Linear Programming Problem

25/11/2024

1. Maximizing Profit for a Factory.

Table

Product	Constraint		Profit
	Machine Time	Raw Material	
A	2	1	3
B	3	2	4
Max available	12	8	

Solution

let Product A = x

let Product B = y

Max $Z = 3x + 4y$

Contraint equation

$2x + 3y \leq 12$ --- (1)

$x + 2y \leq 8$ --- (2)

Non negativity Constraint

$x \geq 0$

$y \geq 0$

Extreme Points.

$2x + 3y = 12$ --- (1)

for $x = 0$

$2(0) + 3y = 12$

$3y = 12, \quad y = 4$

for $y = 0$

$2x + 3(0) = 12$

$2x = 12$

$x = 6$.

Extrem Points for equation (2)

$x + 2y = 8$

for $x = 0$

$0 + 2y = 8$

$2y = 8$

$y = 4$

for $y = 0$

$x + 2(0) = 8$

$x = 8$,,

Extreme Points :

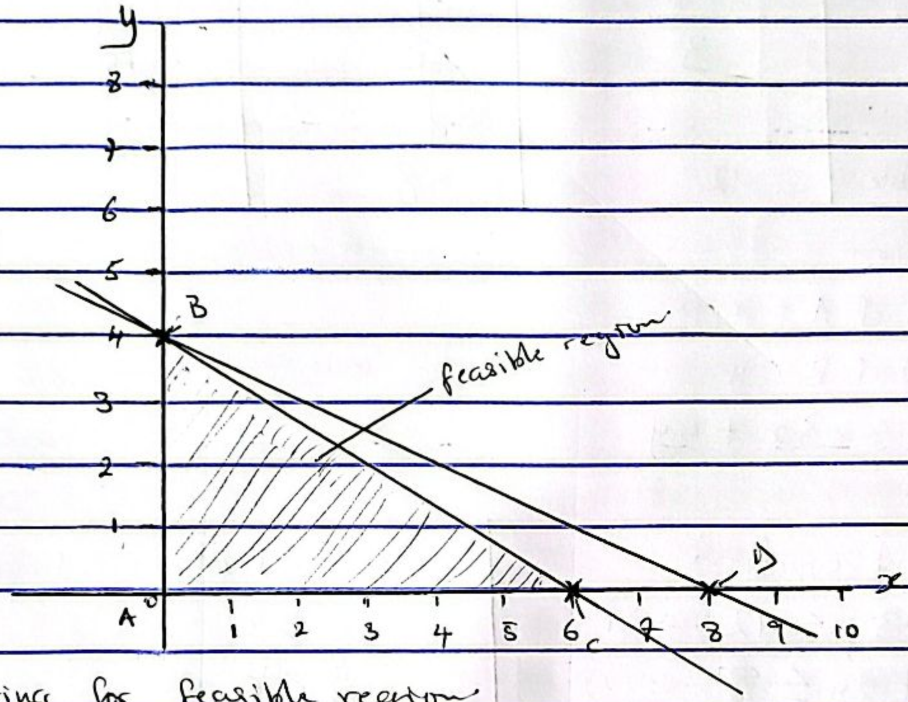
(0, 4)

(6, 0)

(0, 4)

(8, 0)

Graph :



Solving for feasible region

A (0, 0)

B (0, 4)

C (6, 0)

D (8, 0)

Objective function

$Z = 3x + 4y$

finding the Optimal solution.

Extreme Points	Cordinates	Z Objective function.
A	(0, 0)	$3(0) + 4(0) = 0$
B	(0, 4)	$3(0) + 4(4) = 16$
C	(6, 0)	$3(6) + 4(0) = 18$
D	(8, 0)	$3(8) + 4(0) = 24$.

∴ The optimal solution to Maximize Profit is to produce 8 A and 0 B.

2. Minimizing Cost for a Manufacturer.

Table

Product	Constraint		Cost
	Machine time	Raw Material	
x	1	2	2
y	2	1	5
Max available	6	5	

Objective function:

Min $Z = 2x + 5y$.

Constraint equations. — Non negativity Contraint.

$x + 2y \leq 6$ ---① — $x \geq 0$

$2x + y \leq 5$ ---② — $y \geq 0$

Solving for extreme points:

$x + 2y = 6$ -①	$2x + y = 5$ ---②
when $x = 0$	when $x = 0$
$0 + 2y = 6, \; y = 3$	$2(0) + y = 5, \; y = 5$
when $y = 0$	when $y = 0$
$x + 2(0) = 6, \; x = 6$	$2(x) + 0 = 5, \; x = 2.5$

Extrem Points:

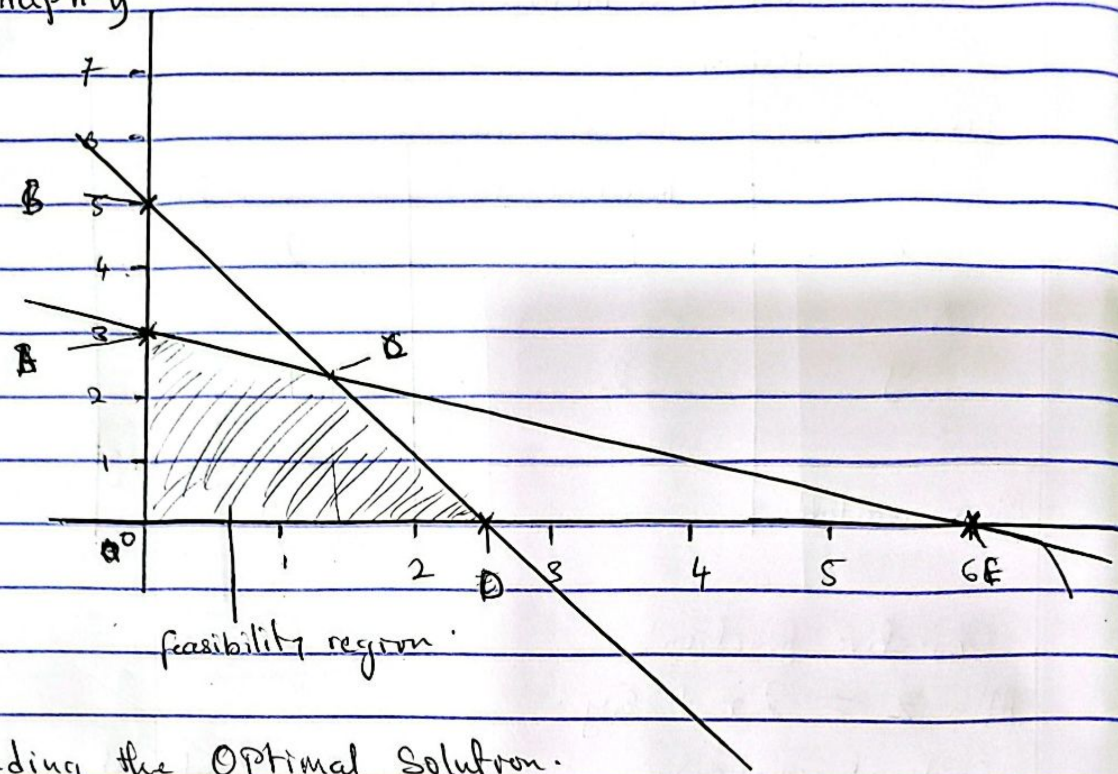
$(0, 3)$

$(6, 0)$

$(0, 5)$

$(2.5, 0)$

Graph.



Finding the Optimal Solution.

Extreme Point	Cordinates	Z objective function
O	(0, 0)	$2(0) + 5(0) = 0$
A	(0, 3)	$2(0) + 5(3) = 15$
B	(0, 5)	$2(0) + 5(5) = 25$
C	(1.4, 2.4)	$2(1.4) + 5(2.4) = 14.8$
D	(2.5, 0)	$2(2.5) + 5(0) = 5$
E	(6, 0)	$2(6) + 5(0) = 12$

∴ The Optimal solution to Minimize Cost with a non-zero Constraint ~~too~~ is D. to produce 2.5 of x and 0 of y.

Vug/csc/23/9561

Maximizing Production with Multiple Resources.

3. Table

Product	Constraint			Profit
	Labour	Material	Machine Time	
A	2	3	1	5
B	1	2	2	4
Max avail	20	30	18	

Solution

Let Product A $= x$

Let Product B $= y$

Max $z = 5x + 4y$

Constraint equations	Non-negativity Constraint
$2x + y \leq 20$ --- ①	$x \geq 0$
$3x + 2y \leq 30$ --- ②	$y \geq 0$
$x + 2y \leq 18$ --- ③	

Solving for extreme Points:

$2x + y = 20$ ... ①	for $y = 0$
for $x = 0$	$2x + 0 = 20$
$2(0) + y = 20,\ y = 20$	$2x = 0,\ x = 10$

equation 2.

$3x + 2y = 30$	for $y = 0$
for $x = 0$	$3x + 2(0) = 30$
$3(0) + 2y = 30$	$3x = 30$
$2y = 30,\ y = 15$ //	$x = 10$ //

Equation --- 3

$x + 2y = 18$

for $x = 0$

$0 + 2y = 18$

$y = 9$

for $y = 0$

$x + 2(0) = 18$

$x = 18$

Coordinates (Extreme Points)

(0, 20)

(10, 0)

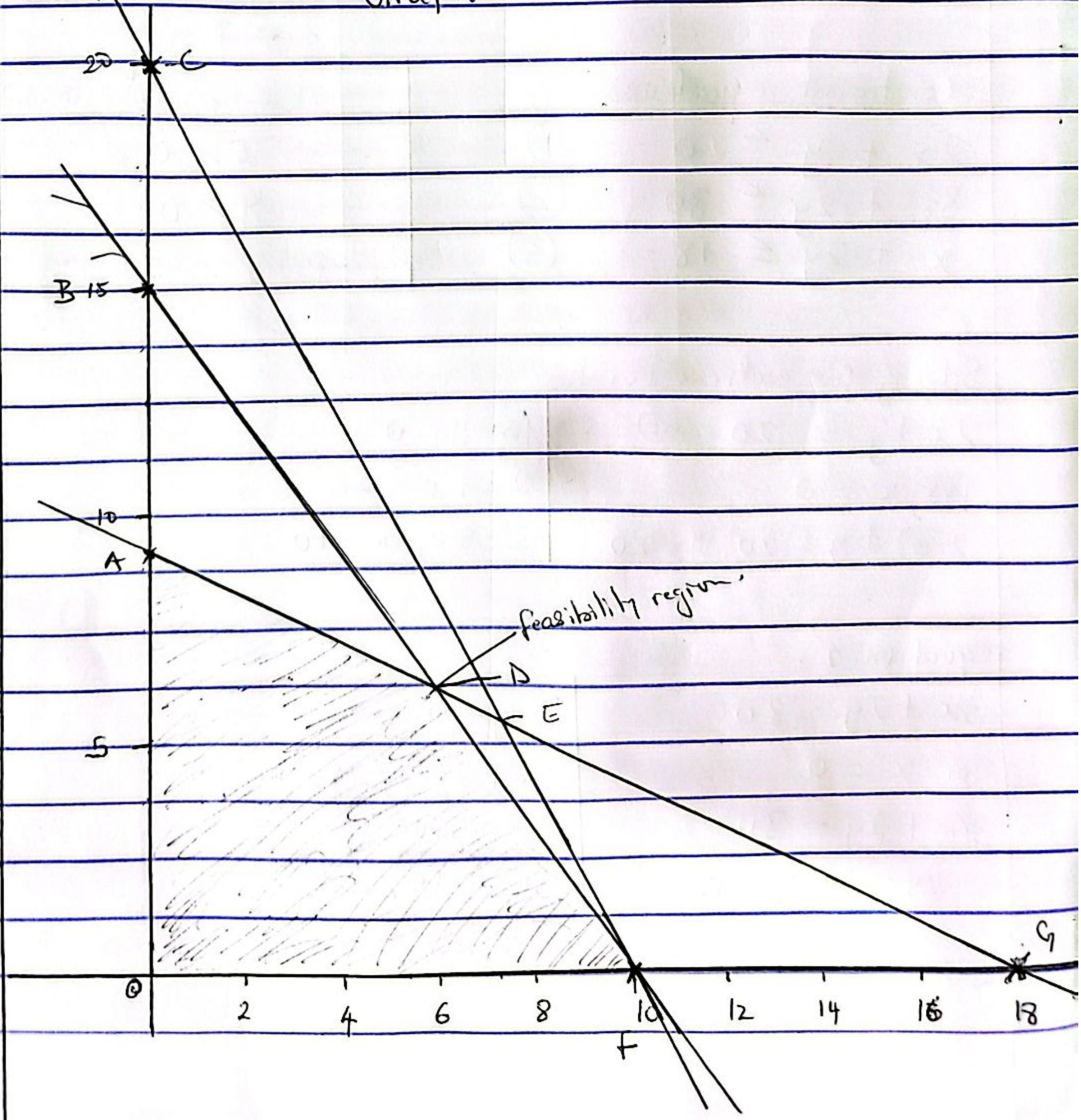
(0, 15)

(10, 0)

(0, 9)

(18, 0)

Graph.



Solving for feasible region

O (0, 0)
A (0, 9)
B (0, 15)
C (0, 20)
D (   ) ? = ~~(0, 6.4)~~ = (6.4, 6)
E (   ) ? = (5.7, 7.2)
F (10, 0)
G (18, 0)

Finding the Optimal Solution

Extreme Points	Coordinates	Z Objective function
O	(0, 0)	5(0) + 4(0) = 0
A	(0, 9)	5(0) + 4(9) = 36
B	(0, 15)	5(0) + 4(15) = 60
C	(0, 20)	5(0) + 4(20) = 80
D	(6.4, 6)	5(6.4) + 4(6) = 49.6
E	(5.7, 7.2)	5(5.7) + 4(7.2) = 57.3
F	(10, 0)	5(10) + 4(0) = 50
G	(18, 0)	5(18) + 4(0) = 90

∴ The optimal solution to maximize total profit is 18 A and 0 B.

Table.

4.

Product	Constraint		~~Profit~~ Revenue.
	Advert	Production Capacity	
A	1	1	4
B	2	2	5
Max availab	20	15	

Max $Z = 4x + 5y$

let Product A = x

let Product B = y

Constraint Equation.	Non negativity Constraint
$x + 2y \leq 20$ --- ①	$x \geq 0$
$x + 2y \leq 15$ --- ②	$y \geq 0$

Solving for Extreme points:

~~w~~equation --- ①	equation -- ②
when $x = 0$	when $x = 0$
$x + 2y = 20, = 2y = 20$	$0 + 2y = 15$.
$y = 10$	$y = 7.5$
when $y = 0$	when $y = 0$
$x + 2(0) = 20$	$x + 2(0) = 15$
$x = 20$	$x = 15$

Coordinates

(0, 10)

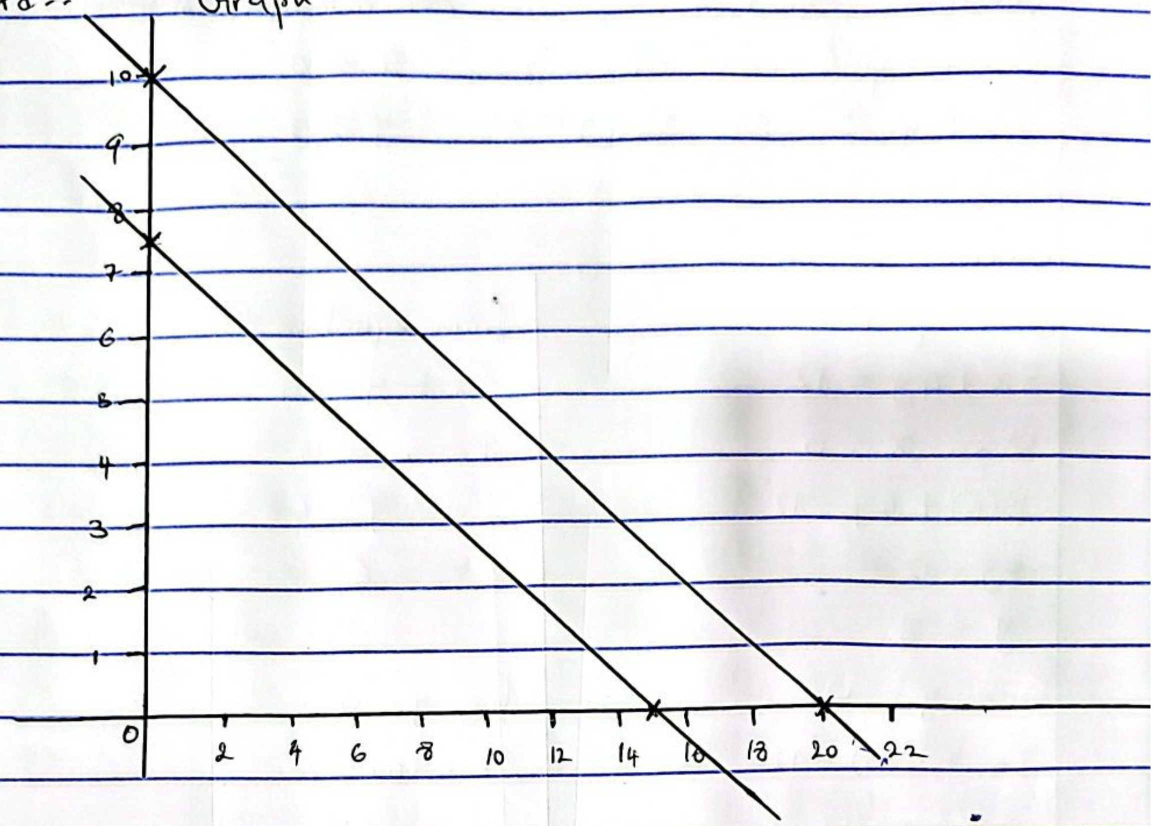
(20, 0)

(0, 7.5)

(15, 0)

Vug/csc/23/95G1

(4) Contd.. Graph:



looking at the graph, since the two lines are parallel, there is no feasible region where both constraints are satisfied.
This implies that the problem has no solution

5. Table:

Projects	Constraints		Profit.
	Labour hours	Capital Investment.	~~Profit~~.
P_1	3	2	8
P_2	4	1	7
Max available	12	6	

Solution

let $P_1 = x$

let $P_2 = y$

Max $z = 8x + 7y$

Constraint Equation:

$3x + 4y \le 12$ --- ①

$2x + y \le 6$ --- ②

Non-negativity Constraint

$x \geq 0$

$y \geq 0$

Extreme points

from equation ①

$3x + 4y = 12$,

When $x = 0$

$3(0) + 4y = 12$

$4y = 12$

$y = 3$

When $y = 0$

$3x + 4(0) = 12$

$3x + 0 = 12$

$3x = 12$

$x = 4$

from equation ⑪

$2x + y = 6$

When $x = 0$

$2(0) + y = 6$

$y = 6$

When $y = 0$

$2x + (0) = 6$

$2x = 6$

$x = 3$

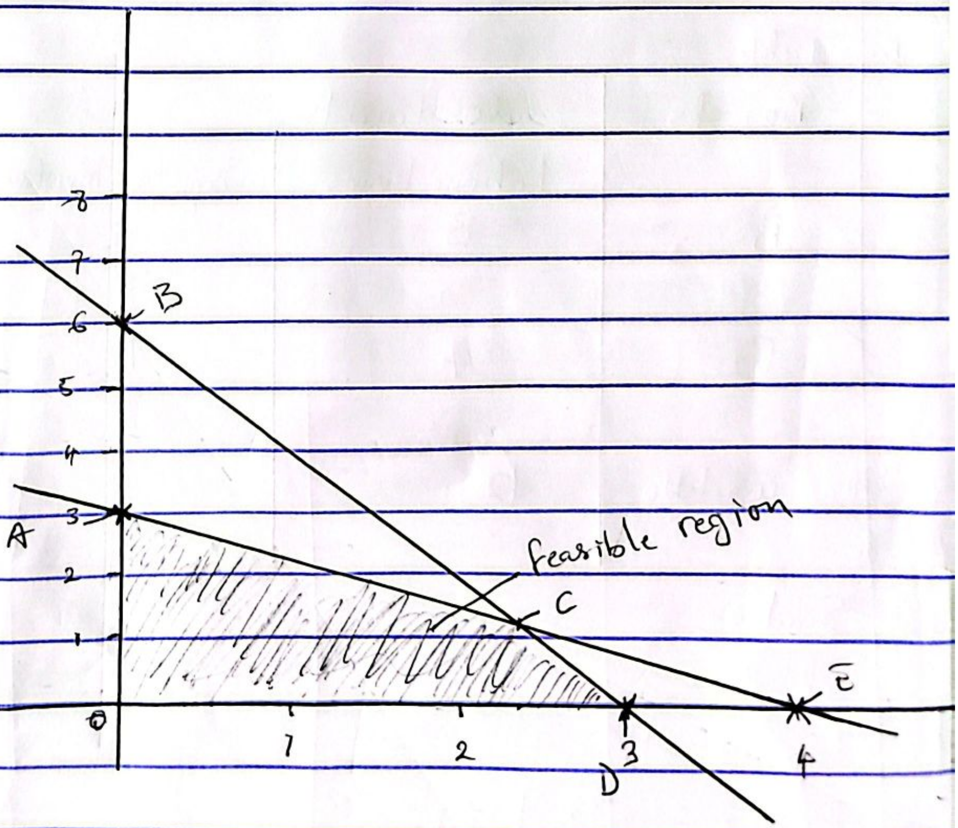
Extrem Points:

(0, 3)

(4, 0)

(0, 6)

(3, 0)



Graph.

Solving for feasible region.

O (0, 0)	from the graph,	Objective function:
~~A~~ A (0, 3)	C = (2.5, 1.3)	$Z = 8x + 7y$
B (0, 6)		
C ( ) ??		
D (3, 0)		
E (4, 0)		

Finding the Optimal Solution.

Extreme Point	Coordinates	Z Objective function
O	(0, 0)	$8(0) + 7(0) = 0$
A	(0, 3)	$8(0) + 7(3) = 21$
B	(0, 6)	$8(0) + 7(6) = 42$
C	(2.5, 1.3)	$8(2.5) + 7(1.3) = 29.1$
D	(3, 0)	$8(3) + 7(0) = 24$
E	(4, 0)	$8(4) + 7(0) = 32.$

∴ The optimal solution to Maximize Profit within the feasible region is to ~~Produce~~ use 2.5 hours of labour hours and 1.3 of Capital Investment on the Products. Which is option C.

6 Table

Product	Constraints		Profit
	Baking time	Flour.	
Chocolate Cake	1	3	5
Vanilla Cake	2	2	3
Max available	8	12	

$Z\ Max = 5x + 3y.$

Solution:

Let Chocolate Cake $= x$

Let Vanilla Cake $= y$

Constraint Equations:

$x + 2y \leq 8$ --- (1)

$3x + 2y \leq 12$ --- (2)

Non-negativity Constraint

$x \geq 0$

$y \geq 0$

Solving for Extreme Points:

From equation (1)

$x + 2y = 8$

When $x = 0$

$0 + 2y = 8$

$y = 4$

When $y = 0$

$x + 2(0) = 8$

$x = 8$

From equation (2)

$3x + 2y = 12$

When $x = 0$

$3(0) + 2y = 12$

$y = 6$

When $y = 0$

$3x + 2(0) = 12$

$x = 12/3 = 4$

Extreme Points:

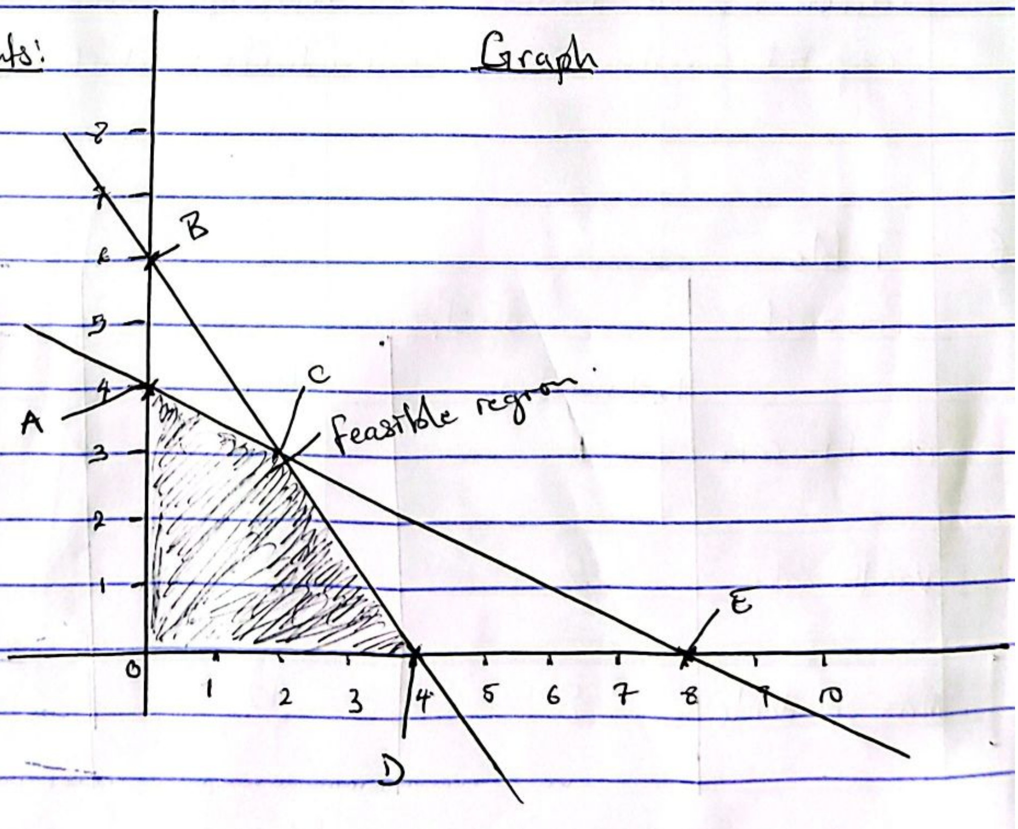
(0, 4)

(8, 0)

(0, 6)

(4, 0)

Graph



VUG/CSC/23/9561

Solving for feasible region.

O (0, 0)	From the Graph	Objective function
A (0, 4)	C = (2, 3)	$Z = 5x + 3y$.
B (0, 6)		
C ( ) ??		
D (4, 0)		
E (8, 0)		

Finding the Optimal Solution.

Extreme Points	Coordinates	Z objective function
O	(0, 0)	$5(0) + 3(0) = 0$
A	(0, 4)	$5(0) + 3(4) = 12$
B	(0, 6)	$5(0) + 3(6) = 18$
C	(2, 3)	$5(2) + 3(3) = 19$
D	(4, 0)	$5(4) + 3(0) = 20$
E	(8, 0)	$5(8) + 3(0) = 40$

∴ the general Maximum profit will be Made by producing 8 units of Chocolate cakes and 0 units of Vanilla.

7. Table:

Vehicles	Constraints		Cost.
	Fuel	Driver time	
x	3	2	6
y	4	1	7
Max available	18	10	

let Vehicle x = x

let Vehicle y = y

Z Max $= 6x + 7y$

Constraint Equation:

$3x + 4y \leq 18$ --- ①

$2x + y \leq 10$ --- ②

Non-negativity Constraint:

$x \geqslant 0$

$y \geqslant 0$

Solving for Extreme Points:

From equation ①; $3x + 4y = 18$	From equation ⑪ $2x + y = 10$
when $x = 0$	When $x = 0$
$3(0) + 4y = 18$	$2(0) + y = 10$
$4y = 18$	$y = 10$
$y = 4.5$	
When $y = 0$	When $y = 0$
$3x + 4(0) = 18$	$2x + (0) = 10$
$3x = 18.$	$2x = 10$
$x = 18/3 = 6$ //	$x = 5$ //

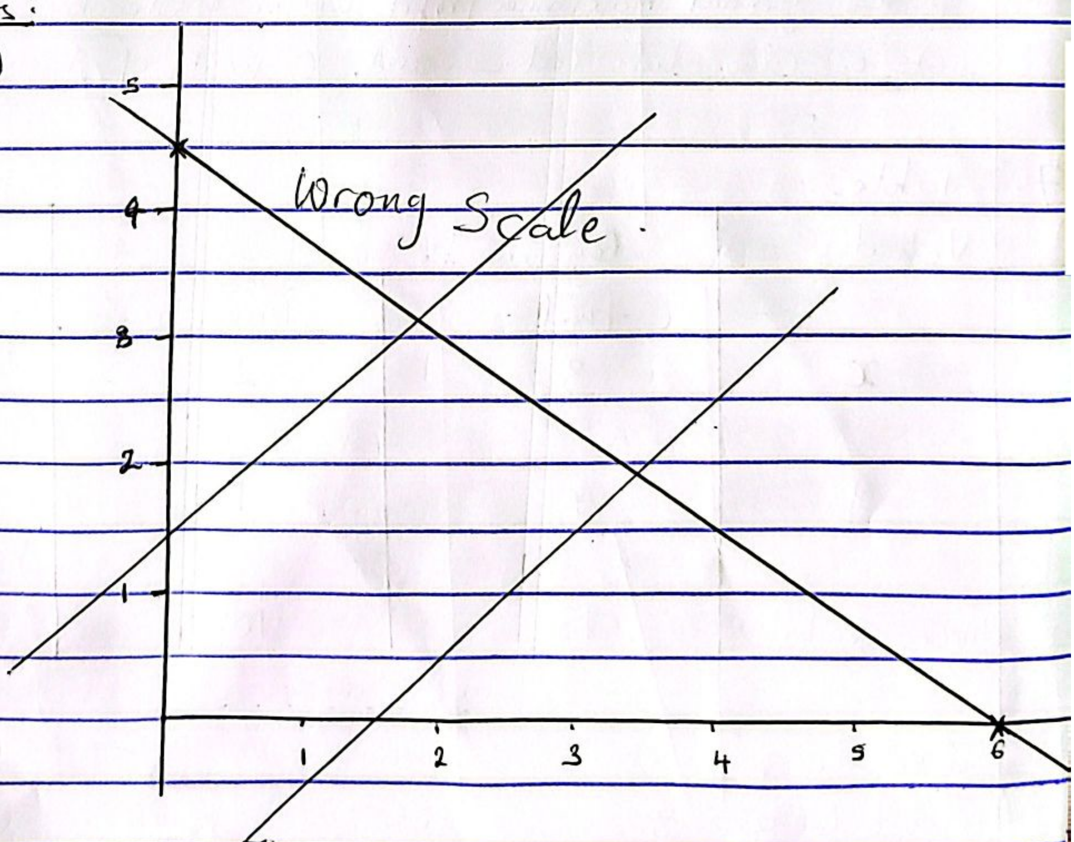
Extreme Points:

$(0, 4.5)$

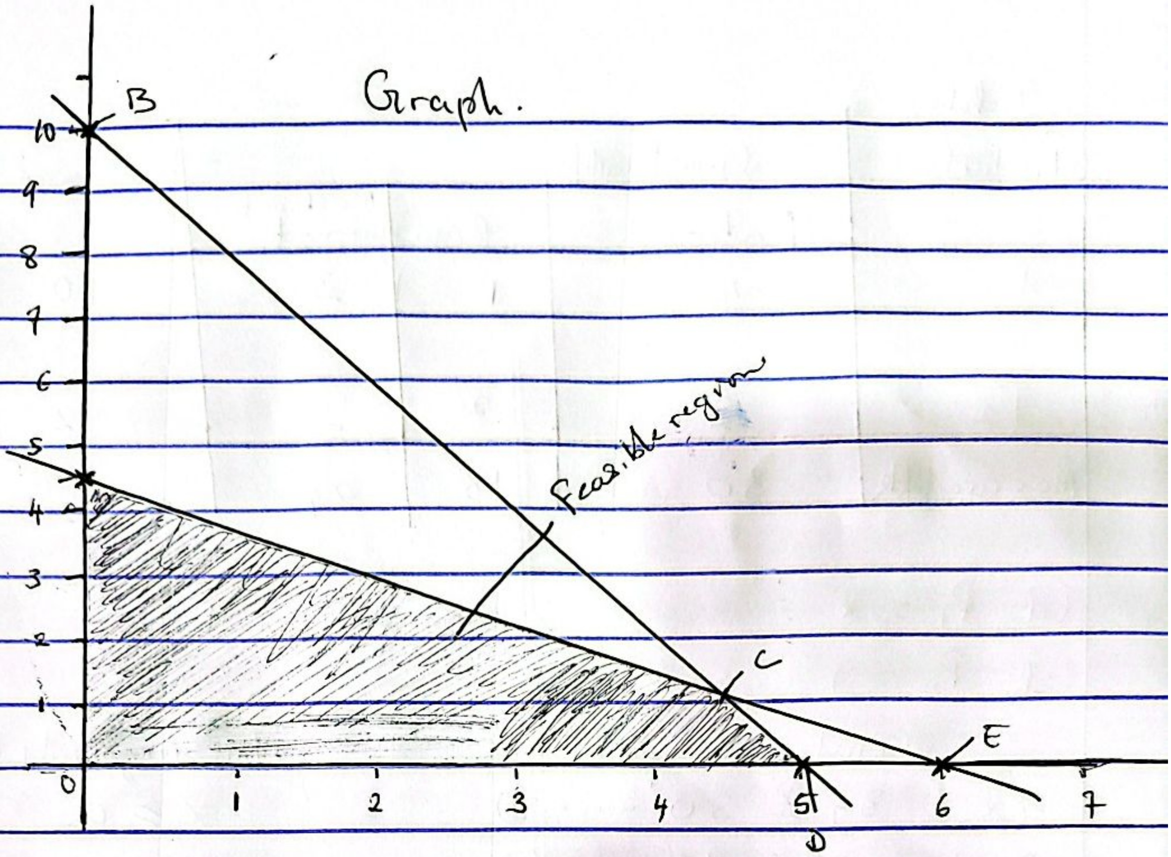
$(6, 0)$

$(0, 10)$

$(5, 0)$



Graph.



Solving for feasible region.

O (0, 0)	From the Graph	Objective function
A (0, 4.5)	C = (4.5, 1.2)	$Z = 6x + 7y$
B (0, 10)		
C ( ) ?		
D (5, 0)		
E (6, 0		

Finding the Optimal Solution.

Extreme Points	Coordinates	Z objective function
O	(0, 0)	$6(0) + 7(0) = 0$
A	(0, 4.5)	$6(0) + 7(4.5) = 31.5$
B	(0, 10)	$6(0) + 7(10) = 70$
C	(4.5, 1.2)	$6(4.5) + 7(1.2) = 35.5$
D	(5, 0)	$6(5) + 7(0) = 30$
E	(6, 0)	$6(6) + 7(0) = 36$

∴ The Optimal Solution to minimize the total cost will be to make use of Car x 6 times and Car y 0 times to minimize cost with a non-zero constraint. Otherwise it will be t use non.

8. Table.

Products	Constraints			Revenue.
	Labour	Raw material	Machine time	
P_1	4	1	3	10
P_2	3	2	2	12
Max available	30	18	24	

Let $P_1 = x$ $\qquad Z \max = 10x + 12y$

Let $P_2 = y$

Constraint Equation

$4x + 3y \leq 30$ --- ①

$x + 2y \leq 18$ --- ②

$3x + 2y \leq 24$ --- ③

Non-negativity constraint

$x \geq 0$

$y \geq 0$

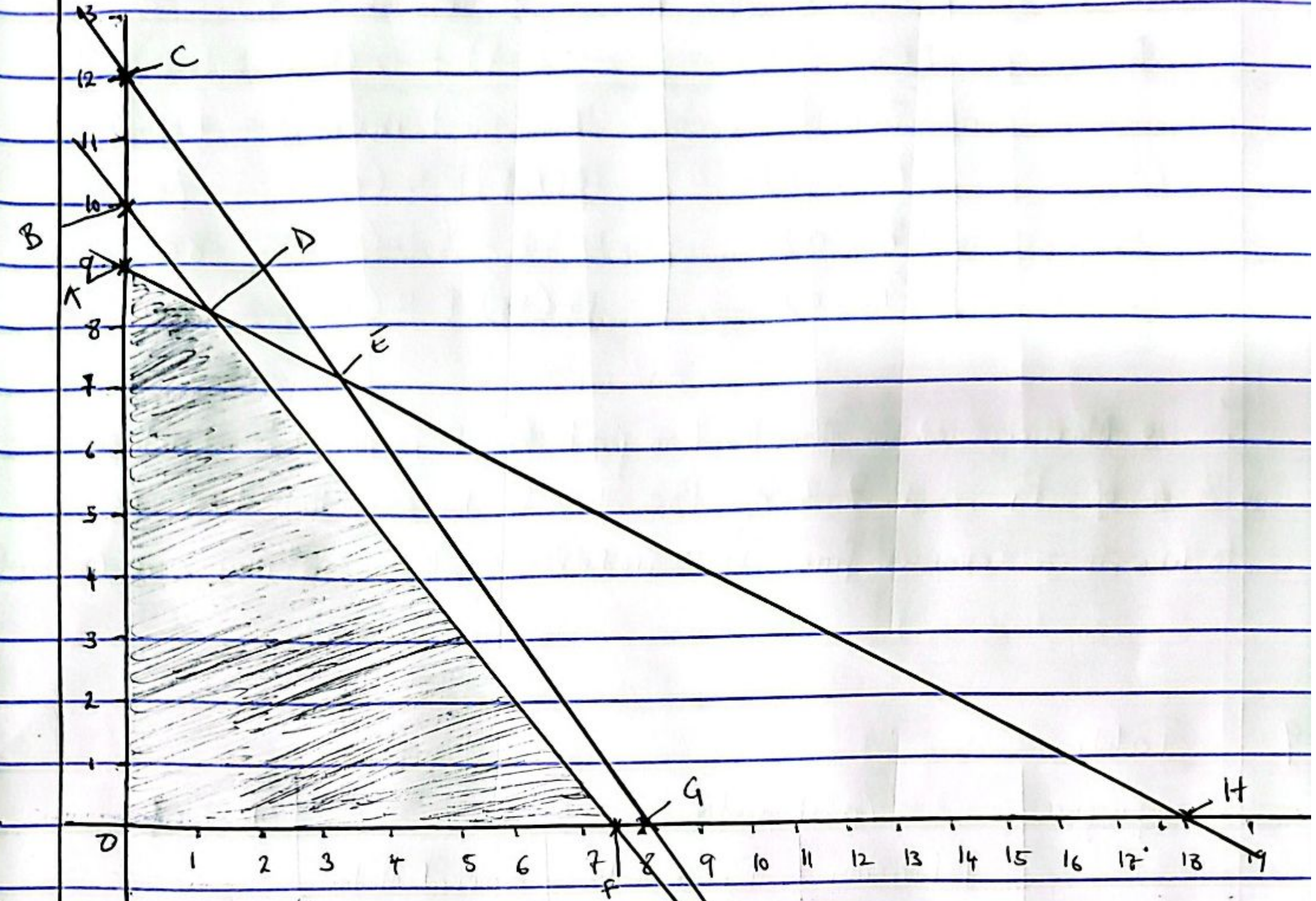
Extreme Points of the Equations

From Equation ①	From Equation ②	From Equation ③
$4x + 3y = 30$	$x + 2y = 18$	$3x + 2y = 24$
when $x = 0$	when $x = 0$	when $x = 0$
$4(0) + 3y = 30$	$0 + 2y = 18$	$3(0) + 2y = 24$
$3y = 30$	$2y = 18$	$2y = 24$
$y = 10$	$y = 9$	$y = 12$
when $y = 0$	when $y = 0$	when $y = 0$
$4x + 3(0) = 30$	$x + 2(0) = 18$	$3x + 2(0) = 24$
$4x = 30$	$x = 18$	$3x = 24$
$x = 7.5$		$x = 8$

Extreme Points

$(0, 10)$	$(0, 9)$	$(0, 12)$
$(7.5, 0)$	$(18, 0)$	$(8, 0)$

Vug/csc/23/9561



Solving for Feasible Region

	From the Graph.	Objective function.
O (0, 0)	D = (1·3, 8·3)	$Z = 10x + 12y$
A (0, 9)	E = (3·4, 7·3)	
B (0, 10)		
C (0, 12)		
D ( ) ??		
E ( ) ??		
F (7·5, 0)		
G (8, 0)		
H (18, 0)		

Finding the Optimal Solution

Extreme Points	Coordinates	Z objective function
O	(0, 0)	10(0) + 12(0) = 0
A	(0, 9)	10(0) + 12(9) = 108
B.	(0, 10)	10(0) + 12(10) = 120
C	(0, 12)	10(0) + 12(12) = 144

Extreme Points	Coordinates	Z objective function:
D	(1·3, 8·3)	10(1.3) + 12(8·3) = 112.6
E	(3·4, 7·3)	10(3.4) + 12(7·3) = 121.6
F	(7·5, 0)	10(7·5) + 12(0) = 75
G	(8, 0)	10(8) + 12(0) = 80
H	(18, 0)	10(18) + 12(0) = 180

∴ To Maximize revenue from the two products it is wise to Produce 3·4 of P_1 and 7·3 of P_2. This is because the task is to maximize revenue from Two products. So, there is a non-zero constraint

9. Table

Campaigns	Constraints			Reach
	Television	Print media	Social Media	
A	4,000	2,000	1,000	500,000
B	3,000	2,500	1,500	400,000
Max Available.	5,000	4,500	3,000	

Let Campaign A = x

Let Campaign B = y

Z Max = 500,000x + 400,000y

Constraint Equation.

$4{,}000x + 3{,}000y \leq 5000$ --- ①

$2{,}000x + 2{,}500y \leq 4{,}500$ --- ②

$1{,}000x + 1{,}500y \leq 3{,}000$ --- ③

Non-negativity Constraint.

$x \geq 0$

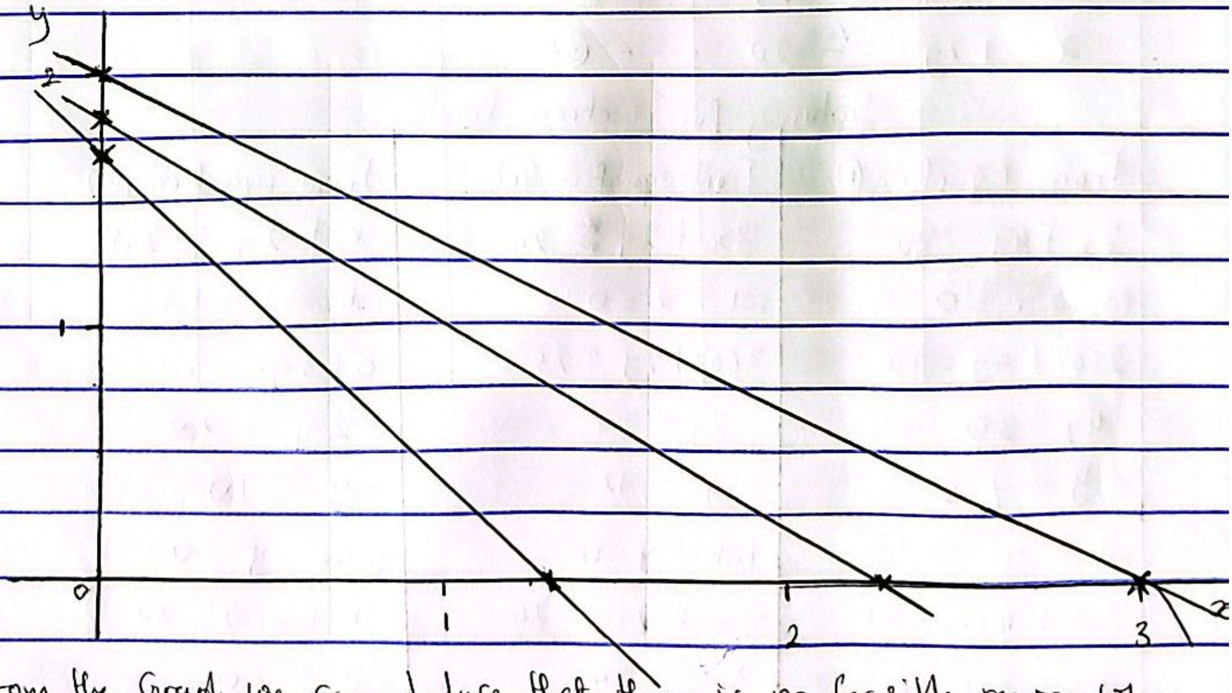
$y \geq 0$

Extreme Points of the Equations

From equation ①	From equation ②	From equation ③
$4{,}000x + 3{,}000y = 5{,}000$	$2000x + 2500y = 4{,}500$	$1000x + 1500y = 3000$
When $x = 0$	When $x = 0$	When $x = 0$
$4{,}000(0) + 3{,}000y = 5000$	$2000(0) + 2500y = 4500$	$1000(0) + 1500y = 3000$
$3{,}000y = 5{,}000$	$2500y = 4500$	$1500y = 3000$
$y = 1.7$	$y = 1.8$	$y = 2$
When $y = 0$	When $y = 0$	When $y = 0$
$4{,}000x + 3{,}000(0) = 5000$	$2000x + 2500(0) = 4500$	$1000x + 1500(0) = 3000$
$4000x = 5000$	$2000x = 4500$	$1000x = 3000$
$x = 1.3$	$x = 2.3$	$x = 3$

Extreme Points:

$(0, 1.7)$	$(0, 1.8)$	$(0, 2)$
$(1.3, 0)$	$(2.3, 0)$	$(3, 0)$



From the Graph, we can deduce that there is no feasible region where all the constraints are satisfied.

Hence the problem has no optimal solution.

10 Table:

Meals	Constraints			Revenue
	Meat	Vegetables	rice	
A	2	3	1	6
B	4	2	2	5
Max available	30	24	20	

let Meal A $= x$

let Meal B $= y$

$Z \max = 6x + 5y$

Constraints Equation:

$2x + 4y \leq 30$ --- (1)

$3x + 2y \leq 24$ --- (2)

$x + 2y \leq 20$ --- (3)

Non-negativity Constraint:

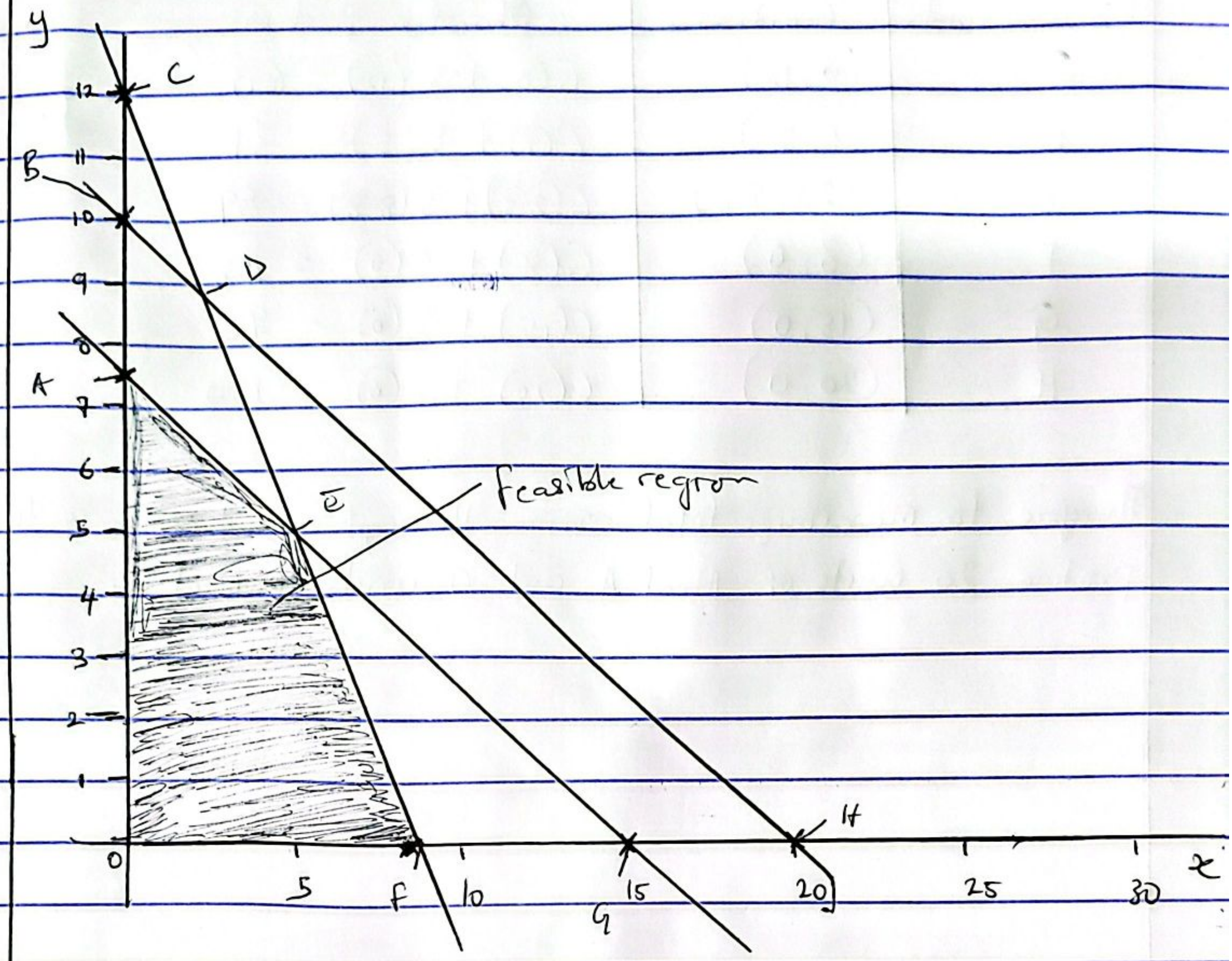
Solving for Extreme Points

From Equation (1)	From equation (11)	From Equation (3)
$2x + 4y = 30$	$3x + 2y \leq 24$	$x + 2y = 20$
when $x = 0$	When $x = 0$	when $x = 0$
$2(0) + 4y = 30$	$3(0) + 2y = 24$	$0 + 2y = 20$
$4y = 30$	$2y = 24$	$2y = 20$
$y = 7.5$	$y = 12$	$y = 10$
When $y = 0$	When $y = 0$	When $y = 0$
$2x + 4(0) = 30$	$3x + 2(0) = 24$	$x + 2(0) = 20$
$2x = 30$	$3x = 24$	$x = 20$ //
$x = 15$	$x = 8$	

Extreme points

(0, 7.5)	(0, 12)	(0, 10)
(15, 0)	(8, 0)	(20, 0)

Vug/csc/23/9561



Solving for feasible region

O (0, 0)	From the Graph	Objective function
A (0, 7.5)	D = (1, 5)	Z = 6x + 5y
B (0, 10)	E (2.5, 8.8)	
C (0, 12)		
D ( ) ??		
E ( ) ?		
F (8, 0)		
G (15, 0)		
H (20, 0)		

Finding the Optimal Solution

Extreme Points	Coordinates	Objective function
O	(0, 0)	6(0) + 5(0) = 0
A	(0, 7.5)	6(0) + 5(7.5) = 37.5
B	(0, 10)	6(0) + 5(10) = 50

Extreme Points	Coordinates	Objective function.
C	(0,12)	$6(0)+5(12)=60$
D.	(1,5)	$6(1)+5(5)=31$
E	(2.5, 8.8)	$6(2.5)+5(8.8)=59$
F	(8,0)	$6(8)+5(0)=48$
G	(15,0)	$6(15)+5(0)=90$
H	(20,0)	$6(20)+5(0)=120$

Therefore, to maximize total revenue, the optimal decision is to produce 20 units of Meal A and 0 units of Meal B.