Discrete Mathematics: Homework 2

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1. Let p and q be two propositional variables. Up to logical equivalence (i.e., if A $\equiv$ B, then we consider A and B has the same formula), how many different formulas in p, q are there? Write down all of them.

Solution:

There are 15 different formulas.

$\neg(\neg p) \equiv p$	$p \wedge p \equiv p$	$p \vee p \equiv p$
$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$	$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$
$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$	$p \vee (p \wedge q) \equiv p$	$p \vee (p \wedge q) \equiv p$
$p \rightarrow q \equiv \neg p \vee q$	$p \rightarrow q \equiv \neg q \wedge \neg q$	$p \leftrightarrow q \equiv (\neg p \vee q) \wedge \neg(p \vee \neg q)$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$

2. Let ↑ be a binary operation defined by the following truth table:

p	q	p ↑ q
T	T	F
T	F	T
F	T	T
F	F	T

Represent every formula in Question 1 with an expression where the only operation is ↑. For example, for the formula ¬p ∨ ¬q, we have the representation ¬p ∨ ¬q ≡ p ↑ q, where the only operation in the right-hand expression p ↑ q is ↑ and none of the other operations such as ¬, ∧, ∨, → or ↔ appears.

Solution:

1	$\neg(\neg p) \equiv p$	$\neg(\neg p) \equiv p$
2	$p \wedge p \equiv p$	$p \wedge p \equiv p$
3	$p \vee p \equiv p$	$p \vee p \equiv p$
4	$p \wedge q \equiv q \wedge p$	$p \wedge q \equiv (p \uparrow q) \uparrow (p \uparrow q)$
5	$p \vee q \equiv q \vee p$	$p \vee q \equiv (p \uparrow p) \uparrow (q \uparrow q)$
6	$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$	$\neg(p \wedge q) \equiv p \uparrow q$
7	$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$	$\neg(p \vee q) \equiv (p \uparrow p) \uparrow (q \uparrow q) \uparrow (p \uparrow p) \uparrow (q \uparrow q)$
8	$p \vee (p \wedge q) \equiv p$	$p \vee (p \wedge q) \equiv p$
9	$p \vee (p \wedge q) \equiv p$	$p \vee (p \wedge q) \equiv p$
10	$p \to q \equiv \neg p \vee q$	$p \to q \equiv p \uparrow (q \uparrow q)$
11	$p \leftrightarrow q \equiv (\neg p \vee q) \wedge \neg(p \vee \neg q)$	$p \leftrightarrow q \equiv (p \uparrow q) \uparrow (p \uparrow p) \uparrow (q \uparrow q)$

3. Show that (P ∧ Q ∧ S) ∨ (P ∧¬Q ∧ ¬R) ∨ (P ∧ Q ∧ ¬ S) ∨ ¬ (P ∧ R → Q) ≡ P using the logical equivalences on page 8-10 of lec3.pptx. (Hint: see page 11, 12 for an example)

Solution:

$$\begin{aligned}
&(P \wedge Q \wedge S) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg S) \vee \neg(P \wedge R \to Q) \\
&\equiv (P \wedge Q \wedge S) \vee (P \wedge Q \wedge \neg S) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge R \wedge \neg Q) \\
&\equiv (P \wedge Q) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge R \wedge \neg Q) \\
&\equiv (P \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q) \\
&\equiv (P \wedge Q) \vee (P \wedge \neg Q) \\
&\equiv P
\end{aligned} \tag{1}$$

4. Show that $(P \vee Q \to R \wedge S) \wedge (S \vee W \to U) \wedge P \Rightarrow U \vee V$ using the tautological implications (and the resulting valid argument forms) on page 3 of lec4.pptx. (Hint: see page 12, 13 for an example)

Solution:

$$\begin{aligned}
&(P \vee Q \to R \wedge S) \wedge (S \vee W \to U) \wedge P \\
&\equiv (\neg(P \vee Q) \vee (R \wedge S) \wedge (\neg S \wedge \neg W \vee U)) \\
&\equiv (\neg P \wedge \neg Q \wedge P) \vee (R \wedge S \wedge P) \wedge (\neg S \wedge \neg W \vee U) \\
&\equiv (R \wedge S \wedge P \wedge \neg S \wedge \neg W) \vee (R \wedge S \wedge P \wedge U) \\
&\equiv R \wedge S \wedge P \wedge U \\
&\Rightarrow U \\
&\Rightarrow U \vee V
\end{aligned} \tag{2}$$

5. Use the tautological implications (and the resulting valid argument forms) on page 3 of lec4.pptx to show that the premises If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," If the sailing race is held, then the trophy will be awarded," and " The trophy was not awarded" imply "the conclusion It rained."

   Solution:

   p: It rained.

   q: It's foggy.

   r: The sailing race will be held.

   s: The lifesaving demonstration will go on.

   t: The trophy will be awarded.

   so, we have

$$\begin{gathered} \neg p \vee \neg q \rightarrow r \wedge s \\ r \rightarrow t \\ \neg t \Rightarrow p \end{gathered} \tag{3}$$

   so,

$$\begin{aligned} & (\neg p \vee \neg q \rightarrow r \wedge s) \wedge (r \rightarrow t) \wedge (\neg t) \\ \equiv\ & (p \wedge q \vee r \wedge s) \wedge (\neg r \vee t) \wedge \neg t \\ \equiv\ & (p \wedge q \vee r \wedge s) \wedge (\neg r \wedge \neg t) \\ \Rightarrow\ & p \end{aligned}$$

6. Suppose that the following two premises are true:

   (a) Math is hard or Leibniz doesn't like Math";

   (b) If SI120 is easy, then Math is not hard".

   Which of the following conclusions are true under the above premises?

   (a) If Leibniz likes Math, then SI120 is not easy."

   (b) If SI120 is not easy, then Leibniz doesn't like Math."

   (c) Math is not hard or SI120 is not easy."

   (d) Leibniz doesn't like Math, then either SI120 is not easy or Math is not hard."

   Justify your answers.

   Solution:

p: Math is hard.

q: Leibniz like Math.

r: SI120 is easy.

so, we have

$$p \vee \neg q$$

$$r \to \neg p$$

(a)

$$(p \vee \neg q) \wedge (r \to \neg p) \Rightarrow (q \to \neg r)$$

$$let \quad A = (p \vee \neg q) \wedge (r \to \neg p), B = (q \to \neg r)$$

$$A^{-1}(T) = (T, F, F), (T, F, T), (T, T, F), (F, F, F), (F, F, T)$$

$$B^{-1}(T) = (T, T, F), (F, T, F), (F, F, F), (T, F, F), (T, F, T), (F, F, T)$$

so the conclusion is ***true*** *.*

(b)

$$(p \vee \neg q) \wedge (r \to \neg p) \Rightarrow (\neg r \to \neg q)$$

$$(p \wedge \neg q) \wedge (\neg r \vee \neg q) \wedge \neg r \wedge q$$

$$\equiv (p \wedge \neg q) \wedge ((\neg r \wedge \neg r) \vee (\neg q \wedge \neg r)) \wedge q$$

$$\equiv (p \wedge \neg q) \wedge q$$

$$\equiv F$$

so the conclusion is ***true*** *.*

(c)

$$(p \vee \neg q) \wedge (r \to \neg p) \Rightarrow (\neg p \vee \neg r)$$

$$let \quad A = (p \vee \neg q) \wedge (r \to \neg p), B = (\neg p \vee \neg r)$$

$$A^{-1}(F) = (T,T,T),(T,F,T),(F,T,F),(F,T,T)$$

$$B^{-1}(F) = (T,T,T),(T,F,T)$$

*so the conclusion is **true** .*

(d)

$$(p \vee \neg q) \wedge (r \to \neg p) \Rightarrow \neg q \to (\neg r \vee \neg p)$$

$$\neg q \to (\neg r \vee \neg p) \equiv q \vee \neg r \vee \neg p$$

$$let \quad A = (p \vee \neg q) \wedge (r \to \neg p), B = q \vee \neg r \vee \neg p$$

$$A^{-1}(F) = (T,T,T),(T,F,T),(F,T,F),(F,T,T)$$

$$B^{-1}(F) = (T,F,T)$$

*so the conclusion is **true** .*