University of Cape Town

Applied Mathematics

SQUID SIMULATION PROJECT

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Abstract

We endeavour to determine a suitable catch rule of acquiring squid from a squid population. Our approach is to develop a model that estimate the impact of different catch rules on the population. Particularly, we seek minimum risk to the population and a satisfactory squid-biomass yield.

Introduction

Firstly, we model a scenario of acquiring squid from a squid population. Specifically, our squid population is measured by biomass. A recurrence rule is then used to determine the evolution of the resource-biomass by considering several factors. In this project we are interested in determining a catch rule that will produce a suitable yield. Particularly , a rule that will yield plentiful resources without rendering the squid population extinct.

The recurrence relation to used considers the biomass, the recruitment and the annual catch of the previous year to determine the biomass of following year. This model will simulate a 20 years period with a catch and recruitment at the beginning of every year.

Analytical interpretation

We consider our modeling equation, $B_{t+1} = (B_t + R(B_t) - C_t)e^{-M}$. B_t is the biomass at time "t", $R(B_t)$ is the amount of biomass that is added every year (Recruitment), C_t is the annual harvest at the beginning of every year (catch) and M is the instantaneous rate of natural mortality.

This equation sums up recruitment and biomass, then subtracts the catch. Thereafter, there is a natural exponential decay of the squid population due to death and predation, we account for this by multiply our equation with e^{-M}.

Method

Analytic solutions

We explore varies cases to establish the fundamentals of our recurrence relation. Firstly, let us consider the case with no harvesting ($C_t = 0$ for all "t"). This implies our recurrence rule becomes $B_{t+1} = (B_t + R(B_t))e^{-M}$.

Then to determine the stability of the recurrence relation we let; $B_{t+1} = B_t = B_0 = K$.

Therefore $K = (K + \frac{\alpha K}{\beta + K})e^{-M}$
$\implies K = (\frac{K^2 + K(\alpha+\beta)}{\beta+K})e^{-M}$
$\implies K^2 + K\beta = e^{-M}K^2 + K(\beta+\alpha)e^{-M}$
$\implies K^2(1-e^{-M}) + K(\beta - (\beta+\alpha))e^{-M}) = 0$
$\implies K(K(1-e^{-M}) + (\beta - (\beta+\alpha))e^{-M})) = 0$
$\implies K = 0$ or $K(1-e^{-M}) + (\beta - (\beta+\alpha)e^{-M}) = 0$

We know that the carry capacity K is not zero. Therefore:
$K(1-e^{-M}) + (\beta - (\beta+\alpha)e^{-M}) = 0$

$\implies K(1-e^{-M}) = -(\beta - e^{-M}\beta) + e^{-M}\alpha$
$\implies K = \frac{-\beta(1-e^{-M}}{1-e^{-M}} + \frac{e^{-M}\alpha}{1-e^{-M}}$
$\implies K = -\beta + \frac{\alpha}{e^m - 1}$

Then if the stock recruit function is $R(B_t) = \frac{\alpha B_t}{\beta + B_t}$ and the steepness of the stock recruit function is defined as $R(0.2K) = hR(K)$.

Therefore $R(0.2K) = \frac{\alpha 0.2K}{\beta + 0.2K} = hR(K)$
But $R(K) = \frac{\alpha K}{\beta + K}$
$\implies \frac{\alpha 0.2K}{\beta+0.2K} = h\frac{\alpha K}{\beta+K}$
$\implies \frac{0.2}{\beta+0.2K} = \frac{h}{\beta+K}$
$\implies (\beta+K)0.2 = h(\beta+0.2K)$
$\implies (h-0.2)\beta = 0.2K(1-h)$
$\implies \beta = \frac{0.2K(1-h)}{h-0.2}$

Basic Plots

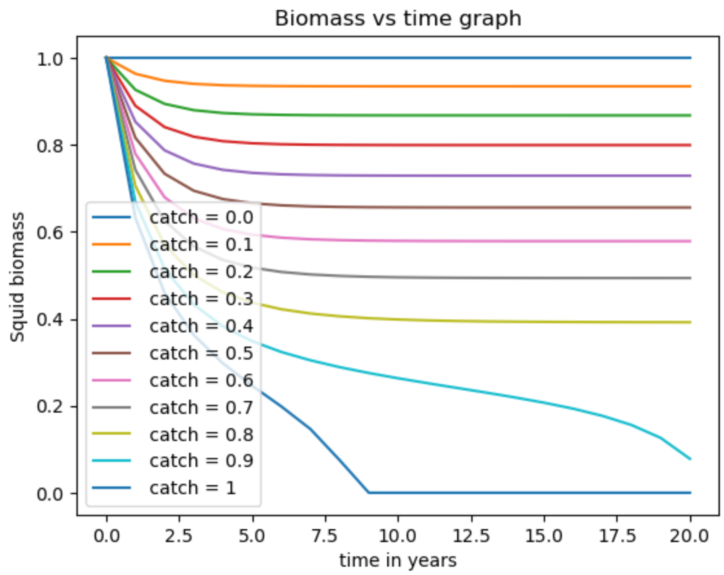


Figure 1: Simulation with no random factor, with constant catches from 0, 0.1, ... to 1. B_0 = K = 1, M = 1, h = 0.7.

It is apparent that when we have a zero catch through out our 20-year period the squid biomass is stable at K= 1 as previously demonstrated analytically. When the catch is 0.1 the biomass drops exponentially and stabilises slightly below the initial biomass (K=1). Increasing the catch decrease the biomass even more, which is expected. Thereafter, the biomass stabilizes further below K=1 indirectly proportional to the catch size. When the catch reaches 0.9 the biomass no longer stabilise, it keeps decreasing until it seems to deplete. When the catch is 1 the squid population depletes almost half-way before the end of the simulation.

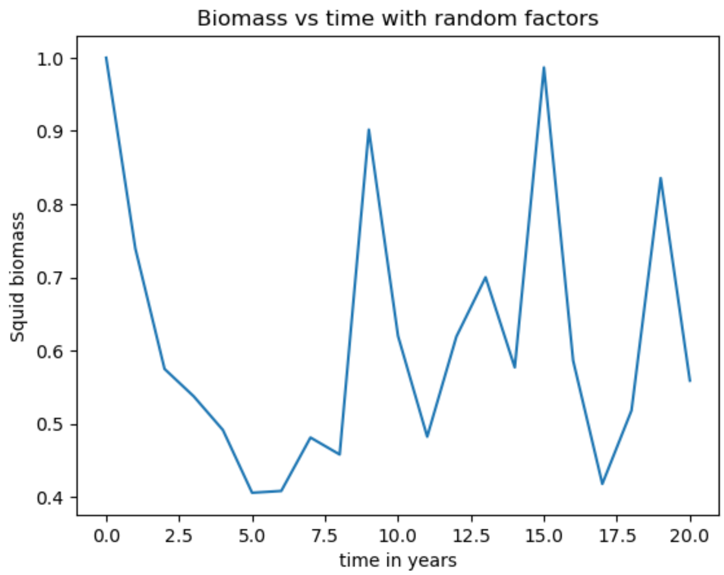


Figure 2: Simulation with random factor, with constant catch = 0.7. B_0 = K =1, M =1, h = 0.7.

The random factor adds unpredictable increases and decreases, however the long term behaviour of the simulation is decreasing. Furthermore, the population has difficultly in reaching stability.

Algorithm

Analytically the algorithm uses the current biomass B_t to determine the biomass-survey at time "t". Then the biomass survey at that same time "t" is used to determine the catch at time "t". Particularly, $B_t^{survey} = B_t e^{n_t - \frac{\sigma_s^2}{2}}$, then $C_t = C(B_t^{survey})$. The biomass survey is subjected to random factors and we incorporate this by multiplying B_t by $e^{n_t - \frac{\sigma_s^2}{2}}$, where n_t is a random number from a normal distribution with standard deviation σ_s.

Thereafter, we determine B_{t+1} using our original formula $B_{t+1} = (B_t + R(B_t) - C_t)e^{-M}$. The recruitment is also subjected to random factors and they are incorporated by defining the recruitment function as $R(B_t) = \frac{(\alpha B_t)}{(\beta + B_t)} e^{t - \frac{\sigma_r^2}{2}}$. E_t is a random number from a normal distribution with standard deviation σ_r.

This simulation provided a plot of biomass vs time over a 20-year period. In order to optimize our accuracy and consistency we run 1000 simulations. Then take the average of each number in each year of the simulations and use that to represent our overall biomass vs time model.

Catch-Rule

The primary goal is to determine a catch rule that provides a fair yield without any drastic negative consequences on the squid population.

The catch depends on a survey that is done at the beginning of every year. Therefore, we propose two catch rules to contrast and determine a suitable one. That is $(biomss^{survey} * 0.25) + 0.4$ (1st catch rule) and $\sqrt{biomss^{survey}}$ (2nd Catch rule).

Results

Summary Statistics

In order to analyse the proposed catch rules, we use summary statistics. These statistics provide insight into effects of the catches to the squid population .

Capacity management CM - We are interested in the highest catch over the 20 year period. For any given year we have 1000 catch values due to the structure of our simulation. We then take the median these 1000 values for each year. Thereafter, we interpret the highest of these medians as the CM. The significance of tracking this value is to avoid acquiring squid that is beyond our processing or carrying capacity, as this could cost unnecessarily resources.

Average Annual Catch (Cave) - The average annual catch provides a measure of what catch should be expected each year. It's deduced by taking the average catch over the 20-year period that our model runs. Then, we will have a thousand values of these due to our 1000 simulation. Thereafter, we will take the median of these values to be our Cave.

Average annual catch variation (AAV) - The average annual catch variation seeks to enumerate the extent of the difference between all catches over the 20-year period. It is calculated by firstly getting a read of the variation of catches between every consecutive two years, beginning from between year 0 and 1 to year 19 and 20. This is done using the formula $(\frac{|c_t - c_{t-1}|}{c_{t-1}})(100)$. Then, we have 19 values for each simulation, and we take the average for each. Thereafter, we interpret the median of these values of the 1000 simulations as the AAV, which is a percentage. To be efficient we want consistency in the catch that we take each year, so a low AAV percentage is preferable.

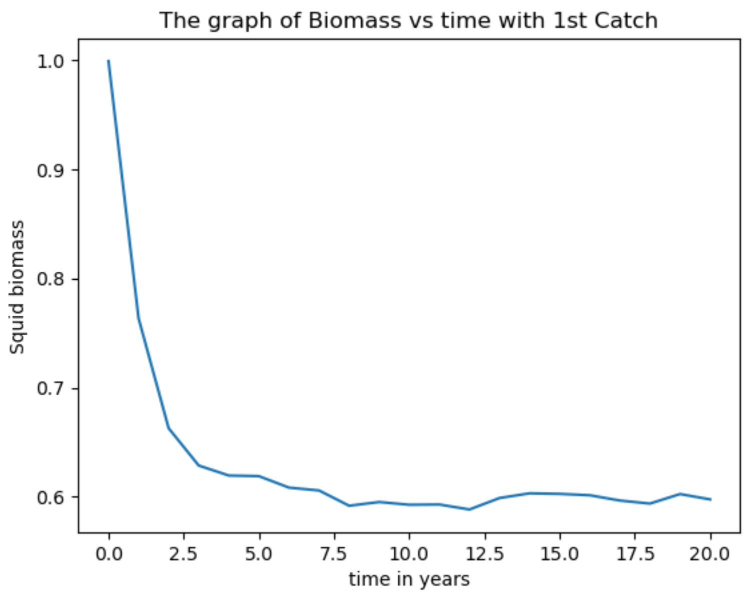
Risk to resources (B_{20} lower 5) - This metric seeks to analysis the final biomass B_{20}. Specifically, for the 1000 simulations we are going record the last value (B_{20}). Then order these from lowest to highest and look at the 5th percentile of these number. This metric evaluates the impact we will have at the end of over endeavour, particularly the risk to render the squid population extinct.

Risk to resources (Biomass Lower) - Throughout the 20-year period we want to see the lowest biomass that the squid population could possibly get. Specifically, we record the lowest value over the 20-year period for each simulation. Then we interpret the median of these values as the Biomass lower. This value indicates how close we can get to rendering the population extinct over the 20 year period. Therefore, we want to keep this number high.

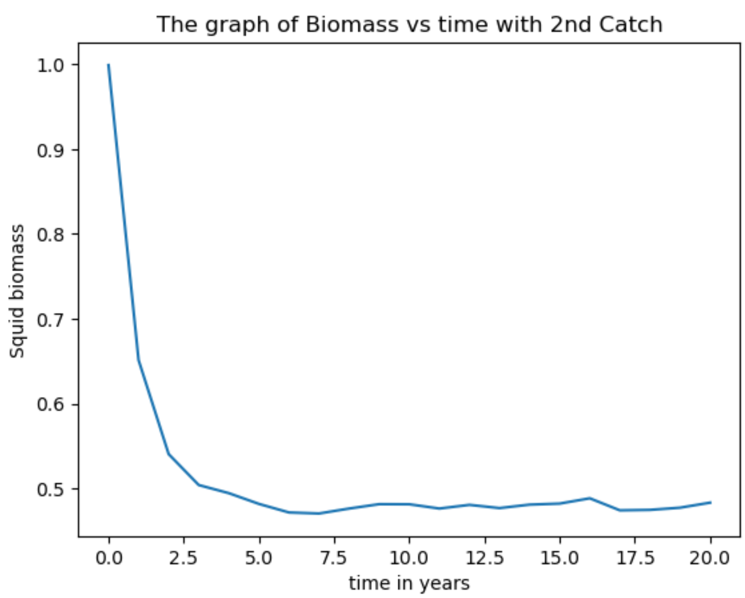
Results of Summary Statics

	Cave	AAV	B_{20} lower 5 percent	Biomass Lower	CM
Catch 1	0.5580	9.8057	0.3237	0.3283	1.0196
Catch 2	0.7020	21.6826	0.2321	0.2250	1.0

Graphs for different catch rules



(a) 1st catch Rule



(b) 2nd Catch Rule

Figure 3: Biomass vs time graph for both catch rules

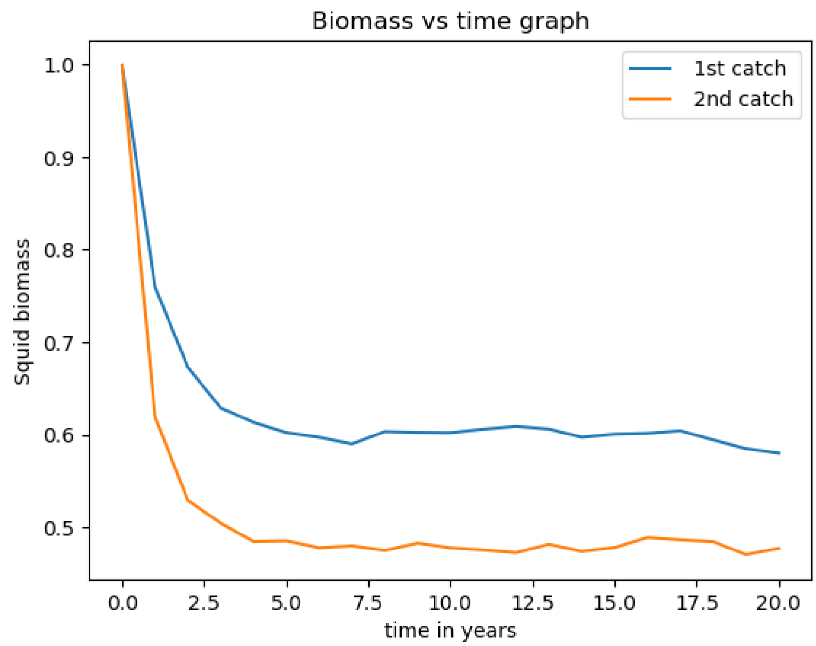


Figure 4: Biomass vs time graph of both catches

The biomass of both graphs stabilize after a certain point though there are still perturbations in their stability.

The 1st graph stabilizes at a higher level of biomass than the second graph. The second graph reached stability at 5 years while the first catch graph reached stability at 7.5. Furthermore, there is more smoothness with respect to disturbance from stability of the second graph than on the first graph.

Conclusion

Our first catch rule falls back when it comes to average annual yield. However, is has lower variations, and has a lower risk of rendering the squid population extinct. In terms of the highest yield per any given year both catch rules rank the same.

Rendering the squid population extinct is a much bigger problem as this would cause no yield for the upcoming years. Moreover, this poses a danger to marine ecosystem as it is inter-dependent. The variation of catch each year is quite important as this will affect the means of production. Particularly, having a low yield one year and then having a very high yield the following year could mean that the system of production does not have the capacity to process such a yield. This leads us to conclude that the first catch rule $((biomss^{survey} * 0.25) + 0.4)$ is the most ideal for harvesting squid.