Discrete Mathematics: Homework 10

Deadline: 15/05/2020

May 8, 2020

120 points in total, try to show you working and explain clearly.

1. (20 points) How many was to reaarange the letters of the word "lexicographic"?

2. (20 points) How many bit strings are there of length 10, not containing 2 consective zero's and not starting or ending in 111?

3. (30 points) Consider permuatations of the set $\{1, 2, 3, 4, 5, 6\}$, how many permuatations come after $\alpha = 245136$ in lexicgraphic order?

4. (30 points) Suppose we have a reguar octahedron with side lengths equal to $\sqrt{2}$. Given 28 points in the octahedron, prove that there is a collection of 4 points such that the distance between any two in the collection is at most 2.

5. (10 points) Consider a finite set $A = \{1, 2, 3, \ldots, m\}$. Recall that $\mathcal{P}(A)$ is the set of subsets of A with $|\mathcal{P}(A)| = 2^m$. Count the number of functions $\tau : \mathcal{P}(A) \to \{0, 1, \ldots, m\}$ satisfying

   - a). For any non-empy subset B, $\tau(B) > 0$.
   - b). For any subsets C and D: $\tau(C \cup D) = \tau(C \cap D) + \tau((C \cup D) \setminus (C \cap D))$.

6. (10 points) Suppose a person delivers packages to a street (the street only has houses on one side, on the other side is a large building, which does not receive packages). One day, the person delivers the packages and the two following conditions occur

   - There is no n consecutive houses with no packages for $n \geq 2$.
   - There are no m consecutive houses with packages for $m \geq 3$.

   If the street has 14 houses, and each house receives at most 1 package. how many possibile ways to deilver packages on this day?