# Discrete Mathematics: Homework 11

## Deadline: 22/05/2020

## May 15, 2020

1. (10 points) Chapter 8 question 16 in the text book.

2. (10 points) Suppose that a park has 10 ponds and 17 birds sitting on the ponds. No two ponds are exactly the same and no two birds are exactly the same, so we count them separately. Each pond has at least one bird. How many possibilities? You may leave your answer in reduced form.

3. (20 points) Suppose we have just 3 yuan coins, 4 yuan coins , 7 yuan coins and 9 yuan coins. How many ways to make 23 yuan?

4. (30 points)Solve the recurrence relation $h_n = h_{n-1} + 9h_{n-2} - 9h_{n-3}$, $h_0 = 0, h_1 = 1$ and $h_2 = 2$.

5. (30 points) Solve the recurrence relation $h_n = 3h_{n-2} - 2h_{n-3}$. $h_0 = 1$, $h_1 = 0$ and $h_2 = 0$.

6. (10 points) Supposse $a_1, \ldots, a_{65}$ is a seqence of numbers. Suppose that for $i \neq j$ then $a_i \neq a_j$. Then prove that either there is an increasing subsequence of length 10 or a decreasing subsequence of length 8.

7. (10 points) Consider the set $S = \{1, \ldots, n\}$, and suppose $S_1$ and $S_2$ are non-empty subsets with $S = S_1 \cup S_2$ with $S_1 \cap S_2 = \emptyset$. Let $A \subset S \times S \times S$ be the subset consisting of elements $(x, y, z) \in S \times S \times S$ satisfying both of the following 2 conditions:

   (a) All of the elements $x, y, z$ are in $S_1$ or all of the elements $x, y, z$ are in $S_2$.

   (b) $x + y + z$ is divisible by $n$.

   *Question:* Determine all the possible values of $n$ for which there exists non-empty subsets $S_1, S_2$ such that $\{1, \ldots, n\} = S_1 \cup S_2$, $S_1 \cap S_2 = \emptyset$ and with the above definition $|A| = 36$?