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A Natural Adaptive Control Law for Robot Manipulators

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***Abstract*— Existing adaptive robot control laws typically require an engineering choice of a constant adaptation gain matrix, which often involves repeated and time-consuming trial and error. Moreover, physical consistency of the estimated inertial parameters or the uniform positive definiteness of the estimated robot mass matrix cannot in general be guaranteed without nonsmooth corrections, e.g., projection to the boundary of the feasible parameter set. In this paper we present a natural adaptive control law that mitigates many of these difficulties, by exploiting the coordinate-invariant differential geometric structure of the space of physically consistent inertial parameters. Our approach provides a more generalizable and physically consistent adaptation law for the robot parameters without significant additional computations compared to existing methods. Simulation results showing markedly improved tracking error convergence over existing adaptive control laws are provided as validation.**

I. INTRODUCTION

Adaptive control methods have a long history in robotics. The two most important classes of globally convergent adaptive controllers are *adaptive inverse dynamics control* (or *adaptive computed torque control*), proposed by Craig et al [7], and *passivity-based adaptive control*, proposed by Slotine and Li [8]. The distinguishing feature of these two methods compared to previous works is that they are proven to be globally convergent without relying on any linear approximation of the dynamics; instead, they utilize the explicit linear decomposition of the inertial parameters in the robot dynamics.

An underlying assumption of adaptive inverse dynamics control [7] and other similar approaches [15], [16], [17] in this category is the requirement of a uniformly positive-definite estimated mass matrix, since the inverse of the estimated mass matrix is explicitly used in the parameter adaptation law. An appealing advantage of passivity-based adaptive control methods [8], [11] is that uniform positive-definiteness of the estimated mass matrix is not required in their implementation.

The requirement that the estimated mass matrix be positive-definite is directly related to the physical consistency of the estimated inertial parameters of a robot. That is, it can be shown that the necessary condition for physical consistency of the inertial parameters (i.e., all links have positive mass, and positive-definite rotational inertia matrix) guarantees that the corresponding robot mass matrix is positive-definite. Exploiting this property together with the convex characterization of the set of physically consistent inertial parameters, projection-based algorithms [13], or parameter resetting algorithms, have been applied for adaptive robot controllers; these essentially switch the parameter update law on the boundary of the feasible set to enforce the estimated parameters to be feasible, which in turn produces a nonsmooth adaptive control when the parameter values lie on the user-specified boundary of the feasible set [7], [10], [14], [18].

Another critical liability that direct adaptive robot control methods based on [7] and [8] have in common is the requirement that users must choose a valid initial adaptation gain matrix Γ. This can be a time-consuming process requiring repeated trial and error, as the number of constant adaptation gains varies as the square of the number of adaptation parameters, which is problem-dependent (e.g., adapting the entire set, or a subset of the link inertial parameters, or only the end-effector link for compensating unknown payloads).

In this paper we propose a new adaptive control law that mitigates many of the above difficulties, in the form of a parameter adaptation law that guarantees physical consistency of the estimated parameters in a smooth manner without relying on any parameter projection or resetting procedure. Whereas existing adaptation laws can be viewed as a gradient update law on a flat Euclidean space with constant metric Γ, our method can be viewed as a natural gradient-like update law on a curved space endowed with a Riemannian metric [6], realized in part by the one-to-one linear mapping between a link's inertial parameters and the space of 4×4 symmetric positive-definite matrices first pointed out in [4]. It does not require additional computation beyond existing methods, and is directly applicable to any globally convergent Lyapunov-based adaptive controller that exploits the linear decomposition of the inertial parameters in the dynamics.

Our natural adaptation law, which like [6] is coordinate-invariant and further respects the underlying Riemannian geometry of the space of inertial parameters, considerably reduces the degree to which engineering choices must be made in the constant adaptation gain; only the choice of a scalar constant gain γ is required for adjusting the speed of adaptation, without regard to the number of parameters for adaptation. Our method and claims are validated via extensive simulation experiments involving a seven-dof robot manipulator performing trajectory tracking tasks with and without unknown end-effector payloads.

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II. Previous Results on Adaptive Control for Robot Manipulators

In this section, we briefly revisit the two main classes of Lyapunov-based globally convergent adaptive controllers developed for robot manipulators in [7] and [8]. The dynamic equations for a general n-dof open chain manipulator are of the form

$$M(q,\Phi)\ddot{q} + C(q,\dot{q},\Phi)\dot{q} + g(q,\Phi) = u, \tag{1}$$

where $q \in \mathbb{R}^n$ is the vector of joint angles, $M(\cdot) \in \mathbb{R}^{n\times n}$, $C(\cdot) \in \mathbb{R}^{n\times n}$, and $g(\cdot) \in \mathbb{R}^n$ respectively denote the mass matrix, Coriolis matrix, and the gravitational force vector, $u \in \mathbb{R}^n$ is the motor torque input, and $\Phi = [\phi_1^T, \cdots, \phi_n^T]^T \in \mathbb{R}^{10n}$ is the complete set of inertial parameters for the n links. The inertial parameter vector $\phi_i \in \mathbb{R}^{10}$ for link i is commonly expressed in the form

$$\phi_i = [m_i, h_i, I_i^{xx}, I_i^{yy}, I_i^{zz}, I_i^{xy}, I_i^{yz}, I_i^{zx}] \in \mathbb{R}^{10},$$

where the scalar m_i is the mass, $p_i \in \mathbb{R}^3$ is the center for mass, and $I_i \in \mathcal{S}(3)$ is the symmetric 3×3 rotational inertia matrix, all represented with respect to a body-fixed reference frame.

The technical objective of adaptive control is to design a control input u that ensures global convergence of the trajectory tracking error in the presence of model uncertainties. We confine our interest in this paper to the class of adaptive controllers that assumes only model uncertainty in the inertial parameters Φ. Furthermore, as the term "adaptive" implies, time-varying estimates of the true model parameters are obtained with the trajectory tracking controller, distinct from the class of robust controllers that make use of fixed parameter estimates with known uncertainty bound.

A. Adaptive Computed Torque Control [7]

The control input for adaptive computed torque control, also referred to as adaptive inverse dynamics, is given by

$$u = M(q,\hat{\Phi})\{\ddot{q}_d - K_v\dot{\tilde{q}} - K_p\tilde{q}\} + C(q,\dot{q},\hat{\Phi})\dot{q} + g(q,\hat{\Phi}), \tag{2}$$

where $K_p \in \mathbb{R}^{n\times n}$ and $K_v \in \mathbb{R}^{n\times n}$ are diagonal matrices of positive gains, q_d is the given reference trajectory, $\tilde{q} = q - q_d$, and $\hat{\Phi}$ is the estimate of the true inertial parameter Φ, whose update law is clarified later. Using the property that the inertial parameters can be linearly factored from the dynamic equations, the closed loop dynamics becomes

$$\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q} = M(q,\hat{\Phi})^{-1}Y(q,\dot{q},\ddot{q})\tilde{\Phi}, \tag{3}$$

where $Y \in \mathbb{R}^{n\times 10n}$ is the regressor function that satisfies

$$M(q,\Phi)\ddot{q} + C(q,\dot{q},\Phi)\dot{q} + g(q,\Phi) = Y(q,\dot{q},\ddot{q})\Phi, \tag{4}$$

and $\tilde{\Phi} = \hat{\Phi} - \Phi$. The state space formulation of (3) with augmented state vector defined as $e = [\tilde{q}^T, \dot{\tilde{q}}^T]^T$ is given by

$$\dot{e} = Ae + BM(q,\hat{\Phi})^{-1}Y(q,\dot{q},\ddot{q})\tilde{\Phi}, \tag{5}$$

where $A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$ is a Hurwitz matrix and $B = \begin{bmatrix} 0 & I \end{bmatrix}^T \in \mathbb{R}^{2n\times n}$. One may then choose some constant symmetric $2n\times 2n$ positive-definite matrix Q, and let P be the unique symmetric positive-definite matrix satisfying the Lyapunov equation, $A^TP + PA = -Q$. The Lyapunov function candidate V is defined as follows:

$$V = e^TPe + \tilde{\Phi}^T\Gamma\tilde{\Phi},$$

where $\Gamma \in \mathbb{R}^{10n\times 10n}$ is a constant symmetric positive-definite matrix. The time derivative of the Lyapunov function candidate $\dot{V}$ is given by

$$\dot{V} = -e^TQe + 2\tilde{\Phi}^T\{Y(q,\dot{q},\ddot{q})^TM(q,\hat{\Phi})^{-1}B^TPe + \Gamma\dot{\hat{\Phi}}\},$$

which makes use of the fact that $\dot{\tilde{\Phi}} = \dot{\hat{\Phi}}$ since Φ is constant. Defining the parameter update law (or adaptation law) as

$$\dot{\hat{\Phi}} = -\Gamma^{-1}Y(q,\dot{q},\ddot{q})^TM(q,\hat{\Phi})^{-1}B^TPe, \tag{6}$$

we have $\dot{V} = -e^TQe \le 0$. From the stability analysis provided in [7] it follows that the position tracking error e converges to zero asymptotically, and the parameter estimate error $\tilde{\Phi}^T\Gamma\tilde{\Phi}$ remains bounded, provided that the estimated mass matrix $M(q,\hat{\Phi})$ is uniformly bounded and invertible.

In fact, $M(q,\hat{\Phi})$ must be invertible in order to implement the adaptation law (6), but the existence of $M(q,\hat{\Phi})^{-1}$ is not rigorously justified. In [7], the authors provide a switching scheme to reset the parameters to always reside inside some feasible bound, defined by a set of linear inequality constraints on $\hat{\Phi}$ that lead to sufficient conditions guaranteeing the boundedness and invertibility (or equivalently, positive-definiteness) of $M(q,\hat{\Phi})$. Another important issue in implementing the adaptation law (6) is the requirement of the joint acceleration feedback term $\ddot{q}$, which is difficult to obtain accurately from position and velocity measurements. The adaptive controller presented below proposed by Slotine and Li [8] removes both of these difficulties.

B. Passivity-based Adaptive Control [8]

The control input for passivity-based adaptive control is given by

$$\begin{aligned} u &= M(q,\hat{\Phi})a + C(q,\dot{q},\hat{\Phi})v + g(q,\hat{\Phi}) - Kr, \\ &= Y(q,\dot{q},a,v)\hat{\Phi} - Kr \end{aligned} \tag{7}$$

where the vectors $v, a, r \in \mathbb{R}^n$ are defined as

$$v = \dot{q}_d - \Lambda\tilde{q}, \quad a = \dot{v} = \ddot{q}_d - \Lambda\dot{\tilde{q}}, \quad r = \dot{q} - v = \dot{\tilde{q}} + \Lambda\tilde{q},$$

and K and Λ are diagonal matrices of constant positive gains. The closed-loop dynamics is given by

$$M(q,\Phi)\dot{r} + C(q,\dot{q},\Phi)r + Kr = Y(q,\dot{q},a,v)\tilde{\Phi}.$$

The following Lyapunov function candidate is introduced:

$$V = \frac{1}{2}r^TM(q,\Phi)r + \frac{1}{2}\tilde{\Phi}^T\Gamma\tilde{\Phi}, \tag{8}$$

where Γ as before is set to be a constant symmetric positive-definite matrix. Using the fact that $\dot{M} - 2C$ as constructed is skew-symmetric [8], $\dot{V}$ reduces to

$$\dot{V} = -r^TKr + \tilde{\Phi}^T\{\Gamma\dot{\hat{\Phi}} + Y^Tr\}.$$

Choosing the parameter update law as

$$\dot{\hat{\Phi}} = -\Gamma^{-1} Y(q, \dot{q}, a, v)^T r, \tag{9}$$

we have $\dot{V} = -r^T K r \leq 0$. It can also be shown that the position tracking error e converges to zero asymptotically from the convergence of r, and the parameter estimate error $\tilde{\Phi}^T \Gamma \tilde{\Phi}$ remains bounded. Note that neither the acceleration measurements $\ddot{q}$ nor the inverse of $M(q, \hat{\Phi})$ is required in the control input (7) and the parameter adaptation law (9).

Remark 1. *As noted in [11], a practical choice for the gain matrices K, Λ is given by $K = \lambda \cdot M(q, \hat{\Phi})$, and $\Lambda = \lambda \cdot I$ for some scalar constant λ. The above choice implies that higher gain values are used for joints with higher inertias. It is further shown in [11] that even with such a time-varying choice of K, global convergence can be guaranteed with only a slight modification of a in the regressor function Y in the adaptation law (9), to $a - \lambda r$.*

III. GEOMETRY OF RIGID BODY INERTIAL PARAMETER

In this section we review some Riemannian geometric preliminaries of the space of rigid body inertial parameters.

A. Physically Consistent Rigid Body Inertial Parameters

There are exactly ten independent parameters in $\phi_b \in \mathbb{R}^{10}$ that capture the mass and inertial properties of a single rigid body with respect to some body-fixed frame $\{b\}$. However, the space of feasible values for ϕ_b does not span the entire $\mathbb{R}^{10}$, but only a subset; as pointed out in [1], [3], the dual requirements that a rigid body's mass m be positive, and its inertia matrix with respect to the center of mass, $I_b^C = I_b - m[p_b][p_b]^T$, be positive-definite is in fact a necessary, but not sufficient, condition. An additional requirement, referred to as the triangular inequality condition, is that the three eigenvalues of I_b^C must satisfy a set of triangle inequalities in order for a physically realizable nonnegative mass density function $\rho : \mathbb{R}^3 \to \mathbb{R}_0^+$ to exist. In [4], this physical consistency condition is expressed in the following alternative but equivalent form: first define the 4×4 symmetric matrix $P_b \in \mathcal{S}(4)$ as

$$P_b = \int \begin{bmatrix} \vec{r}_b \\ 1 \end{bmatrix} \begin{bmatrix} \vec{r}_b \\ 1 \end{bmatrix}^T \rho(\vec{r}_b) dV_b = \begin{bmatrix} \Sigma_b & h_b \\ h_b^T & m \end{bmatrix}, \tag{10}$$

where the second moment matrix $\Sigma_b \in \mathcal{S}(3)$ is given by $\Sigma_b = \int \vec{r}_b \vec{r}_b^T \rho(\vec{r}_b) dV_b$. It is then shown in [4] that the condition that P_b be positive-definite, i.e. $P_b \succ 0$, is equivalent to the physical consistency conditions described in [3]. To make this identification more explicit, from the relation $\Sigma_b = \frac{1}{2}\mathrm{tr}(I_b)\mathbf{1} - I_b$, which holds for any choice of body frame $\{b\}$, define the one-to-one linear mapping $f : \mathbb{R}^{10} \to \mathcal{S}(4)$ follows:

$$f(\phi_b) = P_b = \begin{bmatrix} \frac{1}{2}\mathrm{tr}(I_b) \cdot \mathbf{1} - I_b & h_b \\ h_b^T & m \end{bmatrix} \in \mathcal{S}(4) \tag{11}$$

$$f^{-1}(P_b) = \phi_b(m, h_b, \mathrm{tr}(\Sigma_b) \cdot \mathbf{1} - \Sigma_b) \in \mathbb{R}^{10}. \tag{12}$$

The physical consistency conditions on $\phi_b \in \mathbb{R}^{10}$ can be identified via the mapping f with the requirement that the symmetric matrix $P_b = f(\phi_b) \in \mathcal{P}(4)$ be positive-definite.

Based on the above, we define the manifold $\mathcal{M}$ of the set of physically consistent inertial parameters for a single rigid body as follows:

$$\begin{aligned} \mathcal{M} &\simeq \{\phi_b \in \mathbb{R}^{10} : f(\phi_b) \succ 0\} \subset \mathbb{R}^{10} \\ &\simeq \{P_b \in \mathcal{S}(4) : P_b \succ 0\} = \mathcal{P}(4). \end{aligned}$$

The elements can be identified in both $\mathbb{R}^{10}$ and $\mathcal{P}(4)$, also for different choices of body-fixed reference frame $\{b\}$. For a multibody system with n rigid links, the space of physically consistent inertial parameters is given by the product space $\mathcal{M}^n \simeq \mathcal{P}(4)^n$.

B. Riemannian Geometry of $\mathcal{M} \simeq \mathcal{P}(4)$

A Riemannian geometric structure can be defined on the manifold $\mathcal{M}$ with a natural choice of Riemannian metric, which consequently provides a valid distance metric between two elements connected by a minimal geodesic path.

In order for a distance to be naturally defined on a manifold, it is desirable for it to be invariant with respect to choice of coordinate frames or physical units/scale. It would also be desirable if the distance were to possess a physical meaning that corresponds to our intuition. The standard Euclidean metric on $\mathbb{R}^{10}$ under the vectorized representation ϕ_b is found to satisfy none of these desiderata [5]. In [5] a coordinate-invariant distance metric is defined on $\mathcal{M}$ that is essentially inherited from the affine-invariant Riemannian geometric structure of $\mathcal{P}(4)$. That is, for $P \in \mathcal{P}(4)$ and tangent vector $X, Y \in T_P\mathcal{P}(4)$, the Riemannian metric invariant under the group action $G * P = GPG^T$, where $G \in GL(4)$ is any 4×4 non singular matrix, is given by

$$\langle X, Y \rangle_P = \frac{1}{2}\mathrm{tr}(P^{-1} X P^{-1} Y). \tag{13}$$

The resulting geodesic distance between two arbitrary elements $P_1, P_2 \in \mathcal{P}(4)$ is then given by

$$d_{\mathcal{P}(4)}(P_1, P_2) = \left(\sum_{i=1}^{n} \big(\log(\lambda_i)\big)^2 \right)^{1/2},$$

where λ_i are the eigenvalues of $P_1^{-1/2} P_2 P_1^{-1/2}$, or equivalently, those of $P_1^{-1} P_2$. Now, applying a one-to-one mapping f from the vectorized inertial parameter ϕ_b to the matrix form $P_b = f(\phi_b)$, a distance metric on $\mathcal{M}$ can be constructed as

$$d_{\mathcal{M}}({}^1\phi_b, {}^2\phi_b) = d_{\mathcal{P}(4)}({}^1P_b, {}^2P_b). \tag{14}$$

This distance metric is shown to be invariant to choice of coordinate frames $\{b\}$ and also physical units/scale. It also possesses further interesting features consistent with our physical intuition, and also has close connections to the Fisher information metric as pointed out in [5].

C. Natural Gradient Descent on $\mathcal{M} \simeq \mathcal{P}(4)$

Natural gradient descent is a generalization of steepest descent in Euclidean space to a general curved Riemannian space [6]. Let $h : \mathcal{P}(4) \to \mathbb{R}$ be a differentiable function defined on $\mathcal{M}$. Then the steepest descent direction of $h(P)$ at $P \in \mathcal{P}(4)$ is defined by the tangent vector $dP \in \mathcal{S}(4)$

that minimizes $h(P+dP) \simeq h(P)+\mathrm{tr}(\nabla h(P)\cdot dP)$, where $\|dP\|_P$ has a fixed length in terms of the local Riemannian metric ($[\nabla h(P)]_{ij} \triangleq \partial h/\partial P_{ij}$). The ensuing optimization problem can be formulated as follows:

$$dP = \arg\min_W \mathrm{tr}(\nabla h(P)\cdot W) \tag{15}$$

$$\text{s.t. } \|W\|_P^2 = \frac{1}{2}\mathrm{tr}(P^{-1}WP^{-1}W) = 1. \tag{16}$$

The Lagrangian for the above is $L(W,\lambda) = \mathrm{tr}(\nabla h(P)\cdot W)+\lambda(1-\mathrm{tr}(P^{-1}WP^{-1}W))$; the corresponding first-order necessary condition $\nabla_W L = 0$ yields the following optimal W:

$$W = -\gamma\cdot P\nabla h(P)P, \tag{17}$$

where the constant positive scalar γ is determined from the constraint (16). Here, the natural gradient descent direction $P\nabla h(P)P$ differs from the classical steepest descent direction $\nabla h(P)$ in Euclidean space.

D. Bregman Divergence as a Pseudo-Distance Metric on $\mathcal{M}$

As will be elucidated in the following sections, the exact Riemannian geodesic distance as defined in (14), because of its algebraic nonlinearity, is not particularly useful in constructing valid Lyapunov function candidate as needed in standard robot adaptive control formulations. In this section we propose an alternative pseudo-distance metric on the inertial parameters, derived from the Bregman divergence of a log-det function on $\mathcal{P}(4)$, that still preserves coordinate-invariance, but is only nonnegative and fails to be symmetric or satisfy the triangular inequality. These latter shortcomings in fact do not prevent their use in our natural adaptive control framework as shown later.

The Bregman divergence associated with a function $F:\Omega\to\mathbb{R}$ for points $p,q\in\Omega$, is defined by the difference between the value of F at point p and the value of the first-order Taylor expansion of F around point q evaluated at point p:

$$D_{F(\Omega)}(p||q) = F(p) - F(q) - \langle\nabla F(q), p-q\rangle.$$

When $\Omega = \mathcal{P}(4)$, the Bregman divergence associated with a log-det function F, i.e. $F(P) = -\log|P|$ for $P\in\mathcal{P}(4)$, is given by

$$D_{F(\mathcal{P}(4))}(P||Q) = \log\frac{|Q|}{|P|} + \mathrm{tr}(Q^{-1}P) - 4 \tag{18}$$

$$= \sum_{i=1}^{4}(-\log(\lambda_i)+\lambda_i-1), \tag{19}$$

where λ_i are the eigenvalues of $Q^{-1}P$, or equivalently, $Q^{-1/2}PQ^{-1/2}$. Note that $D_{F(\mathcal{P}(4))}$ is affine-invariant (that is, invariant under the $GL(4)$ group action $*$).

Again using the one-to-one mapping f from ϕ to $P = f(\phi)$, we can define a distance metric on $\mathcal{M}$ as

$$D_{\mathcal{M}}({}^1\phi_b, {}^2\phi_b) = D_{F(\mathcal{P}(4))}({}^1P_b||{}^2P_b). \tag{20}$$

This distance metric is coordinate-invariant since the coordinate transformation on P follows the exact form of the $GL(4)$ group action $*$ as described in [5]. Henceforth we omit all coordinate frame subscripts for expressing the inertial parameters ϕ and P.

Remark 2. *The divergence measure $D_{F(\mathcal{P}(n))}$ approximates the affine-invariant Riemannian metric up to second-order; that is, for two infinitesimally close positive-definite matrices P and $P+dP$, the following holds:*

$$D_{F(\mathcal{P}(n))}(P||P+dP) = \langle dP, dP\rangle_P + o(\lambda_i(P^{-1}dP)^3).$$

As an interesting side remark, the above relation is exactly identical to the one between the Fisher information metric and the Kullback-Leibler (KL) divergence on the statistical manifold of probability density functions.

IV. Main Contribution: A Natural Adaptation Law

Lyapunov stability analysis provides a way of assessing the stability of a closed-loop system by a suitable choice of Lyapunov function. In some cases what at first appears to be a natural Lyapunov function candidate can complicate the design of a stabilizing control law. Particularly for a large class of mechanical systems including robot manipulators, a branch of passivity-based control methods that exploits the skew-symmetry of $\dot{M}-2C$ have been developed for a class of energy-like Lyapunov functions, e.g., a system's kinetic and elastic energy terms as reflected in (8). Such methods allow for the closed-loop system to inherit some of the intrinsic physical properties of the original system [9], rather than replacing entirely the original dynamics with that of a virtual spring-damper system via feedback linearization as done in computed-torque control. As described in the previous section, Slotine and Li have shown that such physically motivated control designs can also be successfully applied to adaptive manipulator control.

The general class of globally convergent adaptive robot control laws as discussed in the previous section consider a Lyapunov function candidate V that is the sum of a tracking error term V_t and a parameter error term V_p:

$$V = \underbrace{V_t(q,\dot{q},q_d,\dot{q}_d,\Phi)}_{\text{(tracking error)}} + \underbrace{V_p(\Phi,\hat{\Phi})}_{\text{(parameter error)}}. \tag{21}$$

Here, as opposed to the choice of tracking error term V_t, the natural choice of parameter error term V_p is often overlooked; a quadratic parameter error of the form $\tilde{\Phi}^T\Gamma\tilde{\Phi}$ with constant positive-definite matrix Γ is often considered. As the inverse of Γ serves as an adaptation gain in (6), (9), selecting an appropriate choice of Γ can be a time-consuming and arduous process. Moreover, as discussed in [5], different choices of metric on the inertial parameters, for example the standard Euclidean metric corresponding to $\Gamma = \gamma I$, can lead to fatal scaling problems and physically inconsistent estimators, especially when the excitation trajectories fail to be persistently exciting. Such problems are pervasive in adaptive control, where only limited trajectory data is available for online adaptation purposes.

There have been several approaches or modifications toward more robust parameter adaptation under the framework of adaptive computed-torque control and passivity based adaptive control. For the case of adaptive computed-torque control, several projection or resetting based modifications of the adaptation law have been proposed to ensure the estimated inertial parameters are physically consistent [7], [10], [13], [14],[18], by sufficiently guaranteeing the uniform positive-definiteness of estimated mass matrix $M(q, \hat{\Phi})$. We believe that such switching-based schemes are more often than not ad hoc remedies to the issue of physical inconsistency: with a poor choice of Γ, the estimated parameters are highly prone to converge to the boundary of a predefined feasible set, which leads to a trivial adaptation of the parameters over a large portion of the operation time. On the other hand, Slotine and Li have suggested indirect [10] or composite versions [11] of their passivity-based method. These take into account additional filtered torque prediction errors in a least-squares sense, which essentially updates the matrix Γ to a more well-conditioned value over the time.

However, to the best of our knowledge, none of the adaptation laws so far respect the physical consistency of the estimated parameters in an intrinsic and smooth manner. In the following sections, we argue that these goals can in fact be achieved by consideration of the natural and coordinate-invariant choice of a smooth parameter error term V_p. This in turn significantly reduces the human burden of having to select an excessive number of adaptation gains compared to existing methods.

A. Bregman Divergence as a Lyapunov Function Candidate

One possible choice for V_p is the geodesic distance metric as defined in [5], i.e.,

$$V_p(\Phi, \hat{\Phi}) = \sum_{i=1}^{n} d_{\mathcal{M}}(\phi_i, \hat{\phi}_i). \tag{22}$$

It turns out, however, that a negative $\dot{V}$ cannot be ensured due to the nonlinearity of (22). The time derivative of the tracking error term V_t is of the form

$$\dot{V}_t \leq \tilde{\Phi}^T b, \tag{23}$$

where $b = Y^T \hat{M}^{-1} B^T P e$ for adaptive computed torque control, and $b = Y^T r$ for the passivity-based adaptive control. In order to ensure that time derivative of $V = V_t + V_p$ is negative, the time derivative of V_p must be of the form

$$\dot{V}_p(\Phi, \hat{\Phi}) = \tilde{\Phi}^T g(\dot{\hat{\Phi}}, \hat{\Phi}), \tag{24}$$

with the adaptation law $g(\dot{\hat{\Phi}}, \hat{\Phi}) = -b$ (when V_p is set to be the conventional quadratic error, i.e., $V_p = \frac{1}{2}\tilde{\Phi}^T \Gamma \tilde{\Phi}$, then $g(\dot{\hat{\Phi}}, \hat{\Phi}) = \Gamma \dot{\hat{\Phi}}$). This eventually enforces the negativity of $\dot{V}$, i.e., $\dot{V} = \dot{V}_t + \dot{V}_p \leq \tilde{\Phi}^T(b + g(\dot{\hat{\Phi}}, \hat{\Phi})) = 0$. It is important here to note that g should be a function of only the estimated parameters $\hat{\Phi}$ and their time derivatives $\dot{\hat{\Phi}}$, and not include the true parameter Φ, since it is not known a priori, but rather a desired value we wish to estimate. With V_p chosen as in (22), its time derivative cannot be explicitly factored in the form (24).

We now claim that the Bregman divergence-based distance measure discussed in (20) allows for a valid Lyapunov function candidate for V_p whose time derivative is of the form (24).

Proposition 1. *A function V_p defined as*

$$V_p(\Phi, \hat{\Phi}) = \gamma \sum_{i=1}^{n} D_{F(\mathcal{P}(4))}(f(\phi_i)||f(\hat{\phi}_i)) \tag{25}$$

for constant positive scalar gain γ and true parameter vector Φ is a valid Lyapunov function candidate of $\hat{\Phi}$. Its time derivative satisfies the form given in (24).

Proof. First, observe that V_p is a symmetric function of the eigenvalues λ_i^j $(j = 1, 2, 3, 4)$ of the matrices $f(\hat{\phi}_i)^{-1} f(\phi_i)$, which implies coordinate invariance of V_p. Explicitly, V_p is given by

$$V_p = \gamma \sum_{i=1}^{n} \sum_{j=1}^{4} [-\log(\lambda_i^j) + \lambda_i^j - 1].$$

Since the scalar function $-\log(x) + x - 1$ is always non-negative, $V_p \geq 0$ is a valid Lyapunov function candidate, becoming zero only when $\lambda_i^j = 1$ for all i, j, i.e., $\hat{\Phi} = \Phi$.

The time derivative of V_p is given by

$$\begin{aligned}
\dot{V}_p =& \gamma \frac{d}{dt} \sum_{i=1}^{n} \left[\log\left(\frac{|f(\hat{\phi}_i)|}{|f(\phi_i)|} \right) + \mathrm{tr}(f(\hat{\phi}_i)^{-1} f(\phi_i)) - 4 \right] \\
=& \gamma \sum_{i=1}^{n} \mathrm{tr}\Big(f(\hat{\phi}_i)^{-1} f(\dot{\hat{\phi}}_i) - f(\hat{\phi}_i)^{-1} f(\dot{\hat{\phi}}_i) f(\hat{\phi}_i)^{-1} f(\phi_i) \Big) \\
=& \gamma \sum_{i=1}^{n} \mathrm{tr}\Big(f(\hat{\phi}_i)^{-1} f(\dot{\hat{\phi}}_i) f(\hat{\phi}_i)^{-1} (f(\hat{\phi}_i) - f(\phi_i)) \Big) \\
=& \gamma \sum_{i=1}^{n} \mathrm{tr}\Big(\big[f(\hat{\phi}_i)^{-1} f(\dot{\hat{\phi}}_i) f(\hat{\phi}_i)^{-1} \big] f(\tilde{\phi}_i) \Big)
\end{aligned} \tag{26}$$

where we used the fact that $\frac{d}{dt} f(\hat{\phi}) = f(\dot{\hat{\phi}})$ and $f(\hat{\phi}) - f(\phi) = f(\hat{\phi} - \phi) = f(\tilde{\phi})$, which holds from the linearity of the mapping f. Again from the linearity of f, $\dot{V}_p$ calculated above can be expressed in the form (24). □

B. Natural Adaptation Law

Based on the choice of V_p as in (25), we now propose a novel adaptation law, which we refer to as a *natural adaptation law*, applicable to any Lyapunov-based adaptive control law in which the time derivative of the tracking error-related Lyapunov function candidate V_t is expressible in the form (23).

Proposition 2. *Given a control law that results in the time derivative of V_t of the form (23), the following adaptation law*

$$\dot{\hat{P}}_i = -\frac{1}{\gamma} \hat{P}_i B_i \hat{P}_i, \qquad i = 1, \cdots, n \tag{27}$$

where $\hat{P}_i = f(\hat{\phi}_i)$ and B_i are unique symmetric matrices that satisfy the relation $\tilde{\Phi}^T b = \sum_{i=1}^n \mathrm{tr}(f(\tilde{\phi}_i)B_i)$, guarantees the asymptotic convergence of the tracking error to zero, bounded parameter error, and also physical consistency of the estimated parameter $\hat{\Phi}$ over the time frame in which the initial estimate is chosen to be physically consistent.

Proof. Consider the valid Lyapunov function candidate

$$V = V_t + V_p,$$

where V_p is defined by (25). Then

$$\begin{aligned}
\dot{V} &= \dot{V}_t + \dot{V}_p \\
&\le \tilde{\Phi}^T b + \gamma \sum_{i=1}^n \mathrm{tr}\big(\hat{P}_i^{-1}\dot{\hat{P}}_i\hat{P}_i^{-1}\tilde{P}_i\big) \\
&= \sum_{i=1}^n \mathrm{tr}(\tilde{P}_i B_i) + \gamma \sum_{i=1}^n \mathrm{tr}\big(\hat{P}_i^{-1}\dot{\hat{P}}_i\hat{P}_i^{-1}\tilde{P}_i\big) \\
&= \sum_{i=1}^n \mathrm{tr}\big([B_i + \gamma\hat{P}_i^{-1}\dot{\hat{P}}_i\hat{P}_i^{-1}]\tilde{P}_i\big)
\end{aligned}$$

holds from (23), (26) and $\tilde{P}_i = f(\tilde{\phi}_i) = \hat{P}_i - P_i$. From the adaptation rule defined by (27), $\dot{V} \le 0$ holds, and asymptotic convergence of the tracking error can be shown in the same manner as before. Given that the initial parameter estimate is set to be physically consistent, i.e. $\hat{P}_i(0) \in \mathcal{P}(4)$, $V_p(0)$ is bounded. Moreover, since $\dot{V} \le 0$, V is bounded, and therefore $V_p = V - V_t \le V$ is always bounded. If there exists some time instance $T > 0$ where $\hat{P}_i(T)$ is not positive-definite, then by a continuity argument there exists time instance $0 < t_0 \le T$ such that $\hat{P}_i(t_0)$ is on the boundary of $\mathcal{P}(4)$, i.e., at least one of the eigenvalues of $\hat{P}_i(t_0)$ is zero. This contradicts the fact that $V_p(t)$ remain bounded for all $t > 0$, since $V_p(t_0)$ is infinity. The present adaptation law therefore always guarantees physical consistency of $\hat{\Phi} \sim \{\hat{P}_i\}_{i=1}^n$. □

From an optimization perspective, the existing parameter update rule (9) can be viewed as a steepest gradient descent-like update [2, page 414] in Euclidean space with constant metric $\Gamma = \gamma I$, for some constant scalar γ. On the other hand, our method is analogous to the natural gradient descent method, which generalizes the steepest descent method to general curved Riemannian manifolds (Note that the natural gradient on $\mathcal{P}(4)$, derived as $dP \sim -P\nabla hP$ in (17), has an identical form as our natural adaptation law of (27), $\dot{P}_i \sim -P_iB_iP_i$). The natural gradient is not only coordinate-invariant, but also known to provide a more efficient direction of descent in terms of the metric predefined on the space [6]. Moreover, the resulting natural gradient-like realization of the adaptation law actually turns out to guarantee physical consistency of the adapted parameters, which we proved using the Bregman divergence in a Lyapunov-based analysis rather than by using the exact Riemannian distance metric.

Remark 3. *Our natural adaptation law (27) can also be expressed in vector form as follows:*

$$\dot{\hat{\Phi}} = -\frac{1}{\gamma}\Gamma(\hat{\Phi})^{-1}b. \tag{28}$$

Here the matrix $\Gamma(.) \in \mathbb{R}^{10n\times 10n}$ is a coordinate representation of the pullback F^*G of the Riemannian metric $G = g_1 \otimes \cdots \otimes g_n$, where g_i is the Riemannian metric on $\mathcal{P}(4)$ defined by (13), and $F : \mathbb{R}^{10n} \to \mathcal{P}(4)^n$ is defined by n copies of the linear mapping f (11), i.e., $F(\Phi) = (f(\phi_1), \cdots, f(\phi_n))$. It can be observed that a state-dependent, time-varying choice of gain matrix Γ is admissible, as opposed to an arbitrary constant one as in conventional approaches. As a by-product, one only has to determine a single tuning variable γ, which is essentially the adaptation speed.

Remark 4. *In principle, tracking error in the passivity-based direct adaptive controllers [8], [2] is guaranteed to asymptotically converge to zero with arbitrary constant choice of gain matrix Γ without regarding to the physical consistency of the estimated parameters. However, as will be shown in the following section, the transient performance of a direct adaptive controller can in fact be largely improved by the proposed natural choice of $\Gamma(\hat{\Phi})$ that also ensures the physical consistency of the estimated parameters.*

V. Simulation Results

In this section, we present extensive simulation experiments to assess the tracking performance of our natural adaptive controller vis-a-vis existing adaptive controllers. The simulations are conducted on a torque-controllable Barett WAM seven-DOF manipulator arm model under the passivity-based adaptive control framework of [8]. The modified recursive Newton-Euler algorithm is used to implement the adaptive controller based on [12]. The simulations are all implemented in matlab. Euler integration with a fixed stepsize of 1e-3 seconds is used for the parameter adaptation implementation, since the estimated inertial parameters will be updated digitally in real implementation. For improved simulation fidelity, the forward dynamics is updated at a faster rate of 10kHz.

The four different adaptation laws to be compared are labelled as follows: 'No-adaptation', 'Euclidean', 'Const. pullback' and 'Natural'. 'No-adaptation' indicates a pure passivity-based tracking controller with a fixed parameter estimate, i.e. $\hat{\Phi}(t) = \hat{\Phi}(0)$. 'Euclidean' and 'Const. pullback' are passivity-based adaptive controllers with the constant adaptation gain matrix respectively chosen as $\Gamma = \gamma \cdot I$ and $\Gamma = \gamma \cdot \Gamma(\hat{\Phi}(0))$ (a constant pullback Riemannian metric evaluated on the initial estimate $\hat{\Phi}(0)$), for some constant scalar γ. When manually tuning the constant adaptation gain matrix, for reasonably close initial estimates, a non-identity or non-diagonal Γ for 'Const. pullback' is a reasonable choice that balances the update law with respect to the scales for each of the parameters. Finally, 'Natural' indicates a passivity-based adaptive controller using our natural adaption law. The adaptation speed factor γ^{-1} is set at a scale as large as possible without accruing large numerical integration

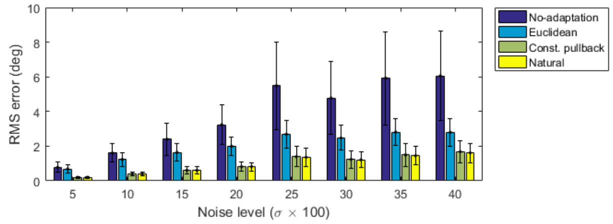


Fig. 1. Plot of joint tracking RMS error when adapting the entire set of the inertial parameters with various levels of noise in the initial estimate

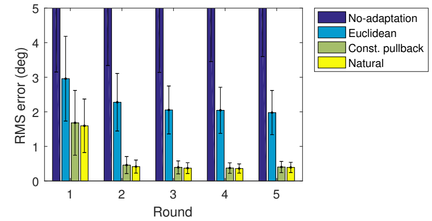


Fig. 2. Plot of joint tracking RMS error when adapting the entire set of the inertial parameters on a varied sequence of desired trajectories, with a fixed noise level for the initial estimate; $\sigma = 0.4$. RMS errors for 'No-adaptation' all exceeded 7 deg

errors for the given stepsize; for 'Euclidean', $\gamma^{-1} = 5$e-5, while for 'Const. pullback' and 'Natural', $\gamma^{-1} = 1.0$. As noted in the Remark 1, the control gains are set to $K = \lambda \cdot M(q, \hat{\Phi})$, $\Lambda = \lambda \cdot I$, and $\lambda = 10.0$.

The common task in our experiments is to track a periodic joint trajectory of the form $q_{d,i}(t) = A_i[\cos(2\pi t/T_i) - 1]$ for $i = 1, \cdots, n$, where $q_{d,i}$ is the desired joint angle of the i^{th} joint, A_i is the amplitude, and T_i is the period.

A. Adaptation of the Complete Set of Inertial Parameters

We adapt the entire set of inertial parameters for each of the links. First, to compare the robustness of the controllers with respect to various levels of model uncertainty, initial estimates of the inertial parameters are deliberately perturbed from the true parameters, i.e., $\hat{\phi}_i(0) = \phi_i \cdot (1 + \epsilon_i)$, where ϵ_i is drawn 100 times from a zero-mean Gaussian distribution with standard deviation σ evenly spaced at 0.05 intervals, from 0.05 to 0.40 (ϵ_i is truncated to be within the range $[-\sigma, \sigma]$ to prevent the initial estimate from being physically inconsistent). The desired trajectory parameters are fixed with amplitude $A_i = 0.8$ and period $T_i = 5$ sec.

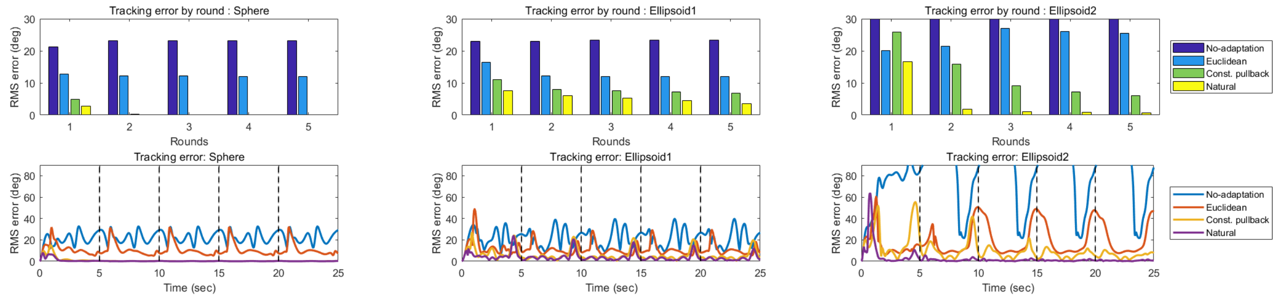
As shown in Figure 1. our method 'Natural' and 'Const. pullback' outperform other methods in tracking performance at all noise levels. For the 'Euclidean' method, the estimated parameters were physically inconsistent for over 90 % of the operation time, which implies that using projection or resetting methods for 'Euclidean' merely enforce the physical consistency of the estimated parameters to be on the boundary of a user-specified (and ad hoc) feasible set. On the other hand, 'Natural' and 'Const. pullback' were shown to always provide physically consistent estimation parameters. Although the 'Const. pullback' method is not yet theoretically guaranteed to always ensure physical consistency, our results highlight the fact that a reasonable choice of metric or adaptation gain matrix Γ plays a more critical role in physically consistent estimation than only considering physical consistency on the boundary of the feasible region.

We now fix the model uncertainty to $\sigma = 0.40$ and consider trajectory tracking for a varied sequence of desired trajectories, each having different amplitudes A_i evenly spaced at 0.2 from 0.4 to 1.2. As can be observed in Figure 2., 'Natural' and 'Const. pullback' considerably reduce the tracking error after the first round compared to 'Euclidean' and continues to maintain this level of tracking performance. The adapted parameters from 'Natural' and 'Const. pullback', while achieving physical consistency, are more generalizable to varied task situations. A lack of generalizability implies that the robot could behave unexpectedly in unexpected task situations. In this regard, 'Natural' and 'Const. pullback' methods show more promise with respect to robustness.

B. Adaptation for an Unknown Load

We now consider adaptation of only the last end-effector link, to emulate situations where an unknown load or tool is loaded on the end-effector. The repeated sequence of constant amplitude 0.8 is used as the desired trajectory. The inertial parameters (except for the end-effector link) are fixed as true parameters. For the initial estimates, we assume a blind guess of the unknown loads assuming a 1kg sphere of radius 0.1m. The unknown loads considered for the experiment are of three types: 'Sphere', 'Ellipsoid1', and 'Ellipsoid2', respectively indicating a uniform sphere with radius 0.05m, a uniform ellipsoid with aspect ratio [0.0323m: 0.3227m: 0.0323m], and a uniform ellipsoid with aspect ratio [0.1375m: 0.2165m: 0.1082m] and center of mass offset from the initial estimate by $p =$ (0.05m, 0.04m, -0.07m), $\|p\| =$ 0.095m. All the unknown loads weigh 3kg.

In Figure 3, our method, 'Natural', is shown to outperform other methods in terms of transient behavior and tracking error convergence. However, for the case of 'Ellipsoid1', our method did not show superior error convergence to other methods, compared to the cases of 'Sphere' and 'Ellipsoid2'. In fact, the initial estimate, sphere, is highly biased from the pipe-shaped load, 'Ellipsoid1', which makes the adaptation most challenging out of the three cases. We believe that such limitations arising from bad initial estimates are an inherent feature of direct adaptive control methods, since the adaptation is done in a local gradient-like sense, directly compensating the model uncertainties with the instant *tracking error*. Indirect [10], [16] or modularized approaches [24], in which the accumulated past trajectory information is preserved in a more consistent way via propagation of the covariance or observation matrix from, e.g., least-squares estimation, may perform better when there are significant errors in the initial estimate. However, they often may not satisfy the asymptotic convergence condition, e.g. uniform invertibility of the estimated mass matrix, or require additional torque and joint acceleration measurements. A composite approach [11] involving setting the initial covariance matrix to be



(a) Sphere (b) Ellipsoid1 (c) Ellipsoid2

Fig. 3. Joint tracking RMS error plots for adaptation of three kinds of unknown loads; 3(a) Sphere, 3(b) Ellipsoid1, and 3(c) Ellipsoid2. The plots above show mean RMS tracking error for repeated rounds of periodic desired trajectories, while the bottom plot shows a time plot of RMS tracking error. The color legends for all the plots are depicted in 3(c). Mean RMS tracking error for 'No-adaptation' in 3(c) all exceed 70 deg.

the inverse of the pullback Riemannian metric on the initial estimate, could be a viable solution in this regard.

VI. CONCLUSION

We have proposed a new type of smooth and physically consistent adaptation law for direct adaptive control of robot manipulators. The robustness and practical utility of our algorithm, which has the added advantage of being implementable without excessive prior tuning, are both traceable in large part to exploiting the coordinate-invariant geometric structure of the inertial parameter space. To highlight, our geometric approach does not impose any additional computational burden compared to existing methods. Hardware experiments are currently being conducted for a more complete evaluation of our algorithm, in addition to the inclusion of joint friction parameters adaptation into our adaptation law.

Adaptive control methods are currently being developed for more advanced robotic tasks, e.g., interaction, constrained or hybrid position/force control [19], [20], or for complex structures, e.g., underactuated [21], flexible-joint [22], and spacecraft [23], with most existing works adopting the Lyapunov-based framework used in [7], [8]. Applying our natural adaptation laws to these more advanced tasks appears to be potentially profitable and direction for future work.

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