Cartesian Products

1) When is $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$ true?
2) Define $F : P(B) \to P(A \times B)$ to be $F(B_s) = A \times B_s$. Prove F is injective if $A \neq 0$.
   Why does it matter that $A \neq 0$?

Set Theory

Operators: $\cap, \cup, \setminus, \overline{A}$,

1) Let U be a universe set. All complements are taken with respect to U. Prove if true, give counterexample if false:
   a) $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$
   b) $A \subseteq B \Rightarrow \overline{B} \subseteq \overline{A}$
   c) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
   d) $\overline{A \cap B} = \overline{A} \cup \overline{B}$
2) Suppose $|U| = n$ and $|A| = k$. Compute $|\mathcal{P}(A)|$ and $|\mathcal{P}(U \setminus A)|$. Determine for which n, k it is possible that $\mathcal{P}(A) = \mathcal{P}(U \setminus A)$.

Functions

1) Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$.
   a) Show that no function $f : A \to B$ can be injective.
   b) Show that a surjective function $f : A \to B$ can exist, and give one explicit example.
2) Let $f : A \to B$. Prove from the definition that if f is injective and A is finite, then $|A| \leq |B|$.
3) Let A, B be finite with $|A| = |B| = n$. Prove: $f : A \to B$ is injective iff f is surjective.

Number Theory

Divisbility, GCD, The Division Theorem
$\text{Div}(n) = \{a : a \mid n\}$

1) Let $\gcd(m, n)$ be defined as the largest element of $\text{Div}(m) \cap \text{Div}(n)$.
   1. Prove directly from the definition that $\gcd(m, n) \mid m$ and $\gcd(m, n) \mid n$.
   2. Prove: if $d \mid m$ and $d \mid n$, then $d \mid \gcd(m, n)$.
2) Prove: $a \mid b \Rightarrow \text{Div}(a) \subseteq \text{Div}(b)$.