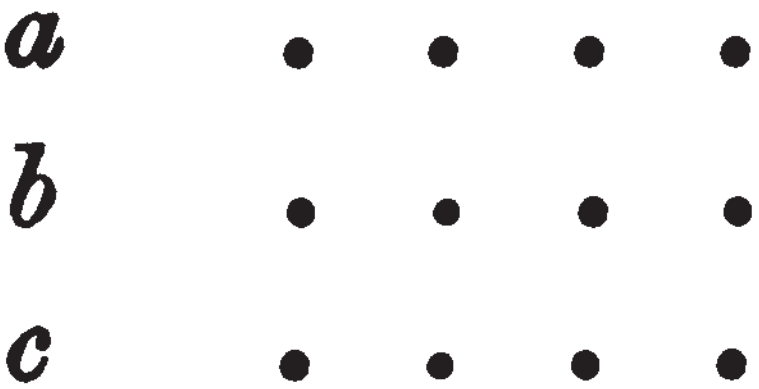
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This brief bulletin is excerpted from *The Psychology of Number* by John Dewey and James McLellan, a small book intended as a guide for teaching children. It was first published in 1895, 40 years before Dewey's best known work, *Art as Experience*. McLellan was a Canadian pedagogue whose previous work was already indebted to Dewey's thinking.

When this book was published, psychology as a discipline distinct from philosophy was being articulated by William James. Psychology was directly concerned with reception, affect, and perception of the human subject. At the same time, James and Dewey were also constructing the emerging American philosophy of Pragmatism, which placed its focus on results, outcomes, and empirical evidence. *The Psychology of Number* attempts to apply a similarly concrete approach to the abstract idea of number in order to set a child on a productive course of thinking.

Some of the original orthography has changed to make for easier reading through modern eyes.

Cover image: A diagram from the original edition: "This rhythmic process of parting and wholing which leads to all definite quantitative ideas, and involves the correlation of times and parts, may be illustrated by simple intuitions. In measuring a certain length we find it, let us suppose, to contain four parts of three feet each; then the relation between parts (measuring units) and numerical value (times of repetition) may be perceived in the following, where the dots symbolize both times and units of quantity."

DEFINITION OF NUMBER: The simplest expression of quantity in numerical terms involves two components:

1. A *Standard Unit*: This is itself a magnitude necessarily of the same kind as the quantity to be measured. Or, as it may be otherwise expressed, the unity of quantity to be measured and the unit of quantity which measures it are HOMOGENEOUS quantities. Thus, inch and foot (measuring unit and measured unity), pound and ton, minute and hour, dime and dollar, are pairs of homogeneous quantities.

2. *Numerical Value*: This expresses HOW MANY of the standard units make up, or construct, the quantity needing measurement. Examples of numerical value are: the yard of cloth costs SEVENTEEN cents; the box will hold THIRTY-SIX cubic inches; the purse contains EIGHT ten-dollar pieces. The seventeen, thirty-six, eight represent just SO MANY units of measurement, the cent, the cubic inch, the ten-dollar piece; they express the numerical values of the quantities; they are pure NUMBERS, the results of a purely mental process. The numerical value alone represents the relative value or ratio of the measured quantity to the unit of measurement. The numerical value and the unit of measurement taken together express the absolute value (or magnitude) of the measured quantity.

In the teaching of arithmetic, much confusion arises from the mistake of identifying numerical value with absolute magnitude — that is, NUMBER, the instrument of measurement, with measured quantity. Number is the product of the mere repetition of a unit of measurement; it simply indicates HOW MANY THERE ARE; it is purely abstract, denoting the series of acts by which the mind constructs defined parts into a unified and definite whole. Absolute value (quantity numerically defined) is represented by the application of this HOW MANY to magnitude, to quantity — that is, to limited quality. To take an example of the confusion referred to: we are told that division is dividing a (1) number into a (2) number of equal (3) numbers. This definition as it stands has absolutely no meaning; there is confusion of NUMBER with measured QUANTITY. Doubtless the definition is intended to mean: division is dividing a certain definite quantity into a number of definite quantities equal to one another. Only in (2), in the definition as quoted, is the term number correctly used; in both (1) and

(3), it means a measured magnitude. A measured or numbered quantity may be divided into a number of parts, or taken a number of times; but no number can be multiplied or divided into parts. Number SIMPLY as number always signifies how many times one "so much," the unit of measurement, is taken to make up another "so much," the magnitude to be measured. It is, as already said, due to the fundamental activities of mind, discrimination, and relation, working upon a qualitative whole; and we might as well talk of multiplying hardness and redness, or of dividing them into hard and red things, as to talk of multiplying a number or of dividing it into parts.

It may be observed that the problems constantly used in our arithmetics, multiply 2 by 4, divide 8 by 4, are legitimate enough provided they are properly interpreted, if not orally at least mentally, but taken literally are absurd. The first expression means, of course, that a quantity having a value of two units of a certain kind is to be taken four times; and similarly $8 \div 4$ MEANS that a total quantity of a certain kind is measured by four units or by two units of the same kind. Of course, in all mathematical calculations, we ultimately operate with pure symbols, and the operations do not affect the unit of measure; but in the beginning we should make constant reference to measured quantity, and always should be prepared to interpret the symbols and the processes.

Number, then, as distinct from the magnitude which is the unit of reference, and from the magnitude which is the unity or limited quality to be measured, is:

The repetition of a certain magnitude used as the unit of measurement to equal or express the comparative value of a magnitude of the same kind. It always answers the question: "How many?"

This "how many" may assume two related aspects: either how many times one part as unit has to be taken or repeated to make up the whole quantity; or how many parts as units, each taken once, compose the whole. In the first case, the times of repetition of the measuring unit is mentally the more prominent; in the second, the actual number of measuring parts; e.g. in thinking of forty yards, we may at one time dwell on the forty TIMES the unit is repeated; at another time, on the actual forty parts making the unified WHOLE.

As already said, the number and the measuring unit together give the absolute magnitude of the quantity. The number by itself indicates its RELATIVE value. It ALWAYS expresses ratio—i.e. the relation which the magnitude to be measured bears to the unit of reference. Seven, as pure number, expresses equally the ratio of 1 foot to 7 feet, of 1 inch to 7 inches, of 1 day to 1 week, of $1,000 to $7,000, and so on indefinitely. Simply as seven it has no meaning, no definite value at all; it only states a possible measurement.

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