Discrete Mathematics: Homework 3

Name ID: Number

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1. Let P(x) =" x is a person", L(x, y) = "x loves y"and E(x, y) ="x=y".

   Translate the following statements into formulas:

   (a) "Every person loves some person."

   (b) "Every person loves some other person."

   (c) "There is a person who is loved by every person."

   (d) "There is a person who is loved by every other person."

   Solution:

   (a)

   $$\forall x P(x) \wedge \exists y P(y) \wedge L(x, y)$$

   (b)

   $$\forall x P(x) \wedge \exists y P(y) \wedge \neg E(x, y) \wedge L(x, y)$$

   (c)

   $$\exists x P(x) \wedge \forall y P(y) \wedge L(y, x)$$

   (d)

   $$\exists x P(x) \wedge \forall y P(y) \wedge \neg E(x, y) \wedge L(y, x)$$

2. Determine if the following formulas are logically valid, satisfiable or unsatisfiable.

   (a) $(\exists x P(x) \leftrightarrow \exists x Q(x)) \to \exists x(P(x) \leftrightarrow Q(x))$

   (b) $\exists x(\mathbf{T} \vee P(x) \to \mathbf{F})$

   (c) $\forall x((P(x) \vee \neg\exists y(Q(y) \wedge \neg Q(y))))$

   (d) $\exists x P(x) \to P(0)$

   Solution:

   (a) if $\exists x P(x) \leftrightarrow \exists x Q(x)$ *is True, which indicates* $P(x) = T, Q(x) = T$ *or* $P(x) = F, Q(x) = F$*. Hence,* $\exists x(P(x) \leftrightarrow Q(x))$ *is T. So it's T in every interpretation.*

   It's logically valid.

   (b) $T \vee P(x) \equiv T$ *,so* $\exists x(\boldsymbol{T} \vee P(x) \to \boldsymbol{F})$ *is F in every interpretation.*

   It's unsatisfiable.

   (c) $\neg\exists y(Q(y) \wedge \neg Q(y))$*is always T.* $\forall x((P(x) \vee \neg\exists y(Q(y) \wedge \neg Q(y))))$ *is always T.*

   It's logically valid.

   (d) when $P(0) = T, \exists x P(x) \to P(0) \equiv T$*,when* $P(0) = F, \exists x = 0$*, such that,* $P(x) = F$ *. So it's T in every interpretation.*

   It's logically valid.

3. Show that $\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

   Solution:

   (1) ***Show that*** $\exists x(P(x) \vee Q(x)) \rightarrow \exists x P(x) \vee \exists x Q(x)$ ***is logically valid.***

   Suppose that $\exists x P(x) \vee \exists x Q(x)$ *is F in an interpretation I*

   $P(x)$ *is F and* $Q(x)$ *is F in an x in I*

   $x(P(x) \vee Q(x))$ *is F in the x in I*

   $\exists x(P(x) \vee Q(x))$ *is F in I*

   (2) ***Show that*** $\exists x P(x) \vee \exists x Q(x) \rightarrow \exists x(P(x) \vee Q(x))$ ***is logically valid.***

   Suppose that $\exists x P(x) \vee \exists x Q(x)$ *is F in an interpretation in I*

   $P(x) \vee Q(x)$ *is F in an x in I*

   $P(x)$ *is F and* $Q(x)$ *is F in the x in I*

   $P(x)$ *is F in an x in I and Q(x) is F in an x in I*

   $\exists x P(x) \vee \exists x Q(x)$ *is F in I*

4. Show that $\forall x P(x) \vee \forall x Q(x) \Rightarrow \forall x((P(x) \vee Q(x)))$

   Solution:

   Suppose that the left hand side is true in an interpretation I

   $\forall x P(x) \vee \forall x Q(x)$ *is T*

   P(x) is T, Q(x) is T or P(x) is T, Q(x) is F or P(x) is F, Q(x) is T in every x in I

   P(x) $\vee$ *Q(x) is T in every x in I*

   $\forall x(P(x) \vee Q(x))$ *is T*

5. Show that $\exists x P(x) \wedge \exists x Q(x) \Rightarrow \exists x(P(x) \wedge Q(x))$

   Solution:

   Suppose that the left hand side is true in an interpretation I

   $\exists x P(x) \wedge \exists x Q(x)$ *is T*

   P(x) is T and Q(x) is T for the x in I

   $P(x) \wedge Q(x)$ *is T for the x in I*

   $\exists x(P(x) \wedge Q(x))$ *is T in I*

6. Show that $\forall x(P(x) \leftrightarrow Q(x)) \Rightarrow \exists P(x) \leftrightarrow \exists x Q(x)$

   Solution:

   Suppose that the left hand side is true in an interpretation I

   $\forall x(P(x) \leftrightarrow Q(x))$ *is T*

   P(x) is T , Q(x) is T or P(x) is F, Q(x) is F in every x in I

   P(x) $\leftrightarrow$ *Q(x) is T in an x in I*

   $\exists P(x) \leftrightarrow \exists x Q(x)$ *is T in I*