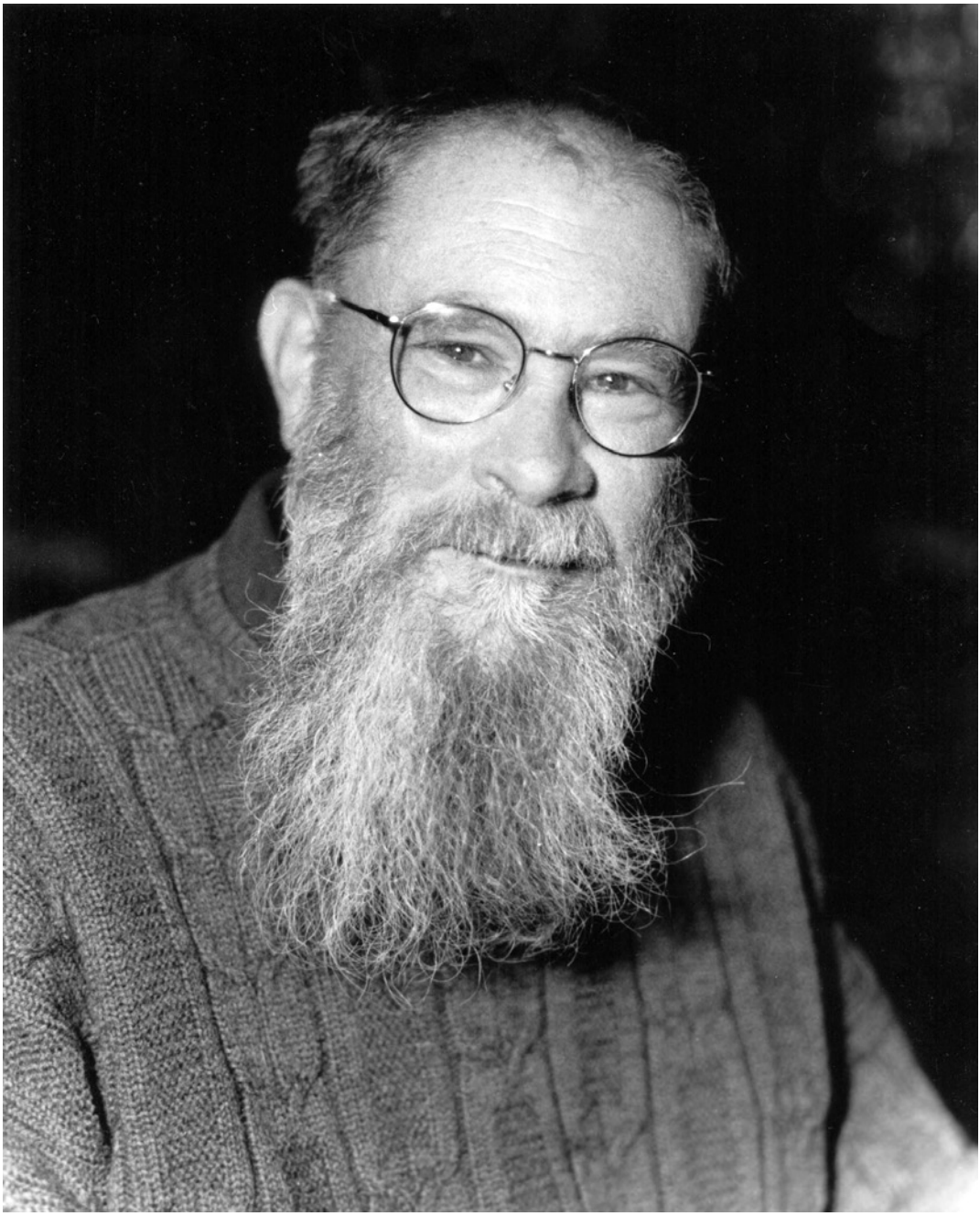

The following text includes some abstruse philosophy (which might make it unsuitable for rationally-minded readers), some formal logics (which might make it unsuitable for humanists), and some gross generalizations in both fields (which might make it unsuitable for both categories). It is also quite long.

But don't let this scare you. Its structure is modular, like a textbook in geometry or a string of sausages. Any group of parts can be skipped or, with some background, read independently. It relies on as little logic as possible.

Parts 0 and 1 are a general introduction to what we're talking about (the summoning of truths out of thin air), and why there could be a point in doing so. Parts 2, 3 and 4 are about a theological dispute in the 12th and 13th century, that revolved around morons and islands. Parts 5 and 6 discuss Immanuel Kant's pieces of eight and David Lewis's incredulous stares. If you know what those are, you can skip them. Parts 7 to 11 cover one of the instances in which David Lewis had to face the most intense of said stares, prompted by his 1970 paper "Anselm and Actuality." Parts 12 through 15 deal with one of Kurt Gödel's most perplexing achievements. Part 16 sums it all up and offers candy to those who put up with us that long.

Cover image: David Lewis's stare

0. ALL THIS SHOULD NOT SOUND SO WEIRD

I devoted a good part of my academic life to the analytical study of magic.

The spells I studied were formulated not by sorcerers but by philosophers, mathematicians, and theologians; they were strings of words that, once uttered, would bring into being a divinity. They were called "ontological proofs": rational demonstrations of the existence of god. A half dozen or so such proofs have been sketched out over the course of the past thousand years, by thinkers ranging from Anselmus Candiae Genevae, Doctor of the Church (a title roughly equivalent to a PhD for saints), to Kurt Gödel, widely regarded as the greatest mathematician of the 20th century.

It should be remarked that all this should not sound so weird. On the surface, it seems hard to consider these proofs as a plausible intellectual undertaking. It's magic, after all: the summoning of a being out of thin air. And yet they fit our way of thinking so tightly that it's hard to tell them apart from mental procedures we are used to from a very young age.

Think of a spell as a set of instructions: a sequence of words and glyphs, which might or might not include gestures. The words must be formulated with the utmost precision; various versions are admitted, but the slightest alteration of any one of them will jeopardize the result.

No interaction with reality is needed. The procedure unfolds in your mind only; it has no bearing on anything in the tangible world. You can chant the words aloud in a crowded lecture hall, or silently pass over them in the mind's eye high up on a cliff some stormy night. The result will not be affected. If carried out correctly, this procedure will yield an absolute truth: an insight that consistently holds in any circumstance, revealing previously unknown facts about objects and situations scattered throughout the universe.

The above is a very general way to describe divination. It is also a very general way to describe Pythagoras's theorem.

1. TRUTHS OUT OF THIN AIR: A BLUEPRINT

At a young age, we are inoculated against the belief in magic by grown-ups telling us some of its most bewildering results are, really, "just math." But try to think of Pythagoras's theorem with no previous knowledge of what it's supposed to do, or even of the fact that it's possible to do it.

You analyze a concept in your mind: the right-angled triangle. You don't need to refer to any actually existing triangle; you might draw one if it helps you visualize the steps, but it's irrelevant to the proof. You start from a characteristic you decide in your mind (I want to think about a figure with three angles, one of which is 90) and end up determining a property (the square on the longer side is the sum of the squares on the two other sides) that has nothing to do with what you knew about it to begin with. You discovered it; and you did so only by a process of thought.

What's more, this discovery does not hold of one triangle in particular: it holds of every triangle in the universe. You learned something about countless physical objects without needing to see or touch one. For all you know, triangles mightn't even exist; but if they do, they'll be just as you've proved them to be.

The second biggest mystery in this is that we've come to see it as no mystery at all. Why theorems such as this work (that is: why they allow us to consistently predict facts) has been one of the crucial questions of Western philosophy for the past 2,000 years. What seems to be involved is some affinity between what with dangerous generalizations we might call the mind and the world. On the surface, the explanation goes something like this: the world has rules, which the mind can discover and use to make predictions. This, very loosely, is something similar to the so-called scientific method; only in this case, the rules are determined not by observation and conjecture, but by pure thought.

This is why the impact of proofs such as Pythagoras's extended well beyond the boundaries of the theory of knowledge. For thousands of years, they seemed to provide a blueprint for successful abstract speculation. If, by analyzing the concept of a triangle, we can come up with previously unknown facts about triangles, who's to say we cannot do the

same with other, equally abstract concepts that nonetheless might have very concrete ramifications? There would be nothing weird about it; it's just the extension of a demonstrably valid procedure of thought.

With something like this on their minds, logicians and theologians set out to rationally prove the existence of god.

2. A SINNER, BUT NOT A MORON

The first ontological proof of the existence of god was put forth by a man born in Vallée d'Aôte around 1033 AD, known in Italy as Anselm of Aosta (from his birthplace), in France as Anselm of Bec (from his alma mater), and in the English-speaking world as Anselm of Canterbury (from his main employer). In the Christian world he is also known as Saint Anselm, and in some denominations as a Doctor of the Church. So much for naming.

Anselm sought to find an unbeatable way to convince an atheist to change his or her mind. This was usually accomplished with a moral system of threats and rewards (both otherworldly or cast-iron), but left the theoretical loophole of evil. Accepting the existence of god was presented as an ethical choice: whoever was willing to face the consequences could legitimately refuse to believe in it. He could be called a sinner, but not a moron.

Which is precisely what Anselm sought to change. Stating that god doesn't exist wasn't merely wrong in the moral sense: he saw it as logically wrong, a contradiction in terms. An unbeliever wasn't evil, he was simply ignorant, or misinformed: but logical analysis and intellectual honesty would be more than enough to make him change his mind.

His proof, which he put forth in a section of his 1077 bestseller (the *Proslogion*) titled "An Argument Against Morons," goes more or less like this:

Ok, so you say god doesn't exist. That's fine, for the moment, even if I dissent. Surely, in order to say this, you must accept that the notion of "god" makes sense: you can understand it, and you understand it in the

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same way I do. Only, I think it describes something that exists, while you think that nothing fits its description.

Let's take a closer look at this notion. A lot of things have been said to be implied by it (trinity, goodness, creator-of-the-universe), but it ultimately boils down to this: the concept of god is the concept of the greatest conceivable entity. Everything else you can think of is less perfect, for some meaning of "great" that is not literal (that is, that doesn't refer to physical size). You may take this in the moral sense, or merely in terms of power. It doesn't matter. Since we understand the concept of god in the same way, we both agree that the realm of conceivable entities, when ordered by greatness, is a set with an upper bound.

By "entity" I'm not saying god exists; only that it makes sense, that it's conceivable. Of course, not all conceivable entities correspond to actual things — for instance, we can conceive of a 100-km-tall bell tower, but we know it doesn't exist in the real world. So we also both accept that reality is a subset of the set of conceivable entities. We might say it includes all entities with the property of actual existence. What you're saying is that the greatest conceivable entity is not among them. For the moment, I'm willing to concede this. I'm assuming, following you, that the greatest conceivable entity does not have the property of ACTUAL existence.

Let's turn to the whole "conceivable" issue. A little conceiving can go a long way. For starters, you can produce variations on a conceivable entity by adding or removing properties: it'll still be conceivable. For instance, if you can conceive of a square tower, you can conceive of a red square tower, or of a square tower made of chocolate. I suppose you agree with this.

We can also produce variations on the concept of god. For instance, we can conceive of god as being, say, unable to cause firestorms, defining another conceivable entity — let's call it god2. god2, you will agree, is less great than the original god. Even without a strict definition of greatness, I guess that firestorms could somehow be involved. But this actually confirms our thesis: god, which we originally defined as the upper bound of our set of conceivable entities, is undoubtedly greater than that other conceivable entity called god2.

Well, let's try this again. We can conceive of god as something that actually exists. You yourself accepted it, when we said that things with the property of actual existence are a subset of conceivable things. So we can also conceive of a variation of god that has the property of *actual existence*—let's call it god3.

Now, now. Since I accepted your position—that the greatest conceivable entity does not have the property of actual existence, it must follow that god3, which is also conceivable, is less great than god, just like god2 because of that firestorm business.

But can you really claim that? By pretty much any standard, something which does exist is greater than something which doesn't. Something which actually exists is much better at causing firestorms, for instance. A hundred bucks that exist in your pocket are greater than a hundred bucks that don't.

By the same line of reasoning, god3 is greater than god as you define it. But since you agree to define god as the greatest conceivable entity, what you have on your hands is a contradiction, since we have just conceived of something greater than it.

It follows that your position is untenable. If you understand the concept of "god," you cannot claim that it does not actually exist. Hence, it does.

3. IN DEFENSE OF THE MORON

Even to its supporters (there've been several—and there still are), Anselm's argument smacks of wordplay. And yet, as is the case with many ontological proofs, it's very hard to pinpoint exactly what is wrong with it.

Before attempting to do so, it might be interesting to remark what ISN'T wrong. In its structure, Anselm's argument is actually surprisingly similar to Pythagoras's. Both aim to analyze a concept by making a minimal assumption about it and lead, through logical reasoning alone, to a conclusion seemingly not included in the premises. Both require no empirical experience and hence have a very high degree of generality

(either they're universally true, or universally false). Anselm probably had geometry in mind when devising his proof.

The most evident way to question the argument's conclusion would be to state it in IF-THEN form. What it actually proves (even if we take it as valid) is that, if the notion of god makes sense, it follows that god necessarily exists.

But the "if" part would have been considered self-evident in Anselm's times, when even hypothetically accepting the unbeliever's position meant brushing shoulders with heresy. Moreover, in many ways Anselm's definition does capture a crucial aspect of divinity shared by every monotheistic religion — and one curiously devoid of ambiguity or theological infighting, at that: it characterizes god as the "greatest" being there is, leaving priests and monks to figure out what that greatness actually entails.

The argument's other weak points seem to be two. On the one hand, the concept of "greatness" it employs seems both quite generic and a bit too strict. It seems to require us to be able to form an ordering of every concept we can think of according to some standard of greatness. But while we can (more or less) understand what it means when applied to a theoretical divinity, what of other conceivable things? How great is my laptop? Is it greater than your aunt? Would it be greater if I painted it pink?

On the other hand, there seems to be a lot of confusion between THINKING of something as existing, and the thing actually existing out there. Of course, a hundred bucks in my pocket are greater than a hundred bucks which aren't, but a hundred bucks that I think are in my pocket won't be enough to pay the tab.

Both of these objections were first put forth around the end of the 11th century by Gaunilo of Marmoutiers, in a booklet titled *In Defense of the Moron*. It's the only text of his that survives to this day, and it does so only because, in a fit of theologian's brag, Anselm insisted it be appended to every future edition of his own book.

4. OPINIONS ABOUT ISLANDS

Gaunilo doesn't really point at a specific flaw in Anselm's argument. Rather, he seeks to show that there must be one somewhere, by proving that, if we accept it, undesirable consequences will follow. If Anselm's proof works, Gaunilo writes, we'll end up proving the existence of a perfect instance of pretty much everything.

Consider the notion of a perfect island — an island so great (in a sense not simply synonymous with "big") that no greater island can be conceived. By paralleling Anselm's line of reasoning, we could prove that such an island necessarily exists, since otherwise we could conceive of a greater island — one which shares all of the former's traits, plus existence. The same, of course, could be said of the greatest ice cream parlor, the greatest mass murderer, or the greatest Higgs boson. This is what makes this objection so troubling. If it holds, Anselm's argument just populated the universe with a lot of perfection.

The objection is answered in a second appendix of Anselm's treatise. It is extremely short, and vague to the point of evanescence. The best way to make sense of it is through an example. Let's assume some criterion of what is great about islands — for instance, palm trees and beaches. It would follow that an island with any given amount of those will not be as great as an island with more.

This would seem to point at an obvious recursive demonstration that the notion of the greatest conceivable island doesn't make sense, since it involves a numerical attribute which itself can always increase; Anselm wants us to go there, but the path he chooses is more puzzling. It is indeed possible to conceive of an island with an infinite number of beaches and palm trees. This would be the greatest island: only, it wouldn't be an island. An island is by definition enclosed. But the perfect island would have infinite beaches, hence infinite extension, hence it would be mainland. Thus, Gaunilo's parallel fails because there is no such notion as that of a "perfect island."

The main reason for Anselm's strategy here is that he wants to blame the failure of the "perfect island" counterexample on the "island" part of

the phrase — so as to avoid questioning the notion of greatness he himself needs for his proof to go through. This might seem even more farfetched than what we've already seen. But surprisingly enough, once rephrased in the mathematical terms of set theory (which would be developed over six centuries later), it actually makes some sense. Imagine the set of all real numbers inferior to 6. The set has no greatest element; since for any "great" number inside it, such as 5.999, we could always add a few more 9s after the comma and make it even greater. And yet there is a number greater than all of them (8, for instance, or for that matter 6 itself); only, such number is not part of "the set of all numbers inferior to 6."

In this example, then, "being an island" behaves similarly to "being a number inferior to 6." Greatness, Anselm seems to say, applies in different ways to different notions. And while a single notion (god) can be greater than anything else, maybe the same isn't true within some specific category of notions such as islands or bosons. His conclusions would be generalized, theologically, in Leibniz's idea that countable attributes are a mark of imperfection and hence do not pertain to divinity — which sounds both sensible and an easy way out.

In any event, the philosophical tradition sided with Anselm against Gaunilo, and promoted the former to theological glory while the latter languished in an appendix of the *Proslogion*, condemned to be remembered as the guy for whom just a few palm trees were more than enough.

If the argument still sounds rather less than conclusive, this is probably because — to a contemporary mind — the notion of "greatness" it involves seems fuzzy to say the least. A pretty solid attempt at clarification would be made by Kurt Gödel some 800 years later.

But before we worry about that, there's a stronger objection to be dealt with — the one that involved paying your bills with money that you thought existed. It was formulated at the end of the 18th century, when steam textile mills had just started hissing all across northern Europe, producing money more solid than anything Anselm might have conceived. Its author, in his capitalist naiveté, was Immanuel Kant.

5. THIS AND EVERYTHING ELSE

One of the most enjoyable sections of Kant's *Critique of Pure Reason*, per se not a landmark of enjoyable reads, is devoted to what he calls the "paralogisms of pure reason." It's a treasure trove of philosophical conundrums that had puzzled thinkers for the previous 2,000 years, and which, in Kant's opinion, all find a final solution within Kant's own theory of knowledge. Among them, of course, is the proof of the existence of god, which persisted more or less unchallenged until then. (The fact that it still puzzles us today should say something about this last point.)

Kant finds little to object to Anselm's ideas about how we can conceive of entities by combining properties — a strategy with roots stretching back to Aristotle and onward all the way up to 20th century constructivist psychology. Only, Kant maintained, while Anselm's "greatness" was a legitimate property, existence wasn't.

Kant drew a sharp distinction between TRUTHS WE LEARN BY THOUGHT (such as those offered by a theorem) and TRUTHS WE LEARN BY EXPERIENCE (such as those offered by bumping into a previously unseen glass door). The former have to do with *concepts* — that is, combinations of properties; the latter with the occurrence of *instances of such concepts in real life* — that is, combinations of properties you perceive by smashing into them.

Concepts are characterized by *properties,* such as "transparency" or "hostility." These properties add information to the concept, by grouping it with other concepts that share the same property and separating it from those that don't. This mirrors how properties are formed: you see a bunch of things with a common characteristic that distinguishes them from everything else, and you group them together in your mind. Both aspects (what something is like and what it isn't) are crucial to the property's informative value.

The key here is precisely that "everything else." For existence to be a property, it would need to be formed in the way just described — by contrast and comparison. But existence is the shared characteristic of everything you're bound to perceive. Nothing exists that doesn't exist

(ha). Hence, as a property, the informational value of existence would amount to zero. Hence, existence is NOT a property, but rather simply *the difference between a concept and something you encounter in real life.*

This, he added, also explains the difference between a proof such as Pythagoras's and one such as Anselm's. Since they are *conceptual* truths, both are only significant insofar as their object exists. But this cannot be stated within the concept itself, as EXISTENCE IS NOT A PROPERTY. The correct form of stating them would thus be conditional: "IF right triangles exist, then the square on their hypotenuse is the sum of the squares on the two sides." This is a perfectly reasonable, if slightly redundant way of stating a geometrical theorem. But it turns Anselm's conclusion into a banality: "If god exists, then it exists." Duh.

Kant's objections to Anselm have lived way beyond his theories about everything else; some version of the "existence is not a property" idea has been widely accepted well into the 20th century. The most compelling argument against it was put forth in the 1970s by American philosopher David Lewis, whom some encyclopedias refer to as "the greatest American metaphysician." Lewis also wrote a version of the ontological proof (which he himself would sabotage), as well as a treatise that included the sentence: "The only objection I cannot refute is an incredulous stare." His expression while saying this can be seen on this bulletin's cover.

6. THINGS THAT MIGHT HAVE BEEN THE CASE

David Lewis's research focused on the analysis of modality, a family of logical concepts having to do with possibility and necessity. He is best known for a position called "modal realism," which, in its barest form, states that infinite possible worlds exist. One of the many ways he argued for modal realism was through Anselm's 1,000-year-old proof of the existence of god.

But first, some background. The notion of "possible world" originated in the philosophy of language as a theoretical construct to account for the meaningfulness of hypothetical statements. Normal declarative statements

(of the form "a is b") are considered to be meaningful if they can be true or false: understanding them involves understanding what the world is like if they are true. This theory of meaning seems pretty self-evident. But what about a statement such as "Kim Dotcom could be slim"?

This statement is true: the larger-than-life web tycoon could go on a diet and lose whatever excess weight he has. On the other hand, a statement such as "Kim Dotcom could be a slab of granite," strikes us as false, at least if we understand the name "Kim Dotcom" to refer to a human being.

The "what-the-world-is-like" theory is not enough to account for statements such as these, for there is nowhere in the world that we might turn to in order to check whether they are true or false. They do not describe things as they are, but only things that might have been the case.

"Possible world" is, then, a fancy name for a list of such THINGS THAT MIGHT HAVE BEEN THE CASE. In their simplest version, possible worlds are defined as all the combinations of properties and entities that do not entail a logical contradiction when conceived. For instance, in the real world Kim Dotcom is:

German-born	×
Fat	×
A slab of granite	
French-born	
Wanted by the law	×

We can hypothetically conceive of him as French-born, or slim, or both; but if we give the words their current meaning it would be a contradiction to conceive of him, for instance, as both German-born and French-born. By combining all the properties not involving contradictions, we can create a list of possible scenarios:

	@	1	2	3	4	5	6	7	8	9	10	11
German-born	×		×	×	×							
Fat	×			×		×	×	×	×			
A slab of granite												
French-born							×	×		×	×	
Wanted by the law	×				×			×	×		×	×

Every statement of the form "Kim Dotcom could have been" followed by any of the above properties will be determined as true or false by this table. Since the @ column describes the real world, columns 1 to 11 have been referred to, by analogy, as possible worlds. But, at least in this minimal version, these "worlds" are little more than a semantic device.

Two things are interesting to notice here. On the one hand, we defined this very loose sense of "possibility" by using the same notion of conceivability Anselm based his proof on, when he defined god as "the greatest conceivable entity."

On the other hand, and more importantly, once we accept this theory of modality, Kant's killer argument against existence-as-a-predicate sort of flops. Consider a more complex hypothetical statement, such as "I could have been as fat as Kim Dotcom," which refers to two entities: the real Kim Dotcom and a possible, fat Vincenzo Latronico. If possible entities are legitimate elements of a meaningful, true statement, all of a sudden we have just what we needed to define existence as a property: we can refer to something that doesn't exist.

Both points were what had been preventing, in the two centuries between Immanuel Kant and David Lewis, Anselm's old argument to go through. One might wonder where this left god.

7. IS ONE WORLD ENOUGH?

David Lewis did exactly this bit of wondering in his paper "Anselm and Actuality." The paper tries to rephrase Anselm's proof in modern, logical terms, within his general theory of possible worlds.

Lewis's rendition is to Anselm's original proof what Euclidean geometry was to Pythagoras's drawings on the white sand of Samos. On the one hand, you have an informal, loose summoning of a truth. On the other hand, you have formal theoretical frameworks — geometry and possible worlds — within which a proof can be written as a set of premises from which a conclusion (the theorem) can be derived. It's like a grown-up

engineer's attempt to make sense of the fantastic fortresses he drew as a kid. Some might stand.

Anselm's argument involved three premises. The first was a general definition of possibility. The second was a very vague definition of greatness. The third was a definition of god as a conceivable entity as great as it gets. This was enough to reach its conclusion. Lewis's task was then merely to find a formal translation for the premises; a proofing algorithm, or a red-eyed grad student, could then check whether the argument was valid.

He had no problem with the first two premises. The first, after all, was as close to a definition of possibility as we have dealt with so far: if some combination of properties is conceivable (that is, not contradictory), then it's possible.

The second involved the problem of defining Anselm's very vague "greatness." Lewis mostly didn't. He understood it as a rule that calculated a number for every entity (its greatness) in every possible world—a bit like a metaphysical credit check. We needn't specify all the parameters involved in determining this number, only the minimal requirement that something that exists in a world is greater than something that doesn't. This doesn't seem too farfetched.

The third premise—the definition of god—was more problematic. Anselm defined it as the greatest conceivable entity. But an entity's greatness varies from world to world (since it is higher where it exists). So, suppose we find some entity we believe to be the greatest one around. In which world should one go check whether there's anything greater? Is one world enough? And which one? Or all of them?

Different answers to this question lead to different versions of the third premise: different completed transcripts of the spell for the summoning of god. Of course, as every reader of *Harry Potter* knows, not all versions of a spell will work with equal strength.

A first, stronger version of the premise would say all of them:

‹ANY› There is an understandable being whose greatness *in any world* is higher than the greatness of any other being *in any world.*

A second version would say just one world is enough, provided it's ours:

‹REAL› There is an understandable being whose greatness *in the real world* is higher than the greatness of any other being *in any world.*

A third, less strong version would say just one, any one:

‹SOME› There is an understandable being whose greatness *in some world* is higher than the greatness of any other being *in any world.*

A fourth would say even less:

‹SPECIFIC› There is an understandable being whose greatness *in some world* is higher than the greatness of any other being *in that specific world.*

Anselm's text was much more vague than this — maybe because of the different writing standards in his times (the paragon was Medieval theology, after all), or because some arguments, like love letters, just sound more convincing when some detail is casually omitted.

The premises of any proof can be pictured as a sequence of steps in a ladder. The top (the conclusion: in our case, the existence of god) seems unreachable from the bottom (where we, morons in Anselm's eyes, are); each step brings us a bit further. As in a ladder, steps must not be too close to each other (otherwise the conclusion will still be unreachably high), nor too far apart (otherwise we still won't be able to reach them). This means that premises must be (a) effective — allowing us to prove the conclusion — and (b) plausible enough to be more credible, closer to the ground, than the conclusion itself. In its vagueness, Anselm's version seemed to be both.

What we're trying to understand now, then, is if any of Lewis's shiny, modern, aluminum steps can replace the old, carved wood one Anselm left us with.

8. A WORLD WHERE ONLY MUD AND IKEA CATALOGS EXIST

Let's take a closer look at how possible worlds would be if we accept any one of these premises.

‹SPECIFIC› has every entity being as great as it gets in some world, which is both fair-minded and useless. It merely ends up selecting whatever out-greats everything else in some world, and designates it as god. But since worlds are built by exhausting combinations, everything will out-great everything else, in some world. For instance, if we assume (as there might be reason to) that the late writer Kurt Vonnegut was, in his lifetime, as great as it gets, then the premise **‹SPECIFIC›** allows us to prove that Kurt Vonnegut is god, or was, until 2007. Something's wrong with this version—it clearly doesn't allow the proof to go through—and Lewis dropped it. That spell didn't work.

A premise such as **‹SOME›** also clearly wouldn't work. What it allows Lewis to prove is that god exists in some possible world, but he has no information about whether it's the real one or not. **‹SOME›** might be easy to defend, but it doesn't allow us to derive the conclusion. This spell, too, is not good for us.

Lewis is left with **‹ANY›** and **‹REAL›**. Either of them makes the proof valid—that is, either of them justifies the conclusion that "god exists." (We could have reached the same result by formal derivation, but it would have been less fun.) The success of Anselm's proof thus depends on whether either of these premises can be successfully argued for.

‹ANY› is the strongest version of Anselm's third premise. It effectively proves the existence of a necessary being—an entity that exists in every world; because if, in some world, it didn't exist, then by our definition of greatness, whatever was in that world would end up being greater than it.

The way Lewis defined possible worlds, however, excludes the possibility of necessary beings. He assumed that every conceivable entity is granted possible existence. This would also be true, say, of a world where only mud and IKEA catalogs exist—there is no logical contradiction in that. In such a world, god would then be either a chunk of mud or an IKEA

catalog, or part thereof. This hardly seems to befit any known concept of divinity.

The only option left is ‹REAL›. It does indeed seem reasonable — at least with the somewhat stretchy standards of reasonability we're upholding here — insomuch as it both fits whatever notion one might have of god, and leads to the conclusion that it exists. And yet, there's something suspicious, almost circular, in it referring precisely to the actual world, among the uncountable, infinite possible worlds we defined in our model. Many of them will, in fact, be better worlds, for any possible notion of "better" (no war, free alcohol, immortality — so long as it's not a logical contradiction, it's in there). Why would the greatest possible being then exist specifically in ours? What's so special about this world?

Well, for one, it's the real world, while the others are, uh, lists of statements. This must mean something.

As a matter of fact, it does. Lewis here summons the persona of what he calls an "ontological arguer," because the proof's case sounds stronger when argued for in first person, or maybe because speaking of "seats of perfection" is indeed kind of awkward if you do it on your own behalf. Let's follow him on this and give our ontological arguer a gender-neutral name. Sam will do.

9. SAM SAYS

Well, now. You agreed to accept modal logic as a sound tool to analyze the implications of our ordinary statements about possibility. You also agreed that, if you are to believe in the meaningfulness of everyday statements such as "I could be fatter than Kim Dotcom," you must accept believing whatever this might be shown to entail. Starting from these considerations, and from a very basic assumption about the notion of god — which you agree to be at least conceivable — I can almost prove his or her existence. The only step I miss is a reason to justify ‹REAL› — a reason to accept the idea that his or her greatness must be at its greatest in the real world among all possible ones.

But you said it yourself. The real world is, well, the real world. It's the only vivid, tangible one. It's the only one where causation happens. It's the only one that is not a list of statements. If any one world must be mentioned in the definition of the greatest conceivable being, it follows that ours should be it. Its realness is a unique quality that sets it apart as the only appropriate seat for perfection.

If you agree to that, there's nothing you can do to refute Anselm's argument's conclusion, except reject the very possibility of speaking meaningfully about, well, possibility. That sounds like a steep price to pay. You're better off with god, after all.

I win.

10. AHA-cum-WTF

This is where Lewis drops the mask and shows what all this is really about. He did not actually believe a philosopher could prove the existence of anything, especially that of an almighty, invisible creator. Which means that, if one such proof seems to go through, the only option sensible philosophers are left with is to question their assumptions. Revising the latter is surely an easier thing to do than accepting Sam's proof. And so it happens that Lewis was interested in the ontological argument only because showing it to be successful would allow him, in an aha-cum-wtf moment, to question the nature of reality. Or, rather, of "reality."

Under scrutiny, Sam's argument didn't prove the existence of god, but only that the existence of god derives from the recognition of the uniqueness of the real world among all possible ones. Lewis does not deny such uniqueness. Only, he understands it in different terms.

Think of "today" — the term referring to the day you're reading this. It seems to refer to something unique. There have been and will be countless Wednesdays, but only one is today. This seems to mark it as a unique day among all the others — a fitting moment, say, for doing something grand or particularly foolish. Isn't it?

Of course not. Linguists call words such as "today" (and "now" and "I" and "here") *indexicals*: words that do not describe a property of an object, but its relation to the utterer. It's not that they don't have a meaning; only, their meaning is determined by — indexed to — the time and place where they are pronounced. "Anselm and Actuality" was Lewis's way of arguing that we should consider "real" to be just one such word, and "this is the real world," to be as uninformative and tautological as "this day is today."

Just like "today" or "here," Lewis maintained, "real" refers to something in relation to the utterer, stating that the two are in the same possible world. This sounds weird, because we've only ever heard the term as referring to our world. But this amounts to saying that we've only ever heard "today" as referring to the day in which it was uttered. Of course: THAT'S THE WAY IT'S USED.

Considering "real" to be an indexical term allows Lewis to do away with the paradox of the almost-identical world: Sam's statement, "This is the real world," will now be true there, too, since it will refer to that place.

This also allows Lewis to do away with Anselm's argument completely, since the reasoning behind premise **‹REAL›** no longer works.

11. LEWIS SAYS

The argument from **‹REAL›** is valid, but **‹REAL›** derives its credibility entirely from the illusion that because our world alone is real, therefore our world is radically different from all other worlds — special in a way that makes it a fitting place for greatest greatness. But once we recognize the indexical nature of reality, the illusion is broken and the credibility of **‹REAL›** evaporates. It is true of any world, at that world but not elsewhere, that that world alone is real. The world an ontological arguer calls "real" is special only in that the ontological arguer resides there — and it is no great distinction for a world to harbor an ontological arguer.

Think of an ontological arguer in some dismally mediocre world — there are such ontological arguers — arguing that his world alone is real, hence

a fitting place of greatest greatness, hence a world wherein something exists that which no greater can be conceived. He is wrong to argue thus. So are we.

Let's leave some blank space for the rhetoric to reverberate.[1]

David Lewis thus used Anselm's ontological argument as a bogeyman, to scare philosophy majors into accepting his theory of possibility (that "real" is an indexical term). This, he maintained, was the only way to ensure Anselm's proof couldn't go through and, well, prove the existence of god.

To a certain extent, he succeeded. But one might argue that his reconstruction of Anselm's argument — just like the original version, according to Gaunilo — was plagued by a misunderstanding of the concept of greatness. Come to think of it, his whole line of reasoning pivoted precisely on the definition of that term.

What if we did away with it altogether?

12. A PRETTY GOOD YEAR FOR GOD IN NEW JERSEY

1970 was a pretty good year for god in New Jersey. David Lewis had just begun teaching at Princeton when his paper came out in May. Meanwhile, in his room at the Institute for Advanced Studies, a half hour's walk from Lewis's office at the Department of Philosophy, Kurt Gödel finally decided

to go public with his own ontological proof, which he had devised several decades earlier but chose to divulge only now because he sensed death was nigh.

It wasn't — Gödel would live until 1978 — but that fear was enough to lead him to ask Dana Scott, himself one of the greatest logicians of the century, to keep a copy of the proof in case anything happened. Scott would only publish it a decade after Gödel's death. Speculation about whether its author truly believed in it hasn't stopped since, both because sources differ as to whether Gödel was religious or not, and because, by Gödel's standards, it is quite a messy bit of logic. It's actually the sketch of a proof, a precise formalization of which logicians have been trying find for over 20 years without settling on a universally accepted solution.

Gödel's proof originated from a somewhat cryptic remark by 17th-century philosopher Wilhelm Gottfried Leibniz. Leibniz noted that Anselm's ontological argument could be stripped to the mere demonstration that "all perfections are compatible." It boiled down to this: in Leibniz's extremely complicated metaphysics — arcane even by the somewhat tolerant standards of, well, metaphysics — the universe was the effect of a sort of combinatorial mechanism, much like possible worlds as we defined them before. Every noncontradictory sequence of properties was granted a "logical" form of existence, whatever that might mean. Some of these combinations, called complete notions, corresponded to actual, tangible, existing things.

To Leibniz, the universe had an intrinsically ethical structure; properties, just like actions in a traditional moral system, could be classified according to their goodness. When this was maximal, a property was a "perfection." By the same line of reasoning, necessary existence was clearly a perfection, just like omnipotence, omniscience and whatnot. This meant that a combination of all perfections — which could be an appropriate definition of the Christian god — would be omnipotent, omniscient and exist necessarily.

In his system, it followed that the only thing preventing god's existence from being a theorem was the eventuality that perfections might not be compatible — that the conjunction of two or more of them, that is, might

generate a contradiction. A proof of the existence of god would then simply be a proof of the compatibility of perfections. Leibniz tried to offer one, but it was so deeply enmeshed in his metaphysics that anyone rejecting the latter (which is what almost every philosopher after him did) would end up unable to support the former.

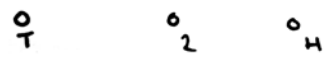
Kurt Gödel sought to change this.

13. SOMETHING THAT FEELS METAL AND POINTY

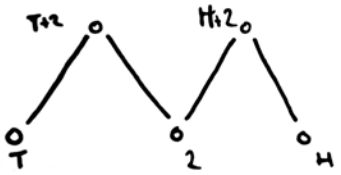
Gödel's proof is carried out in an extremely powerful and complex formal language, known by the alluring nickname of "third-order modal predicate calculus with unrestricted lambda-extraction." The best way to understand it is to visualize dealing with language as a whole, as opposed to individual entities or propositions.

We have seen that possible worlds exhaust every non-contradictory combination of properties. In this case, if god exists, there must be in some possible world a combination of properties that describes it. Gödel's problem consisted of how, exactly, to identify this combination.

Of course, we cannot sift through possible worlds until we find the entity —god— we're looking for (the logician Saul Kripke once dismissed Lewis's theory by saying he "seems to believe possible worlds are discovered through powerful telescopes"); specifying what those properties are (omnipotence, omniscience, etc) is not within the logician's job description. Gödel thus sought a way to univocally characterize the structure of properties an entity such as god might enjoy —the relations that must be holding god together. It's akin to the way you check whether your keys are in a canvas bag not by rummaging in it (too much stuff in there) but by fingering its surface, looking for something that feels metal and pointy. So, what would the structure of god's properties feel like? Well, picture properties as a set of points on a grid. Suppose T means "being a tree," H means "being human," and 2 means "being two meters tall"; a small section of the grid would then look like this:

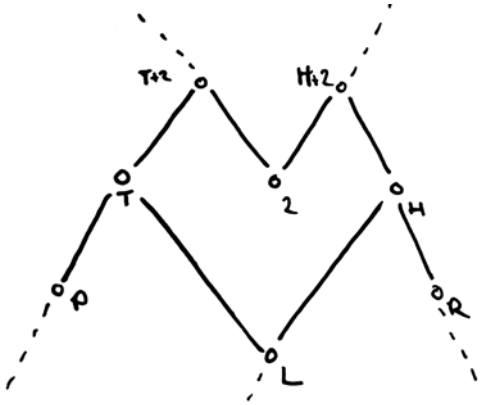


If the conjunction of any two properties is not contradictory, it, too, will be a property. It can be depicted as being one step above its constituents. In this case, T+2 means "being a 2-meter-tall tree," and H+2 means "being a 2-meter-tall human" :



This diagram can, of course, also be read from the top down; the link would then be read as entailment rather than conjunction, since anything which is a 2-meter tall tree would obviously be both a tree and two meters tall.

This graph's vertical development ends here. T+2 and H+2 cannot be conjoined, for "being a tree" while "being a human" would be contradictory. It could go further down, though. For sure, T entails several more general properties (such as R, "having roots"), and so does H (such as P, "being a primate"). Some of them will be shared by both (such as L, "being a lifeform"). This would form a lattice-like structure, as follows:



Note that T+2 and H+2, here, are not final stops of the conjunction rule — they might be conjoined with other properties, for instance "being in Turin" (which might apply both to a tree and a human). A hypothetical single peak in this graph would be a maximal set of properties: that is, a set of properties that conjoins every possible property of an entity, in a way that any other addition would either be redundant (that is, already entailed) or contradict something that has already been specified about it.

Gödel reasoned this would be precisely the case of god. He thus went fingering the canvas bag of language looking for the spiky feel of such a closed pyramid — what in set-theoretical terms is called a "complete lattice." He called the properties he was looking for "positive properties." The requisites for the set of positive properties to form a lattice, under this perspective, are that:

a. If a property is positive, its opposite is not;
b. If a property is positive, it ONLY entails positive properties.

Keeping in mind the structure we just saw, it's easy to see the reason behind these conditions. Why did our example stop at two peaks? Because "being a tree" entailed — among other things — "not being a human," so that the two couldn't be conjoined. Gödel's definition of positive properties ensures that everything they entail is compatible; hence, it ensures that they form a single peak. They also meet at the base, for every property trivially entails the universal property of being identical to oneself, A=A (rather than a pyramid, then, the lattice would look more like an oddly skewed diamond, or, actually, a double-truncated cone). From any point in the diamond, there is always a one-stop path to the bottom point (A=A), since every property entails it, whereas the path from any point to the peak might be longer.

Imagine it as a scheme of phone connections in the U.S. administration: Barack Obama has the direct phone number of every civil servant, while a post office manager in Butte, Montana will probably need several — several — relays and secretaries to get in touch with Barack.

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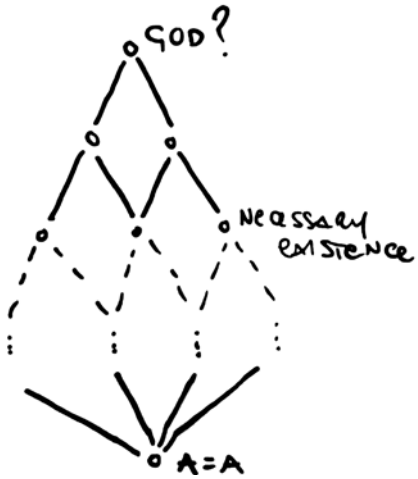
911, on the other hand, can be reached by literally everyone without any intermediaries. In this scheme, Obama would be the top of the diamond —fittingly enough, that's where we placed god; 911 would be the bottom; the post office manager would be somewhere in-between, since perhaps the cops would need to go at least through her secretary in order to reach her.

Gödel's structure of positive properties can be proved, with a few minor additions, to be logically noncontradictory. It can also be proved that, if we accept "necessary existence" as a property (defined as existence in every possible world), and we stipulate it to be positive, no contradiction ensues.

This amounts to saying, of course, that this set of properties actually applies to at least one entity in at least one possible world. But since it includes necessary existence, this amounts to saying this entity exists in every possible world, including the real one.

Gödel seems to have found the keys.



14. WHAT DOES THE PROOF PROVE?

Technically, this:

If we accept necessary existence as a property, there is a maximal set of properties ("positive properties") of which it is part, and all of which are

mutually compatible, and therefore there is an individual they are true of, and this individual exists necessarily.

To say that this proves the existence of god is a bit like saying that the U.S. constitution guarantees the right to the pursuit of happiness: it does, but there are a few details to figure out.

What we're saying is that Gödel's spell DID bring something into being; it's sitting there all shiny and moist with arcane juices in our circle of runes. But we have yet to understand if it's what we wanted to summon to begin with (god) and if we're happy with it being around. Creepy stuff can creep out of that portal.

One thing that follows from Gödel's proof is that the entity it describes is unique (which is sort of the point), and it enjoys no concrete attribute whatsoever — that is, every property of physical objects (such as having a certain size, being somewhere, etc.) is NOT POSITIVE. Whether this is good or bad is for theologians to discuss, but in any case, there's much more interesting stuff to consider here.

The language required to formulate Gödel's proof has basically unrestricted expressive powers: not only can it turn everything we say into formal expressions, it can also define properties that regular language has a very hard time capturing (for instance: "the property of being the property of a property" — ugh). "Expressive power" sounds like a good thing, but more often than not, it isn't — a bit like the requirement for "firepower" on a vacation in Sierra Leone. On paper, it should make you feel safe, but it's actually a sign that you aren't. Among these powers is the ability to turn any true statement — such as "England won World War Two" — into a property. This property would read something like "existing while England won WW2," and would trivially apply to any object in existence while the statement is true.

This, of course, means it applies to the god we just proved the existence of. In which case, it must be a positive property, since we defined god as the entity enjoying the combination of all such properties, and we required such a combination to be maximal (the pyramid's peak!).

But the positiveness of properties doesn't change across worlds, which means that "existing while England won WW2" is positive — and hence applies to god — in every possible world. A straightforward consequence of this is that England demonstrably won WW2 in every possible world: it could not have lost, which makes its victory a logical necessity — just like, say, the fact that 2+2=4.

The same argument, of course, can be made for any true statement: that I am writing this while eating butter cookies in Turin, that I am not as fat as Kim Dotcom (but currently working towards it), etc. This situation is called "modal collapse" — when all possible worlds collapse on the real one, all sentences with "could" become false. It's a huge problem.

It might not look like one. After all, a theologian might argue the idea that everything happens because of god's wish. Predestination and the lack of free will are indeed part of some religious perspectives. Leibniz, perhaps unsurprisingly, even went as far as maintaining that, if we believe in the Christian tradition, ours cannot but be the best of possible worlds.

But we are not theologians. When we originally introduced modal logic back with David Lewis, we did so because it allowed us to express truths (such as the fact that I could be fatter than Kim Dotcom, or that England could have lost WW2) that traditional logic could not account for. The same modal logic now turns out to show that such truths were actually no truths at all — because from Gödel's proof it follows that nothing could have been different from what it is.

This isn't merely an undesirable consequence, because — as Kripke pointed out — possibilities are not discovered with telescopes; *they're part of the way we think about the world.* And if a formal tool we created to express them (such as modal logic) does NOT allow us to express them, well, it loses its reason to be acceptable. But without modal logic, the whole of Gödel's proof would be impossible to express. As a sort of slapstick sidekick to Baron von Münchhausen, the proof thus pulled the carpet from under its own feet.

So what's under the carpet?

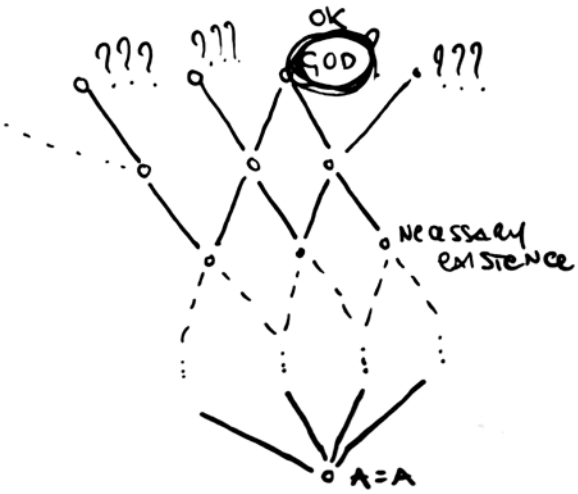
15. AN INFINITE ARRAY OF ALMOST-PERFECT DEMIGODS

There's still a way for Gödel's spell to work. The modal collapse objection we just saw is only possible because we are forced to decide whether "existing while England won WW2" is a positive or negative property.

But while Gödel only spoke of positive and non-positive properties, he didn't explicitly rule out the possibility of a third category. He merely said that if a property is positive, its opposite is not. This is still compatible with the idea that, alongside the opposing groups of positive and non-positive properties, we might consider a third group of "neutral" properties, whose opposites also are neutral. Think of how numbers behave when multiplied by -1. Some of them switch sides: positive numbers become negative, and vice versa. 0, on the other hand, remains 0.

If we accept this three-group partition on properties, we could reasonably argue that properties such as "existing while England won WW2" are neutral, and their opposite also is: and so god might enjoy them in some worlds, but it needn't do so in others. Neutral properties wouldn't be the crucial attributes of a divine entity such as omnipotence, but the more mundane, contingent ones, such as having heard that prayer I uttered in 1993 that Cecilia would finally kiss me (the god of our world, needless to say, didn't hear it). God could very well enjoy these properties in some worlds, and their opposites in others, while still retaining a core of divine attributes across every world. This would prevent modal collapse.

Unfortunately, modal collapse isn't the only thing it would prevent. A crucial aspect of Gödel's proof was its ability to prove the uniqueness of the peak of positive properties. But this corollary required precisely the positive-negative binary split we just dropped in order to nail the carpet on the ground under our feet. For, once we accept negative properties that can be compatible with the whole set of positive properties (including necessary existence, of course) without generating a contradiction, there's no way to rule out a structure like this:



In other words, in this case the spell becomes too powerful: and summons god, yes, alongside an infinite array of almost-perfect demigods all around. But after all, we could say they're saints.

This is where ontological arguers on both sides of the struggle — some forced to choose between modal collapse and infinite demigods, others having spent hours or years discussing saints and lambda-extraction — usually give up.

16. WHAT ABOUT THE MAGIC?

In the end, there really isn't much else to say for, or against, Gödel's proof. The objections we just saw (alongside some others, more complex) do not ultimately undermine it. The result remains valid, if one is willing to accept modal collapse, or else if one isn't troubled by the existence of a potentially countless number of necessarily existing, immaterial beings.

Yet, a final arrow lingers in the most passionate discussant's quiver. As a consequence of the very high expressive powers of the language required by Gödel's proof, not every theory that can be proven within it has an interpretation. That is: it is possible that a theory is demonstrably consistent, but no set of actual entities can ever fulfil its assumptions. If this were the case, Gödel's proof would seem to make sense but actually be meaningless.

My only scholarly contribution to this topic was a short demonstration that this arrow doesn't hit its mark. Gödel's proof does, after all, make sense.

So, what sense is it?

Alas, not the intended one. I don't believe anyone takes these proofs seriously, if "seriously" is taken to mean that whoever hears them, and finds them conclusive, changes his or her mind and all of a sudden turns to religion. Most often, they are studied as limit cases in language analysis; the underlying principle being that if your language allows you to prove the existence of some supernatural being, there's something wrong with your language. All well and good, but what about the magic?

There seems to be none; at least not the kind of magic that drew us to this to begin with. Here, too, philosophy proves faithful to the adage that has it concerned with questions rather than answers — if for no other reason than because the latter, once found, are immediately fed back into the questions so as to fine-tune them and make them more astute. An ontological proof's success can always be undermined by a change in axioms, definitions, inference rules — any principle can be changed in order to make a proof valid or failed. As a matter of fact, this usually goes the other way around: a change in axioms, definitions or inference rules requires a strong case as to what the undesirable consequences could be. A proof FOR the existence of god would be one such consequence.

Interestingly enough, this ends up incarnating the OPPOSITE of Wittgenstein's famous soundbite, which claims that since every question that can be formulated in language must necessarily have an answer — "there is no enigma" — then what's most important and relevant to us cannot even be said. When formulating that proposition, Wittgenstein had in mind a language and a logical system whose rules were set in stone (or in neurons, or in Kantian categories — it's pretty much the same anyway).

But as we have seen, we can have our logical system do pretty much whatever we wish it to do — including materializing deities or possible worlds. What's important to us can indeed be asked, and answered, but the truth of such an answer will be exactly what we want it to be.

The point is not that there's no enigma, but rather that *there's nothing to discover*: You pick open the lock and realize the vault holds exactly what you left in it.

And so it turns out that the only thing proofs such as this prove — the only truth we can materialize out of thin air — is *whatever we require of our logical system,* whatever kinds of things we want our language to admit (gods, worlds, possibilities). We can indeed discover something with an ontological proof, only this discovery will not be ontological in nature, "merely" moral: the tacit assumptions we have about how a language should be. And one of these tacit assumptions is that no truth, really, can be materialized out of thin air.

Or can't they? I never finished my PhD thesis on how to prove existence with logic. I write novels now.

*

1. This whole section, with minor changes in terminology, is excerpted from D. Lewis, "Anselm and Actuality," *Noûs,* vol. 4, no. 2, May, 1970.