**МІНІСТЕРСТВО ОСВІТИ І НАУКИ, МОЛОДІ ТА СПОРТУ УКРАЇНИ**

**НАВЧАЛЬНО-НАУКОВИЙ КОМПЛЕКС**

**«ІНСТИТУТ ПРИКЛАДНОГО СИСТЕМНОГО АНАЛІЗУ»**

**НАЦІОНАЛЬНОГО ТЕХНІЧНОГО УНІВЕРСИТЕТУ УКРАЇНИ**

**«КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ»**

**КАФЕДРА МАТЕМАТИЧНИХ МЕТОДІВ СИСТЕМНОГО АНАЛІЗУ**

**Лабораторна робота №6**

**з курсу «Чисельні методи»**

**Тема: Обчислення власних чисел та відповідих власних векторів матриць**

Виконала:

студентка 3 курсу

групи КА-83

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**Київ – 2020**

**Варіант – 18**

**Мета роботи:** набути вміння та досвід використання методів розв’язання часткової та повної проблеми власних значень. Закріпити та поглибити теоретичні знання методів розв’язання часткової та повної проблеми власних значень.

**Завдання на роботу:**

**Завдання1.** Знайти найбільше та власне число та відповідний власний вектор степеневим методом або методом скалярних добутків з точністю ε=0,0001 (матриця з таблиці 1).

**Завдання 2.** Розв'язати повну проблему власних значень:

1. методом Якобі (матриця з таблиці 1);
2. QR або LR методом (матриця з таблиці 2)

**Матриця з таблиці 1:**

**Матриця з таблиці 2:**

**Текст програми**

import numpy as np

import scipy.linalg

def Power\_method(matrix, eps = 1e-4):

    """

    This function finds max eigenvalue and eigenvector by using power method.

    :param matrix: given matrix

    :param eps: accuracy of computation

    :return: cortege of eigenvalue and eigenvector

    """

    k, n = 1, len(matrix)

    x = 1

    Xn = np.random.random(n) #eigenvector

    yn = matrix @ Xn

    l\_n = yn / Xn #lambda

    Xn = yn / np.max(yn) #normalization

    dl = np.copy(l\_n) #delta lamda: lambda[k + 1] - lambda[k]

    with open('Result\_1(6).txt', 'a') as file:

        file.write('------------------------------POWER METHOD------------------------------\n')

        file.write('\nGiven matrix =\n' + str(matrix) + '\n')

        while all(abs(i) > eps for i in dl):

            k+=1

            l\_o = np.copy(l\_n)

            yn = matrix @ Xn

            l\_n = yn / Xn

            Xn = yn / np.max(yn)

            dl = np.array([abs(l\_n[i] - l\_o[i]) for i in range(n)])

            file.write('\ny[' + str(k) + '] =' + str(yn) + '\n')

            file.write('lambda[' + str(k) + '] =' + str(l\_n) + '\n')

            file.write('X[' + str(k) + '] =' + str(Xn) + '\n')

            x = matrix @ Xn.transpose() - np.mean(l\_n) \* Xn.transpose()

        file.write('\nAnswer:\neigenvalue:' + str(np.mean(l\_n)))

        file.write('\neigenvector:\n' + str(Xn))

        file.write('\n\nResidual vector:\n')

        file.write('Ax - lx = ' + str(x) + '\n')

    return np.mean(l\_n), Xn

def Jacobi\_method(matrix, eps = 1e-4):

    """

    This function solves full problem of eigenvalues

    :param matrix: given matrix

    :param eps: accuracy of computation

    :return: eigenvalues and eigenvectors

    """

    temp , count = 1, 0

    check = np.zeros((len(matrix), len(matrix[1])))

    Eigenvectors = np.eye(5)

    A = np.copy(matrix)

    with open('Result\_1(6).txt', 'a') as file:

        file.write('\n------------------------------Jacobi Method------------------------------')

        while temp > eps:

            count += 1

            file.write('\n\nIteration: ' + str(count))

            file.write('\nA =\n' + str(matrix))

            mask = np.ones(matrix.shape, dtype=bool)

            np.fill\_diagonal(mask, 0)

            key\_element = max( matrix[mask], key = abs) #find the biggest value

            a = np.argwhere(matrix == key\_element) #find indices with the biggest value

            file.write('\n\nIndices with max absolute value: i = ' + str(a[0, 0]) + ' j = ' + str(a[0, 1]))

            t = (matrix[a[0, 0], a[0, 0]] - matrix[a[0, 1], a[0, 1]]) / (2 \* key\_element)

            if t == 0:

                c = s = 1 / np.sqrt(2)

            else:

                tau = np.sqrt(1 + t \*\* 2)

                if t > 0:

                    t = -t + tau

                else:

                    t = -t - tau

                c = 1 / np.sqrt(1 + t \*\* 2)

                s = c \* t

            file.write('\nAngular parameters: c = ' + str(c) + ' s = ' + str(s))

            file.write('\nCheck: c^2 + s^2 == ' + str(c \*\* 2 + s\*\* 2))

            Temp = np.array([i \*\* 2 for i in matrix])

            omega\_2 = np.sum(Temp) - np.trace(Temp)

            check\_o = np.sum(check) - np.trace(check)

            check = np.copy(Temp)

            file.write('\nDelta = ' + str(np.trace(Temp)))

            file.write('\n2 \* Omega = ' + str(omega\_2))

            if count != 1:

                file.write('\nCheck delta = ' + str(np.trace(Temp) - np.trace(check)))

                file.write('\nCheck 2 \* Omega = ' + str(omega\_2 - check\_o))

            else:

                file.write('\nCheck delta = 0')

                file.write('\nCheck 2 \* Omega = 0')

            file.write('\nDelta + 2 \* Omega = ' + str(np.sum(Temp)))

            T = np.eye(matrix.shape[1])

            T[a[0, 0], a[0, 0]] = c

            T[a[0, 1], a[0, 1]] = c

            T[a[0, 0], a[0, 1]] = -s

            T[a[0, 1], a[0, 0]] = s

            B = T.transpose() @ (matrix @ T)

            matrix = np.copy(B)

            Eigenvectors = Eigenvectors.dot(T)

            Temp = np.array([i \*\* 2 for i in matrix])

            temp = np.sum(Temp) - np.trace(Temp)

        file.write('\n\nAnswer:\nEigenvalues:' + str(np.diag(matrix)))

        file.write('\nEigenvectors:\n' + str(Eigenvectors.transpose()))

        file.write('\n\nResidual vectors:\n')

        for j in range(len(matrix)):

            x = A @ Eigenvectors.transpose()[j] - matrix[j, j] \* Eigenvectors.transpose()[j]

            file.write('x[' + str(np.diag(matrix)[j]) + '] = ' + str(x) + '\n')

    return np.diag(matrix), Eigenvectors.transpose()

def LU\_decomposition(matrix, eps = 1e-4):

    """

    This function solves full problem of eigenvectors and eigenvalues using LU-decomposition

    :param matrix: given matrix

    :param eps: accuracy of computation

    :return: eigenvalues and eigenvectors

    """

    A = np.copy(matrix)

    LL = UU = np.eye(len(matrix))

    count = 0

    x, y = np.linalg.eig(matrix)

    with open('Result\_1(6).txt', 'a') as file:

        file.write('\n------------------------------LU-algorithm------------------------------\n')

        file.write('Given matrix = \n' + str(matrix) + '\n')

        while abs(np.linalg.det(matrix) - np.prod(np.diag(matrix)))> eps:

            P, L, U = scipy.linalg.lu(matrix)

            L = P @ L

            matrix = U @ L

            LL = LL @ L

            UU = np.linalg.inv(L) @ UU

            count += 1

            if count == 1 or count % 5 == 0:

                file.write('\nInteration ' + str(count))

                file.write('\nL = \n' + str(L) + '\n')

                file.write('U =\n' + str(U) + '\n')

        matrix = np.sort(np.diag(matrix))

        matrix = matrix[::-1]

        file.write('\n\nAnswer:\nEigenvalues: ' + str(matrix))

        file.write('\nEigenvectors:\n' + str(y.transpose()))

        file.write('\n\nResidual vectors\n')

        for j in range(len(matrix)):

            x = A @ y.transpose()[j] - matrix[j] \* y.transpose()[j]

            file.write('x[' + str(matrix[j]) + '] = ' + str(x) + '\n')

    return matrix, y.transpose()

matrix\_1 = np.loadtxt('matrix\_1.txt', 'f')

matrix\_2 = np.loadtxt('matrix\_2.txt', 'f')

eigenvalue\_p, eigenvector\_p = Power\_method(matrix\_1)

print('------------------------------Power method------------------------------\nEigenvalue:\n', eigenvalue\_p)

print('Eigenvector: \n', eigenvector\_p)

eigenvalues\_j, eigenvectors\_j = Jacobi\_method(matrix\_1)

print('\n------------------------------Jacobi method------------------------------\nEigenvalues: \n', eigenvalues\_j)

print('Eigenvectors:\n', eigenvectors\_j)

eigenvalues\_lu, eigenvectors\_lu = LU\_decomposition(matrix\_2)

print('\n------------------------------LU algorithm------------------------------\nEigenvalues: \n', eigenvalues\_lu)

print('Eigenvectors: \n', eigenvectors\_lu)

**Текстовий файл matrix\_1.txt**

931 585 351 -75 390

585 867 -14 129 306

351 -14 801 -308 -32

-75 129 -308 356 -175

390 306 -32 -175 839

**Текстовий файл matrix\_2.txt**

-973 -378 -195 683 -487

905 -779 -69 676 362

707 223 -868 724 135

715 630 128 -917 408

308 596 -572 -204 -54

**Результати роботи програми**

------------------------------Power method------------------------------

Eigenvalue

1807.509782582055

Eigenvector

[ 1. 0.81718438 0.35760705 -0.13614489 0.67365449]

------------------------------Jacobi method------------------------------

Eigenvalues

[1807.50900619 195.01967057 1019.25754891 95.57503608 676.63873826]

Eigenvectors

[[ 0.66401316 0.542621 0.23745611 -0.09040216 0.44731534]

[-0.66779667 0.63032902 0.24667846 -0.30783007 0.03351597]

[ 0.1038325 -0.391686 0.79648724 -0.40911244 -0.18448845]

[-0.23892253 -0.08107463 0.46833826 0.76672504 0.35935347]

[-0.21276503 -0.38504126 -0.17038689 -0.37660035 0.79725451]]

------------------------------LU algorithm------------------------------

Eigenvalues

[ -94.78896 -261.58487 -551.79395 -1297.6287 -1385.2035 ]

Eigenvectors

[[-0.4128942 -0.01850755 -0.1918212 0.03821078 0.88933706]

[ 0.3987725 -0.26899958 -0.22199246 -0.34829584 -0.7733234 ]

[-0.45127386 0.2880226 0.42270043 0.4257158 0.59454626]

[ 0.69283634 -0.43206507 0.06005434 -0.5733523 -0.03095425]

[ 0.70856494 -0.27461526 0.06993608 -0.6370592 -0.10856643]]

**Текстовий файл Result\_1(6).txt**

------------------------------POWER METHOD------------------------------

Given matrix =

[[ 931. 585. 351. -75. 390.]

[ 585. 867. -14. 129. 306.]

[ 351. -14. 801. -308. -32.]

[ -75. 129. -308. 356. -175.]

[ 390. 306. -32. -175. 839.]]

y[2] =[1910.15693641 1562.68534304 629.85975181 -277.22385588 1432.91074919]

lambda[2] =[1910.15693641 1925.6241214 1700.09524101 3601.67656166 1516.08622442]

X[2] =[ 1. 0.81809265 0.32974241 -0.14513146 0.75015342]

y[3] =[1828.76847507 1500.49491793 624.36595233 -253.97035662 1284.56131258]

lambda[3] =[1828.76847507 1834.13813171 1893.49605415 1749.93323282 1712.39813983]

X[3] =[ 1. 0.82049474 0.34141334 -0.13887507 0.70241878]

y[4] =[1815.18445908 1488.61441521 633.28128046 -246.67429877 1243.77865452]

lambda[4] =[1815.18445908 1814.28879334 1854.88147972 1776.2316327 1770.70815622]

X[4] =[ 1. 0.82008989 0.34887985 -0.13589489 0.68520786]

y[5] =[1810.63259162 1483.27677603 638.90047661 -244.95335553 1228.45434875]

lambda[5] =[1810.63259162 1808.67585646 1831.29085263 1802.52067757 1792.8199961 ]

X[5] =[ 1. 0.81920362 0.35286036 -0.13528606 0.67846694]

y[6] =[1808.83666676 1480.46847522 642.12946695 -244.89727773 1222.29359783]

lambda[6] =[1808.83666676 1807.2045052 1819.78349283 1810.21807889 1801.552192 ]

X[6] =[ 1. 0.81846443 0.35499583 -0.13538938 0.67573464]

y[7] =[1808.09594185 1478.94829136 643.96957571 -245.20898496 1219.72475638]

lambda[7] =[1808.09594185 1806.97930582 1814.02013265 1811.13896866 1805.03511317]

X[7] =[ 1. 0.81795897 0.35615896 -0.13561724 0.67459073]

y[8] =[1807.77946932 1478.11433844 645.0151111 -245.51336913 1218.61299504]

lambda[8] =[1807.77946932 1807.07638837 1811.03152821 1810.34037743 1806.44788875]

X[8] =[ 1. 0.81764085 0.35679967 -0.13580936 0.67409383]

y[9] =[1807.63887638 1477.65272624 645.60784025 -245.73318031 1218.11187529]

lambda[9] =[1807.63887638 1807.21490336 1809.44070727 1809.39799688 1807.03607169]

X[9] =[ 1. 0.81744907 0.35715532 -0.13594152 0.67386904]

y[10] =[1807.57376557 1477.39564304 645.94330331 -245.87517176 1217.87633792]

lambda[10] =[1807.57376557 1807.32438192 1808.57814019 1808.68338024 1807.2893465 ]

X[10] =[ 1. 0.81733629 0.35735377 -0.13602497 0.67376301]

y[11] =[1807.54235223 1477.25187333 646.13293853 -245.96199648 1217.76111943]

lambda[11] =[1807.54235223 1807.39786526 1808.10443078 1808.21206546 1807.40275812]

X[11] =[ 1. 0.81727096 0.3574649 -0.13607537 0.67371097]

y[12] =[1807.52662283 1477.17124901 646.24005121 -246.01348672 1217.70273457]

lambda[12] =[1807.52662283 1807.44370158 1807.84199754 1807.92074724 1807.45569065]

X[12] =[ 1. 0.81723347 0.35752727 -0.13610504 0.67368454]

y[13] =[1807.51849584 1477.125952 646.30051906 -246.04346933 1217.67227666]

lambda[13] =[1807.51849584 1807.47119558 1807.69575078 1807.74691302 1807.48141189]

X[13] =[ 1. 0.81721208 0.35756233 -0.13612224 0.67367071]

y[14] =[1807.51419106 1477.10047142 646.33464214 -246.0607312 1217.65602421]

lambda[14] =[1807.51419106 1807.48731594 1807.61392839 1807.64530739 1807.49437063]

X[14] =[ 1. 0.81719993 0.35758206 -0.13613212 0.67366333]

y[15] =[1807.51186767 1477.08612623 646.35389367 -246.07059797 1217.64720486]

lambda[15] =[1807.51186767 1807.49663699 1807.56802912 1807.58667046 1807.50109943]

X[15] =[ 1. 0.81719304 0.35759317 -0.13613775 0.67365931]

y[16] =[1807.51059647 1477.07804569 646.36475319 -246.07621157 1217.64236092]

lambda[16] =[1807.51059647 1807.50197951 1807.54223604 1807.55310237 1807.50467718]

X[16] =[ 1. 0.81718915 0.35759943 -0.13614095 0.67365711]

y[17] =[1807.50989423 1477.07349233 646.37087822 -246.0793957 1217.63967782]

lambda[17] =[1807.50989423 1807.50502449 1807.52772467 1807.53398494 1807.50661358]

X[17] =[ 1. 0.81718695 0.35760295 -0.13614277 0.67365589]

y[18] =[1807.50950369 1477.0709259 646.37433264 -246.0811982 1217.63818293]

lambda[18] =[1807.50950369 1807.50675366 1807.51955415 1807.52313402 1807.50767516]

X[18] =[ 1. 0.8171857 0.35760494 -0.13614379 0.6736552 ]

y[19] =[1807.5092855 1477.06947915 646.37628078 -246.08221725 1217.63734674]

lambda[19] =[1807.5092855 1807.50773328 1807.51495143 1807.51698873 1807.50826242]

X[19] =[ 1. 0.817185 0.35760606 -0.13614437 0.67365482]

y[20] =[1807.50916324 1477.06866349 646.37737941 -246.08279286 1217.63687776]

lambda[20] =[1807.50916324 1807.50828738 1807.51235768 1807.51351349 1807.50858932]

X[20] =[ 1. 0.81718461 0.3576067 -0.1361447 0.67365461]

y[21] =[1807.50909458 1477.06820361 646.37799896 -246.08311782 1217.63661424]

lambda[21] =[1807.50909458 1807.50860047 1807.51089571 1807.51155008 1807.50877207]

X[21] =[ 1. 0.81718438 0.35760705 -0.13614489 0.67365449]

Answer:

eigenvalue:1807.509782582055

eigenvector:

[ 1. 0.81718438 0.35760705 -0.13614489 0.67365449]

Residual vector:

Ax - lx = [-7.26616863e-04 -8.21534466e-04 1.03336846e-04 -8.97119699e-05

-6.11717829e-04]

------------------------------Jacobi Method------------------------------

Iteration: 1

A =

[[ 931. 585. 351. -75. 390.]

[ 585. 867. -14. 129. 306.]

[ 351. -14. 801. -308. -32.]

[ -75. 129. -308. 356. -175.]

[ 390. 306. -32. -175. 839.]]

Indices with max absolute value: i = 0 j = 1

Angular parameters: c = 0.726160863859085 s = 0.6875248357691722

Check: c^2 + s^2 == 0.9999999999999998

Delta = 3090708.0

2 \* Omega = 1720274.0

Check delta = 0

Check 2 \* Omega = 0

Delta + 2 \* Omega = 4810982.0

Iteration: 2

A =

[[ 1.48487456e+03 5.25747894e-14 2.45257116e+02 3.42286390e+01

4.93585337e+02]

[ 2.58652807e-14 3.13125440e+02 -2.51487469e+02 1.45239114e+02

-4.59294616e+01]

[ 2.45257116e+02 -2.51487469e+02 8.01000000e+02 -3.08000000e+02

-3.20000000e+01]

[ 3.42286390e+01 1.45239114e+02 -3.08000000e+02 3.56000000e+02

-1.75000000e+02]

[ 4.93585337e+02 -4.59294616e+01 -3.20000000e+01 -1.75000000e+02

8.39000000e+02]]

Indices with max absolute value: i = 0 j = 4

Angular parameters: c = 0.8796297088079835 s = 0.4756590957633232

Check: c^2 + s^2 == 1.0

Delta = 3775157.999999999

2 \* Omega = 1035824.0

Check delta = 0.0

Check 2 \* Omega = -684450.0

Delta + 2 \* Omega = 4810981.999999999

Iteration: 3

A =

[[ 1.75178046e+03 -2.18467662e+01 2.00514354e+02 -5.31318140e+01

-6.17753989e-14]

[-2.18467662e+01 3.13125440e+02 -2.51487469e+02 1.45239114e+02

-4.04009189e+01]

[ 2.00514354e+02 -2.51487469e+02 8.01000000e+02 -3.08000000e+02

-1.44806928e+02]

[-5.31318140e+01 1.45239114e+02 -3.08000000e+02 3.56000000e+02

-1.70216363e+02]

[-4.50347880e-14 -4.04009189e+01 -1.44806928e+02 -1.70216363e+02

5.72094105e+02]]

Indices with max absolute value: i = 2 j = 3

Angular parameters: c = 0.8903894208773017 s = -0.4551996036792907

Check: c^2 + s^2 == 0.9999999999999999

Delta = 4262410.9691125965

2 \* Omega = 548571.0308874035

Check delta = 0.0

Check 2 \* Omega = -487252.9691125965

Delta + 2 \* Omega = 4810982.0

Iteration: 4

A =

[[ 1.75178046e+03 -2.18467662e+01 2.02721440e+02 4.39660494e+01

-6.17753989e-14]

[-2.18467662e+01 3.13125440e+02 -2.90034569e+02 1.48423743e+01

-4.04009189e+01]

[ 2.02721440e+02 -2.90034569e+02 9.58460853e+02 2.27504397e-14

-5.14521364e+01]

[ 4.39660494e+01 1.48423743e+01 4.46661154e-14 1.98539147e+02

-2.17474905e+02]

[-4.50347880e-14 -4.04009189e+01 -5.14521364e+01 -2.17474905e+02

5.72094105e+02]]

Indices with max absolute value: i = 1 j = 2

Angular parameters: c = 0.9337328002507004 s = 0.3579707498329797

Check: c^2 + s^2 == 1.0

Delta = 4452138.9691125965

2 \* Omega = 358843.0308874026

Check delta = 0.0

Check 2 \* Omega = -189728.00000000093

Delta + 2 \* Omega = 4810981.999999999

Iteration: 5

A =

[[ 1.75178046e+03 5.21693038e+01 1.97108161e+02 4.39660494e+01

-6.17753989e-14]

[ 5.21693038e+01 2.01933146e+02 -4.77508944e-14 1.38588117e+01

-5.61420230e+01]

[ 1.97108161e+02 -5.34755999e-14 1.06965315e+03 -5.31313585e+00

-3.35802002e+01]

[ 4.39660494e+01 1.38588117e+01 -5.31313585e+00 1.98539147e+02

-2.17474905e+02]

[-4.50347880e-14 -5.61420230e+01 -3.35802002e+01 -2.17474905e+02

5.72094105e+02]]

Indices with max absolute value: i = 3 j = 4

Angular parameters: c = 0.9087177598294037 s = 0.4174111078668488

Check: c^2 + s^2 == 1.0

Delta = 4620379.072084397

2 \* Omega = 190602.92791560292

Check delta = 0.0

Check 2 \* Omega = -168240.10297179967

Delta + 2 \* Omega = 4810982.0

Iteration: 6

A =

[[ 1.75178046e+03 5.21693038e+01 1.97108161e+02 3.99527299e+01

-1.83519174e+01]

[ 5.21693038e+01 2.01933146e+02 -4.77508944e-14 -1.08405557e+01

-5.68020753e+01]

[ 1.97108161e+02 -5.34755999e-14 1.06965315e+03 -1.88448895e+01

-2.82971624e+01]

[ 3.99527299e+01 -1.08405557e+01 -1.88448895e+01 9.86440583e+01

5.47583827e-14]

[-1.83519174e+01 -5.68020753e+01 -2.82971624e+01 2.12821502e-14

6.71989193e+02]]

Indices with max absolute value: i = 0 j = 2

Angular parameters: c = 0.9658703351409492 s = 0.2590260521544324

Check: c^2 + s^2 == 1.0000000000000004

Delta = 4714969.740614537

2 \* Omega = 96012.25938546285

Check delta = 0.0

Check 2 \* Omega = -94590.66853014007

Delta + 2 \* Omega = 4810982.0

Iteration: 7

A =

[[ 1.80464071e+03 5.03887830e+01 5.17282795e-14 3.37078393e+01

-2.50552749e+01]

[ 5.03887830e+01 2.01933146e+02 -1.35132088e+01 -1.08405557e+01

-5.68020753e+01]

[ 8.88774253e-14 -1.35132088e+01 1.01679290e+03 -2.85505176e+01

-2.25777650e+01]

[ 3.37078393e+01 -1.08405557e+01 -2.85505176e+01 9.86440583e+01

5.47583827e-14]

[-2.50552749e+01 -5.68020753e+01 -2.25777650e+01 2.12821502e-14

6.71989193e+02]]

Indices with max absolute value: i = 4 j = 1

Angular parameters: c = 0.9929791109958173 s = -0.11828983526050145

Check: c^2 + s^2 == 1.0000000000000002

Delta = 4792672.99514241

2 \* Omega = 18309.004857592285

Check delta = 0.0

Check 2 \* Omega = -77703.25452787057

Delta + 2 \* Omega = 4810982.000000002

Iteration: 8

A =

[[ 1.80464071e+03 4.70712246e+01 5.17282795e-14 3.37078393e+01

-3.08398454e+01]

[ 4.70712246e+01 1.95166530e+02 -1.60890542e+01 -1.07644454e+01

-6.98905944e-14]

[ 8.88774253e-14 -1.60890542e+01 1.01679290e+03 -2.85505176e+01

-2.08207737e+01]

[ 3.37078393e+01 -1.07644454e+01 -2.85505176e+01 9.86440583e+01

1.28232755e+00]

[-3.08398454e+01 -7.63744201e-14 -2.08207737e+01 1.28232755e+00

6.78755809e+02]]

Indices with max absolute value: i = 0 j = 1

Angular parameters: c = 0.9995733291660824 s = 0.029208896244720253

Check: c^2 + s^2 == 1.0000000000000002

Delta = 4799125.946670248

2 \* Omega = 11856.05332975369

Check delta = 0.0

Check 2 \* Omega = -6452.951527838595

Delta + 2 \* Omega = 4810982.000000002

Iteration: 9

A =

[[ 1.80601619e+03 -5.59148565e-13 -4.69943514e-01 3.33790396e+01

-3.08266869e+01]

[-5.85085350e-13 1.93791045e+02 -1.60821894e+01 -1.17444213e+01

9.00797844e-01]

[-4.69943514e-01 -1.60821894e+01 1.01679290e+03 -2.85505176e+01

-2.08207737e+01]

[ 3.33790396e+01 -1.17444213e+01 -2.85505176e+01 9.86440583e+01

1.28232755e+00]

[-3.08266869e+01 9.00797844e-01 -2.08207737e+01 1.28232755e+00

6.78755809e+02]]

Indices with max absolute value: i = 3 j = 0

Angular parameters: c = 0.9998091003460464 s = -0.019538752908752465

Check: c^2 + s^2 == 1.0

Delta = 4803557.347034693

2 \* Omega = 7424.652965311892

Check delta = 0.0

Check 2 \* Omega = -4431.400364441797

Delta + 2 \* Omega = 4810982.000000005

Iteration: 10

A =

[[ 1.80666850e+03 -2.29471345e-01 -1.02769531e+00 7.38031839e-14

-3.07957470e+01]

[-2.29471345e-01 1.93791045e+02 -1.60821894e+01 -1.17421793e+01

9.00797844e-01]

[-1.02769531e+00 -1.60821894e+01 1.01679290e+03 -2.85358852e+01

-2.08207737e+01]

[ 8.33670648e-14 -1.17421793e+01 -2.85358852e+01 9.79917490e+01

1.88439777e+00]

[-3.07957470e+01 9.00797844e-01 -2.08207737e+01 1.88439777e+00

6.78755809e+02]]

Indices with max absolute value: i = 0 j = 4

Angular parameters: c = 0.9996280270423864 s = -0.02727283541082302

Check: c^2 + s^2 == 0.9999999999999999

Delta = 4805785.667604493

2 \* Omega = 5196.332395510748

Check delta = 0.0

Check 2 \* Omega = -2228.3205698011443

Delta + 2 \* Omega = 4810982.000000004

Iteration: 11

A =

[[ 1.80750870e+03 -2.53953300e-01 -4.59471500e-01 -5.13928703e-02

-5.08076533e-13]

[-2.53953300e-01 1.93791045e+02 -1.60821894e+01 -1.17421793e+01

8.94204437e-01]

[-4.59471500e-01 -1.60821894e+01 1.01679290e+03 -2.85358852e+01

-2.08410571e+01]

[-5.13928703e-02 -1.17421793e+01 -2.85358852e+01 9.79917490e+01

1.88369683e+00]

[-4.96867940e-13 8.94204437e-01 -2.08410571e+01 1.88369683e+00

6.77915609e+02]]

Indices with max absolute value: i = 2 j = 3

Angular parameters: c = 0.9995189839282224 s = -0.0310129129088848

Check: c^2 + s^2 == 1.0000000000000002

Delta = 4807682.423675697

2 \* Omega = 3299.576324304566

Check delta = 0.0

Check 2 \* Omega = -1896.7560712061822

Delta + 2 \* Omega = 4810982.000000002

Iteration: 12

A =

[[ 1.80750870e+03 -2.53953300e-01 -4.57656645e-01 -6.56176991e-02

-5.08076533e-13]

[-2.53953300e-01 1.93791045e+02 -1.57102945e+01 -1.22352866e+01

8.94204437e-01]

[-4.57656645e-01 -1.57102945e+01 1.01767830e+03 -1.93397403e-12

-2.08894512e+01]

[-6.56176991e-02 -1.22352866e+01 -1.91654726e-12 9.71063422e+01

1.23644885e+00]

[-4.96867940e-13 8.94204437e-01 -2.08894512e+01 1.23644885e+00

6.77915609e+02]]

Indices with max absolute value: i = 4 j = 2

Angular parameters: c = 0.9981293701571575 s = 0.061137226218366605

Check: c^2 + s^2 == 0.9999999999999998

Delta = 4809311.01716591

2 \* Omega = 1670.9828340923414

Check delta = 0.0

Check 2 \* Omega = -1628.5934902122244

Delta + 2 \* Omega = 4810982.000000003

Iteration: 13

A =

[[ 1.80750870e+03 -2.53953300e-01 -4.56800538e-01 -6.56176991e-02

-2.79798578e-02]

[-2.53953300e-01 1.93791045e+02 -1.57355755e+01 -1.22352866e+01

-6.79521148e-02]

[-4.56800538e-01 -1.57355755e+01 1.01895782e+03 -7.55930529e-02

-1.03715615e-13]

[-6.56176991e-02 -1.22352866e+01 -7.55930529e-02 9.71063422e+01

1.23413591e+00]

[-2.79798578e-02 -6.79521148e-02 -1.17943352e-13 1.23413591e+00

6.76636093e+02]]

Indices with max absolute value: i = 2 j = 1

Angular parameters: c = 0.9998183573749538 s = -0.019059177738008307

Check: c^2 + s^2 == 0.9999999999999999

Delta = 4810183.755508027

2 \* Omega = 798.2444919748232

Check delta = 0.0

Check 2 \* Omega = -872.7383421175182

Delta + 2 \* Omega = 4810982.000000002

Iteration: 14

A =

[[ 1.80750870e+03 -2.62613413e-01 -4.51877423e-01 -6.56176991e-02

-2.79798578e-02]

[-2.62613413e-01 1.93491083e+02 1.03790833e-12 -1.22345049e+01

-6.79397718e-02]

[-4.51877423e-01 1.03091857e-12 1.01925778e+03 1.57615181e-01

1.29511143e-03]

[-6.56176991e-02 -1.22345049e+01 1.57615181e-01 9.71063422e+01

1.23413591e+00]

[-2.79798578e-02 -6.79397718e-02 1.29511143e-03 1.23413591e+00

6.76636093e+02]]

Indices with max absolute value: i = 1 j = 3

Angular parameters: c = 0.9922837113963048 s = -0.12398804820455472

Check: c^2 + s^2 == 1.0

Delta = 4810678.972180537

2 \* Omega = 303.0278194639832

Check delta = 0.0

Check 2 \* Omega = -495.21667251084

Delta + 2 \* Omega = 4810982.000000001

Iteration: 15

A =

[[ 1.80750870e+03 -2.52451202e-01 -4.51877423e-01 -9.76722986e-02

-2.79798578e-02]

[-2.52451202e-01 1.95019811e+02 -1.95423986e-02 -5.72912860e-14

-2.20433632e-01]

[-4.51877423e-01 -1.95423986e-02 1.01925778e+03 1.56398976e-01

1.29511143e-03]

[-9.76722986e-02 -4.66829332e-14 1.56398976e-01 9.55776137e+01

1.21618924e+00]

[-2.79798578e-02 -2.20433632e-01 1.29511143e-03 1.21618924e+00

6.76636093e+02]]

Indices with max absolute value: i = 3 j = 4

Angular parameters: c = 0.9999978095799773 s = -0.002093044492503851

Check: c^2 + s^2 == 1.0000000000000002

Delta = 4810978.338401915

2 \* Omega = 3.6615980863571167

Check delta = 0.0

Check 2 \* Omega = -299.36622137762606

Delta + 2 \* Omega = 4810982.000000001

Iteration: 16

A =

[[ 1.80750870e+03 -2.52451202e-01 -4.51877423e-01 -9.76135215e-02

-2.81842290e-02]

[-2.52451202e-01 1.95019811e+02 -1.95423986e-02 4.61377398e-04

-2.20433149e-01]

[-4.51877423e-01 -1.95423986e-02 1.01925778e+03 1.56395923e-01

1.62245861e-03]

[-9.76135215e-02 4.61377398e-04 1.56395923e-01 9.55750681e+01

-4.94008085e-12]

[-2.81842290e-02 -2.20433149e-01 1.62245861e-03 -4.97530737e-12

6.76638638e+02]]

Indices with max absolute value: i = 0 j = 2

Angular parameters: c = 0.9999998356832164 s = -0.0005732656804221699

Check: c^2 + s^2 == 1.0

Delta = 4810981.296634454

2 \* Omega = 0.7033655475825071

Check delta = 0.0

Check 2 \* Omega = -2.9582325387746096

Delta + 2 \* Omega = 4810982.000000002

Iteration: 17

A =

[[ 1.80750896e+03 -2.52439958e-01 -2.31265161e-11 -9.77031619e-02

-2.81851545e-02]

[-2.52439958e-01 1.95019811e+02 -1.96871170e-02 4.61377398e-04

-2.20433149e-01]

[-2.30909027e-11 -1.96871170e-02 1.01925752e+03 1.56339939e-01

1.60630129e-03]

[-9.77031619e-02 4.61377398e-04 1.56339939e-01 9.55750681e+01

-4.94008085e-12]

[-2.81851545e-02 -2.20433149e-01 1.60630129e-03 -4.97530737e-12

6.76638638e+02]]

Indices with max absolute value: i = 1 j = 0

Angular parameters: c = 0.9999999877455853 s = 0.00015655295955241446

Check: c^2 + s^2 == 0.9999999999999999

Delta = 4810981.705020866

2 \* Omega = 0.29497913643717766

Check delta = 0.0

Check 2 \* Omega = -0.4083864111453295

Delta + 2 \* Omega = 4810982.000000003

Iteration: 18

A =

[[ 1.80750900e+03 1.70580065e-10 3.08205331e-06 -9.77032329e-02

-2.81506447e-02]

[ 1.70553617e-10 1.95019772e+02 -1.96871168e-02 4.46081674e-04

-2.20437558e-01]

[ 3.08205334e-06 -1.96871168e-02 1.01925752e+03 1.56339939e-01

1.60630129e-03]

[-9.77032329e-02 4.46081674e-04 1.56339939e-01 9.55750681e+01

-4.94008085e-12]

[-2.81506447e-02 -2.20437558e-01 1.60630129e-03 -4.97530737e-12

6.76638638e+02]]

Indices with max absolute value: i = 4 j = 1

Angular parameters: c = 0.9999998952548418 s = -0.0004577011091325725

Check: c^2 + s^2 == 0.9999999999999999

Delta = 4810981.8324727295

2 \* Omega = 0.1675272723659873

Check delta = 0.0

Check 2 \* Omega = -0.12745186407119036

Delta + 2 \* Omega = 4810982.000000002

Iteration: 19

A =

[[ 1.80750900e+03 -1.28844107e-05 3.08205331e-06 -9.77032329e-02

-2.81506417e-02]

[-1.28844107e-05 1.95019671e+02 -1.96863795e-02 4.46081627e-04

3.74427402e-11]

[ 3.08205334e-06 -1.96863795e-02 1.01925752e+03 1.56339939e-01

1.61531194e-03]

[-9.77032329e-02 4.46081627e-04 1.56339939e-01 9.55750681e+01

-2.04177017e-07]

[-2.81506417e-02 3.74404212e-11 1.61531194e-03 -2.04177052e-07

6.76638739e+02]]

Indices with max absolute value: i = 3 j = 2

Angular parameters: c = 0.9999999856759951 s = -0.00016925722851219552

Check: c^2 + s^2 == 0.9999999999999999

Delta = 4810981.929658163

2 \* Omega = 0.07034183759242296

Check delta = 0.0

Check 2 \* Omega = -0.09718543477356434

Delta + 2 \* Omega = 4810982.000000001

Iteration: 20

A =

[[ 1.80750900e+03 -1.28844107e-05 -1.34549252e-05 -9.77032321e-02

-2.81506417e-02]

[-1.28844107e-05 1.95019671e+02 -1.96863037e-02 4.49413682e-04

3.74427402e-11]

[-1.34549251e-05 -1.96863037e-02 1.01925755e+03 -1.21342035e-11

1.61531188e-03]

[-9.77032321e-02 4.49413682e-04 -1.21182240e-11 9.55750417e+01

-4.77580236e-07]

[-2.81506417e-02 3.74404212e-11 1.61531188e-03 -4.77580271e-07

6.76638739e+02]]

Indices with max absolute value: i = 0 j = 3

Angular parameters: c = 0.9999999983714023 s = -5.7071845638168827e-05

Check: c^2 + s^2 == 1.0000000000000002

Delta = 4810981.978542517

2 \* Omega = 0.021457483991980553

Check delta = 0.0

Check 2 \* Omega = -0.04888435360044241

Delta + 2 \* Omega = 4810982.000000001

Iteration: 21

A =

[[ 1.80750901e+03 -1.29100595e-05 -1.34549251e-05 -9.68509789e-10

-2.81506416e-02]

[-1.29100596e-05 1.95019671e+02 -1.96863037e-02 4.49412946e-04

3.74427402e-11]

[-1.34549251e-05 -1.96863037e-02 1.01925755e+03 -7.80031613e-10

1.61531188e-03]

[-9.68503321e-10 4.49412946e-04 -7.80015636e-10 9.55750361e+01

-2.08418931e-06]

[-2.81506416e-02 3.74404212e-11 1.61531188e-03 -2.08418935e-06

6.76638739e+02]]

Indices with max absolute value: i = 0 j = 4

Angular parameters: c = 0.9999999996901718 s = -2.4892899083642983e-05

Check: c^2 + s^2 == 1.0

Delta = 4810981.997634361

2 \* Omega = 0.0023656412959098816

Check delta = 0.0

Check 2 \* Omega = -0.01909184269607067

Delta + 2 \* Omega = 4810982.000000002

Iteration: 22

A =

[[ 1.80750901e+03 -1.29100595e-05 -1.34951349e-05 -9.16628274e-10

-2.18632874e-09]

[-1.29100596e-05 1.95019671e+02 -1.96863037e-02 4.49412946e-04

-2.83926070e-10]

[-1.34951349e-05 -1.96863037e-02 1.01925755e+03 -7.80031613e-10

1.61531155e-03]

[-9.16621806e-10 4.49412946e-04 -7.80015636e-10 9.55750361e+01

-2.08418934e-06]

[-2.18631532e-09 -2.83928388e-10 1.61531155e-03 -2.08418937e-06

6.76638738e+02]]

Indices with max absolute value: i = 2 j = 1

Angular parameters: c = 0.9999999997147713 s = -2.3884251270382814e-05

Check: c^2 + s^2 == 1.0

Delta = 4810981.999219279

2 \* Omega = 0.0007807239890098572

Check delta = 0.0

Check 2 \* Omega = -0.0015849173069000244

Delta + 2 \* Omega = 4810982.000000003

Answer:

Eigenvalues:[1807.50900619 195.01967057 1019.25754891 95.57503608 676.63873826]

Eigenvectors:

[[ 0.66401316 0.542621 0.23745611 -0.09040216 0.44731534]

[-0.66779667 0.63032902 0.24667846 -0.30783007 0.03351597]

[ 0.1038325 -0.391686 0.79648724 -0.40911244 -0.18448845]

[-0.23892253 -0.08107463 0.46833826 0.76672504 0.35935347]

[-0.21276503 -0.38504126 -0.17038689 -0.37660035 0.79725451]]

Residual vectors:

x[1807.5090061862575] = [ 7.22099230e-06 -2.85113799e-06 -1.39332273e-05 9.49522584e-06

2.05486276e-06]

x[195.0196705689863] = [-1.15955599e-04 -4.34565211e-05 2.07405799e-04 3.45728505e-04

1.55753462e-04]

x[1019.2575489102006] = [-0.00035264 -0.00062928 -0.00027844 -0.00060712 0.00128177]

x[95.57503607804681] = [-2.99674832e-04 2.84084531e-04 1.11206225e-04 -1.37553119e-04

1.34025976e-05]

x[676.6387382565093] = [ 0.00016819 -0.0006325 0.00128561 -0.00066245 -0.00029875]

------------------------------LU-algorithm------------------------------

Given matrix =

[[-973. -378. -195. 683. -487.]

[ 905. -779. -69. 676. 362.]

[ 707. 223. -868. 724. 135.]

[ 715. 630. 128. -917. 408.]

[ 308. 596. -572. -204. -54.]]

Interation 1

L =

[[ 1. 0. 0. 0. 0. ]

[-0.9301131 1. 0. 0. 0. ]

[-0.7266187 0.04569491 1. 0. 0. ]

[-0.73484075 -0.31154746 0.09346028 0.3904772 1. ]

[-0.31654677 -0.42132726 0.7405112 1. 0. ]]

U =

[[ -973. -378. -195. 683. -487. ]

[ 0. -1130.5828 -250.37206 1311.2672 -90.965065]

[ 0. 0. -998.24994 1160.3623 -214.70667 ]

[ 0. 0. 0. -294.5872 -87.49166 ]

[ 0. 0. 0. 0. 76.02267 ]]

Interation 5

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 3.5437319e-01 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-2.6155986e-02 2.0161691e-01 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.4123492e-03 -1.1550991e-02 3.2183249e-02 1.0000000e+00

0.0000000e+00]

[-5.3129584e-06 3.0160405e-05 -1.5869109e-04 -7.8109181e-03

1.0000000e+00]]

U =

[[-2358.0212 1411.4507 -1016.565 -662.3399 2111.484 ]

[ 0. -672.41943 -517.4674 -372.40277 316.85306 ]

[ 0. 0. -638.0721 115.256996 565.59344 ]

[ 0. 0. 0. -261.4954 -410.40768 ]

[ 0. 0. 0. 0. -92.95862 ]]

Interation 10

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.2401722e-02 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-1.5531597e-04 7.7693854e-03 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.5076127e-07 -6.8092913e-06 7.7953783e-04 1.0000000e+00

0.0000000e+00]

[-3.3842460e-12 1.1632193e-10 -2.2287054e-08 -4.8129477e-05

1.0000000e+00]]

U =

[[-1526.2444 955.05634 -1057.2661 -687.92914 2111.484 ]

[ 0. -1175.0046 -240.58696 -181.44923 -281.03314 ]

[ 0. 0. -553.3139 165.64787 606.2905 ]

[ 0. 0. 0. -261.50012 -427.96045 ]

[ 0. 0. 0. 0. -94.777245]]

Interation 15

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 8.0452003e-03 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-1.1357916e-06 1.4353648e-04 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 4.3627331e-11 -2.8796838e-09 1.8656001e-05 1.0000000e+00

0.0000000e+00]

[-3.6803912e-18 3.1311274e-16 -3.3311971e-12 -3.0081961e-07

1.0000000e+00]]

U =

[[-1455.0896 940.27124 -1058.2612 -688.0875 2111.484 ]

[ 0. -1235.2651 -176.814 -139.79195 -408.79907 ]

[ 0. 0. -551.8177 166.57736 608.627 ]

[ 0. 0. 0. -261.5823 -428.37796 ]

[ 0. 0. 0. 0. -94.788895]]

Interation 20

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 3.9623962e-03 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-9.5085104e-09 2.3443652e-06 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 8.7438152e-15 -1.1052595e-12 4.4667235e-07 1.0000000e+00

0.0000000e+00]

[-4.6086059e-24 7.5746399e-22 -4.9830911e-16 -1.8794806e-09

1.0000000e+00]]

U =

[[-1426.4469 940.0039 -1058.285 -688.08844 2111.484 ]

[ 0. -1260.1093 -148.89879 -121.63837 -464.50494]

[ 0. 0. -551.79425 166.59135 608.6766 ]

[ 0. 0. 0. -261.5848 -428.38797]

[ 0. 0. 0. 0. -94.78896]]

Interation 25

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 2.25754199e-03 1.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[-8.52046489e-11 3.60905439e-08 1.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 1.87598587e-18 -4.03148428e-16 1.06946745e-08 1.00000000e+00

0.00000000e+00]

[-6.17766104e-30 1.73479603e-27 -7.45423567e-20 -1.17425826e-11

1.00000000e+00]]

U =

[[-1411.5059 939.9996 -1058.2856 -688.08844 2111.484 ]

[ 0. -1273.4484 -133.8819 -111.87444 -494.4667 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 30

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 1.3955300e-03 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.9359056e-13 5.3685018e-10 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 4.1835139e-22 -1.4266303e-19 2.5606253e-10 1.0000000e+00

0.0000000e+00]

[-8.6071922e-36 3.8472740e-33 -1.1150833e-23 -7.3365068e-14

1.0000000e+00]]

U =

[[-1402.7131 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1281.4308 -124.894844 -106.031136 -512.3976 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 35

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 9.0690615e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.5699453e-15 7.8158565e-12 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 9.5546747e-26 -4.9522898e-23 6.1309035e-12 1.0000000e+00

0.0000000e+00]

[-1.2282381e-41 8.3606203e-39 -1.6680595e-27 -4.5836876e-16

1.0000000e+00]]

U =

[[-1397.1686 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1286.5161 -119.169556 -102.3086 -523.8206 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 40

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 6.08745730e-04 1.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[-7.33368471e-17 1.12196656e-13 1.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 2.21627577e-29 -1.69737414e-26 1.46792192e-13 1.00000000e+00

0.00000000e+00]

[ 0.00000000e+00 1.82168800e-44 -2.49525994e-31 -2.86378717e-18

1.00000000e+00]]

U =

[[-1393.5187 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1289.8857 -115.37576 -99.8419 -531.38995]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 45

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 4.1755592e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.1791264e-19 1.5953299e-15 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 5.1945940e-33 -5.7676354e-30 3.5146448e-15 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 -3.7326735e-35 -1.7892314e-20

1.0000000e+00]]

U =

[[-1391.0474 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1292.1772 -112.79585 -98.164474 -536.53735 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 50

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.9069203e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.0781829e-21 2.2535891e-17 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 1.2262533e-36 -1.9481574e-33 8.4151129e-17 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 -5.5837274e-39 -1.1178724e-22

1.0000000e+00]]

U =

[[-1389.3422 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1293.7632 -111.01039 -97.00358 -540.09973]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 55

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.0447001e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.0134392e-23 3.1689219e-19 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.9091657e-40 -6.5529088e-37 2.0148298e-18 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 -8.3517388e-43 -6.9842214e-25

1.0000000e+00]]

U =

[[-1388.1501 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1294.8743 -109.75967 -96.19037 -542.59515]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 60

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 1.4486843e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9736364e-25 4.4416812e-21 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 6.8663625e-44 -2.1976704e-40 4.8241060e-20 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -4.3635879e-27

1.0000000e+00]]

U =

[[-1387.3091 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1295.6593 -108.876076 -95.61586 -544.3581 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 65

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 1.0316848e-04 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9512659e-27 6.2113961e-23 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 -7.2867520e-44 1.1550354e-21 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -2.7262739e-29

1.0000000e+00]]

U =

[[-1386.712 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1296.217 -108.24811 -95.20756 -545.611 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 70

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 7.3740608e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9412062e-29 8.6720653e-25 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 2.7655005e-23 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -1.7033159e-31

1.0000000e+00]]

U =

[[-1386.2864 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1296.6151 -107.79991 -94.916145 -546.50525 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 75

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 5.28445235e-05 1.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[-6.93991082e-31 1.20934195e-26 1.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 6.62143664e-25 1.00000000e+00

0.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 -1.06419432e-33

1.00000000e+00]]

U =

[[-1385.9817 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1296.9001 -107.47906 -94.70753 -547.1454 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 80

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 3.7940736e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9448934e-33 1.6850476e-28 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 1.5853704e-26 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -6.6488519e-36

1.0000000e+00]]

U =

[[-1385.7632 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.1046 -107.24886 -94.55785 -547.6047 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 85

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 2.7276859e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9543910e-35 2.3464667e-30 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 3.7958510e-28 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -4.1540568e-38

1.0000000e+00]]

U =

[[-1385.6063 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.2515 -107.083466 -94.45031 -547.93475 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 90

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 1.9629177e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9671506e-37 3.2660964e-32 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 9.0884040e-30 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -2.5953589e-40

1.0000000e+00]]

U =

[[-1385.4935 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.357 -106.964485 -94.372955 -548.1721 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 95

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 1.4135506e-05 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-6.9822793e-39 4.5447339e-34 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 2.1760359e-31 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 -1.6213023e-42

1.0000000e+00]]

U =

[[-1385.4122 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.4332 -106.878815 -94.31726 -548.343 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 100

L =

[[ 1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 1.01844535e-05 1.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[-6.99920557e-41 6.32252737e-36 1.00000000e+00 0.00000000e+00

0.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 5.21008116e-33 1.00000000e+00

0.00000000e+00]

[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 -9.80908925e-45

1.00000000e+00]]

U =

[[-1385.3538 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.4879 -106.81711 -94.27714 -548.4661 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 105

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 7.3404071e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.0205053e-43 8.7943373e-38 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 1.2474495e-34 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

1.0000000e+00]]

U =

[[-1385.3118 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.5273 -106.772644 -94.24822 -548.5548 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 110

L =

[[ 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 5.2919463e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00

0.0000000e+00]

[-7.0064923e-45 1.2231080e-39 1.0000000e+00 0.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 2.9867675e-36 1.0000000e+00

0.0000000e+00]

[ 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00

1.0000000e+00]]

U =

[[-1385.2812 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.5559 -106.740585 -94.22738 -548.6188 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 115

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[3.8158578e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 1.7008961e-41 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 7.1512156e-38 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2594 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.5763 -106.717476 -94.21236 -548.66486 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 120

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[2.7518668e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 2.3681944e-43 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.7122144e-39 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2435 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.5912 -106.700806 -94.20152 -548.6981 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 125

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[1.9847466e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 4.2038954e-45 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 4.0994987e-41 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2322 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6018 -106.68878 -94.19371 -548.7221 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 130

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[1.4315709e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 9.8231022e-43 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.224 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6095 -106.68011 -94.18807 -548.7394 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 135

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[1.0326249e-06 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 2.3822074e-44 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2181 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.615 -106.67386 -94.18401 -548.75183]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 140

L =

[[1.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00]

[7.44883e-07 1.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00]

[0.00000e+00 0.00000e+00 1.00000e+00 0.00000e+00 0.00000e+00]

[0.00000e+00 0.00000e+00 0.00000e+00 1.00000e+00 0.00000e+00]

[0.00000e+00 0.00000e+00 0.00000e+00 0.00000e+00 1.00000e+00]]

U =

[[-1385.2139 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.619 -106.66935 -94.181076 -548.7608 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 145

L =

[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[5.373348e-07 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[-1385.2108 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6218 -106.666084 -94.17896 -548.76733 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 150

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[3.8762343e-07 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2086 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6239 -106.66374 -94.17744 -548.7719 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 155

L =

[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[2.796282e-07 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[-1385.207 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6254 -106.662056 -94.17633 -548.7753 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 160

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[2.0172335e-07 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2058 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6266 -106.660835 -94.17554 -548.7777 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 165

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[1.4552394e-07 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2051 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6272 -106.65995 -94.174965 -548.7795 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 170

L =

[[1.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[1.04981964e-07 1.00000000e+00 0.00000000e+00 0.00000000e+00

0.00000000e+00]

[0.00000000e+00 0.00000000e+00 1.00000000e+00 0.00000000e+00

0.00000000e+00]

[0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00

0.00000000e+00]

[0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00

1.00000000e+00]]

U =

[[-1385.2045 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6278 -106.6593 -94.17455 -548.78076]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 175

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[7.5735066e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2039 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6284 -106.65884 -94.174255 -548.7817 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 180

L =

[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[5.463626e-08 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[-1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6287 -106.6585 -94.17404 -548.7823 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 185

L =

[[1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[3.941533e-08 1.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00 0.000000e+00]

[0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 1.000000e+00]]

U =

[[-1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6287 -106.658264 -94.17388 -548.7828 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 190

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[2.8434743e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6287 -106.6581 -94.17377 -548.7831 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 195

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[2.0513202e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6287 -106.657974 -94.17369 -548.7834 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818 ]

[ 0. 0. 0. 0. -94.78896 ]]

Interation 200

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[1.4798499e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6287 -106.65789 -94.17363 -548.7837 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Interation 205

L =

[[1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[1.0675836e-08 1.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00 0.0000000e+00]

[0.0000000e+00 0.0000000e+00 0.0000000e+00 0.0000000e+00 1.0000000e+00]]

U =

[[-1385.2035 939.9995 -1058.2856 -688.08844 2111.484 ]

[ 0. -1297.6287 -106.65782 -94.17359 -548.7837 ]

[ 0. 0. -551.79395 166.59155 608.6774 ]

[ 0. 0. 0. -261.58487 -428.38818]

[ 0. 0. 0. 0. -94.78896]]

Answer:

Eigenvalues: [ -94.78896 -261.58487 -551.79395 -1297.6287 -1385.2035 ]

Eigenvectors:

[[-0.4128942 -0.01850755 -0.1918212 0.03821078 0.88933706]

[ 0.3987725 -0.26899958 -0.22199246 -0.34829584 -0.7733234 ]

[-0.45127386 0.2880226 0.42270043 0.4257158 0.59454626]

[ 0.69283634 -0.43206507 0.06005434 -0.5733523 -0.03095425]

[ 0.70856494 -0.27461526 0.06993608 -0.6370592 -0.10856643]]

Residual vectors

x[-94.78896] = [ 1.9073486e-05 -2.3365021e-05 -2.2888184e-05 -3.4093857e-05

-3.8146973e-05]

x[-261.58487] = [ 1.5258789e-05 7.6293945e-06 -7.6293945e-06 -3.8146973e-05

-1.5258789e-05]

x[-551.79395] = [ 1.5258789e-05 -3.0517578e-05 -6.1035156e-05 0.0000000e+00

-6.1035156e-05]

x[-1297.6287] = [-3.6621094e-04 2.4414062e-04 -1.5258789e-05 3.0517578e-04

1.9073486e-05]

x[-1385.2035] = [ 4.2724609e-04 -9.1552734e-05 9.1552734e-05 -3.0517578e-04

-6.1035156e-05]

**Висновок**

Під час лабораторної роботи були набуті та закріпленні вміння та досвід використання методів розв’язання часткової та повної проблеми власних значень.

В ході роботи було реалізовано три методи: степеневий метод, метод Якобі та LR-метод.

За допомогою степеневого методу було знайдене найбільше по модулю власне значення заданої матриці та відповідний цьому значенню вектор. Для цього знадобилось 21 ітерацій. Отриманий вектор нев’язки задовольняє задану точність: .

За допомогою методу Якобі було знайдено всі власні значення матриці та відповідні власні вектори. Для цього знадобилось 22 ітерацій. Отримані вектор нев’язки задовольняють задану точність: .

Для несиметричної матриці був використаний LR-метод. З його допомогою було знайдено всі власні значення заданої матриці. Для цього знадобилось 205 ітерацій.