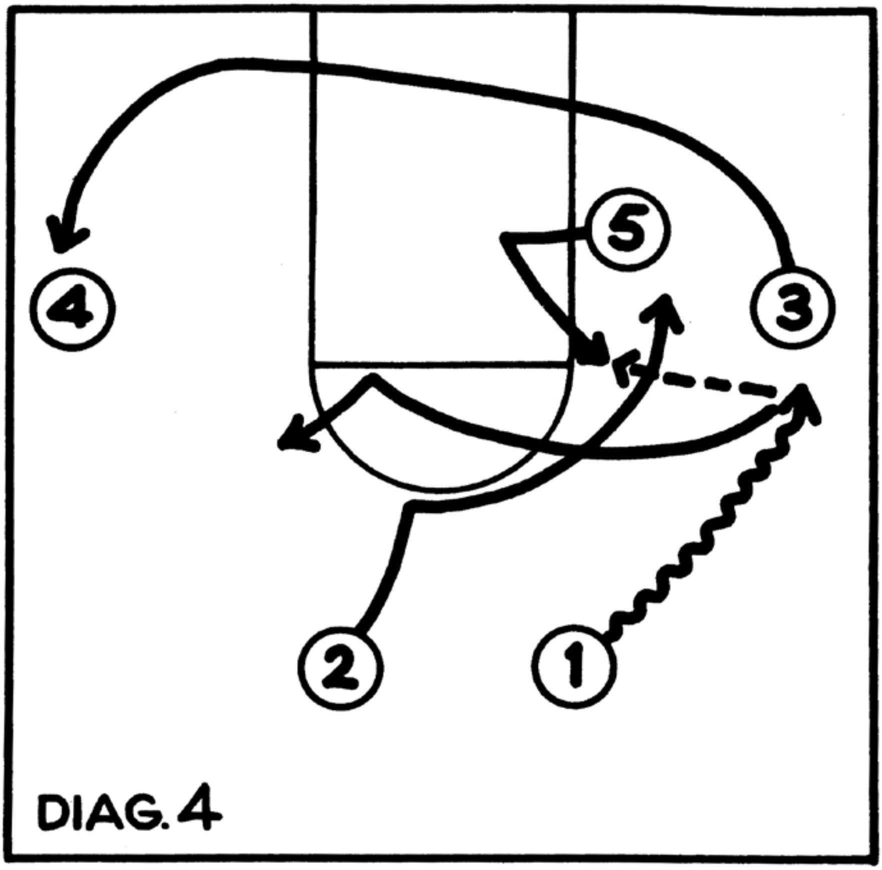

5
4
3
2
1
DIAG. 4

Mathematics is not a spectator sport. The cliché is supposed to mean that in order to understand the game you have to play it, not just watch it happen up on the blackboard. Nevertheless, this Bulletin offers a (highly speculative) mathematical model inspired by watching basketball. It's an attempt to pick apart Phil Jackson's "Triangle Offense," a key strategy that claims some credit for his success as an 11-time NBA championship winning coach. The fundamental principles of the triangle offense are even spacing, interchangeability of all players, and individual player responsibility. There are no set plays, and team members act in response to the defense as well as their teammates' positions. Of course, defense has its habits, so there are several patterns that a triangle offense takes. But these options obscure the basic symmetries behind the triangle outlined here.

Some mathematical notation and terminology is in order at the outset. The exclamation mark denotes the factorial operation, $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$. A group is an abstract algebraic structure that can be thought of, informally, as a set of permutations of things. The key observation is that pairs of permutations can be combined one after another to yield new permutations. More formally, a group is a pair $(G,*)$ that consists of a set G and a function (or operation) $*: G \times G \rightarrow G$ such that: $*$ is associative, G contains an identity element e, and for every element g there is an inverse g^{-1} in G such that $g*g^{-1} = g^{-1}*g = e$. If every element of a group can be expressed as a product of elements in a subset of the group, then the subset is said to generate the group. A presentation of a group consists of a list of generating elements followed by a set of relations on those generators.

Cover image: A basic triangle move, "Split the Post," as diagrammed in: Herman I. Masin, ed., *The Best of Basketball from Scholastic Coach*, (Upper Saddle River, NJ: Prentice-Hall, 1962).

A basketball team has possession of the ball. Each of the five players takes one of five positions: strong-side guard, strong-side forward, center, weak-side guard, and weak-side forward. For each of these 5! or 120 different player-position permutations, the ball is in possession of 1 of the 5 players, yielding 600 offensive configurations in total.

Any given offensive play may be characterized, somewhat coarsely, by a sequence of elements drawn from among this set of configurations. The bilateral symmetry of the court about the centerline connecting the two baskets may be exploited to reduce this by a factor of two. Consider a three-player subset of the team — a triangle.

The set of configurations of 3 given players contains 18 elements (3! player permutations times 3 ball positions), all of which appear in the table below. Players are numbered, and the underlined number indicates the player with the ball. For example, $2\underline{1}3$ represents the triangle configuration in which player 1 is in second position with the ball, player 2 is in first position, and player 3 is in third position.

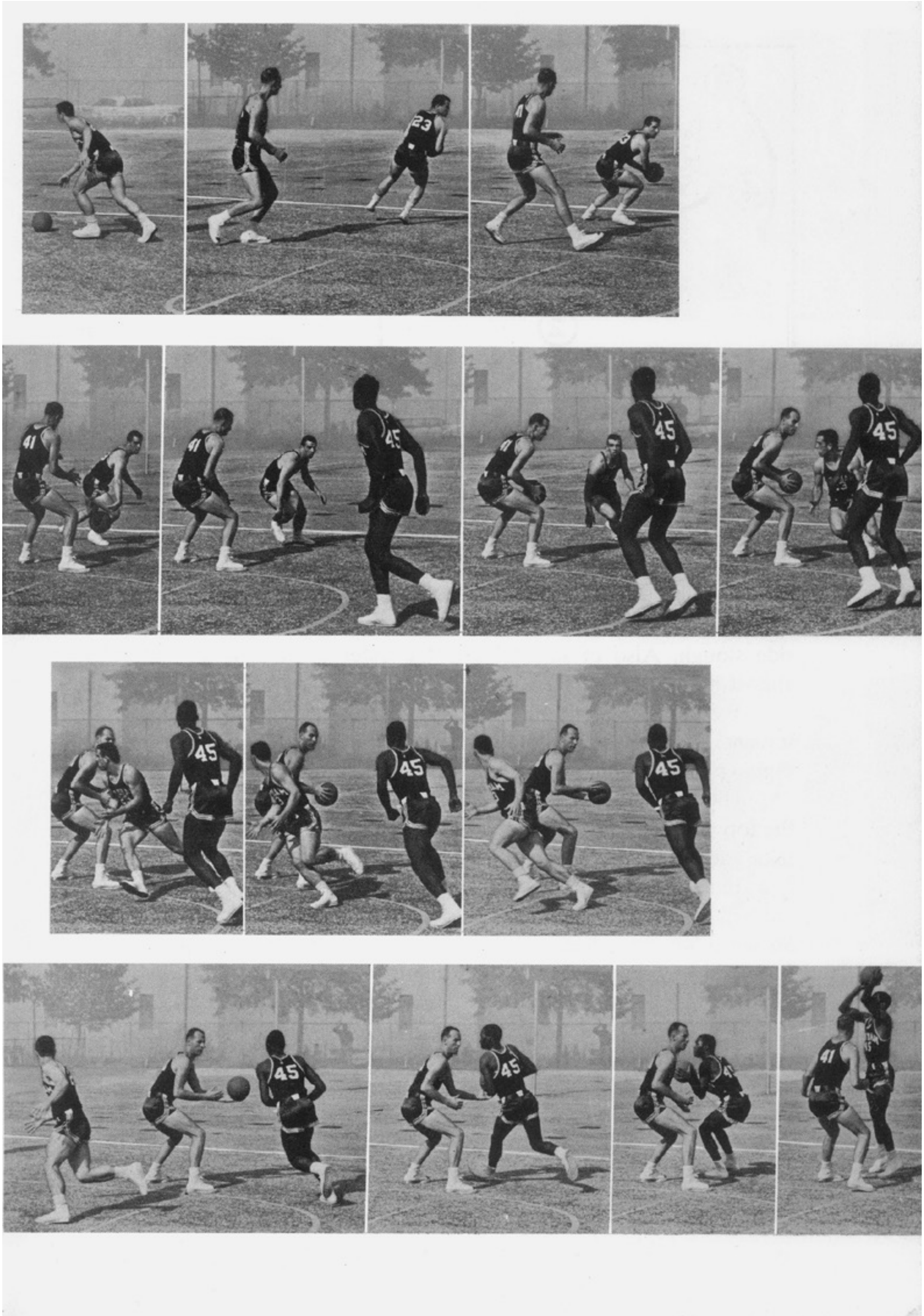
$\underline{1}23$	e	hold	$\underline{2}13$	h	hand-off right
$1\underline{2}3$	p	pass right	$1\underline{3}2$	hp	split pass right
$12\underline{3}$	p^2	pass left	$32\underline{1}$	hp^2	swap left
$3\underline{1}2$	r	rotate right	$3\underline{2}1$	hr	swap left pass right
$23\underline{1}$	r^2	rotate left	$13\underline{2}$	hr^2	split pass left
$31\underline{2}$	rp	pass right rotate right	$21\underline{3}$	hrp	swap right pass left
$\underline{3}12$	rp^2	pass left rotate right	$\underline{1}32$	hrp^2	split
$\underline{2}31$	r^2p	pass right rotate left	$\underline{3}21$	hr^2p	hand-off left
$2\underline{3}1$	r^2p^2	pass left rotate left	$2\underline{1}3$	hr^2p^2	swap right

In relation to the initial state, $\underline{1}23$, each configuration represents a play, which is noted to its right. The composition of two or more plays, one after another, is itself a play, and the set of triangle configurations forms a group isomorphic to the direct product of the symmetric group of degree three and the cyclic group of order three. The symmetric group factor is generated by hand-offs and rotations and the cyclic group factor by passes; altogether, the group admits the following presentation:

$$S_3 \times C_3 = \langle\, h, p, r \mid h^2 = p^3 = r^3 = e,\ hr = rh,\ pr = rp \,\rangle.$$

*

BoTSL#9 2015 JUN 24 2:07 PM



“Split the Post,” image sequence from *The Best of Basketball from Scholastic Coach*