# Day 1

*Definition*: A sequence $\{a_n\}$ is a set of numbers *with a defined order* (not necessarily having a pattern).

Individual values (not continuous)

You should think of a sequence $\{a_n\}$ as a *discrete function* in which the *domain* is the set of natural numbers $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$.

Reminder: a *function* is any relationship in which each input from the *domain* (first element) is paired with only one output from the range (second element).

*Definition*: An ***explicit definition/formula*** is determined solely by the position ($n$) of the term. The *rule* is defined only in terms of the position ($n$) of the term.

*Definition*: A ***recursive definition/formula*** for defining the value of the terms in a sequence is based on previous terms. One must provide . . .

a. the ***initial terms(s)*** of the sequence,
b. the ***rule/equation*** that defines the next term of the sequence, and
c. the ***domain*** over which the rule works.

***Ex. 3***: Write a recursive formula for each sequence below.

a. $\{a_n\} = \{-3, 1, 5, 9, \ldots\}$ b. $\{b_n\} = \{8, 5, 2, -1, \ldots\}$

first term $a_1 = -3$
rule $a_n = a_{n-1} + 4$ or $a_{n+1} = a_n + 4$
domain $n \geq 2$ $n \geq 1$

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## Conclusions

1. A *sequence* is simply a set of numbers.
2. An *arithmetic sequence* is a set of numbers that has a *common difference* between successive terms.
3. The graph of the terms in an *arithmetic sequence* form a linear function.
4. An *explicit formula* for an *arithmetic sequence* is written in terms of the position of the term only and is written as a linear function.
5. A *recursive formula* for an *arithmetic sequence* must have a starting (initial) value, then defines the value of the "next term" by the value of the "previous term".

Day 2

*Definition*: A sequence $\{a_n\}$ is a set of numbers *with a defined order* (not necessarily having a pattern).

You should think of a sequence $\{a_n\}$ as a *discrete function* in which the *domain* is the set of natural numbers $\mathbb{N} = \{1, 2, 3, 4, \ldots.\}$.

Reminder: a *function* is any relationship in which each input from the *domain* (first element) is paired with only one output from the range (second element).

There are many types of sequences, but we will be concerned, for now, with only two types. We will discuss other types later.

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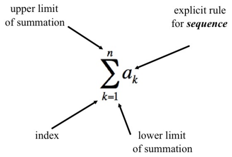
Day 3-4

## Sigma Notation

*Definition*: A ***series*** $S_n = \sum_{k=1}^{n} a_k$ is the *sum* of the terms in a sequence $\{a_n\}$.

The symbol $\sum$ is called sigma (Greek letter S) and represents the sum of terms. Shown below are the specifics about *sigma/summation notation*.




Day 5

$$S_n = \frac{n}{2}(a_1 + a_n)$$

last term

first term

In general to find the sum $S_n$ of the terms in an arithmetic series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

or

$$S_n = \frac{n}{2}(2a_1 + d(n-1))$$

Day 5 ~ 6

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$S_n = \frac{n}{2}(a_1 + a_n)$

or

$S_n = \frac{n}{2}(2a_1 + d(n-1))$

*Definition*: a ***geometric series*** is the sum of the terms in a *geometric sequence*.

Day 7

*Definition*: An infinite series ***converges*** if the sum of the terms in the sequence approach ***a*** *specific value* as the number of terms increases (as $n \longrightarrow \infty$).

*Definition*: An infinite series ***diverges*** if the sum of the terms in the sequence *do not* approach ***a*** specific value as the number of terms increases (as $n \longrightarrow \infty$).

*Note*: if a series is finite, it will converge (think about it), so we will consider only infinite series here.

converges

a. $\sum_{k=1}^{\infty} 2^{-k} = 1$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$

## Infinite Geometric Series

Let $g_n$ be a geometric sequence and define $S = \sum_{k=1}^{\infty} g_k$.

1. If $|r| < 1$, what is $\lim_{k \to \infty} r^k =$ ___0___? Justify your answer.

convergent

2. If $|r| > 1$, what is $\lim_{k \to \infty} r^k =$ ___$\pm\infty$___? Justify your answer.

divergent

So, in summary if $|r| < 1$, then $S = \frac{g_1}{1-r}$ and if $|r| > 1$, then $S$ diverges.

***Ex. 4***: Given each infinite geometric series below, determine whether it converges or diverges. If it converges, then determine the value to which it converges; if it diverges, then explain why it diverges.

a. $64 + 48 + 36 + 27 + \ldots$

converges if if $\left(\frac{48}{64} = \frac{36}{48} = \frac{27}{36} \ldots\right)$

b. $\sum_{k=1}^{\infty} 50 \cdot \left(-\frac{2}{3}\right)^{k-1}$

$r = -\frac{2}{3}$

converges at $\frac{50}{1+\frac{2}{3}} \to \frac{50}{5/3} = 30$

***Ex. 4***: Given each infinite geometric series below, determine whether it converges or diverges. If it converges, then determine the value to which it converges; if it diverges, then explain why it diverges.

a. $64 + 48 + 36 + 27 + \ldots$

converges if if $\left(\frac{48}{64} = \frac{36}{48} = \frac{27}{36} \ldots\right)$

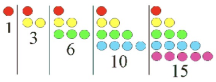
# Day 8

***Ex. 1***: Consider the sequence being defined below.

a. Find the next two terms in the sequence: $t_n = 1, 3, 6, 10, 15,$ 21, 28
Explain how you determined the next two terms.

$t_n = x^2$

These numbers are called "Triangular Numbers" as they can be represented by triangles of dots (as shown below).




b. Is this sequence arithmetic, geometric, or neither? neither

c. Write a recursive formula for this sequence.

$t_n = t_{n-1} + n$

$t_1 = 1;\ t_n \geq 2$

d. Write an explicit formula for this sequence. Explain how you determined your answer.

$t_n = \frac{1}{2}x^2 + \frac{1}{2}x$

$t_n = \frac{n}{2}(1+n)$

$t_n = \frac{n(n+1)}{2}$

$t_1 = 1$

$t_2 = 3 = 1 + 2$

$t_3 = 6 = 3 + 3 = 1 + 2 + 3$

$t_n = 1 + 2 + 3 \ldots + n$

e. Let's consider the difference of the differences. What do you notice?

quadratic   1 1 1 1   Degree 2

(Arithmetic) linear   2 3 4 5 6   Degree 1

1, 3, 6, 10, 15, 21

f. Does your answer to (e) help you to determine what degree polynomial is defining our initial sequence: 1, 3, 6, 10, 15, . . . ?

Amount of "levels" defines degree of sequence

## Conclusion

$a_n = a_1 + d(n-1)$    $g_n = g_1 r^{n-1}$    $S_n = \frac{n}{2}(a_1 + a_n)$    $S_n = g_1 \cdot \frac{1-r^n}{1-r}$    $S = \frac{g_1}{1-r}$

The number of levels it takes to determine a *non-zero* common difference will determine the degree of the polynomial defining the sequence. This is called the ***Method of Finite Differences***.

| | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
|---|---|---|---|---|---|
| $t_1 = 1$ | 2 | 1 | | | |
| $t_2 = 3$ | 3 | 1 | | | |
| $t_3 = 6$ | 4 | 1 | | | |
| $t_4 = 10$ | 5 | | | | |
| $t_5 = 15$ | | | | | |
| | degree 1 Linear | degree 2 Quadratic | degree 3 Cubic | degree 4 Quartic | degree 5 Quintic |

$a_1 = -2$

1   $a_n = n^2 - 5n + 2$

$a_{n+1} = (n+1)^2 - 5(n+1) + 2$

...

$= (n^2 - 5n + 2) + (2n - 4)$

$a_n + (2n - 4)$

2   $a_n = n^2 - 5n + 2$

$a_{n-1} = (n-1)^2 - 5(n-1) + 2$

$= n^2 - 2n + 1 - 5n + 5 + 2$

$= n^2 - 7n + 8$

$a_{n-1} + (2n - 6)$

$a_{n+1} = a_n + (2n-4);\ n \geq 1$

$a_n = a_{n-1} + (2n-6);\ n \geq 2$