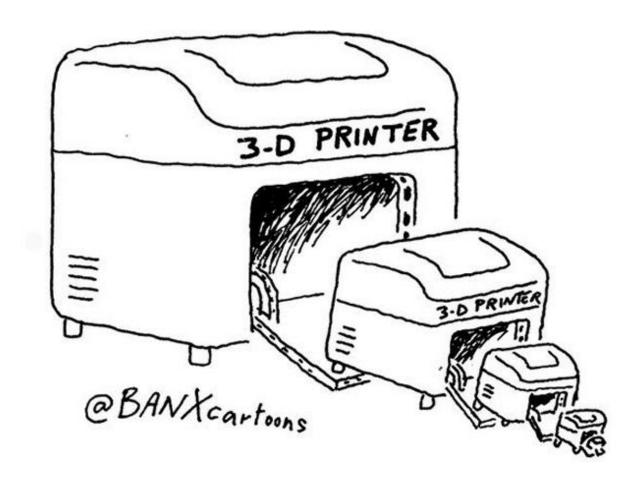
CS100 Introduction to Programming

Lecture 7. Recursion

What Is Recursion?

- The method in which a problem is solved by reducing it to smaller cases of the same problem.
- Recursion is the name for the case when
 - a function calls itself, or
 - calls a sequence of other functions, one of which eventually calls the first function again.
- Two parts
 - Base case (with terminating condition)
 - Recursive case (with recursive condition)







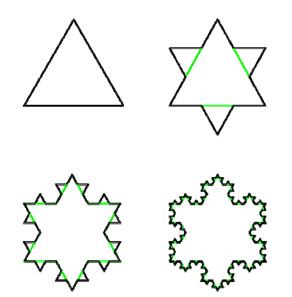
Life Mirror Recursive (1909), by Coles Phillips (1880 – 1927)

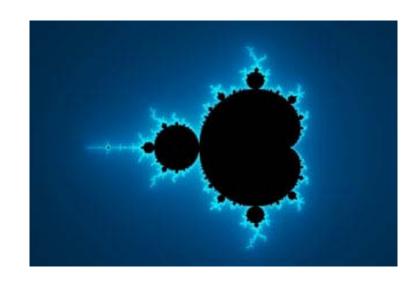
FROM THE MIRROR.



Recursion is similar to fractals

- Fractals tend to appear nearly the same at different levels
 - Can be defined recursively





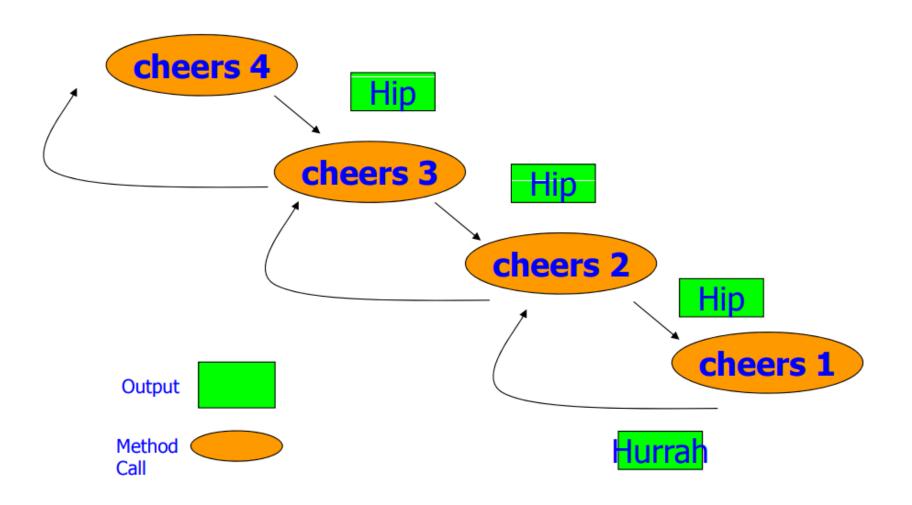
Example 1: Cheers

What does the following recursive method print when called with cheers(4)?

```
void cheers(int n)
{
    if (n <= 1)
        printf("Hurrah \n");
    else {
        printf("Hip \n");
        cheers(n-1);
    }
}</pre>
```

Output: Hip Hip Hip Hurrah

Recursive Cheers – Tracing

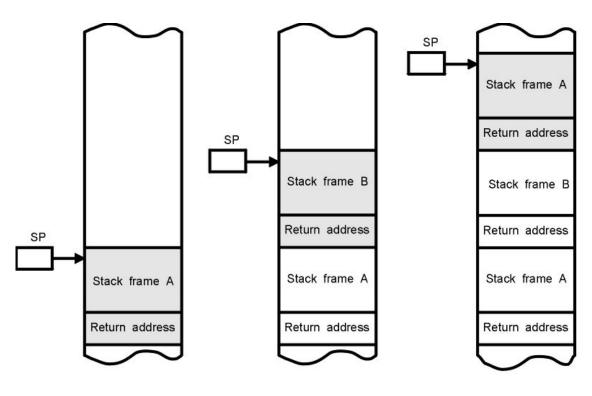


Recursive Cheers

```
void cheers(int n)
{
   if (n <= 1)
      printf("Hurrah \n"); recursive condition
   else {
        printf("Hip \n");
      cheers(n-1);
   }
}</pre>
```

How Recursive Function is Called?

Recall function stack



b. The state of the stack during subroutine B

c. The state of the stack during a second call to subroutine A

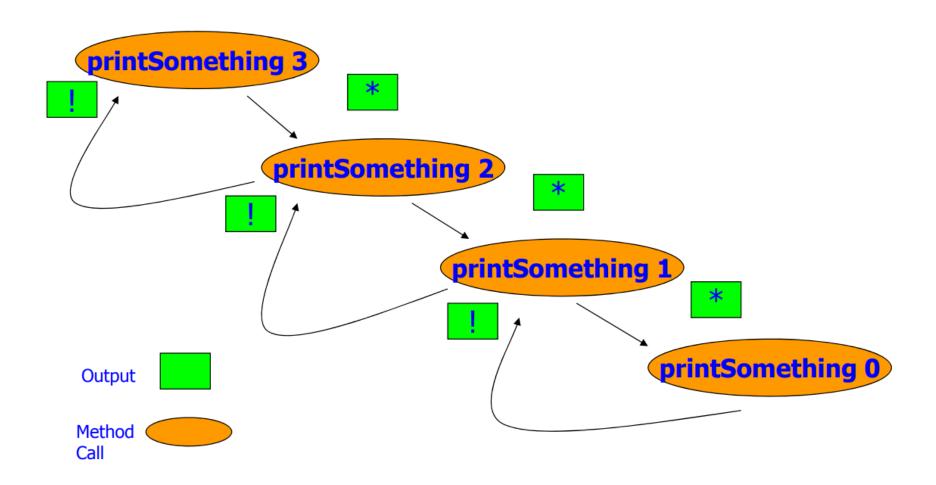
Example 2: PrintSomething

What does the following **recursive** method print when called with printSomething(3)?

```
void printSomething(int n)
{
    if (n > 0) {
        printf("*");
        printSomething(n-1);
        printf("!");
    }
}
What condition is this?
Continue looping condition

Output:
    ***!!!
    ***!!!
}
Where is the other condition?
```

Recursive PrintSomething – Tracing



Example 3: Factorials

- **Problem**: Find the factorial of a non-negative integer number.
- The factorial function of a positive integer is usually defined by the formula

$$n! = n \times (n - 1) \times \dots \times 1$$

• A more precise definition (recursive definition)

$$n! = 1$$
 if $n = 0$
 $n! = n \times (n - 1)!$ if $n > 0$

Non-recursive Factorials

The following program will do the job, using a

for loop:

```
int fact1(int n)
{
    int i;
    int temp = 1;
    for (i=n; i > 0; i--)
        temp *= i;
    return temp;
}
```

```
Output:
Enter an integer: <u>4</u>
n! = 24
```

Recursive Factorials

Suppose we wish to calculate 4!

As
$$4 > 0$$
, $4! = 4 \times 3!$

But we do not know what is 3!

As
$$3 > 0$$
, $3! = 3 \times 2!$
As $2 > 0$, $2! = 2 \times 1!$
As $1 > 0$, $1! = 1 \times 0!$

What is 0! equal to?

$$4! = 4 \times 3!$$

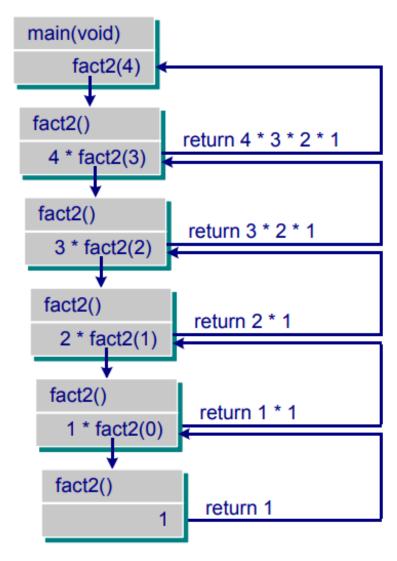
 $= 4 \times (3 \times 2!)$
 $= 4 \times (3 \times (2 \times 1!))$
 $= 4 \times (3 \times (2 \times (1 \times 0!)))$
 $= 4 \times (3 \times (2 \times (1 \times 1)))$
 $= 4 \times (3 \times (2 \times 1))$
 $= 4 \times (3 \times 2)$
 $= 4 \times 6$
 $= 24$

Recursive Factorials

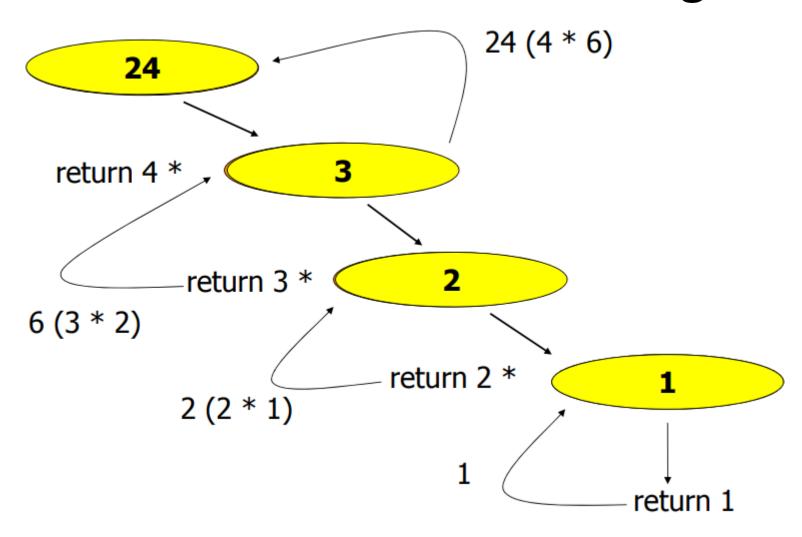
```
int fact2(int n)
{
   if (n == 0) {
     // terminating condition
     return 1;
   } else {
     // recursive condition
     return n*fact2(n-1);
   }
}
```

Output:

Enter an integer: 4 n! = 24



Recursive Factorials: Tracing



Example 4: Multiplication by Addition

The multiplication operation

$$multi(m, n) = m \times n$$

can be defined **recursively** as

```
multi(m, n) = m if n = 1

multi(m, n) = m + multi(m, n-1) if n > 1
```

Using Call by Value

```
/* multiplication by addition, pass parameter using
call by value */
#include <stdio.h>
int multi1(int, int);
int main(void)
   printf("5 * 3 = %d\n", multi1(5, 3));
   return 0;
                                              Output:
                                             5 * 3 = 15
int multi1(int m, int n)
   if (n == 1)
                          // terminating condition
      return m;
   else
                          // recursive condition
      return m + multi1(m, n-1);
```

Using Call by Pointer

```
/* multiplication by addition, pass parameter using call by
pointer */
#include <stdio.h>
void multi2(int, int, int*);
int main(void)
   int result;
   multi2(5, 3, &result);
   printf("5 * 3 = %d\n", result);
                                                  Output:
   return 0;
                                                  5 * 3 = 15
void multi2(int m, int n, int *product)
   if (n == 1)
                             // terminating condition
       *product = m;
   else {
                             // recursive condition
       multi2(m, n-1, product);
       *product = *product + m;
```

Recursive Call by Pointer

```
#include <stdio.h>
void fn(int x, int y, int *z);
int main(void){
    int n1=20, n2=15, n3=0;
    fn(n1, n2, &n3);
    pritnf("n1 = %d, n2 = %d, n3 = %d\n", n1, n2, n3);
    return 0;
                                       Output:
                                       x = 10, y = 10, *z = 100
void fn(int x, int y, int *z){
                                       x = 12, y = 11, *z = 100
    if (x > y) {
                                       x = 14, y = 12, *z = 100
       x = x - 2;
                                       x = 16, y = 13, z = 100
       y = y - 1;
                                       x = 18, y = 14, *z = 100
       *z = x * y;
                                       n1 = 20, n2 = 15, n3 = 100
       fn(x, y, z);
    printf("x = %d, y = %d, *z = %d\n", x, y, *z);
```

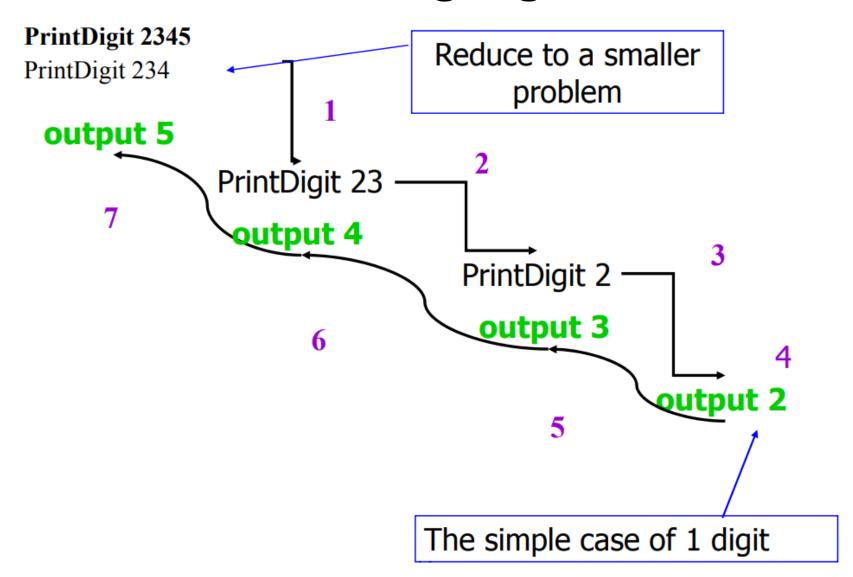
Example 5: Printing Digits

• **Problem**: Given a number, say 2345, print each digit of the number, in the order from left to right, one per line.

Recursive Solution:

- Look for the <u>simplest case</u> (terminating condition). If the number is a single digit, just print that digit.
- Look into <u>reducing</u> the problem into a <u>smaller</u> but same problem (<u>recursive condition</u>).

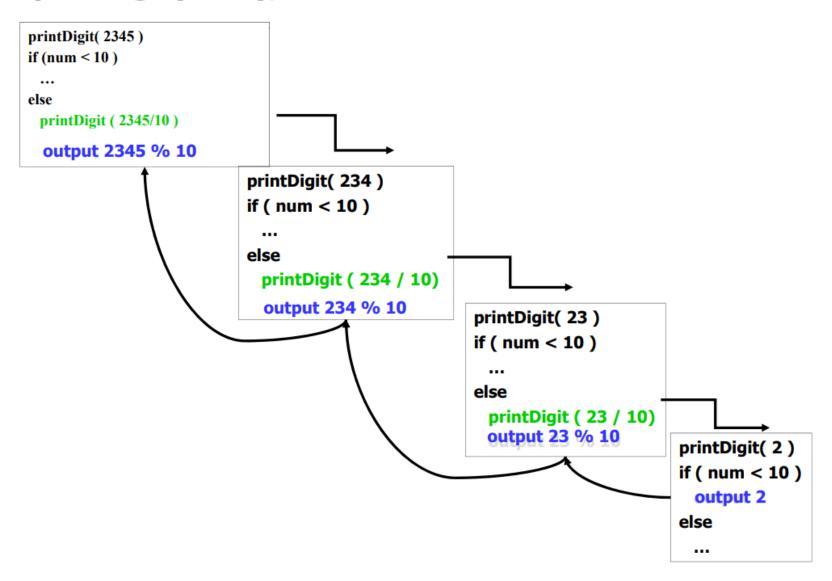
Printing Digits



Printing Digits: Recursive Solution

```
#include <stdio.h>
                                            Output:
void printDigit(int);
                                            Enter a number: <u>2345</u>
int main(void) {
                                            2
   int num;
                                            3
   printf("Enter a number: ");
   scanf("%d", &num);
                                            4
   printDigit(num);
                                            5
   return 0;
void printDigit(int num) {
   if (num < 10)
                              // terminating condition
       printf("%d\n", num);
                              // recursive condition
   else {
       printDigit(num/10);
       printf("%d\n", num%10);
```

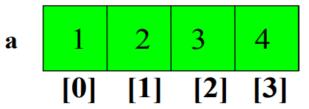
printDigit(2345);



Example 6: Summing Array of Integers

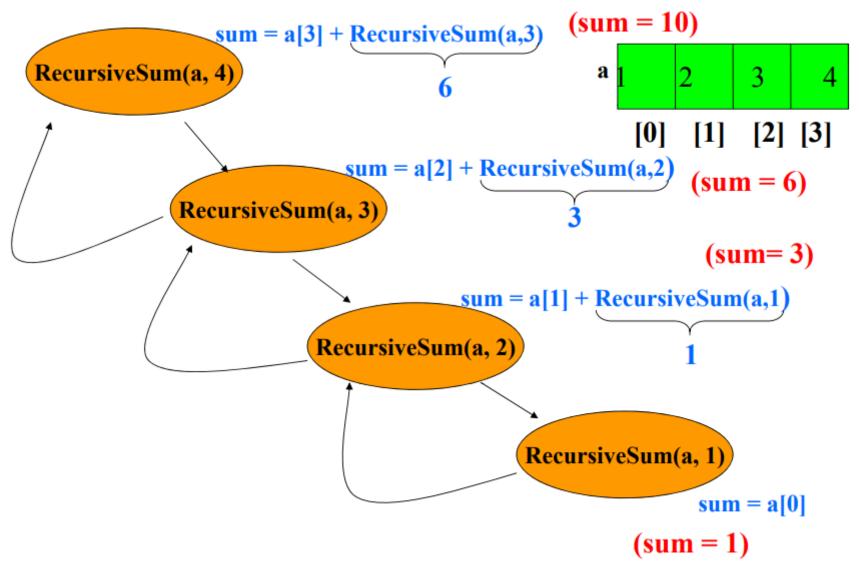
```
int sumArray(int a[], int size)
   int sum = 0;
   for (int i = 0; i < size; i++)
       sum = sum + a[i];
   return sum;
int recursiveSum(int a[], int size)
   if (size == 1)
       return a[0];
   else
       return a[size - 1] +
           recursiveSum(a, size - 1);
```

 Given an array of integers, write a method to calculate the sum of all the integers.



```
Input:
a[] = {1,2,3,4}
Output:
Sum = 10
```

Recursive Sum – Tracing



Recursion: Summary

- To obtain the answer to a larger problem, a general method is used that
 - reduces the large problem to one or more problems of a similar nature but a smaller size.
- Recursion continues until the size of the problem is reduced to the smallest, i.e. base case
 - where the solution is given directly without using further recursion.
- As such, recursive methods consist of two parts:
 - A smallest, base case that is processed without recursion (terminating condition).
 - A general method that reduces a particular case to one or more of the smaller cases (recursive condition).

Recursion: Summary

- Each function makes a call to itself with an argument which is <u>closer</u> to the terminating condition.
- Each call to a function has its own set of values/arguments for the formal arguments and local variables.
- When a recursive call is made, control is transferred from the calling point to the first statement of the recursive function. When a call at a certain level is finished, control returns to the calling point one level up.

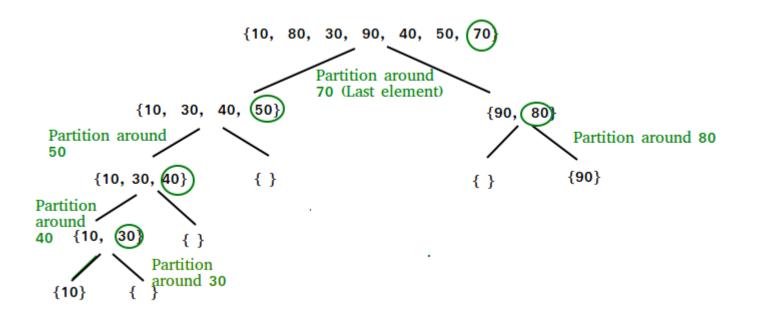
Sorting a Sequence of Data

Quick sort

- a divide-and-conquer algorithm
- pick an element as pivot; partition the data sequence into two sequences around the pivot
 - The left sequence contains element smaller than pivot
 - The right sequence contains element larger than pivot
- This partition is recursively applied to the sequence until no partition can be done

Sorting a Sequence of Data

An example of quick sort



The algorithm

Key part: partition of the data sequence

```
partition(arr[], low, high)
    // pivot (Element to be placed at right position)
    pivot = arr[high];
    i = (low - 1) // Index of smaller element
    for (j = low; j <= high - 1; j++)
         // If current element is smaller than the pivot
         if (arr[j] < pivot)</pre>
              i++; // increment index of smaller element
              if(i != j) swap arr[i] and arr[j]
    swap arr[i + 1] and arr[high])
    return (i + 1)
```

The algorithm

Key part: partition of the data sequence

The algorithm

Key part: partition of the data sequence

```
j = 2 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])</pre>
i = 1
arr[] = \{10, 30, 80, 90, 40, 50, 70\} // We swap 80 and 30
j = 3 : Since arr[j] > pivot, do nothing
// No change in i and arr[]
j = 4 : Since arr[j] <= pivot, do i++ and swap(arr[i], arr[j])</pre>
i = 2
arr[] = \{10, 30, 40, 90, 80, 50, 70\} // 80  and 40 Swapped
j = 5 : Since arr[j] <= pivot, do i++ and swap arr[i] with arr[j]</pre>
i = 3
arr[] = \{10, 30, 40, 50, 80, 90, 70\} // 90  and 50 Swapped
We come out of loop because j is now equal to high-1.
Finally we place pivot at correct position by swapping
arr[i+1] and arr[high] (or pivot)
arr[] = \{10, 30, 40, 50, 70, 90, 80\} // 80 and 70 Swapped
```

The algorithm

– The recursive part

```
quickSort(arr[], low, high)
                                              {10, 30, 40}
{
                                         40 {10, (30)
    if (low < high)
        /* pi is partitioning index, arr[pi] is now
           at right place */
        pi = partition(arr, low, high);
        quickSort(arr, low, pi - 1); // Before pi
        quickSort(arr, pi + 1, high); // After pi
```

```
{10, 80, 30, 90, 40, 50, 70}

Partition around

Partition around

{10, 30, 40, 50}

Partition around

{10, 30, 40}

{10, 30, 40}

Partition around 80

{10, 30, 40}

Partition around 30

{10}

Partition around 30
```

- Constructing a dynamic array
 - Declaring dynamic array structure

```
struct ARRAY
{
    int size;
    int count;
    float* data;
};
```

Note that the size of the array (for storage) could be larger than the real number (count) of the array

- Constructing a dynamic array
 - What functions we need (by design)?

```
bool create_array(int, ARRAY*);
void destroy_array(ARRAY*);

bool array_add_element(ARRAY*, float);
bool array_remove_element(ARRAY*, int);

int get_array_count(const ARRAY*);
int get_array_size(const ARRAY*);

float& get_array_element(int, const ARRAY*);
void set_array_element(int, float, ARRAY*);

void print_array(const ARRAY*);
```

- Constructing a dynamic array
 - Create the array

```
bool create_array(int size, ARRAY* p_array_out)
{
    if (size <= 0)
        return false;

    if (p_array_out != NULL)
    {
        p_array_out->data = (float*)malloc(sizeof(float) * size);
        if (p_array_out->data == NULL)
            return false;

        p_array_out->size = size;
        p_array_out->count = 0;

        return true;
    }
    else
        return true;
}
```

- Constructing a dynamic array
 - Destroy the array

- Constructing a dynamic array
 - Add array element (dynamically)

```
bool array add element(ARRAY* p array, float element value)
    if (p_array == NULL || p_array->data == NULL
                   || p_array->count < 0 || p_array->size <= 0)</pre>
         return false;
    if (p array->count + 1 <= p array->size)
         p array->count++;
         p array->data[p array->count - 1] = element value;
         return true;
    else
    return true;
```

- Constructing a dynamic array
 - Add array element (dynamically)

```
bool array add element(ARRAY* p array, float element value)
     else
          float* data prev = p array->data;
          p array->data = (float*)realloc(data prev, p array->size
                                                    + ARRAY INCREMENT SIZE);
          if (p array->data == NULL)
                    return false;
          p array->size += ARRAY INCREMENT SIZE;
          p array->count++;
          p array->data[p array->count - 1] = element value;
          return true;
     return true;
}
```

- Constructing a dynamic array
 - Remove array element

```
bool array remove element(ARRAY* p array, int i)
     if (i < 0 \mid \mid i >= p_array->count)
          return false;
     int rest count = p array->count - i - 1;
     int copy size = sizeof(float) * rest count;
     float* p temp data = (float*)malloc(copy size);
     if (p temp data == NULL)
          return false;
     memcpy(p temp data, &p array->data[i + 1], copy size);
     memcpy(&p array->data[i], p temp data, copy size);
     p array->data[p array->count - 1] = 0.0f;
     p array->count--;
```

- Constructing a dynamic array
 - Remove array element

- Constructing a dynamic array
 - Remove array element

```
bool array remove element2(ARRAY* p array, int i)
     if (i < 0 \mid | i >= p array -> count)
          return false:
     for (int j = i; j < p_array->count - 1; j++)
          p_array->data[j] = p_array->data[j + 1];
     p array->data[p array->count - 1] = 0.0f;
     p array->count--;
     if (p array->count size - 2 * ARRAY INCREMENT SIZE)
          float* p temp data = p array->data;
          p array->data = (float*)realloc(
                    p temp data, p array->size - ARRAY INCREMENT SIZE);
          if (p array->data == NULL)
               return false;
     return true;
```

- Constructing a dynamic array
 - Other array functions

```
int get array count(const ARRAY* p array)
      return p_array->count;
int get array size(const ARRAY* p array)
      return p array->size;
float& get array element(int i, const ARRAY* p array)
      return p_array->data[i];
void set array element(int i, float data value, ARRAY* p array)
      p_array->data[i] = data_value;
```

- Constructing a dynamic array
 - Print(display) the array on the screen

Implementing quicksort with dynamic array

Partition function

return i+1;

```
void swap_data(float* a, float* b)
{
    float t = *a;
    *a = *b;
    *b = t;
}
```

- Implementing quicksort with dynamic array
 - Recursive function call for partitioning

How to use the quicksort for dynamic array?

```
void test array sort()
    ARRAY a;
    if (!create_array(20, &a))
         printf("unable to create the array!\n");
         return;
    printf("Please input the array elements:\n");
    while (1)
         char str[32];
         scanf("%s", str);
         if (strcmp(str, "quit") == 0)
         break;
         array add element(&a, (float)atof(str));
```

How to use the quicksort for dynamic array?

```
void test_array_sort()
{
    ...
    printf("Your input ARRAY is:\n");
    print_array(&a);

    printf("\nSorting the whole array...\n");
    quick_sort_array(&a, 0, get_array_count(&a) - 1);
    printf("Your sorted array is:\n");
    print_array(&a);

    destroy_array(&a);//don't forget to destroy the array
}
```

- Constructing a double linked list
 - Declaring double linked list structure

```
struct LIST
                        struct LIST NODE
struct LIST DATA
                                                      LIST NODE* p head = NULL;
     float data;
                             LIST DATA data;
                                                      LIST NODE* p tail = NULL;
                             LIST NODE* prev;
};
                                                      LIST NODE* p current = NULL;
                             LIST NODE* next;
                                                      int count = 0:
                        };
                                                 };
        frontPtr
                                         prev node pointer
                                  data
                                                              next node pointer
                                                    List node
                 One-node List
         backPtr
                                                                                52
```

- Constructing a double linked list
 - What functions we need?

```
bool add_list_head_element(LIST*, const LIST_DATA&);
bool remove_list_head_element(LIST*);

bool add_list_tail_element(LIST*, const LIST_DATA&);
bool remove_list_tail_element(LIST*);

bool add_current_list_element_before(LIST*, const LIST_DATA&);
bool add_current_list_element_after(LIST*, const LIST_DATA&);
bool add_list_element_before(LIST*, LIST_NODE*, const LIST_DATA&);
bool add_list_element_after(LIST*, LIST_NODE*, const LIST_DATA&);
bool remove_current_list_element(LIST*);
bool remove_list_element(LIST*, LIST_NODE*);
```

- Constructing a double linked list
 - What functions we need?

```
bool move_list_pointer_to(LIST*, int);
bool destroy_list(LIST*);
int get_list_count(LIST*);
LIST_NODE* get_list_head(LIST*);
LIST_NODE* get_list_tail(LIST*);
LIST_DATA& get_list_element(const_LIST_NODE*);
void set_list_element(LIST_NODE*, const_LIST_DATA&);
void print_list(LIST*);
```

- Constructing a double linked list
 - Add list element from tail

```
bool add list tail element(LIST* p list, const LIST DATA& data)
     if (p list == NULL)
          return false;
     LIST NODE* p node = (LIST NODE*)malloc(sizeof(LIST NODE));
     if (p node == NULL)
          return false;
     p node->data = data;
                                                   frontPtr
                                                             Double Linked List
                                                                            backPtr
     if (p list->p tail == NULL)
          p node->prev = p node->next = NULL;
          p list->p tail = p node;
          p list->p head = p list->p tail;
     else
                                                                                       55
```

- Constructing a double linked list
 - Add list element from tail

- Constructing a double linked list
 - Remove list element from tail

```
bool remove list_tail_element(LIST* p_list)
     if (p list == NULL)
          return false;
    LIST_NODE* p_node = p_list->p tail;
     if (p node == NULL)
          return false;
     p list->p tail = p list->p tail->prev;
     p list->p tail->next = NULL;
                                               frontPtr
                                                       Double Linked List
                                                                      backPtr
    free(p node);
     p list->count--;
     p list->p current = p_list->p_tail;
    return true;
                                                                            57
```

- Constructing a double linked list
 - Remove list element that is currently operating

```
bool remove_current_list_element(LIST* p_list)
{
    if (p_list == NULL)
        return false;

LIST_NODE* p_node = p_list->p_current;

    p_node->prev->next = p_node->next;
    p_node->next->prev = p_node->prev;

    p_list->count--;
    p_list->p_current = p_node->next;

    free(p_node);

    return true;
}
```

- Constructing a double linked list
 - Destroy the whole list

```
bool destroy list(LIST* p list)
    if (p list == NULL)
        return false;
    LIST_NODE* p_node = p_list->p_head;
    while (p_node != NULL)
        LIST NODE* p node temp = p node;
        p_node = p_node->next;
        free(p_node_temp);
    return true;
```

- Constructing a double linked list
 - Other list functions

```
int get_list_count(LIST* p_list)
{
         return p_list->count;
}

LIST_NODE* get_list_head(LIST* p_list)
{
         return p_list->p_head;
}

LIST_NODE* get_list_tail(LIST* p_list)
{
         return p_list->p_tail;
}
```

- Constructing a double linked list
 - Print(display) the array on the screen

```
void print_list(LIST* p_list)
{
    if (p_list == NULL)
        return;

LIST_NODE* p_node = p_list->p_head;

    while (p_node != NULL)
    {
        printf("%f, ", p_node->data.data);
        p_node = p_node->next;
    }
}
```

- Implementing quicksort with linked list
 - Auxiliary list node array

- Implementing quicksort with linked list
 - Partition function

```
int quick_sort_list_partition(LIST_NODE** p_list_node_array, int low, int high)
    float pivot = p list node array[high]->data.data;
    int i = low - 1;
    for (int j = low; j < high; j++)</pre>
         if (p_list_node_array[j]->data.data < pivot)</pre>
              i++;
              if (i != j) swap_data(&p_list_node_array[i]->data.data,
                                      &p list node array[j]->data.data);
    swap_data(&p_list_node_array[i+1]->data.data,
                             &p list node array[high]->data.data);
    return i+1;
```

- Implementing quicksort with linked list
 - Recursive function call for partitioning

- Implementing quicksort with linked list
 - Recursive function call for partitioning

```
bool quick_sort_list(LIST* p_list_in_out, int low, int high)
{
    if (low >= high)
        return false;

    LIST_NODE** p_list_node_array =
        (LIST_NODE**)malloc(sizeof(LIST_NODE*) * p_list_in_out->count);
    if (p_list_node_array == NULL)
        return false;
    ... //previous construction of list node array

    quick_sort_list_with_array(p_list_node_array, low, high);
    free(p_list_node_array);
    return true;
}
```

How to use the quicksort for linked list?

```
void test list sort()
    LIST 1;
    printf("Please input the list elements:\n");
    while (1)
         char str[32];
         scanf("%s", str);
         if (strcmp(str, "quit") == 0)
         break;
         LIST_DATA data;
         data.data = (float)atof(str);
         add_list_tail_element(&l, data);
```

How to use the quicksort for linked list?

```
void test_list_sort()
{
    ...

print_list(&l);
printf("\n\n");

printf("\nSorting the whole list...\n");
quick_sort_list(&l, 0, get_list_count(&l) - 1);
printf("Your sorted list is:\n");
print_list(&l);

destroy_list(&l); //don't forget to destroy the list
}
```