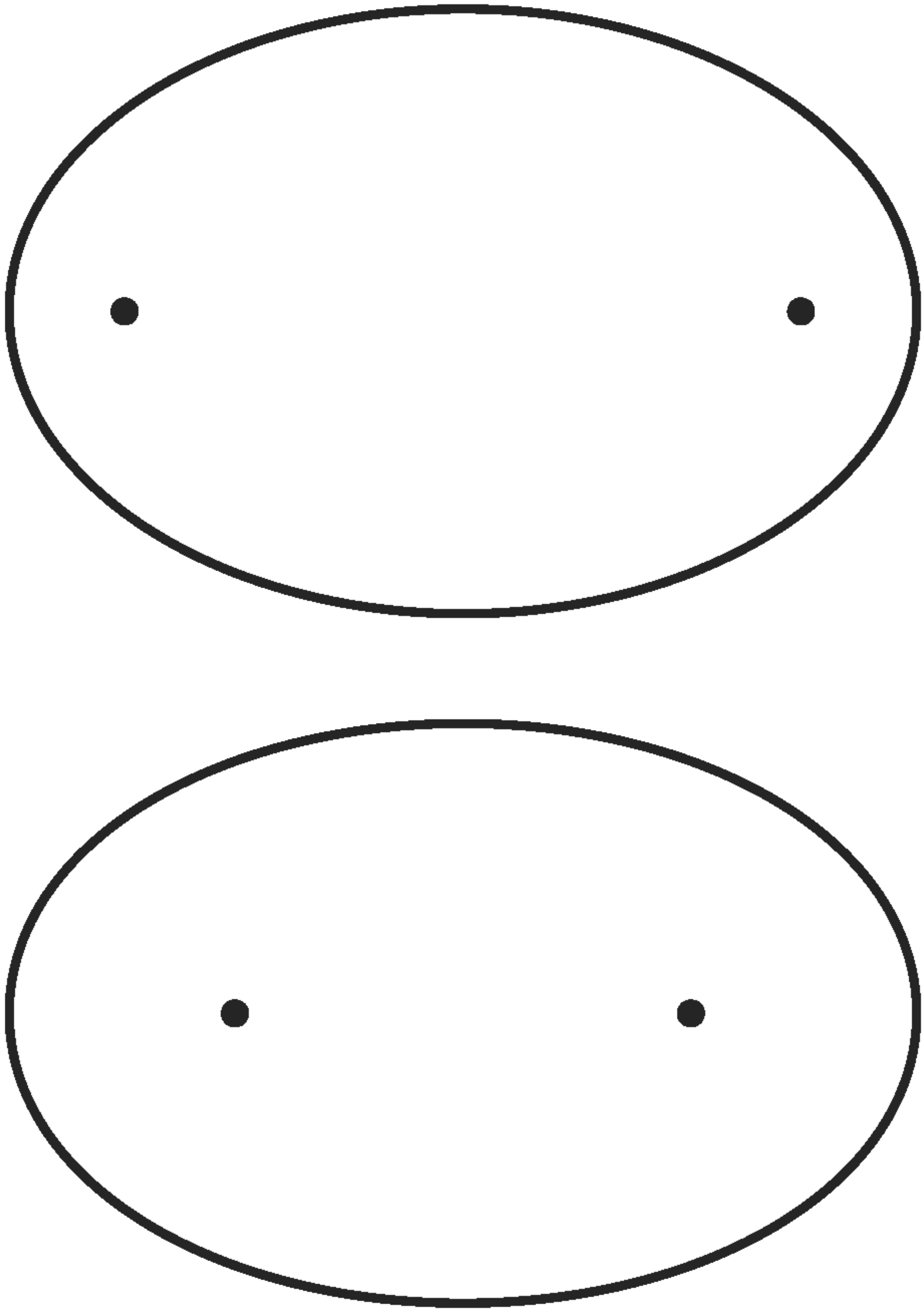
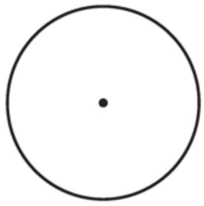


BoTSL#14 2018 SEP 15 5:00 PM

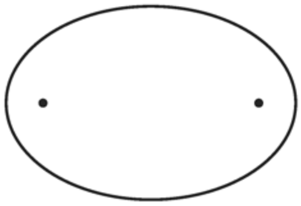
This bulletin emerged from six months at the American Academy in Rome, wandering into churches and trying to *translate* how and what is to be seen inside.

Cover: A regular ellipse and a Borromini ellipse (spot the difference).

The circle is a special case: it has only one center. It's the collection of all points that stand at exactly the same distance from that center (or more precisely, "focus"). This common distance is called the radius and it determines the circle's overall size.



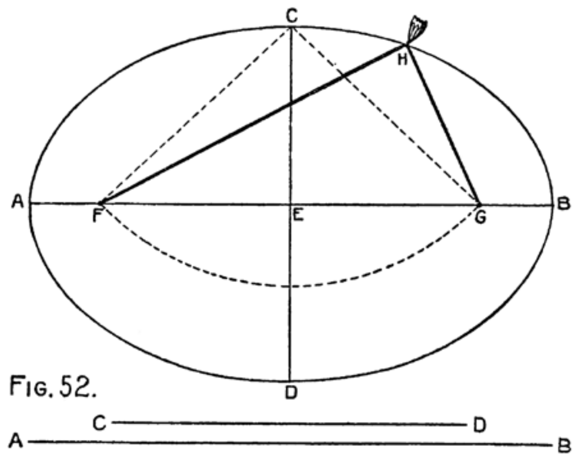
But I'm different: I have TWO so-called centers. These aren't really at my center either, but rather spread a bit. My shape is defined as all the points whose *combined* distance from the two foci is constant. The line that runs from one side of me to the other and passes through both foci is called the major axis. A second line that cuts me in half, running from top to bottom at a right angle to the first one is called the minor axis. In a circle, the major and minor axis are the same, but in an ellipse, like me, they are different and that difference determines my overall shape, from long and skinny to short and plump. The ratio between the two axes is the ellipse's *eccentricity.* As you can see in the self-portrait below, I'm not really that eccentric.



The Greeks (well, Apollonius of Perga in 3 B.C.) named me after the ellipsis, a composite typographic glyph composed of three full stops, one after the other which identifies an ommission. "Ellipse" was coined in Apollonius' *Conics*, where Book 1, Postulate 13 defines the ellipse as a conic section (a slice) that "falls short." We'll come back to conic sections, but for now, just keep in mind that a circle is always an ellipse, but an ellipse is rarely a circle.

Then there's the oval. It's quite common — you find these all over. The oval is a closed form, continuously curved with two axes of symmetry. This just means you can fold an oval evenly in half two ways, across two different lines that are perpendicular to each other. Many, many shapes are ovals, but only a select few qualify as ellipses.

People have known about ellipses for a long time, at least since Euclid wrote his book on geometry in the 4th century B.C. Take a circle, stretch it in any one direction and you'll find me. There's also a very straightforward (and enjoyable) way to construct an ellipse. You'll need two push pins, a pencil and a length of string. Take the two pins, stick them in the paper any distance apart. Connect the pins with a string loop larger than the distance between the two. Then just put your pencil in the loop, and stretch it tight to make the third point of a triangle. Just rotate the pencil around the two pins keeping the string tight and the resulting shape your pencil draws is an ellipse.



String construction of an ellipse from *Radford's Cyclopedia of Construction* (1909)

This is maybe the most obvious way to construct an ellipse, but clearly not the only one, and many constructions are known and used. The first properly formalized construction was published and circulated in Germany at the beginning of the 16th century. *Unterweisung der Messung mit dem Zirkel un Rischtscheyt* or *Course in the Art of Measurement with Compass and Ruler* is a set of four books written, illustrated, and published by Albrecht Dürer in 1525. Each book collects existing knowledge and Dürer's own insights around geometry, i.e. the study of shapes like me.

. . .

Albrecht Dürer

Albrecht Dürer was born in the city of Nuremberg, Germany, in 1471, number three of 18 children to a Hungarian goldsmith. He was a diligent child, a quick study, and soon became his father's favorite. Young Albrecht was sent to school, where he learned reading, writing, and the goldsmith's trade. His deep interest, however, was in art and so his father sent him to study with a leading German painter. Meanwhile, Albrecht's godfather (also a goldsmith) had set up a moveable type printing press. This was brand new technology at the time and Dürer was excited to learn about engraving, woodcutting, typesetting, and the full process of offset printing. After his exceptional education, Dürer began to build a significant reputation for his paintings, engravings, altarpieces, and woodcuts. His work was in demand, and he was in dialog with other artists and scholars across western Europe, traveling frequently, particularly to Italy. Right around the turn of the 16th century, Dürer traveled to Venice, a cosmopolitan center of commerce at the time. There he encountered a new approach to painting, which replaced flat symbolic space with illusionistic, "real" views. With the development of perspective, the picture was no longer just a collection of symbols, but instead, as painter Leon Battista Alberti described, "a transparent window through which we look out onto a section of the visible world." Objects in the painting then appeared smaller as they were farther from the viewer and walls, floors, and ceilings all converged to an imaginary "vanishing point" in the distance. Further, all these relationships could be mathematically determined, calculated even. Dürer, like other painters and artists, knew about perspective empirically, from direct experience, from observing a scene and attempting to *translate* that view to the flat surface of a picture, but he was not aware of the rational, geometric construction of those relationships.

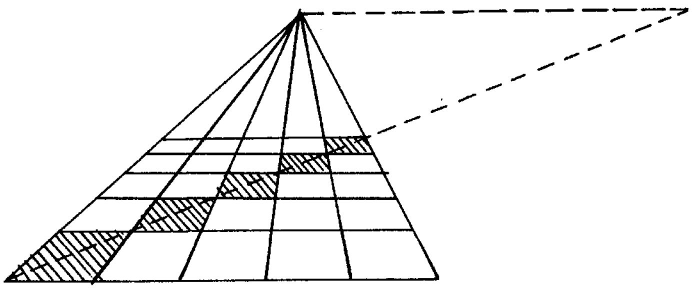
On a return trip to Venice in 1506, Dürer was lured to the university town of Bologna, as he described, "for the sake of 'art' in secret perspective which someone wants to teach me." Dürer wasn't lacking in the practical application of perspective drawing (he'd produced a number of intricate engravings which drafted the illusionistic space), but he was missing the theory—not how perspective is done but rather, how it worked.

No one quite knows the teacher who invited Dürer to Bologna, but there are two convincing, intriguing candidates. The first is Fra Luca Pacioli, a devout scholar, inventor of double-entry accounting, the person who taught mathematics to Leonardo da Vinci, and the designer of a beautiful, well-known set of rational constructions for the uppercase Roman alphabet.

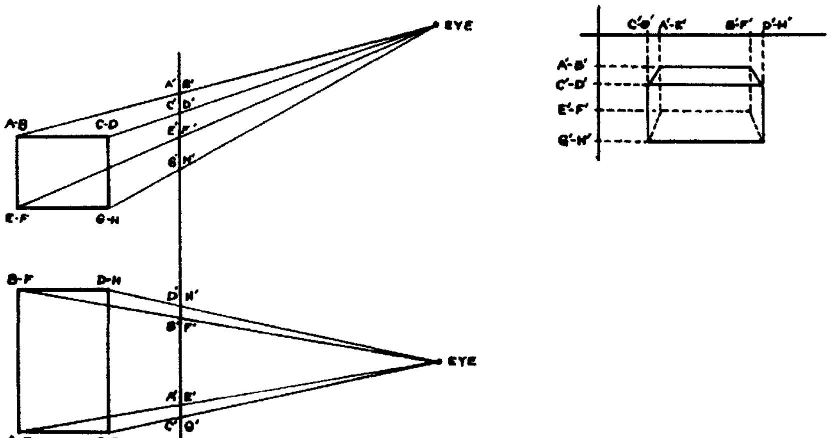
Portrait of Luca Pacioli, attributed to Jacopo de' Barbari, 1495

Pacioli was, literally and figuratively, a Renaissance Man, with a distributed set of skills and interests. He may have been Dürer's instructor or it may have been the architect Donato Bramante. (We'll circle back to Bramante in a few minutes.)

Meanwhile, what Dürer learned was a geometrical method for drafting precise single- or fixed-point perspective. Imagine standing in the doorway of a room and trying to draw its checkerboard floor. The left and right edges of the floor are orthogonal (perpendicular) to your sheet of paper. These two lines eventually meet in a single place in the distance called the scene's "central vanishing point." This is complemented by a second mark, the "lateral vanishing point." This imaginary line is drawn from the bottom left of the checkerboard to a point in the distance which falls on the same line as the central vanishing point. This line then marks out the diagonal corners of the checkerboard squares. The distance between these two vanishing points determines the degree of foreshortening in the scene. A drawing should make this much clearer:



There was a second piece to what Dürer learned. "Parallel projection" is the method of using two diagrams of an object from views at right angles to each other and using these diagrams to construct a perspective drawing. So, for example, a diagram of the top of a rectangular box together with a line of sight and another as a view of its front face can be used to assemble a perspective rendering of the three-dimensional object. This method was known as the *costruzione legittima* (legitimate construction) and together with the idea of vanishing points, this was the "secret" that Dürer took away from Bologna. Again, a drawing helps:



Perspective was a "secret" only in that it hadn't been printed and circulated in a book. Hardly a surprise, as at the time, *printed* books were brand new technology. Only 65 years before in 1439, the idea for moveable type offset printing arrived as a "ray of light" to German goldsmith and alchemist Johannes Gutenberg, while busy trying to invent a magic mirror to capture the holy light of religious relics.

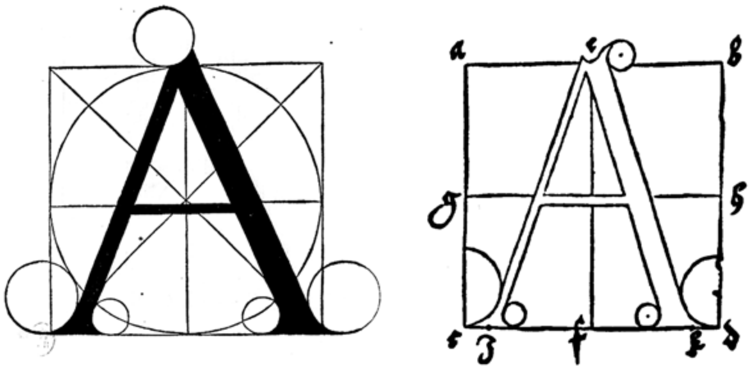
Johannes Gutenberg

Before moveable type, books in western Europe were copied one by one, by hand, whereas now identical copies of books could be produced one hundred or more at once. Gutenberg's idea spread quickly, including to Nuremberg, where Dürer's goldsmith godfather Anton Koberger converted his business to moveable type printing. Koberger quickly became the most widely distributed publisher of printed books in Germany. *The Nuremberg Chronicle* or *Liber Chronicarum* was the most successful of Koberger's books. It was a technical breakthrough, combining illustrations and typography, often on the same page, to which Dürer contributed etchings and woodcuts.

Returning from Bologna with his "secret," a Renaissance disposition to share that "secret," and a godfather who could make it happen, Dürer began his own ambitious four-volume series of books on measurement. *The Course in the Art of Measurement with Compass and Ruler* (1525) was addressed to artists, artisans, architects — "not only the painters, but also goldsmiths, sculptors, stonemasons, carpenters and all those who have to rely on measurement." The material is presented in an accessible form, adopting common language from the trades rather than a more theoretical, learned tone. Illustrations are provided, examples given, and practical problems presented and solved. Although the books convey the theory of perspective, and the wider scope of geometry in general, *The Art of Measurement* was intended to be primarily PRACTICAL. Art historian Erwin Panofsky describes the book as:

a revolving door between the temple of mathematics and the market square. While it familiarized the coopers and cabinetmakers with Euclid and Ptolemy, it also familiarized the professional mathematicians with what may be called "workshop geometry."

The first volume deals with linear geometry and the description of curves. The second book moves from one to two-dimensions, and the geometric constructions of regular two-dimensional figures like the triangle, the pentagon, and (drum roll!) me, the ellipse. The third volume — what Dürer called "the little book" — is purely pragmatic, offering a geometrical construction of the 26 uppercase letters of the Roman alphabet. This is a curious volume, but also fits the decidedly pragmatic approach of the books as a set. Dürer may well have also learned this in Italy, too. One of his possible teachers, Fra Luca Pacioli, had already published a beautiful set of geometrical constructions for the Roman uppercase letters a few years before in *De divina proportione* (1509).



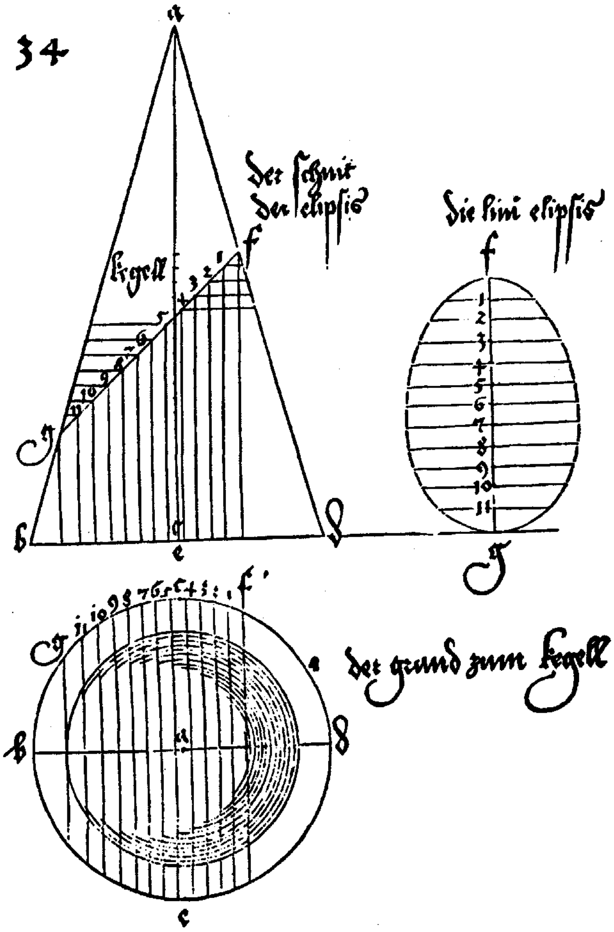
Geometric construction of "A" by Luca Pacioli (1509) and Albrecht Dürer (1525)

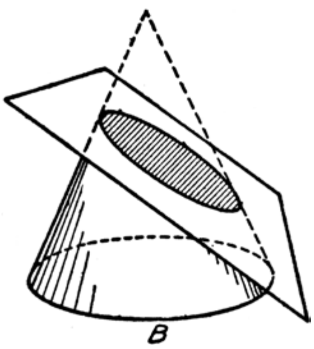
The final book follows the first two in logical series, moving on from one and two to three-dimensional regular geometric forms such as the cylinder, the pyramid, the cube, and my three-dimensional godfather, the cone. The cone, however, had already appeared in the first book with a curious presentation of the regular figures that emerge from slicing it in various ways. Dürer had most likely learned of these regular "conic sections" from Johannes Werner, a mathematician friend in Nuremberg who had published a book on the conic sections three years before in 1522. This is typical of *The Art of Measurement* and very much its spirit — these books are less the production and distribution of new, authored ideas than a collecting, summarizing, formatting, and distributing of *working* knowledge.

Throughout the books, geometry is not mathematically examined, but rather presented as a *construction.* Each figure, from the straight line to the Platonic solids, is introduced to the reader via a recipe that describes how to make the shape. In my case, the ellipse, Dürer presents an accidental construction that I am particularly fond of. He starts with a right-angle, regular cone. Then he describes that a slice taken from the cone and perpendicular to its base will produce a hyperbola. Another from the cone's base but not perpendicular forms a parabola. A third slice taken parallel to the base will produce a circle. He then works out what shape results when a slice is taken at an arbitrary angle to the base without intersecting it.

Relying on a method that any craftsperson would know, the shape he constructed from an oblique slice through a cone is what he called the *Eierlinie* (egg line). Here is Dürer's drawing:

BoTSL#14 2018 SEP 15 5:00 PM





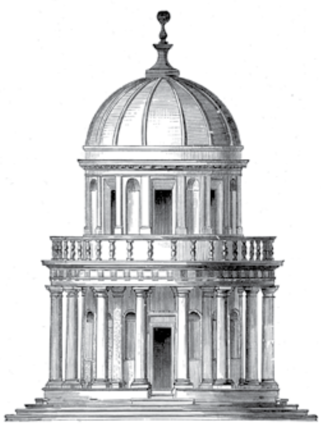
A cone cut by an oblique plane produces an ellipse

Turns out he was wrong: in fact, any oblique slice through a cone produces NOT an egg, but the perfect, shining, regular ellipse. Dürer was working with relatively rough tools and the coarseness of the tradesman's parallel projection method, but also, perhaps more importantly, he just wasn't able to make the abstract imaginative leap to realize the perfect geometry he'd stumbled on and his mistake points to the essential tension between abstract geometrical thought and empirical, intuitive understanding.

Either way, the egg error remained, the books were printed by Koberger in 1525, and they soon found their way into workshops throughout what are now Germany and Italy.

. . .

Now, I said we were going to circle back to Donato Bramante, one of Albrecht Dürer's two possible instructors in the mathematics of perspective. Well, we're here. Bramante began his career as a painter in Milan, where he moved in 1474, and very soon he became interested in perspective via Piero della Francesca, who had published the definitive treatise *De prospectiva pingendi* or *On Perspective for Painting* around that time. From the study of perspective and two-dimensional representations, Bramante ended up being increasingly known for his architectural work and by 1503 he had moved to Rome where he was tapped by Pope Julius II as chief architect for St. Peter's Basilica. This was the symbolic and physical home of the Catholic Church and the most important commission of the time. Steeped in the ideal geometry of the Renaissance, Bramante reworked the plan for the church into the form of a symmetrical Greek cross with each of the four ends to house a chapel. He remained chief architect of St. Peter's until his death in 1514 and his plan became the basis for the final design by Michelangelo Buonarroti.



Bramante's Tempietto (1502)

Soon after arriving in Rome, Bramante was also commissioned to make a small temple at the site of St. Peter's crucifixion on the Gianicolo hill. The Tempietto is set inside an existing courtyard at San Pietro in Montorio, which was commissioned by King Ferdinand and Queen Isabella of Spain a few years prior. The Tempietto is an almost Platonic model of a building — it's petite, as if built at scale, arranged around a circular plan, with a classically symmetric elevation defined by slender Doric columns and a semi-spheric dome. The building is set in the square-plan of the cortile, which further amplifies its regular geometry. Like his plans for St. Peter's Basilica, the Tempietto's plan is purely symmetric, embracing classical, Euclidean geometry and expressing an unfailing conviction in the potency of pure geometric forms.

100 years later, the design and construction of St. Peter's continued, but architecture had moved on from the regular forms of the Renaissance to the more complicated approaches of the early Baroque. If the Renaissance was the time of the perfect circle, the early Baroque was more my era — the age of the ellipse.

Carlo Moderno was now chief architect at St. Peter's and by 1615 he was joined by his distant cousin Francesco Borromini, who worked in Moderno's workshop for 15 years. There Borromini apprenticed as a sculptor and stone mason, and developed formidable design and technical drafting skills. And, by 1624 the young sculptor Gian Lorenzo Bernini was commissioned by Pope Urban VIII to design the baldacchino over the central altar in St. Peter's. Borromini worked closely with Bernini to develop the structural and technical aspects of the project, but Bernini received most of the public credit. By 1629 when Moderno died, Bernini was appointed chief architect both at St. Peter's and at Palazzo Barberini, another one of Moderno's commissions.

Borromini was upset — he knew the details of St. Peter's and Palazzo Barberini intimately and was infuriated that a decorative sculptor was chosen over a technically adept designer. Bernini, however, was charismatic, elegant, and diplomatic, while Borromini was more or less his opposite. Bernini knew he'd need Borromini to complete the projects and Borromini was convinced to stay on, working in Bernini's workshop for the following five years.

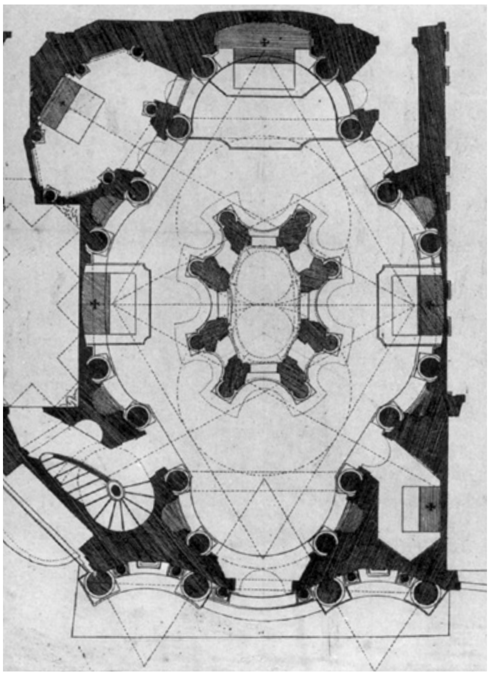
Still, by 1634 Borromini had had enough and quit both high-profile projects abruptly to begin work on a small commission for the Spanish Trinitarians (a Catholic sect) on the Quirinale hill in Rome. The corner site was small — it would fit easily inside a single domed pier of St. Peter's — and the program was big. The site needed to accommodate a church, a cloister, and a collection of monastic buildings. The project was marginal compared to what he had been working on, but Borromini identified with the Trinitarians' ideals and approached the commission vigorously. The church, San Carlo alle Quattro Fontane, was dedicated to San Carlo Borromeo in 1646 and is more typically known by its nickname, San Carlino. Borromini considered this his masterwork and he continued to add to it and to improve it over the course of his lifetime. His portrait, together with an architectural plan for the building, hangs in a small Borromini chapel backstage, behind the sanctuary and near the library.

Francesco Borromini

Like Bramante's Tempietto, San Carlino is small and perfect. But, while Bramante's perfection is rooted in pure, serene, classical geometry, Borromini's at San Carlino is something else. Stepping into the church from the busy intersection of the four fountains, you enter a nearly monochromatic, plastic white space. Everything curves: the walls turn in on themselves, the nearly-elliptical dome stretches improbably upwards, the altar collapses and bends. The tiny space appears many times larger than it can be possibly be. (American artist Richard Serra had the idea for his Torqued Ellipses sculptures while visiting the church, looking up at the oval dome and down at the floor and realizing that the two weren't aligned.) The effect of so much twisted geometry is not visual noise, but rather a surprising, profound silence.

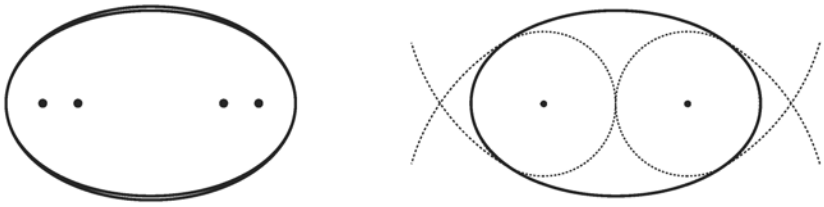
Borromini's plan for San Carlino is derived from two interlocking equilateral triangles. Inscribed into the shared area is a particular oval constructed from two circles. The overlaps produce walls that appear to move in and out of each other, variously suggesting a cross, a hexagon, and an ellipse like me.

The particular oval I just mentioned is known as the "Borromini Ellipse." Its center — which is also the center of San Carlino's dome — is the point of encounter between the circumferences of two circles standing side by side. The shape recurs in many of Borromini's projects including the piazza at St. Peter's, where rows of columns on either side describe the shape and famously act to ease the gigantic scale of Michelangelo's dome. However, despite coming very close, the Borromini Ellipse is not in fact an ellipse at all. Borromini's shape is much easier to draft with a ruler and compass than its perfect counterpart, assembled from the arcs of four




Ground plan of San Carlo alla Quattro Fontane, and looking up into Borromini's central dome

circles — two small and two large. All four circles could be simply drawn with a compass, and the resulting shape could be easily measured and also easily communicated to a builder. As such, the Borromini Ellipse seems to be a kind of "workshop" construction of the perfect ellipse.



The Borromini Ellipse compared to a regular ellipse (left), and its construction (right)

If Bramante was a rationalist, invested in pure geometry and Bernini was a sculptor of flowing, illusionistic forms, then Borromini was a synthesis that surpassed both. He married the two into a new, original alchemy where plastic form emerged from an underlying geometric rigor. And this combination is visceral — the quietness of Borromini's architecture implies a geometric skeleton that lies just behind. The mathematical basis for his architecture also had a theological justification for Borromini. He believed that the universe as well has a deep geometry inscribed by God, legible only to the faithful. The world as we see it is a temporary illusion.

Soon after his initial work with the Trinitarians at San Carlo, Borromini also began working on a series of projects for Cardinal Berndandino Spada at his Palazzo adjoining Via Giulia, next to the Tiber River in Rome. Borromini was involved in many aspects of the renovation and expansion of this important Cardinal's residence, from adjusting the site plan to detailing *trompe-l'oeil* windows, modifying a staircase, and siting a sundial.

One project in the Palazzo completed in 1652 is simply known as La Prospettica, or The Perspective — an architectural folly framed by orange trees and built into one wall of a small courtyard. The Perspective appears to be a 120-foot arched arcade passage lined by columns and framing a life-sized sculpture in the distance. By ramping both the floor and arched roof towards a vanishing point, and adjusting the spacing, sizes, and geometries of the columns, the illusion of a deep corridor is produced. It is actually less than 26 feet deep and the statue at the end is only 2 feet tall. Looking straight into The Perspective leaves the illusion intact, but coming in close and looking from the side (or by actually walking into the conjured hallway!), its considerably warped geometry is clear. Borromini consulted a mathematician on the construction, and the result is a virtuosic motivation of the "secret" of Renaissance perspectival construction, i.e. what either Bramante or Pacioli shared with Dürer in Bologna more than one hundred years before. But now, in the Baroque, the "secret" is turned in on itself. Instead of being a tool for representing in two dimensions the three-dimensional world as we see it, perspective is used to create a 3-D construction that, when viewed from the correct vantage, produces a precisely aligned 2-D picture. And that picture lies. Cardinal Spada offered a moral justification for the work which is also inscribed in it:

… in the same way that illusions may cause small shapes to appear great, worldly matters held to be great may prove to be illusory and insignificant.

La Prospettica

Several years prior, Borromini had situated a sundial on the piano nobile (first floor, noble floor) of the Palazzo. Conceived by a French mathematician, the sundial used a novel mirror device, which focuses sunlight

BoTSL#14 2018 SEP 15 5:00 PM

entering the building through a small, well-placed aperture in the wall to produce a discrete dot of light onto the ceiling. The dot's position reflects the current position of the sun in the sky (and therefore marks the time). On the ceiling, a spiderweb of overlapping ellipses — folded into Borromini's bent ceiling geometry — registers the sun's location at different times of the day and in different seasons.

Astronomer/astrologer Johannes Kepler had also reckoned with patterns of celestial movement in Germany. By 1609, Kepler had published *Astronomia nova*, a book that detailed the workings of the solar system under the relatively new assumption offered by Nicolaus Copernicus that the sun is at its center. The suggestion was heretical, but matched Kepler's theological disposition: he was convinced that a divine order lies behind the movements of the planets. Based on observations of Earth and Mars, Kepler developed the first and second laws of planetary movement. The second one states that "planets sweep out equal areas in equal times." The insight followed Kepler's belief in the Sun as a stand-in for God, motivating the planets' movements in the heavens. Describing these movements was considerably difficult and Kepler attempted several geometries to model the planets' orbits. He rejected the ellipse at first, thinking the form was too simple, too tidy, too perfect. However, extensive observations obtained from his fellow astronomer and patron Tycho Brahe confirmed that hypothesis. Kepler's first law states that "all planets move in ellipses, with the sun at one focus."

Borromini would have been familiar with Kepler's work. He knew astronomer Galileo Galilei in Rome, and both Kepler and Galilei were members of the Accademia dei Lincei. Kepler's cosmology was a good fit for Borromini. Both were convinced that the world is organized around a deep, divine order which is discoverable *only by the faithful* and *only by mathematical induction.* Through the painstaking process of piecing the whole together, they presumed to understand its underlying design.

Now, let me finish with one final story in my elliptical autobiography. Borromini was working also with the Confederazione dell'oratorio di San Filippo Neri, a society of Catholic priests and monks living together in a community bound by no formal vows other than the bond of charity. Borromini was tasked to design a building on a large site adjoining the Chiesa Nuova. At the rear of the Oratorio dei Filippini, Borromini designed a clock tower extending from the building and forming the Piazza dell'Orologio. Borromini's Torre dell'Orologio is monumental: it has a concave façade cut from a circular plan, and its top is detailed with bells and an almost lacy metal crown. The clock face itself, however, is resolutely conventional. It's ringed with Roman numerals I through XII, simple hour and minute hands, and sits confidently within a circle.

BoTSL#14 2018 SEP 15 5:00 PM

Reflected mirror sundial of Emmanuel Maignan on the Borromini ceiling of the piano nobile at Palazzo Spada, Rome

Torre dell'Orologio, c.1637 (2018)

Where almost everything else in a Borromini building tends to be torqued, clipped, or anyway not regular, it is almost jarring to see that the clock face is contained in a simple circle.

But if you climb back down to the ground and onto the Piazza, which is the only place you will likely ever see the clock from unless you're a bird, you'll see that the geometry changes. It all starts to make sense. Go ahead, close one eye and have a good look at the clock face again. What you see is not a circle at all. That's right—it's ME, a perfect ellipse.

I must admit that I have a pretty amazing quality: every time that you encounter a circle from any angle (which is, well, every time you see a circle even as the angle infinitely approaches zero), the resulting shape (the light that hits your eye) is an ellipse. The same is true in three dimensions, too: the projected image of a sphere onto a plane is not merely an oval, much less an egg or any other ordinary curved form, but ALWAYS an exact ellipse.

It makes plenty of sense if you remember that an ellipse is just the oblique slice of a cone. Imagine that cone is your field of vision, with your eye at its tip. Any circle in your view will never be absolutely perpendicular to your line of sight, but rather it will be a distortion and its visible shape will be an ellipse. Renaissance constructed perspective taught us how to see like this and Baroque illusionism taught us not necessarily to believe what we see.

That's me, then: a small pearl of geometric transcendence in your everyday vision. I describe the paths of the planets in the heavens. I am the sphere's shadow. I am the empirical encounter with an ideal form, the particular case in point, and the closest you may ever come to seeing the face of God.

*

BoTSL#14 2018 SEP 15 5:00 PM