1. **[3 pts]** Given vector **v** = < 3, -1, 3 >, find a vector **w** that is orthogonal to **v. Please give a nontrivial solution which is not <0, 0, 0>.**

$$\mathbf{v} = \langle 3, -1, 3 \rangle$$

$$\mathbf{v} \cdot \mathbf{w} = 0$$

$$(3)(w_x) + (-1)(w_y) + (3)w_z = 0$$

$$1.\ \text{let } w_z = 1$$

$$(3)(w_x) + (-1)(w_y) = -3$$

$$2.\ \text{let } w_x = 1$$

$$(3) + (-1)(w_y) = -3$$

$$-6 = (-1)(w_y)$$

$$6 = w_y$$

$$\mathbf{w} = \langle 1, 6, 1 \rangle$$

2) **[7 pts]** Given the following matrix is orthonormal, find x, y, z then find the inverse of the whole matrix: (Note: x, y , z are real numbers)

$$\begin{bmatrix} 2/3 & y & 2/3 \\ x & 2/3 & 1/3 \\ 1/3 & 2/3 & z \end{bmatrix}$$

$$\begin{aligned} q_1^{\mathsf{T}} q_1 &= 1 & q_1^{\mathsf{T}} q_2 &= \varnothing \\ q_2^{\mathsf{T}} q_2 &= 1 & q_1^{\mathsf{T}} q_3 &= \varnothing \\ q_3^{\mathsf{T}} q_3 &= 1 & q_2^{\mathsf{T}} q_3 &= \varnothing \end{aligned}$$

$$\begin{bmatrix} \frac{2}{3} & x & \frac{1}{3} \end{bmatrix} = a_1^{\mathsf{T}}$$

$$\left(\frac{2}{3}\right)^2 + x^2 + \left(\frac{1}{3}\right)^2 = 1$$

$$x^2 = \left(1 - \frac{5}{9}\right)$$

$$x = \pm \frac{2}{3}$$

$$\frac{4}{9} \times y^2 \times \frac{4}{9} = 1$$

$$y = \pm \frac{1}{3}$$

$$\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + z^2 = 1$$

$$1 - \frac{5}{4} = z^2$$

$$z = \pm \frac{2}{3}$$

$$A_1 A_2 = \varnothing$$

$$\frac{2}{3}\,y + x\left(\frac{2}{3}\right) + \left(\frac{2}{9}\right) = 0$$

$$\frac{2}{9} + \left(-\frac{4}{9}\right) + \frac{2}{9} = 0$$

X=-2/3

Y=1/3

$$\frac{2}{3}\,y + \frac{2}{3}\left(\frac{1}{3}\right) + \frac{2}{3}\,z = 0$$

$$\frac{2}{9} + \frac{2}{9} - \frac{4}{9} = \varnothing$$

Z=-2/3

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

Filled in matrix.

$$\left[\begin{array}{ccc|ccc} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 3 & 0 & 0 \\ -2 & 2 & 1 & 0 & 3 & 0 \\ 1 & 2 & -2 & 0 & 0 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 3 & 3 & 3 & 3 & 0 \\ -2 & 2 & 1 & 0 & 3 & 0 \\ 1 & 2 & -2 & 0 & 0 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 1 & 0 \\ -2 & 2 & 1 & 0 & 3 & 0 \\ 1 & 4 & 0 & 2 & 2 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 10 & 1 & 4 & 7 & 6 \\ 1 & 4 & 0 & 2 & 2 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & \frac{7}{9} & \frac{6}{9} & \frac{6}{9} \\ 1 & 4 & 0 & 2 & 2 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} \emptyset & \emptyset & 1 & \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \emptyset & 1 & \emptyset & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 1 & \emptyset & \emptyset & 2-\frac{4}{3} & 2-\frac{8}{3} & 3-\frac{8}{3} \end{array}\right]$$

$$\begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$$

=a^-1

Credits:

Ai was used to transcribe handwritten notes into typed notes, handwritten copy available upon request