2111.484 ]
 [  0.    -1297.6287 -106.65782 -94.17359 -548.7837 ]
 [  0.      0.    -551.79395 166.59155 608.6774 ]
 [  0.      0.      0.    -261.58487 -428.38818]
 [  0.      0.      0.      0.    -94.78896]]
```

Answer:

Eigenvalues: [ -94.78896 -261.58487 -551.79395 -1297.6287 -1385.2035 ]

Eigenvectors:

```
[[ -0.4128942 -0.01850755 -0.1918212  0.03821078  0.88933706]
 [  0.3987725 -0.26899958 -0.22199246 -0.34829584 -0.7733234 ]
 [ -0.45127386  0.2880226  0.42270043  0.4257158  0.59454626]
 [  0.69283634 -0.43206507  0.06005434 -0.5733523 -0.03095425]
 [  0.70856494 -0.27461526  0.06993608 -0.6370592 -0.10856643]]
```

Residual vectors

$$x[-94.78896] = [ 1.9073486e-05 -2.3365021e-05 -2.2888184e-05 -3.4093857e-05 \\ -3.8146973e-05]$$

$$x[-261.58487] = [ 1.5258789e-05 7.6293945e-06 -7.6293945e-06 -3.8146973e-05 \\ -1.5258789e-05]$$

$$x[-551.79395] = [ 1.5258789e-05 -3.0517578e-05 -6.1035156e-05 0.0000000e+00 \\ -6.1035156e-05]$$

$$x[-1297.6287] = [-3.6621094e-04 2.4414062e-04 -1.5258789e-05 3.0517578e-04 \\ 1.9073486e-05]$$

$$x[-1385.2035] = [ 4.2724609e-04 -9.1552734e-05 9.1552734e-05 -3.0517578e-04 \\ -6.1035156e-05]$$

## Висновок

Під час лабораторної роботи були набуті та закріплені вміння та досвід використання методів розв'язання часткової та повної проблеми власних значень.

В ході роботи було реалізовано три методи: степеневий метод, метод Якобі та LR-метод.

За допомогою степеневого методу було знайдено найбільше по модулю власне значення заданої матриці та відповідний цьому значенню вектор. Для цього знадобилось 21 ітерацій. Отриманий вектор нев'язки задовольняє задану точність:  $\varepsilon = 0,0001$ .

За допомогою методу Якобі було знайдено всі власні значення матриці та відповідні власні вектори. Для цього знадобилось 22 ітерацій. Отримані вектор нев'язки задовольняють задану точність:  $\varepsilon = 0,0001$ .

Для несиметричної матриці був використаний LR-метод. З його допомогою було знайдено всі власні значення заданої матриці. Для цього знадобилось 205 ітерацій.