Discrete Mathematics: Homework 10

Name ID: Number

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1. How many was to reaarange the letters of the word "lexicographic" ?

   Solution:

   For the 13 letters, we can have P_{13}^{1} ways to reaarange.

   For the letter 'i' and 'c', which appears twice as the same letter, we devided by P_{2}^{1}

   The arrangement doesn't count in itself, so the answer is,

   $\frac{P_{13}^{13}}{P_2^1 P_2^1} - 1 = 1556755119$

2. How many bit strings are there of length 10, not containing 2 consectivezero's and not starting or ending in 111?

   Solution:

   For the first 3 digits: we have $2^3 - 1$ situations.

   For the rest 7 digits, we have:

   $a_1 = 1$, $b_1 = 1$, $a_{n+1} = a_n + b_n$, $b_{n+1} = a_n$,

   so, we have,

i	a_i	b_i
1	1	1
2	2	1
3	3	2
4	5	3
5	8	5
6	13	8
7	21	13
8	34	21
9	55	34
10	89	55

   so, the answer is $xxxxx - 111xxxx - xxxx111 + 111xxxx111$

   so, the answer is $(a_{10} + b_{10}) - (a_7 + b_7) - (a_8) + (a_4 + b_4) = 84$

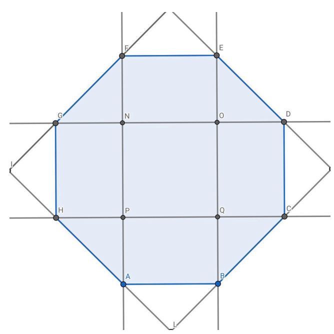
3. Consider permuatations of the set $\{1, 2, 3, 4, 5, 6\}$, how many permuatationscome after $\ = 245136$ in lexicgraphic order?

Solution:

$4 \cdot P_5^5 + 2 \cdot P_4^4 + P_3^3 + 2 \cdot P_2^2 + 1 = 539$

4. Suppose we have a reguar octahedron with side lengths equal to $\sqrt{2}$. Given 28 points in the octahedron, prove that there is a collection of 4 points such that thedistance between any two in the collection is at most 2.

证明.



By the pigeonhole theorem, at least 1 block can have more than $\lfloor \frac{28}{9} + 1 \rfloor = 4$ points.

For the biggest block, the distance between two points can't be larger than 2.

so, there is a collection of 4 points such that thedistance between any two in the collection is at most 2. □

5. Consider a finite set $A = \{1, 2, 3, ..., m\}$. Recall that $P(A)$ is the set of subsets of A with $|P(A)| = 2^m$. Count the number of functions $\tau : P(A) \to \{0, 1, ..., m\}$ satisfying

   (a) For any non-empy subset B, $\tau(B) > 0$.

   (b) For any subsets C and D: $\tau(C \cup D) = \tau(C \cap D) + \tau((C \cup D) \setminus (C \cap D))$.

Solution:

Suppose $C \cap D = \emptyset$, *than* $\tau(C \cup D) = \tau(\emptyset) + \tau((C \cup D) \setminus (\emptyset)) = \tau(\emptyset) + \tau(C \cup D)$

so we have, $\tau(\emptyset) = 0$ *so the rest elements of* $P(A)$, *which is* $2^m - 1$ *can maps to the rest* m *elements.*

And $\tau(C \cup D) = \tau(C \cap D) + \tau((C \cup D) \setminus (C \cap D))$, *so the map is "increasing" or "linear"*

Suppose $x \in P(A) = a$,$y \in P(A) = b$,$x \in P(A) = z$ $\tau(x + y) = \tau(x) + \tau(y) = a + b$

and $1 \leq a + b \leq m$

$|P(A)| = |\tau|P(A)|$ *so there are only 1 function,*

6. Suppose a person delivers packages to a street (the street only has houseson one side, on the other side is a large building, which does not receive packages).One day, the person delivers the packages and the two following conditions occur

   (a) There is no n consecutive houses with no packages for n $\geq$ 2

   (b) There are no m consecutive houses with packages for m$\geq$3

   If the street has 14 houses, and each house receives at most 1 package. how manypossibile ways to deilver packages on this day? If the street has 14 houses, and each house receives at most 1 package. how many possibile ways to deilver packages on this day?

Solution:

$a_1 = 1, b_1 = 2, c_1 = 1$

which represents the i to i+1 digits the number of (0-1) (1-0) (1-1) $a_{n+1} = b_n, b_{n+1} = a_n + c_n, c_{n+1} = a_n$

so we have,

i	a_i	b_i	c_i
1	1	2	1
2	2	2	1
3	2	3	2
4	3	4	2
5	4	5	3
6	5	7	4
7	7	9	5
8	9	12	7
9	12	16	9
10	16	21	12
11	21	28	16
12	28	37	21

so, $a_12 + b_12 + c_12 = 86$ *possibile ways to deilver packages on this day.*