Discrete Mathematics: Homework 9

Deadline: 08/05/2020

May 1, 2020

Here are the questions for this week. We note that it is better to leave your answers in meaningful form and show your working. For example, it is ok to give the answer: $3^7 + 9!$ rather than: 365067. The total number of marks available is 120.

1. Suppose that a language has 38 letters in its alphabet $\mathcal{A}$. Suppose that $A, B \in \mathcal{A}$. The length of a word is the number of letters it has.

   (a) How may words have length 7, having A as the third letter ? (10 points)

   (b) How many words of length 4 such that the letter B appears exacly twice? (10 points)

2. A manager selects a football team from a squad of 20 players. The squad has 17 outfield players and 3 goalkeepers. The team should have 1 goalkeeper and 10 outfield players.

   (a) How many selections are possible? (the selection does NOT include shirt number, the position of outfield players, etc...) (20 points)

3. Suppose we have a bowl with red marbles, green marbles, yellow marbles, purple marbles and blue marbles in it (the number of marbles of a given colour can be 0). Marbles of the same colour are indistinguishable.

   (a) If the bowl has ten marbles, how many possibilities are there? (15 points)

   (b) If the bowl has 13 marbles and the bowl does not contain marbles of all 5 colours, how many possibilitles are there? (15 points)

4. Prove that for any positive integer n, there exists infinitely many positive integers k, such that kn has only 0 and 7 in its decimal expansion (for example: 70700077). Explain your answer with as much detail as possible.(50 points).