Discrete Mathematics: Homework 4

(Deadline: 10:00am, April 3, 2020)

1. (20 points) Show that a set S is infinite if and only if there is a proper subset $A \subset S$ such that $|A| = |S|$.
2. (15 points) Prove or disprove $|\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}| = |\mathbb{R}|$.
3. (30 points) Prove or disprove $|\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}| = |\mathbb{R}|$.
4. (15 points) Prove or disprove $|\{X \subseteq \mathbb{Z}^+ : X \text{ is a finite set}\}| = |\mathbb{Z}^+|$.
5. (20 points) Prove or disprove $|\{(a_1, a_2, a_3, \ldots) : a_1, a_2, a_3, \ldots \in \mathbb{Z}^+\}| = |\mathbb{Z}^+|$.
6. (20 points) Find a countably infinite number of subsets of $\mathbb{Z}^+$, say $A_1, A_2, \ldots \subseteq \mathbb{Z}^+$, such that the following requirements are simultaneously satisfied:
   - A_i is countably infinite for every $i = 1, 2, \ldots$;
   - $A_i \cap A_j = \emptyset$ for all $i \neq j$;
   - $\cup_{i=1}^{\infty} A_i = \mathbb{Z}^+$.