Discrete Mathematics: Homework 3

(Deadline: 10:00am, March 27, 2020)

1. (20 points) Let $P(x)$ = "x is a person", $L(x, y)$ = "x loves y" and $E(x, y)$ ="$x = y$". Translate the following statements into formulas:
   (a) "Every person loves some person."
   (b) "Every person loves some other person."
   (c) "There is a person who is loved by every person."
   (d) "There is a person who is loved by every other person."
2. (20 points) Determine if the following formulas are logically valid, satisfiable or unsatisfiable.
   (a) $(\exists x P(x) \leftrightarrow \exists x Q(x)) \rightarrow \exists x(P(x) \leftrightarrow Q(x))$
   (b) $\exists x(\mathbf{T} \vee P(x) \rightarrow \mathbf{F})$
   (c) $\forall x(P(x) \vee \neg\exists y(Q(y) \wedge \neg Q(y)))$
   (d) $\exists x P(x) \rightarrow P(0)$
3. (15 points) Show that $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$.
4. (15 points) Show that $\forall x P(x) \vee \forall x Q(x) \Rightarrow \forall x(P(x) \vee Q(x))$.
5. (15 points) Show that $\exists x P(x) \wedge \forall x Q(x) \Rightarrow \exists x(P(x) \wedge Q(x))$.
6. (15 points) Show that $\forall x(P(x) \leftrightarrow Q(x)) \Rightarrow \forall x P(x) \leftrightarrow \forall x Q(x)$.