

1) Is the following satisfiable?

$$(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r) \wedge (\neg r \vee s) \wedge (\neg s)$$

2) Simplify the following to only use \neg, \vee, \wedge

i) $(p \implies q) \iff (\neg q \implies \neg p)$

ii) $\neg((p \wedge \neg q) \vee (\neg r \wedge (s \vee \neg t)))$

iii) $\neg((p \implies q) \wedge (\neg r \implies (s \vee t)))$

3) Explain the difference between $\forall x \exists y, f(x, y)$ and $\forall y \exists x, f(x, y)$. Give an example where they are not equivalent.

4) Negate the following statement:

$$\forall x \exists y, f(x, y) \wedge (x = y)$$

5) Prove $|A \times B| = |A| \cdot |B|$

6) True or False. If false, give a concrete example where it is false. If true, prove it.

i) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

ii) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$

iii) $(A \setminus B) \cup C = (A \cup C) \setminus (B \cup C)$