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# **motifcluster**

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A Python package for motif-based spectral clustering of weighted directed networks.



## INTRODUCTION

The **motifcluster** package provides implementations of motif-based spectral clustering of weighted directed networks in Python. These provide the capability for:

- Building motif adjacency matrices
- Sampling random weighted directed networks
- Spectral embedding with motif adjacency matrices
- Motif-based spectral clustering

The methods are all designed to run quickly on large sparse networks, and are easy to install and use.





## INSTALLATION

```
pip install motifcluster
```



## DEPENDENCIES

- Networkx
- Numpy
- Scipy
- Scikit-learn



## DOCUMENTATION

Documentation for the **motifcluster** package is available on [Read the Docs](#).



## TUTORIAL

A tutorial for the **motifcluster** package is available on GitHub. .. at TODO





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CHAPTER  
**SIX**

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## LINKS

- Source code on [GitHub](#)
- Package index page on [PyPI](#)
- Documentation on [Read the Docs](#)



## CONTENTS

### 8.1 Clustering methods

Functions for spectral clustering are in *motifcluster.clustering*.

**cluster\_spectrum** (*spectrum*, *num\_clusts*)

Get cluster assignments from spectrum using k-means++.

Get a list of cluster assignments from a spectrum, using k-means++ and *num\_clusts* clusters.

#### Parameters

- **spectrum** (*dict*) – A dictionary containing “*vects*”: the matrix of eigenvectors to pass to k-means++.
- **num\_clusts** (*int*) – The number of clusters to find.

**Returns** **cluster\_assigns** – A list of integers from 1 to *num\_clusts*, representing cluster assignments.

**Return type** list of int

**run\_motif\_clustering** (*adj\_mat*, *motif\_name*, *motif\_type*='struc', *mam\_weight\_type*='unweighted', *mam\_method*='sparse', *num\_eigs*=2, *type\_lap*='comb', *num\_clusts*=2, *restrict*=True, *gr\_method*='sparse')

Run motif-based clustering.

Run motif-based clustering on the adjacency matrix of a (weighted directed) network, using a specified motif, motif type, weighting scheme, embedding dimension, number of clusters and Laplacian type. Optionally restrict to the largest connected component before clustering.

#### Parameters

- **adj\_mat** (*matrix*) – Adjacency matrix to be embedded.
- **motif\_name** (*str*) – Motif used for the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to use. One of “*func*” or “*struc*”.
- **mam\_weight\_type** (*str*) – Weighting scheme for the motif adjacency matrix. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – The method to use for building the motif adjacency matrix. One of “*sparse*” or “*dense*”.
- **num\_eigs** (*int*) – Number of eigenvalues and eigenvectors for the embedding.
- **type\_lap** (*str*) – Type of Laplacian for the embedding. One of “*comb*” or “*rw*”.
- **num\_clusts** (*int*) – The number of clusters to find.

- **restrict** (*bool*) – Whether or not to restrict the motif adjacency matrix to its largest connected component before embedding.
- **gr\_method** (*str*) – Format to use for getting largest component. One of “*sparse*” or “*dense*”.

#### Returns

- **adj\_mat** (*sparse matrix*) – The original adjacency matrix.
- **motif\_adj\_mat** (*sparse matrix*) – The motif adjacency matrix.
- **comps** (*list*) – The indices of the largest connected component of the motif adjacency matrix (if `restrict=True`).
- **adj\_mat\_comps** (*matrix*) – The original adjacency matrix restricted to the largest connected component of the motif adjacency matrix (if `restrict=True`).
- **motif\_adj\_mat\_comps** (*matrix*) – The motif adjacency matrix restricted to its largest connected component (if `restrict=True`).
- **vals** (*list*) – A length-`num_eigs` list containing the eigenvalues associated with the Laplace embedding of the (restricted) motif adjacency matrix.
- **vects** (*matrix*) – A matrix containing the eigenvectors associated with the Laplace embedding of the (restricted) motif adjacency matrix.
- **clusts** – A vector containing integers representing the cluster assignment of each vertex in the (restricted) graph.

#### Examples

```
>>> adj_mat = np.array(range(1, 10)).reshape((3, 3))
>>> run_motif_clustering(adj_mat, "M1")
```

## 8.2 Adjacency and indicator matrices

Functions for building adjacency and indicator matrices are in `motifcluster.indicators`.

### `_build_G` (*adj\_mat*)

Build sparse adjacency matrix.

Build the sparse adjacency matrix  $G$  from a graph adjacency matrix.

**Parameters** **adj\_mat** (*matrix*) – The original adjacency matrix.

**Returns** **G** – The adjacency matrix in sparse form.

**Return type** sparse matrix

### `_build_Gd` (*adj\_mat*)

Build double-edge adjacency matrix.

Build the sparse double-edge adjacency matrix  $Gd$  from a graph adjacency matrix.

**Parameters** **adj\_mat** (*matrix*) – The original adjacency matrix.

**Returns** **Gd** – A double-edge adjacency matrix in sparse form.

**Return type** sparse matrix

**\_build\_Gp** (*adj\_mat*)

Build product matrix.

Build the sparse product matrix *Gp* from a graph adjacency matrix.

**Parameters** *adj\_mat* (*matrix*) – The original adjacency matrix.

**Returns** *Gp* – A product matrix in sparse form.

**Return type** sparse matrix

**\_build\_Gs** (*adj\_mat*)

Build single-edge indicator matrix.

Build the sparse single-edge adjacency matrix *Gs* from a graph adjacency matrix.

**Parameters** *adj\_mat* (*matrix*) – The original adjacency matrix.

**Returns** *Gs* – A single-edge adjacency matrix in sparse form.

**Return type** sparse matrix

**\_build\_Id** (*adj\_mat*)

Build identity matrix.

Build the sparse identity matrix *Id* from a graph adjacency matrix.

**Parameters** *adj\_mat* (*matrix*) – The original adjacency matrix.

**Returns** *Id* – An identity matrix in sparse form.

**Return type** sparse matrix

**\_build\_J** (*adj\_mat*)

Build directed indicator matrix.

Build the sparse directed indicator matrix *J* from a graph adjacency matrix.

**Parameters** *adj\_mat* (*matrix*) – The original adjacency matrix.

**Returns** *J* – A directed indicator matrix in sparse form.

**Return type** sparse matrix

**\_build\_J0** (*adj\_mat*)

Build missing-edge indicator matrix.

Build the missing-edge indicator matrix *J0* from a graph adjacency matrix.

**Parameters** *adj\_mat* (*matrix*) – The original adjacency matrix.

**Returns** *J0* – A missing-edge indicator matrix.

**Return type** sparse matrix

**\_build\_Jd** (*adj\_mat*)

Build double-edge indicator matrix.

Build the sparse double-edge indicator matrix *Jd* from a graph adjacency matrix.

**Parameters** *adj\_mat* (*matrix*) – The original adjacency matrix.

**Returns** *Jd* – A double-edge indicator matrix in sparse form.

**Return type** sparse matrix

**`_build_Je`** (*adj\_mat*)

Build edge-and-diagonal matrix.

Build the sparse edge-and-diagonal matrix *Ie* from a graph adjacency matrix.

**Parameters** **adj\_mat** (*matrix*) – The original adjacency matrix.

**Returns** **Ie** – An edge-and-diagonal matrix in sparse form.

**Return type** sparse matrix

**`_build_Jn`** (*adj\_mat*)

Build vertex-distinct indicator matrix.

Build the vertex-distinct indicator matrix *Jn* from a graph adjacency matrix.

**Parameters** **adj\_mat** (*matrix*) – The original adjacency matrix.

**Returns** **Jn** – A vertex-distinct indicator matrix.

**Return type** sparse matrix

**`_build_Js`** (*adj\_mat*)

Build single-edge indicator matrix.

Build the sparse single-edge indicator matrix *Js* from a graph adjacency matrix.

**Parameters** **adj\_mat** (*matrix*) – The original adjacency matrix.

**Returns** **Js** – A single-edge indicator matrix in sparse form.

**Return type** sparse matrix

## 8.3 Motif adjacency matrices

Functions for building motif adjacency matrices are in *motifcluster.motifadjacency*.

**build\_motif\_adjacency\_matrix** (*adj\_mat*, *motif\_name*, *motif\_type*='struc',  
*mam\_weight\_type*='unweighted', *mam\_method*='sparse')

Build a motif adjacency matrix.

Build a motif adjacency matrix from an adjacency matrix. Entry (*i*, *j*) of a motif adjacency matrix is the sum of the weights of all motifs containing both nodes *i* and *j*.

- The motif is specified by name and the type of motif instance can be one of:
  - Functional: motifs should appear as subgraphs.
  - Structural: motifs should appear as induced subgraphs.
- The weighting scheme can be one of:
  - Unweighted: the weight of any motif instance is one.
  - Mean: the weight of any motif instance is the mean of its edge weights.
  - Product: the weight of any motif instance is the product of its edge weights.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_name** (*str*) – Motif used for the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build. One of “func” or “struc”.



- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”. The sparse formulation avoids generating large dense matrices so tends to be faster for large sparse graphs.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

## Examples

```
>>> adj_mat = np.array(range(1, 10)).reshape((3, 3))
>>> build_motif_adjacency_matrix(adj_mat, "M1", "func", "mean")
```

**mam\_M1** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M1.

### Parameters

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M10** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif M10.

### Parameters

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M11** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif M11.

### Parameters

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M12** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif M12.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M13** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif M13.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M2** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M2.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M3** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M3.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M4** (*adj\_mat, mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M4.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M5** (*adj\_mat, motif\_type, mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M5.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M6** (*adj\_mat, motif\_type, mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M6.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M7** (*adj\_mat, motif\_type, mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif M7.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M8** (*adj\_mat, motif\_type, mam\_weight\_type, mam\_method*)

Perform the motif adjacency matrix calculations for motif M8.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.

- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_M9** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif M9.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_Mcoll** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif Mcoll.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_Md** (*adj\_mat*, *mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif Md.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_Mexpa** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*, *mam\_method*)

Perform the motif adjacency matrix calculations for motif Mexpa.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.

- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – Which formulation to use. One of “*dense*” or “*sparse*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

**mam\_Ms** (*adj\_mat*, *motif\_type*, *mam\_weight\_type*)

Perform the motif adjacency matrix calculations for motif Ms.

**Parameters**

- **adj\_mat** (*matrix*) – Adjacency matrix from which to build the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to build.
- **mam\_weight\_type** (*str*) – The weighting scheme to use. One of “*unweighted*”, “*mean*” or “*product*”.

**Returns** A motif adjacency matrix.

**Return type** sparse matrix

## 8.4 Network sampling

Functions for random sampling of weighted directed networks are in *motifcluster.sampling*.

**demonstration\_graph** ()

Generate a small graph for demonstrations.

Generate the sparse and dense adjacency matrices of a small weighted directed graph, for demonstrating methods and running tests.

**Returns**

- **adj\_mat\_dense** (*matrix*) – the adjacency matrix in dense form.
- **adj\_mat\_sparse** (*sparse matrix*) – the adjacency matrix in sparse form.

**sample\_bsbm** (*source\_block\_sizes*, *dest\_block\_sizes*, *bipartite\_connection\_matrix*, *bipartite\_weight\_matrix=None*, *sample\_weight\_type='unweighted'*)

Sample a bipartite stochastic block model (BSBM).

Sample the (weighted) adjacency matrix of a (weighted) bipartite stochastic block model (BSBM) with specified parameters.

**Parameters**

- **source\_block\_sizes** (*list of int*) – A list containing the size of each block of source vertices.
- **dest\_block\_sizes** (*list of int*) – A list containing the size of each block of destination vertices.
- **bipartite\_connection\_matrix** (*matrix*) – A matrix containing the source block to destination block connection probabilities.
- **sample\_weight\_type** (*str*) – The type of weighting scheme. One of “*unweighted*”, “*constant*” or “*poisson*”.
- **weight\_matrix** (*matrix*) – A matrix containing the source block to destination block weight parameters. Unused for *sample\_weight\_type* = “*constant*”. Defaults to *None*.

**Returns** **adj\_mat** – A randomly sampled (weighted) adjacency matrix of a BSBM.

**Return type** sparse matrix

### Examples

```
>>> source_block_sizes = [10, 10]
>>> dest_block_sizes = [10, 10, 10]
>>> bipartite_connection_matrix = np.array([0.8, 0.5, 0.1, 0.1, 0.5, 0.8]).
↳reshape((2, 3))
>>> bipartite_weight_matrix = np.array([20, 10, 2, 2, 10, 20]).reshape((2, 3))
>>> sample_bsbm(block_sizes, bipartite_connection_matrix,
... bipartite_weight_matrix, "poisson")
```

**sample\_dsbm** (*block\_sizes*, *connection\_matrix*, *weight\_matrix*=None, *sample\_weight\_type*='unweighted')

Sample a directed stochastic block model (DSBM).

Sample the (weighted) adjacency matrix of a (weighted) directed stochastic block model (DSBM) with specified parameters.

#### Parameters

- **block\_sizes** (*list of int*) – A list containing the size of each block of vertices.
- **connection\_matrix** (*matrix*) – A matrix containing the block-to-block connection probabilities.
- **sample\_weight\_type** (*str*) – The type of weighting scheme. One of “unweighted”, “constant” or “poisson”.
- **weight\_matrix** (*matrix*) – A matrix containing the block-to-block weight parameters. Unused for *sample\_weight\_type* = “constant”. Defaults to None.

**Returns** **adj\_mat** – A randomly sampled (weighted) adjacency matrix of a DSBM.

**Return type** sparse matrix

### Examples

```
>>> block_sizes = [10, 10]
>>> connection_matrix = np.array([0.8, 0.1, 0.1, 0.8]).reshape((2, 2))
>>> weight_matrix = np.array([10, 3, 3, 10]).reshape((2, 2))
>>> sample_dsbm(block_sizes, connection_matrix, weight_matrix, "poisson")
```

## 8.5 Spectral methods

Functions relating to spectral methods are in *motifcluster.spectral*.

**\_get\_first\_eigs** (*some\_mat*, *num\_eigs*)

Compute first few eigenvalues and eigenvectors of a matrix.

Compute the first few eigenvalues (by magnitude) and associated eigenvectors of a matrix.

#### Parameters

- **some\_mat** (*matrix*) – Matrix for which eigenvalues and eigenvectors are to be calculated.
- **num\_eigs** (*int*) – Number of eigenvalues and eigenvectors to calculate.

**Returns**

- **vals** (*list*) – A length-*num\_eigs* list of the first few eigenvalues.
- **vects** (*matrix*) – A *some\_mat.shape[0]* by *num\_eigs* matrix of the associated eigenvectors.

**build\_laplacian** (*adj\_mat*, *type\_lap*='rw')

Build a Laplacian matrix.

Build a Laplacian matrix (combinatorial Laplacian or random-walk Laplacian) from a symmetric (weighted) graph adjacency matrix.

#### Parameters

- **adj\_mat** (*matrix*) – Symmetric adjacency matrix from which to build the Laplacian.
- **type\_lap** (*str*) – Type of Laplacian to build. One of “*comb*” (combinatorial) or “*rw*” (random-walk).

**Returns** The specified Laplacian matrix.

**Return type** sparse matrix

#### Examples

```
>>> adj_mat = np.array(range(1, 10)).reshape((3, 3))
>>> build_laplacian(adj_mat, "rw")
```

**run\_laplace\_embedding** (*adj\_mat*, *num\_eigs*, *type\_lap*='rw')

Run Laplace embedding.

Run Laplace embedding on a symmetric (weighted) adjacency matrix with a specified number of eigenvalues and eigenvectors.

#### Parameters

- **adj\_mat** (*matrix*) – Symmetric adjacency matrix to be embedded.
- **num\_eigs** (*int*) – Number of eigenvalues and eigenvectors for the embedding.
- **type\_lap** (*str*) – Type of Laplacian for the embedding. One of “*comb*” (combinatorial) or “*rw*” (random-walk).

#### Returns

- **vals** (*list*) – The length-*num\_eigs* list of the first few eigenvalues of the Laplacian.
- **vects** (*matrix*) – An *adj\_mat.shape[0]* by *num\_eigs* matrix of the associated eigenvectors.

#### Examples

```
>>> adj_mat = np.array(range(1, 10)).reshape((3, 3))
>>> run_laplace_embedding(adj_mat, 2, "rw")
```

**run\_motif\_embedding** (*adj\_mat*, *motif\_name*, *motif\_type*='struc', *mam\_weight\_type*='unweighted',  
*mam\_method*='sparse', *num\_eigs*=2, *type\_lap*='rw', *restrict*=True,  
*gr\_method*='sparse')

Run motif embedding.

Calculate a motif adjacency matrix for a given motif and motif type, optionally restrict it to its largest connected component, and then run Laplace embedding with specified Laplacian type and number of eigenvalues and eigenvectors.

### Parameters

- **adj\_mat** (*matrix*) – Adjacency matrix to be embedded.
- **motif\_name** (*str*) – Motif used for the motif adjacency matrix.
- **motif\_type** (*str*) – Type of motif adjacency matrix to use. One of “*func*” or “*struc*”.
- **mam\_weight\_type** (*str*) – Weighting scheme for the motif adjacency matrix. One of “*unweighted*”, “*mean*” or “*product*”.
- **mam\_method** (*str*) – The method to use for building the motif adjacency matrix. One of “*sparse*” or “*dense*”.
- **num\_eigs** (*int*) – Number of eigenvalues and eigenvectors for the embedding.
- **type\_lap** (*str*) – Type of Laplacian for the embedding. One of “*comb*” or “*rw*”.
- **restrict** (*bool*) – Whether or not to restrict the motif adjacency matrix to its largest connected component before embedding.
- **gr\_method** (*str*) – Format to use for getting largest component. One of “*sparse*” or “*dense*”.

### Returns

- **adj\_mat** (*sparse matrix*) – The original adjacency matrix.
- **motif\_adj\_mat** (*sparse matrix*) – The motif adjacency matrix.
- **comps** (*list*) – The indices of the largest connected component of the motif adjacency matrix (if restrict=True).
- **adj\_mat\_comps** (*matrix*) – The original adjacency matrix restricted to the largest connected component of the motif adjacency matrix (if restrict=True).
- **motif\_adj\_mat\_comps** (*matrix*) – The motif adjacency matrix restricted to its largest connected component (if restrict=True).
- **vals** (*list*) – A length-*num\_eigs* list containing the eigenvalues associated with the Laplace embedding of the (restricted) motif adjacency matrix.
- **vects** – A matrix containing the eigenvectors associated with the Laplace embedding of the (restricted) motif adjacency matrix.

### Examples

```
adj_mat = np.array(range(1, 10)).reshape((3, 3)) run_motif_embedding(adj_mat, “M1”)
```

## 8.6 Utility functions

Assorted utility functions for the motifcluster module are in *motifcluster.utils*.

**\_a\_b\_one** (*a\_mat*, *b\_mat*)

Compute a right-multiplication with the ones matrix.

Compute  $a * (b @ \text{one\_mat})$  where  $a$ ,  $b$ ,  $\text{ones\_mat}$  are square matrices of the same size, and  $\text{ones\_mat}$  contains all entries equal to one. The product  $*$  is an entry-wise (Hadamard) product, while  $@$  represents matrix multiplication. This method is more efficient than the naive approach when  $a$  or  $b$  are sparse.

**Parameters** **a**, **b** (*matrix*) – Square matrices of the same size.



**Returns** The sparse square matrix  $a * (b @ one\_mat)$ .

**Return type** sparse matrix

**`_a_one_b`** (*a\_mat*, *b\_mat*)

Compute a left-multiplication with the ones matrix.

Compute  $a * (one\_mat @ b)$  where  $a$ ,  $b$ ,  $ones\_mat$  are square matrices of the same size, and  $ones\_mat$  contains all entries equal to one. The product  $*$  is an entry-wise (Hadamard) product, while  $@$  represents matrix multiplication. This method is more efficient than the naive approach when  $a$  or  $b$  are sparse.

**Parameters** **a**, **b** (*matrix*) – Square matrices of the same size.

**Returns** The sparse square matrix  $a * (one\_mat @ b)$ .

**Return type** sparse matrix

**`_drop0_killdiag`** (*some\_mat*)

Set diagonal entries to zero and sparsify.

Set the diagonal entries of a matrix to zero and convert it to sparse form.

**Parameters** **some\_mat** (*matrix*) – A square matrix.

**Returns** **sparse\_mat** – A sparse-form copy of *some\_mat* with its diagonal entries set to zero.

**Return type** sparse matrix

**`_random_sparse_matrix`** (*m*, *n*, *p*, *sample\_weight\_type*='constant', *w*=1)

Build a random sparse matrix.

Build a sparse matrix of size  $m * n$  with non-zero probability  $p$ . Edge weights can be unweighted, constant-weighted or Poisson-weighted.

**Parameters**

- **m**, **n** (*int*) – Dimension of matrix to build is ( $m$ ,  $n$ ).
- **p** (*float*) – Probability that each entry is non-zero (before weighting).
- **sample\_weight\_type** (*str*) – Type of weighting scheme.
- **w** (*float*) – Weight parameter.

**Returns** A random sparse matrix.

**Return type** sparse matrix

**`get_largest_component`** (*adj\_mat*, *gr\_method*)

Get largest connected component.

Get the indices of the vertices in the largest connected component of a graph from its adjacency matrix.

**Parameters**

- **adj\_mat** (*matrix*) – An adjacency matrix of a graph.
- **gr\_method** (*str*) – Format to use before building the graph. One of “sparse” or “dense”.

**Returns** **verts\_to\_keep** – A list of indices corresponding to the vertices in the largest connected component.

**Return type** list

## Examples

```
>>> adj_mat = np.array([0, 1, 0, 0, 0, 0, 0, 0, 0]).reshape((3, 3))
>>> get_largest_component(adj_mat)
```

### **get\_motif\_names()**

Get common motif names.

Get the names of some common motifs as strings.

**Returns** **motif\_names** – A list of names (strings) of common motifs.

**Return type** list

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