Discrete Mathematics: Homework 9

Name ID: Number

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1. Suppose that a language has 38 letters in its alphabet $\mathcal{A}$. Suppose that $A, B \in \mathcal{A}$. Thelength of a word is the number of letters it has.

   (a) How may words have length 7, having A as the third letter ?

   (b) How many words of length 4 such that the letter B appears exacly twice?

   Solution: *(a) For the rest of 6 numbers, each digit has 38 possibilities. And for the third digit, it has only one possibility 'A' .*
   So the answer is, 38^6

   (b) For 4 digits, there has 2 digits which can be 'B', so the possibility is C_4^2*, for the rest 2, each digit has 37 possibilities. (Because they can't be 'B')*
   So the answer is $C_4^2 \cdot 37^2$

2. A manager selects a football team from a squad of 20 players. The squad has 17 outfield players and 3 goalkeepers. The team should have 1 goalkeeper and 10 outfieldplayers.

   (a) How many selections are possible? (the selection does NOT include shirt number,the position of outfield players, etc...)

   Solution:

$$C_{17}^{10} \cdot C_3^1$$

3. Suppose we have a bowl with red marbles, green marbles, yellow marbles, purplemarbles and blue marbles in it (the number of marbles of a given colour can be 0).Marbles of the same colour are indistinguishable.

   (a) If the bowl has ten marbles, how many possibilities are there?

   (b) If the bowl has 13 marbles and the bowl does not contain marbles of all 5 colours,how many possibilitles are there?

证明. (a) For ten marbles, we can consider: | o o o o o | o | o o | o o |,
each '|' has 14 slot, so the answer is

$$C_{14}^4 = 1001$$

(b) The answer is all possibilities without 5

$$C_{17}^4 - C_{12}^4 = 1885$$

□

4. Prove that for any positive integer n, there exists infinitely many positive integers k, such that kn has only 0 and 7 in its decimal expansion (for example: 70700077).Explain your answer with as much detail as possible

证明. For integers 7, 77, 777,$\underbrace{777\ldots777}_{n+1}$, there at least have 2 intergers such that n mod $\underbrace{77\ldots77}_{a}$ and n mod $\underbrace{77\ldots77}_{b}$ by pigeonhole principle.

The same holds for $\underbrace{777\ldots777}_{n+2}\cdots\underbrace{777\ldots777}_{2n+2}$ and $\underbrace{777\ldots777}_{2n+3}\cdots\underbrace{777\ldots777}_{3n+3}$ and $\ldots$

So we can find infinite $a, b \in \{77\cdots77\}$ such that n mod a and n mod b

we also have n mod $|a-b|$ and $|a-b| \in \{7\ldots0\ldots7\}$

So there are infinite k □