ECE7410/ENGI9804 Image Processing and Applications

Laboratory 4

Hu's invariant moments

Introduction

The objective of this exercise is to understand and calculate some features used as image descriptors. Hu's seven moment invariants are insensitive to changes in scale, position and rotation. These moment invariants have been extensively applied to image pattern recognition, image registration, and image reconstruction.

Procedure

The 2-D moment of order *(p+q)* of a digital image *f(x,y)* of size $M \times N$ is defined as

$$m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p y^q f(x, y) \tag{1}$$

where $p = 0, 1, 2, ...$ and $q = 0, 1, 2, ...$ are integers. The corresponding central moment of order $(p + q)$ is defined as:

$$\mu_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \bar{x})^p (y - \bar{y})^q f(x, y) \tag{2}$$

where $\bar{x} = \frac{m_{10}}{m_{00}}$ and $\bar{y} = \frac{m_{01}}{m_{00}}$. The normalized central moments are defined as:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \tag{3}$$

where $\gamma = \frac{p+q}{2} + 1$ for $p + q = 2, 3, ...$

A set of seven invariant moments can be derived from the second and third moments:

$$\phi_1 = \eta_{20} + \eta_{02} \tag{4}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \tag{5}$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \tag{6}$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \tag{7}$$

$$\begin{aligned} \phi_5 = {} & (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\ & + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \end{aligned} \tag{8}$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \tag{9}$$

$$\begin{aligned} \phi_7 = {} & (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\ & - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \end{aligned} \tag{10}$$

Calculating the moment invariants

1. Download the grayscale test image (image.png), *calculate_moments.m*, and *Moment_invariants.m* files from Brightspace under *Lab 04* and save them in your working directory. Read the files and understand the implementation of calculating the seven invariant moments.

2. [5] Read the image, obtain the size of the image and pad the image by one-fourth the image size in all directions with zeros, and display the padded image. Use this as the basis for next steps (referred to as Im1).

3. [8] Create and display the spatial transformation matrices T1 for translation, T2 for scaling to 0.5 (of original size), T3 to rotate the image by 45 degrees, T4 for rotating the image 90 degrees as follows:

```
T1 = maketform('affine', [1 0 0; 0 1 0; Xt Yt 1]);
% Xt, Yt are one-fourth of your original image
T2 = maketform('affine', [0.5 0 0; 0 0.5 0; 0 0 1]);
T3 = maketform('affine', [cos(pi/4) sin(pi/4) 0; -sin(pi/4) cos(pi/4) 0; 0 0 1])
    ;
T4 = maketform('affine', [cos(pi/2) sin(pi/2) 0; -sin(pi/2) cos(pi/2) 0; 0 0 1])
    ;
```

4. [5] Transform Im1 with T1 and display the translated Im2:

```
Im2 = imtransform(Im1, T1, ...
'XData',[1 size(Im1,1)], 'YData',[1 size(Im1,2)]);
```

5. [5] Transform Im1 with T2 and display the scaled Im3:

```
Im3 = imtransform(Im1, T2, ...
'XData',[1 size(Im1,1)], 'YData',[1 size(Im1,2)]);
```

6. [5] Transform Im1 with T3 and display the rotated Im4:

```
Im4 = imtransform(Im1, T3, ...
'XData',[-269 size(Im1,1)-270], 'YData',[+111 size(Im1,2)+110]);
```

7. [5] Transform Im1 with T4 and display the rotated Im5:

```
Im5 = imtransform(Im1, T4, ...
XData',[-539 size(Im1,1)-540], 'YData',[1 size(Im1,2)]);
```

8. [5] Flip the original image Im1 from left to right using following command, generate and display flipped Im6.

```
Im6=flipdim(Im1,2);
```

9. [$6\times 2 = 12$] The file *Moment_invariant.m* contains a function *Moment_invariant()*, which calls another function named *calculate_moments*. Collectively, these functions calculate the moment invariants. Use the function *Moment_invariant()* with images Im1 to Im6 (e.g., `Moment_invariants(Im1);`)to calculate the moment invariants for each image. Report the moments for each image in a table.

10. Discuss the values of the moment invariants computed for each image, including any similarities or differences. What do these similarities or differences mean in terms of image features?

Important notes [20 points]

1. [10] You ***MUST*** include all of your code (such that it can be easily run to verify your results), results and discussion in your lab report, including also any images that are not included with the lab.

2. [10] You ***MUST*** name your lab report G#.pdf using your group number (e.g., G03.pdf), and produce a high-quality report.

3. Submit your work to Brightspace in the appropriate dropbox folder.