Discrete Mathematics: Homework 13

Deadline: 5/06/2020

May 29, 2020

1. (20 points) Is $J(5, 2, 0)$ planar? Give a proof of your answer.
2. ($10 = 5{+}5$ points) 10.5, Question 28. Determine for which values the complete bipartite graph $K_{m,n}$ has a.) an Euler circuit. b.) an Euler path.
3. (15 points) Show that if G is a (connected) biipartite graph and G is planar then $|E| \leq 2|V| - 4$.
4. (20=10+10 points) Prove that a Hamiltonian graph does not contain a cut vertex. Give an example to show the converse is not true.
5. (15 points) Show that if G is a bipartite graph with an even number of vertices then $|E| \leq \frac{|V|^2}{4}$.
6. (a, 10 points, b, 30 points) Let G be a simple graph, and e and edge in G.
   - a.) Let G be a simple graph and e an edge in G. Show that there is a circuit in G containing e, $\iff$ there is a simple circuit in G containing e.
   - b.) Let G be a connected, simple graph. Show that e is a bridge (i.e. $G \setminus \{e\}$ is not connected) $\iff$ e is not contained in any simple circuit in G.