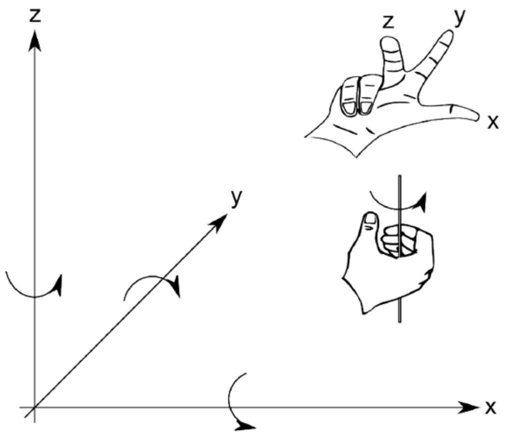
CSCI/ECEN 3302 Introduction to Robotics

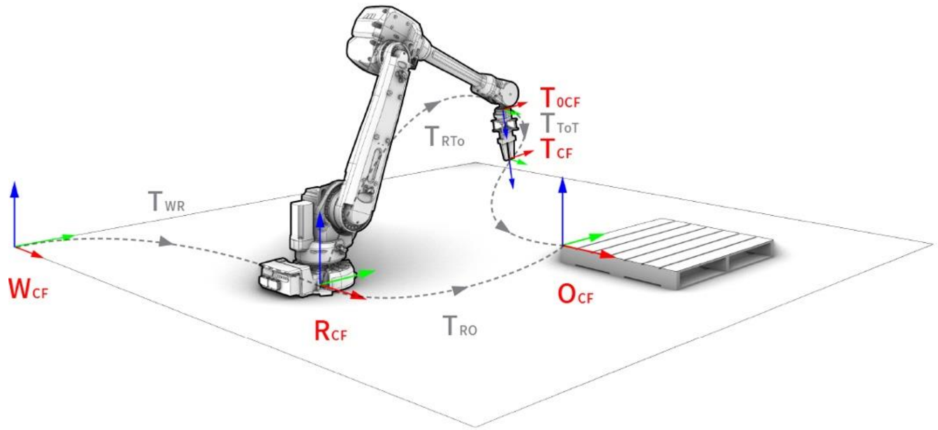
Alessandro Roncone

aroncone@colorado.edu

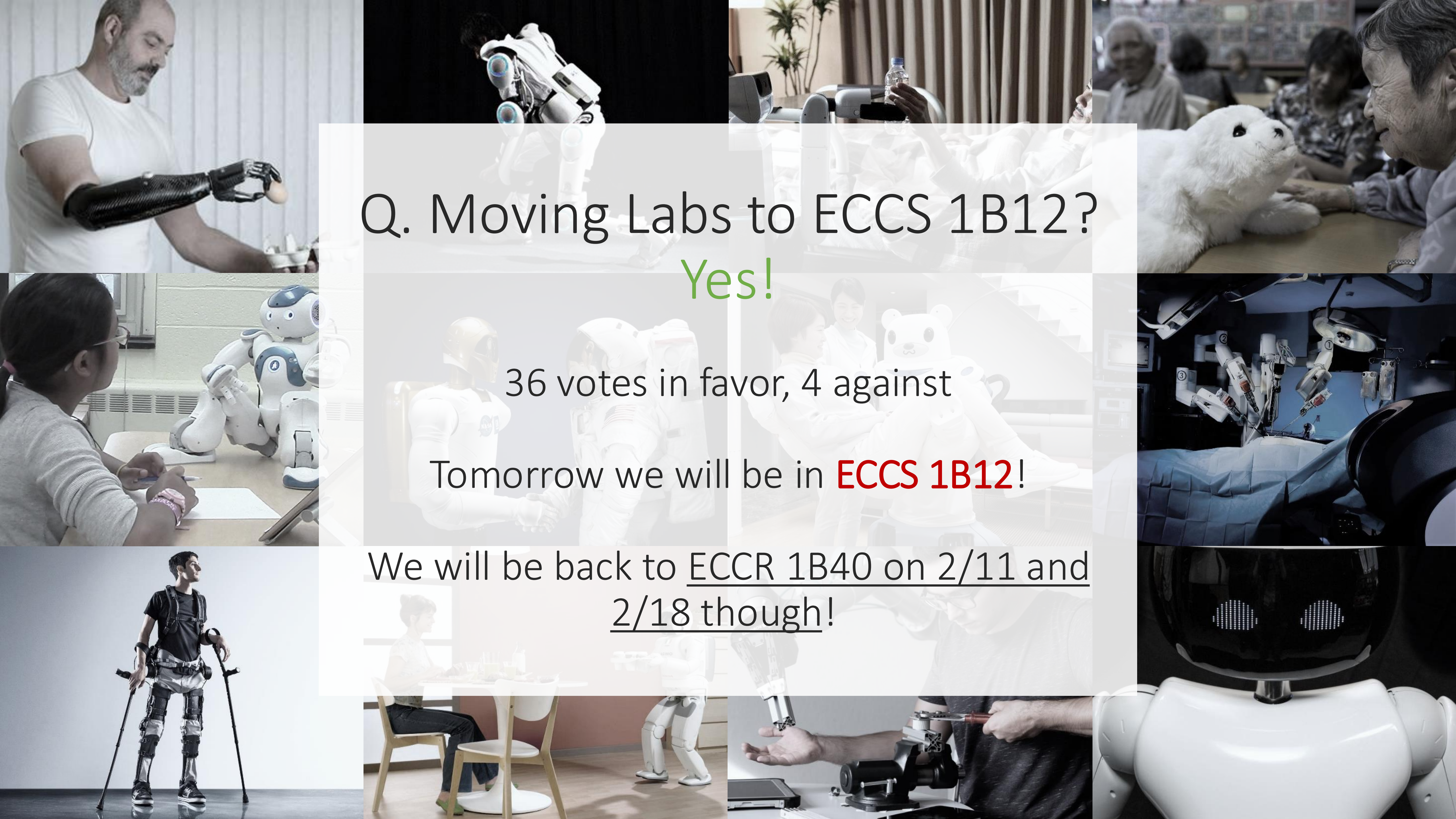
https://hiro-group.ronc.one

Administrivia





- Lab1
  - Grades out in the next couple days
  - Submissions are group submissions – one submission per team!
- HW2 is due tonight! Tuesday 2/3 @ 23:59pm
- Lab2 is due on Tuesday 2/11
  - We will cover odometry again today



Q. Moving Labs to ECCS 1B12?

Yes!

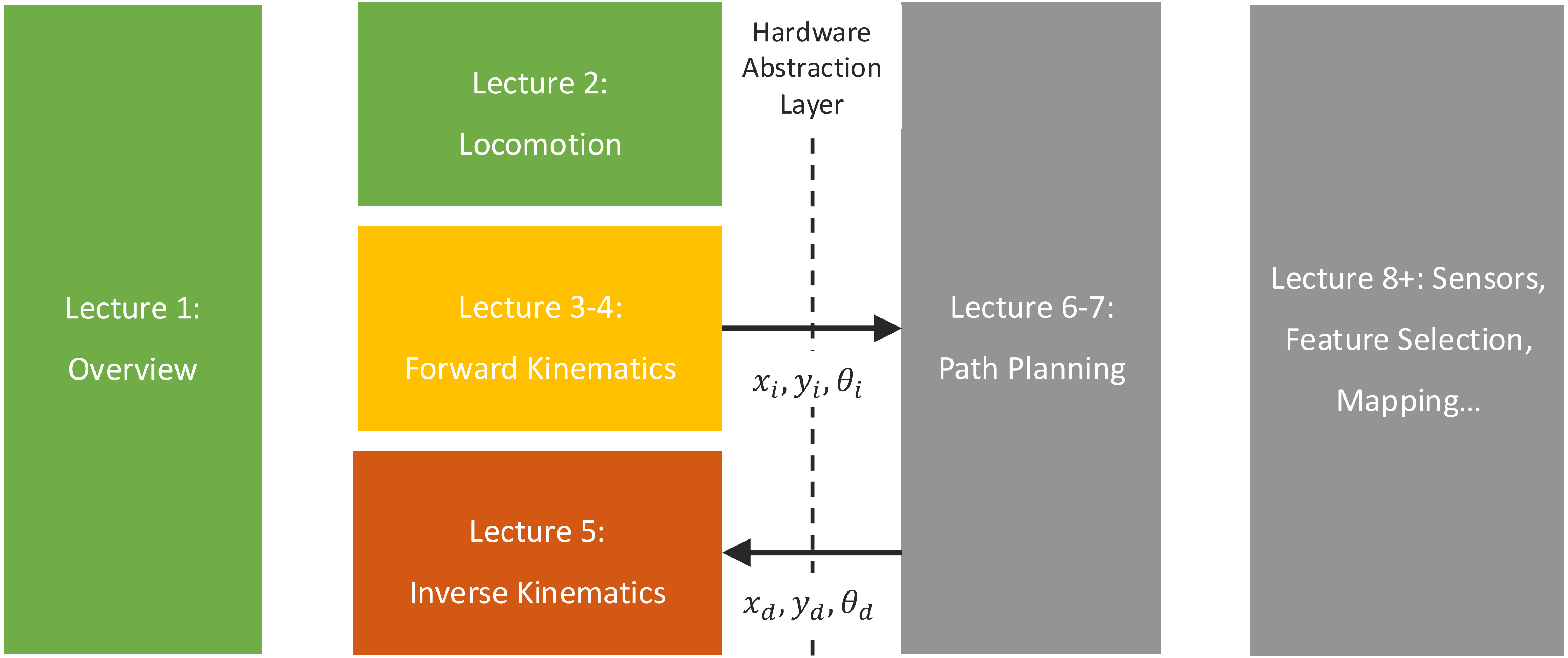
36 votes in favor, 4 against

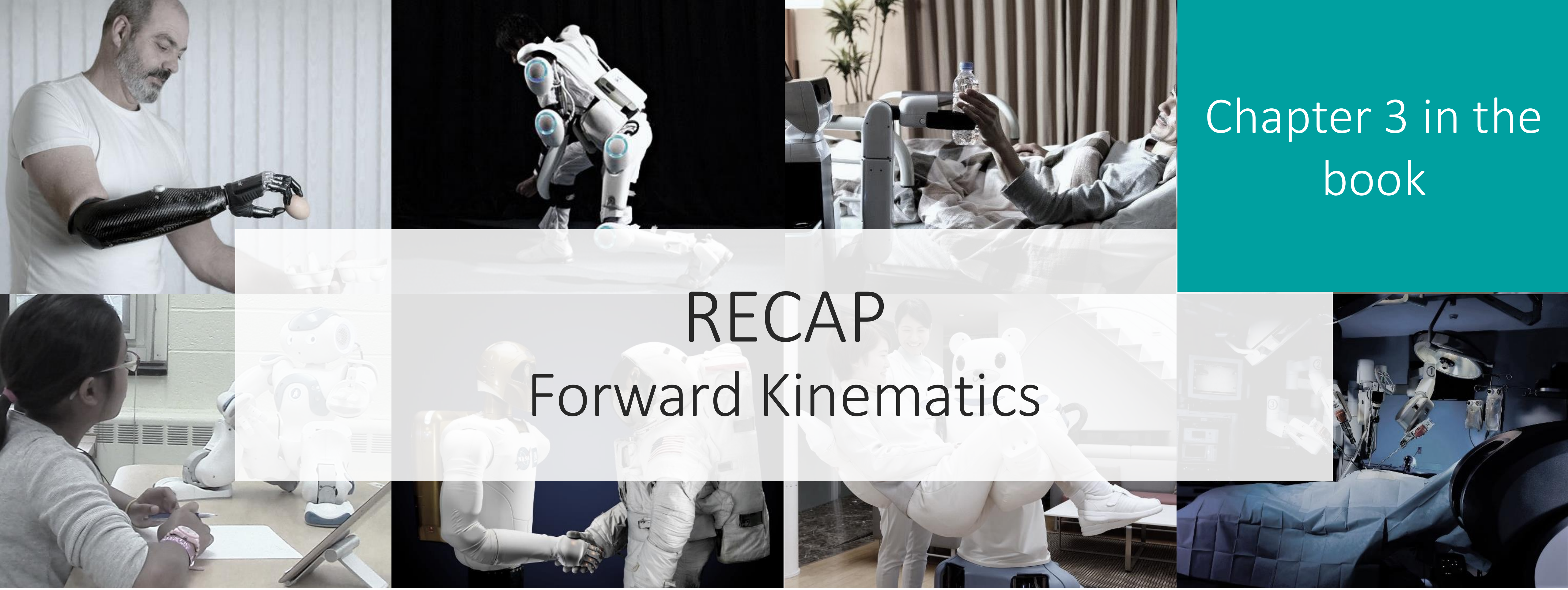
Tomorrow we will be in **ECCS 1B12**!

We will be back to ECCR 1B40 on 2/11 and 2/18 though!

Questions?

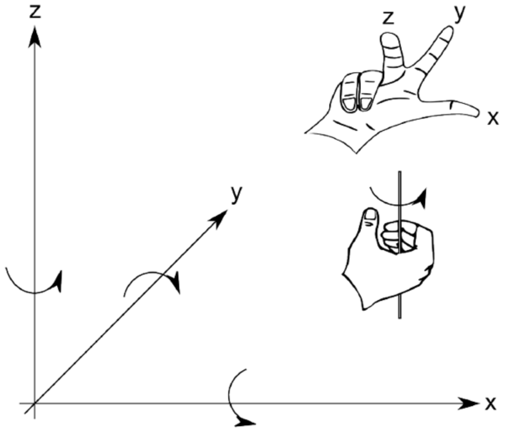
Roadmap

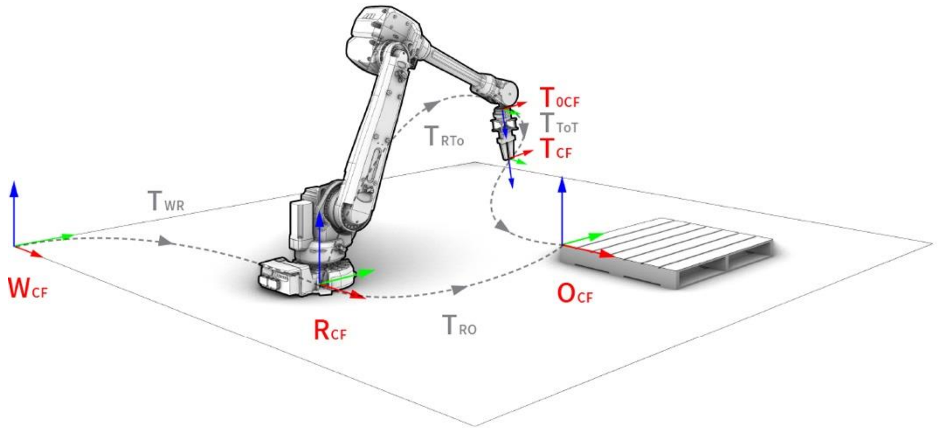

Lecture 1:
Overview
Lecture 2:
Locomotion
Lecture 3-4:
Forward Kinematics
Lecture 5:
Inverse Kinematics
Hardware
Abstraction
Layer
x_i, y_i, θ_i
x_d, y_d, θ_d
Lecture 6-7:
Path Planning
Lecture 8+: Sensors,
Feature Selection,
Mapping…



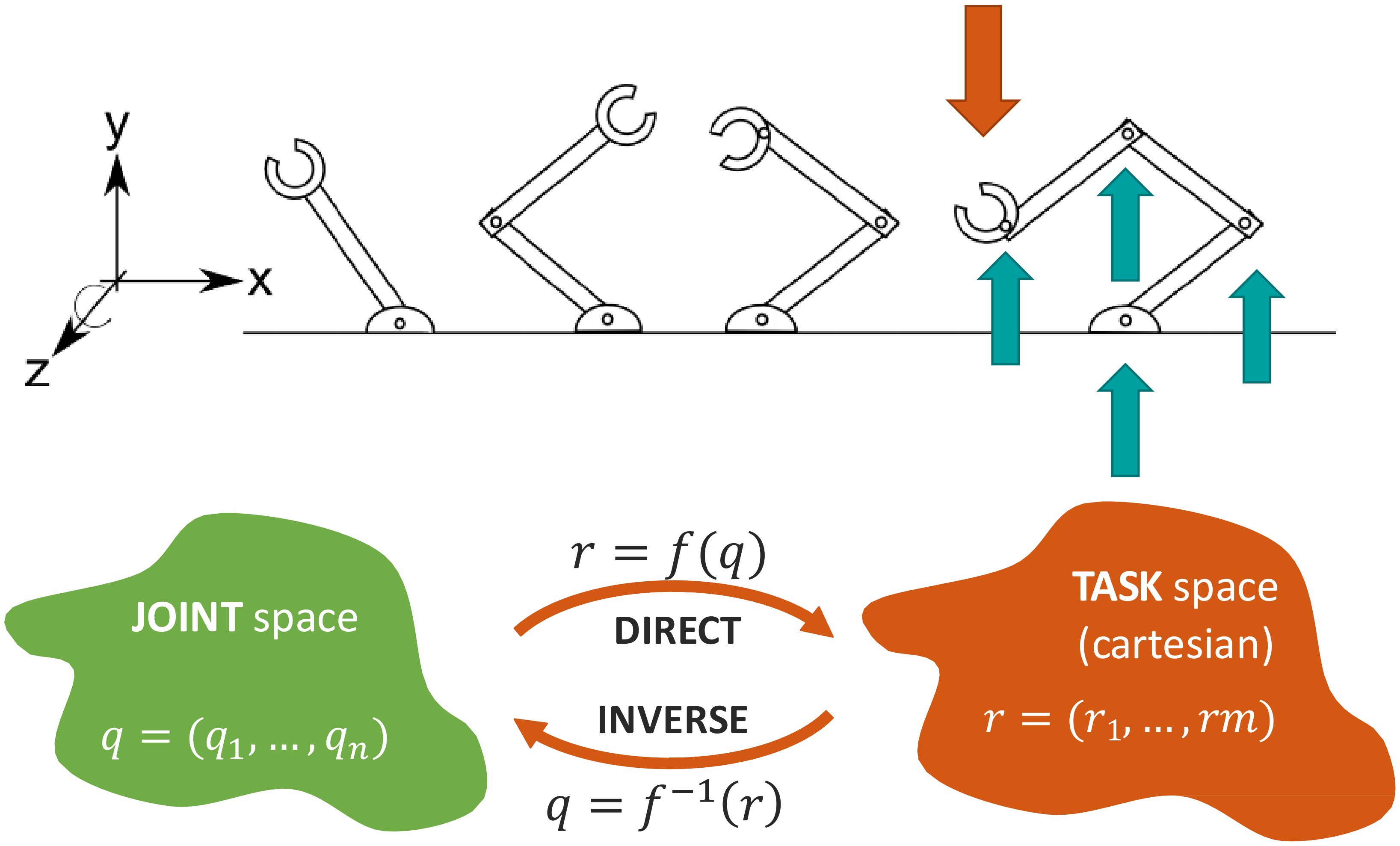
Chapter 3 in the book

RECAP
Forward Kinematics

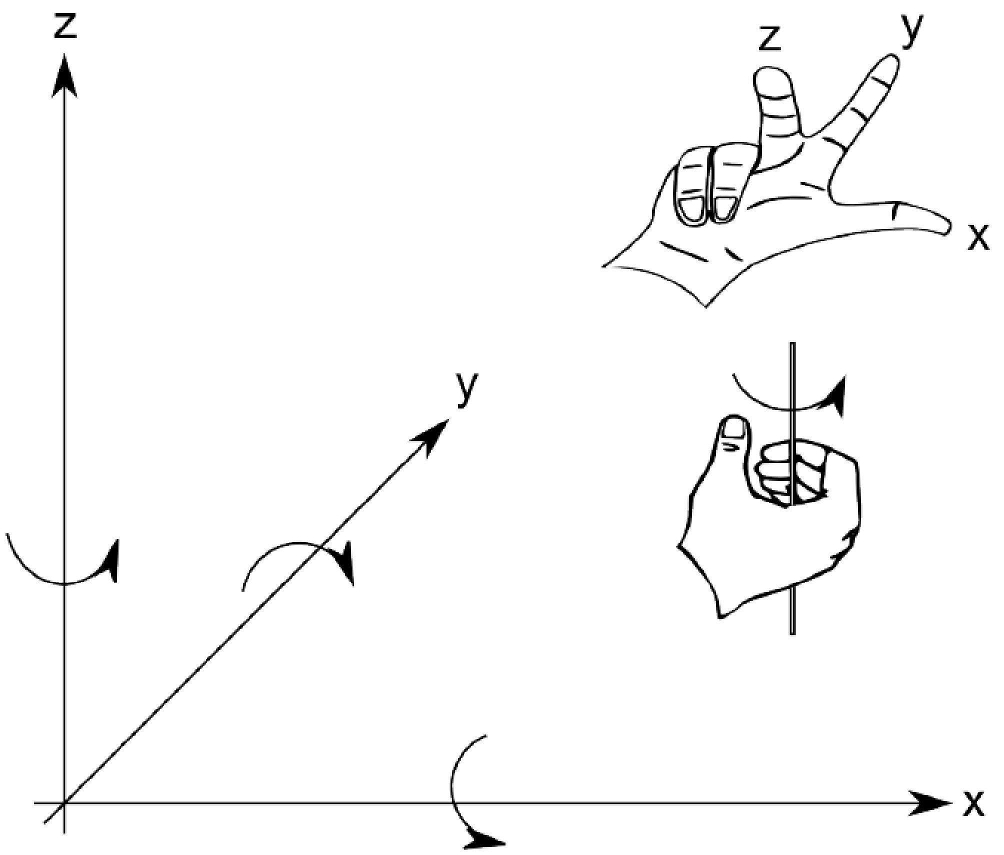




Forward (Direct) Kinematics



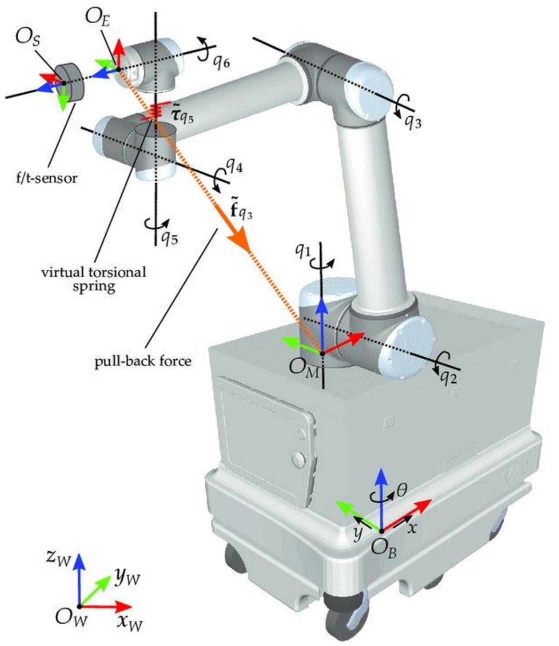
Coordinate Systems and Right Hand Rule



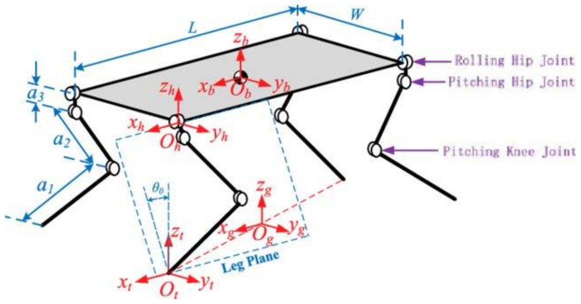
- Thumb along x-axis
- Index along y-axis
- Middle along z-axis

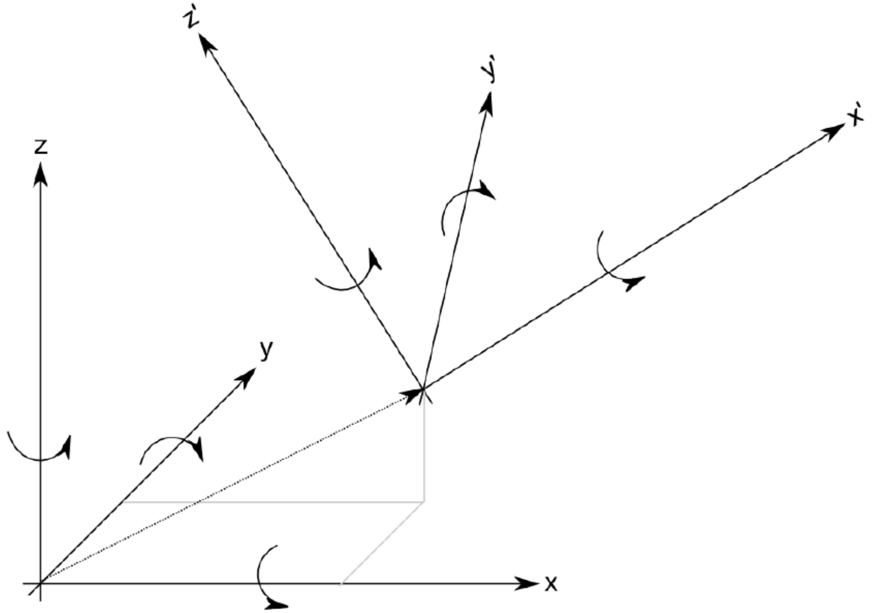
- It’s the order that counts!

Nested Coordinate Systems



- Each DoF / point of actuation presents a new Coordinate System with respect to the previous linkage!!
- Applies to both manipulators and mobile platforms!





Homogenous Transform

- Instead of ${}^{A}Q = {}^{A}_{B}R * {}^{B}Q + {}^{A}P$, we can express the transformation as a single matrix multiplication:

$$\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} = \begin{bmatrix} & {}^{A}_{B}R & & {}^{A}P \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}Q \\ 1 \end{bmatrix}$$

- Inverse Transform:

$$T^{-1} = \begin{bmatrix} R^{T} & -R^{T}\,{}^{A}P \\ 0 & 1 \end{bmatrix} \in SE(3)$$

Homogenous Transform: 2D case

- Instead of ${}^{A}Q = {}^{A}_{B}R * {}^{B}Q + {}^{A}P$, we can express the transformation as a single matrix multiplication:

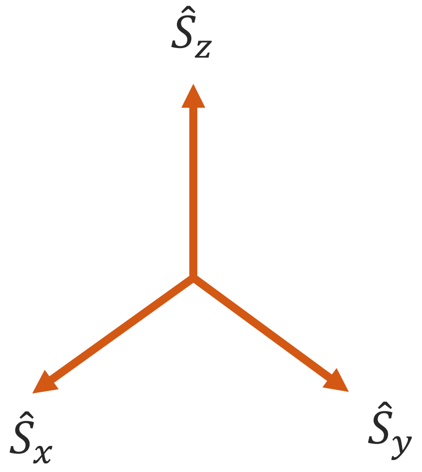
$$\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}Q \\ 1 \end{bmatrix}$$

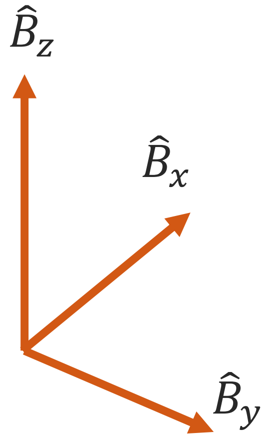
Example

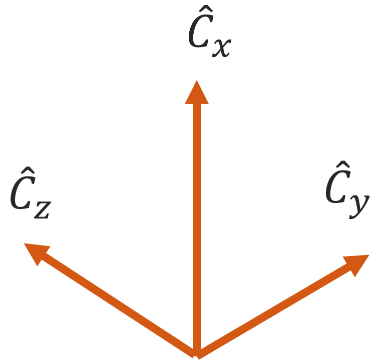
$$^{A}\hat{X}_B = (\hat{X}_B \cdot \hat{X}_A, \hat{X}_B \cdot \hat{Y}_A, \hat{X}_B \cdot \hat{Z}_A)^T$$

$$^{A}\hat{Y}_B = (\hat{Y}_B \cdot \hat{X}_A, \hat{Y}_B \cdot \hat{Y}_A, \hat{Y}_B \cdot \hat{Z}_A)^T$$

$$^{A}\hat{Z}_B = (\hat{Z}_B \cdot \hat{X}_A, \hat{Z}_B \cdot \hat{Y}_A, \hat{Z}_B \cdot \hat{Z}_A)^T$$







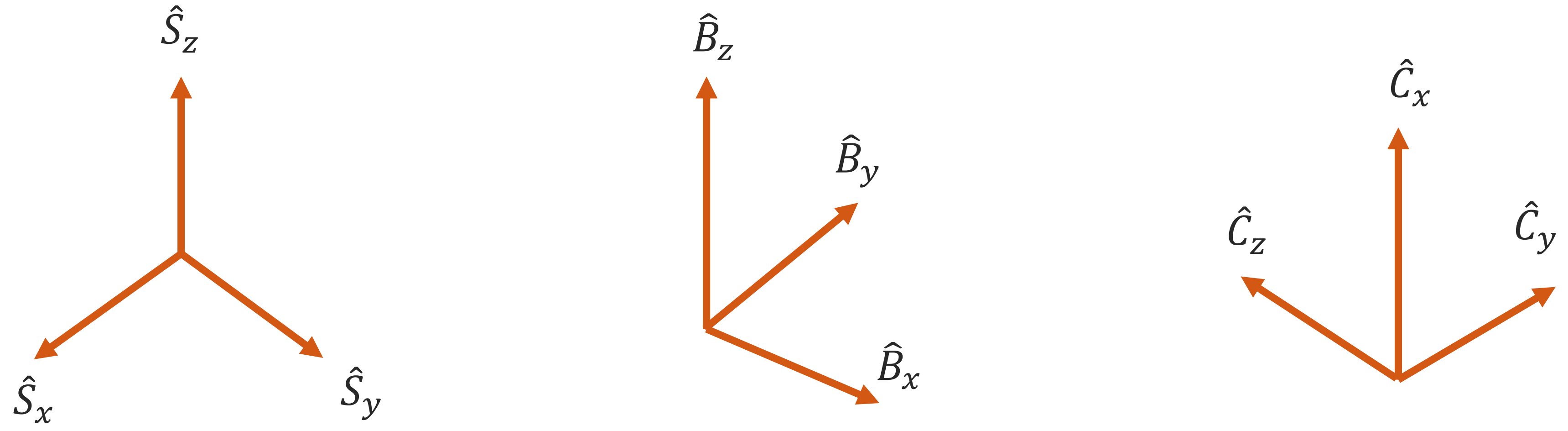
$$^{\hat{S}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

X-axis of $\hat{S}$ is the negative-Y-axis of $\hat{C}$

Y-axis of $\hat{S}$ is the negative-Z-axis of $\hat{C}$

Z-axis of $\hat{S}$ is the X-axis of $\hat{C}$

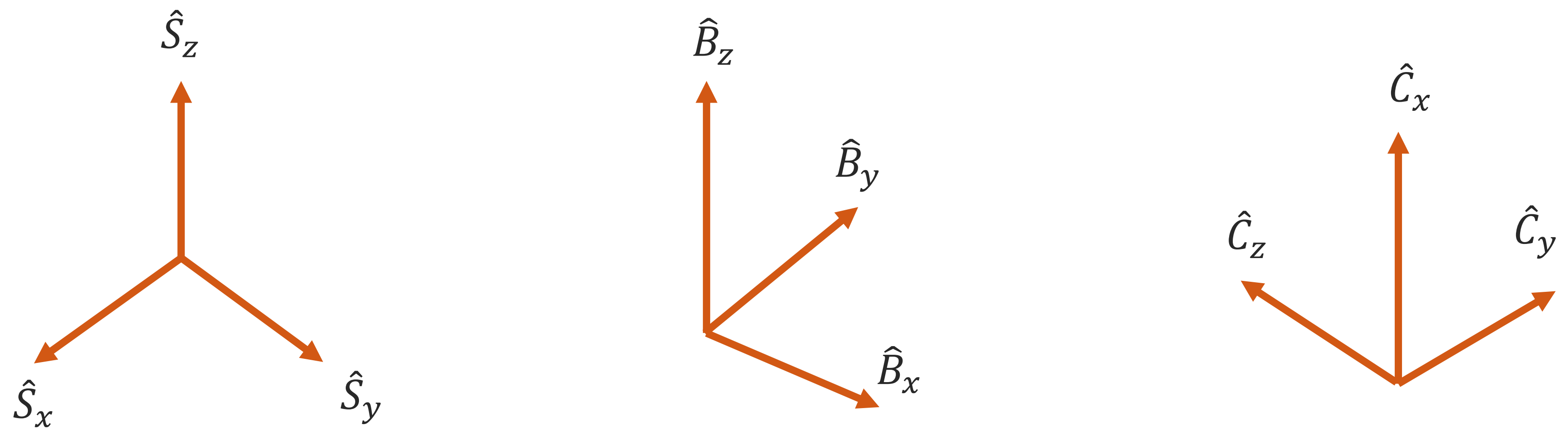
Example: Inverting a Rotation



$${}^{\hat{S}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \qquad {}^{\hat{C}}_{\hat{S}}R = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

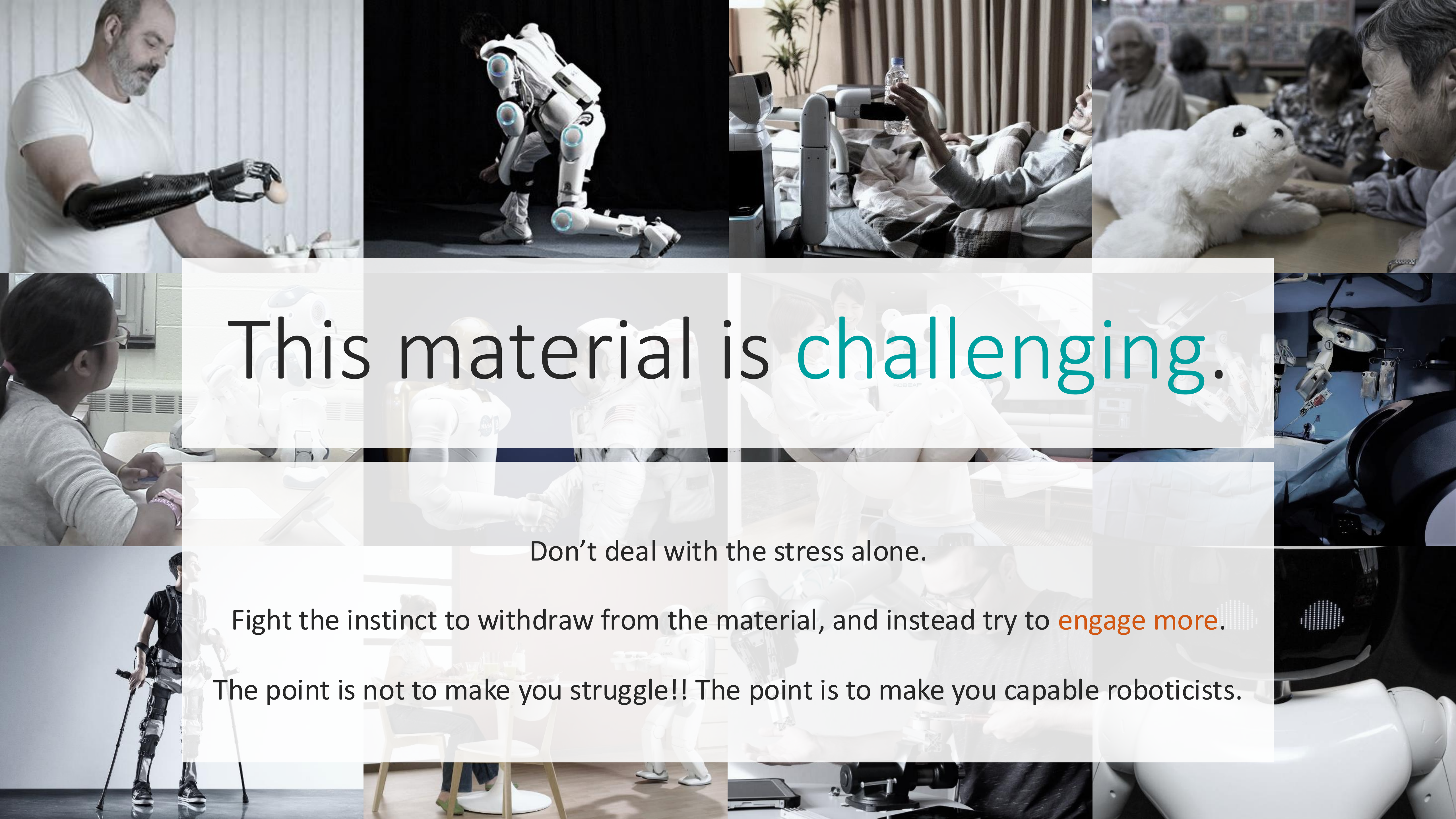
Transpose!

Example: Composing Rotations



$${}^{\hat{S}}_{\hat{B}}R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^{\hat{B}}_{\hat{C}}R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$${}^{\hat{S}}_{\hat{C}}R = {}^{\hat{S}}_{\hat{B}}R\,{}^{\hat{B}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$



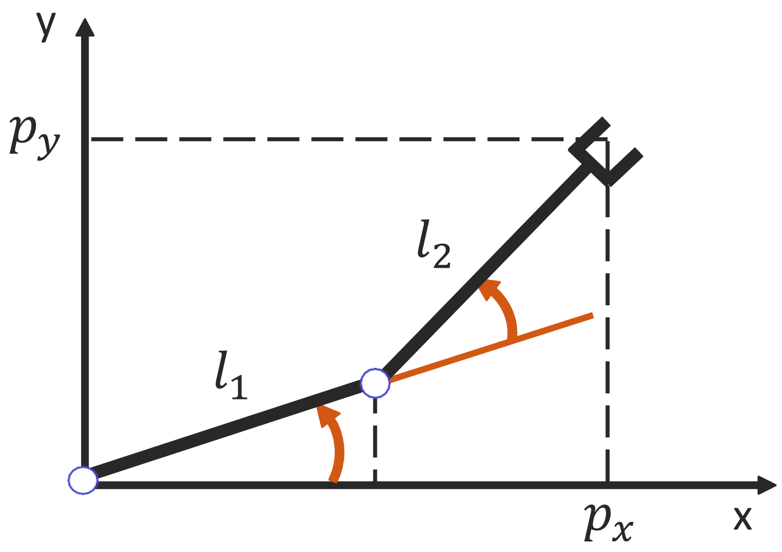
This material is challenging.

Don’t deal with the stress alone.

Fight the instinct to withdraw from the material, and instead try to engage more.

The point is not to make you struggle!! The point is to make you capable roboticists.

Example: direct kinematics of 2R arm



Differential (Forward) Kinematics

- From robot arms to differential wheel robots
- How can we model the forward kinematics of the e-puck?

Chapter 3.3

Forward Kinematics of a differential wheel robot (i.e. the e-puck)

- Manipulator:
  - Forward kinematics is uniquely defined by its joint angles
  - Measure of joint angles through encoders is absolute
- Differential Wheels robot:
  - Measure of joint angles through encoders is relative
  - Encoders' values need to be integrated over time

Please take a look at Section 3.3.2 in the book

Kinematics

- Kinematics – Geometry of Motion
  - The way parts of a robot move with respect to each other and the environment
  - Position (x)
  - Speed (x') <- This is differential kinematics!
- Dynamics – Newtonian Mechanics
  - Acceleration (x'')
  - Jerk (x''')
- Trivia: other derivatives of position?

Abstractions of increasing complexity:

Kinematics

↓

Statics

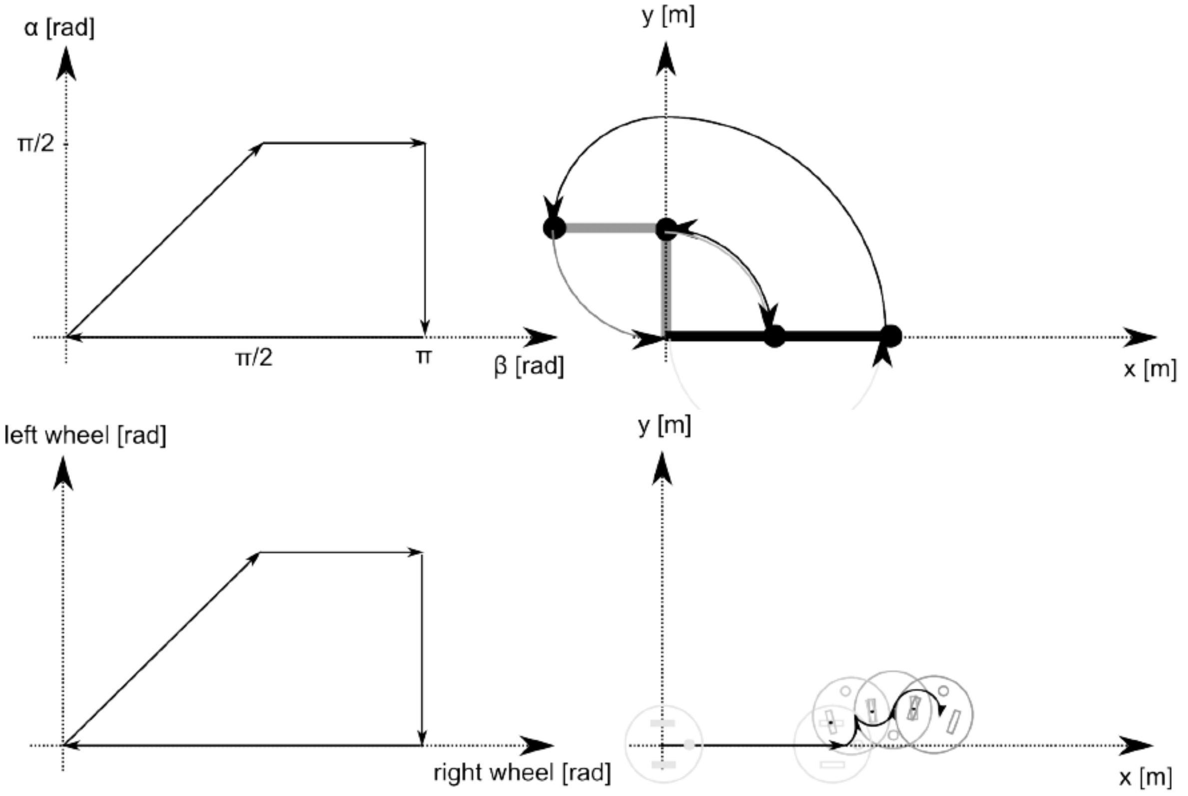
↓

Dynamics

Forward Kinematics of a differential wheel robot (i.e. the e-puck)

Manipulator
[HOLONOMIC]

Differential Wheels
[NON-HOLONOMIC]



Configuration [joint] Space

Work [task] space

Please take a look at Section 3.3.2 in the book

Holonomic or Non-Holonomic on the 2D plane?

Steering wheel is rotated 90 degrees then acceleration is applied for 1 second

vs.

Acceleration is applied for 1 second then steering wheel rotated 90 degrees!

- Different ending configuration = Non-holonomic!

Forward Kinematics – Odometry

- How to model wheel motion
- Wheel motions to position updates
- Position updates to Forward Kinematics in Inertia frame

Chapter 3.3

Odometry

Derived from Greek words for “measure route”

Utilization of sensors to estimate changes in position over time

Useful for position/pose estimation!

Lab 2:

Forward Kinematics and Odometry!

Odometry aka: Where is the e-puck?

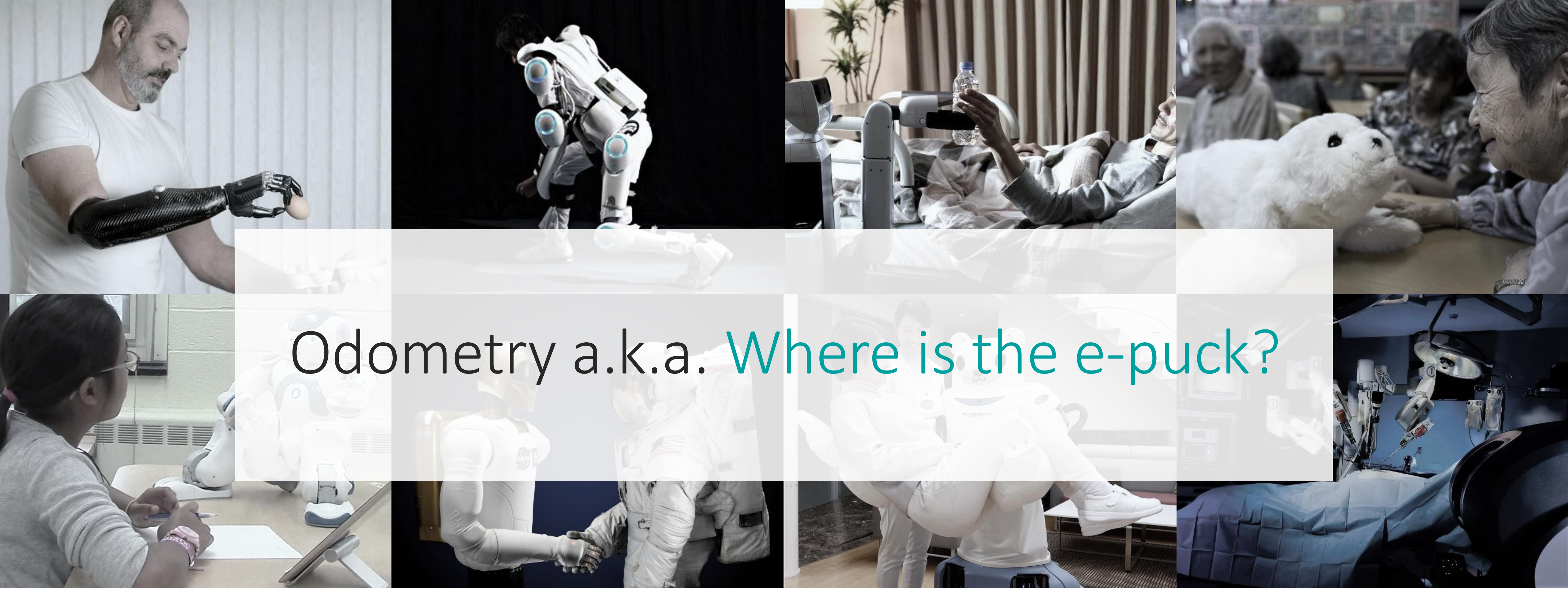
- How do we figure out where the e-puck is in the world?

What is our state vector?

- What are we able to control?
  - [JOINT SPACE DoFs]

- What are we able to measure?

- What variables do we need to measure the robot’s motion in space?
  - [OPERATIONAL SPACE DoFs]



Odometry a.k.a. Where is the e-puck?

- How do we figure out where the e-puck is in the world?
- From forward kinematics to differential kinematics
- Measuring the e-puck's displacement

Chapter 3.3

Odometry aka: Where is the e-puck?

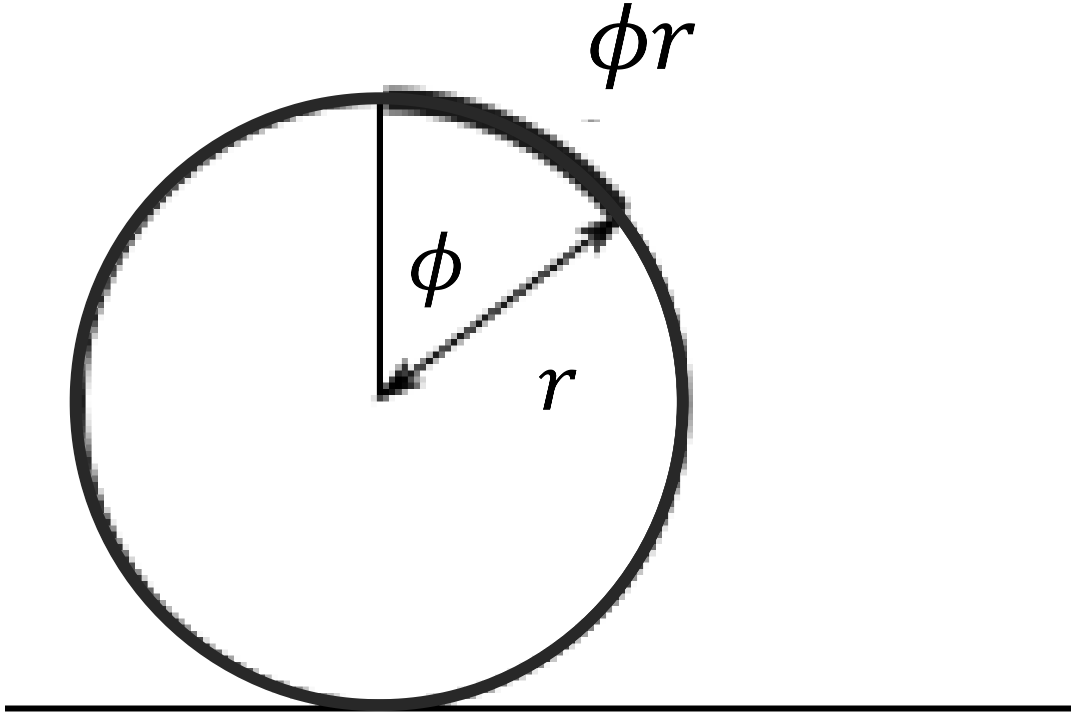
Four step process [and part of Lab2]!

1. Modeling wheel motion
2. From wheel motion to position updates
3. Forward differential kinematics
4. Computing position updates

Symbols

- ϕ → "phi"
- ω → "omega"
- θ → "theta"
- $\dot{}$ → time derivative (e.g. $\dot{\phi}_l$ → derivative (rate of change) of "phi" over time)

1. Wheel motion

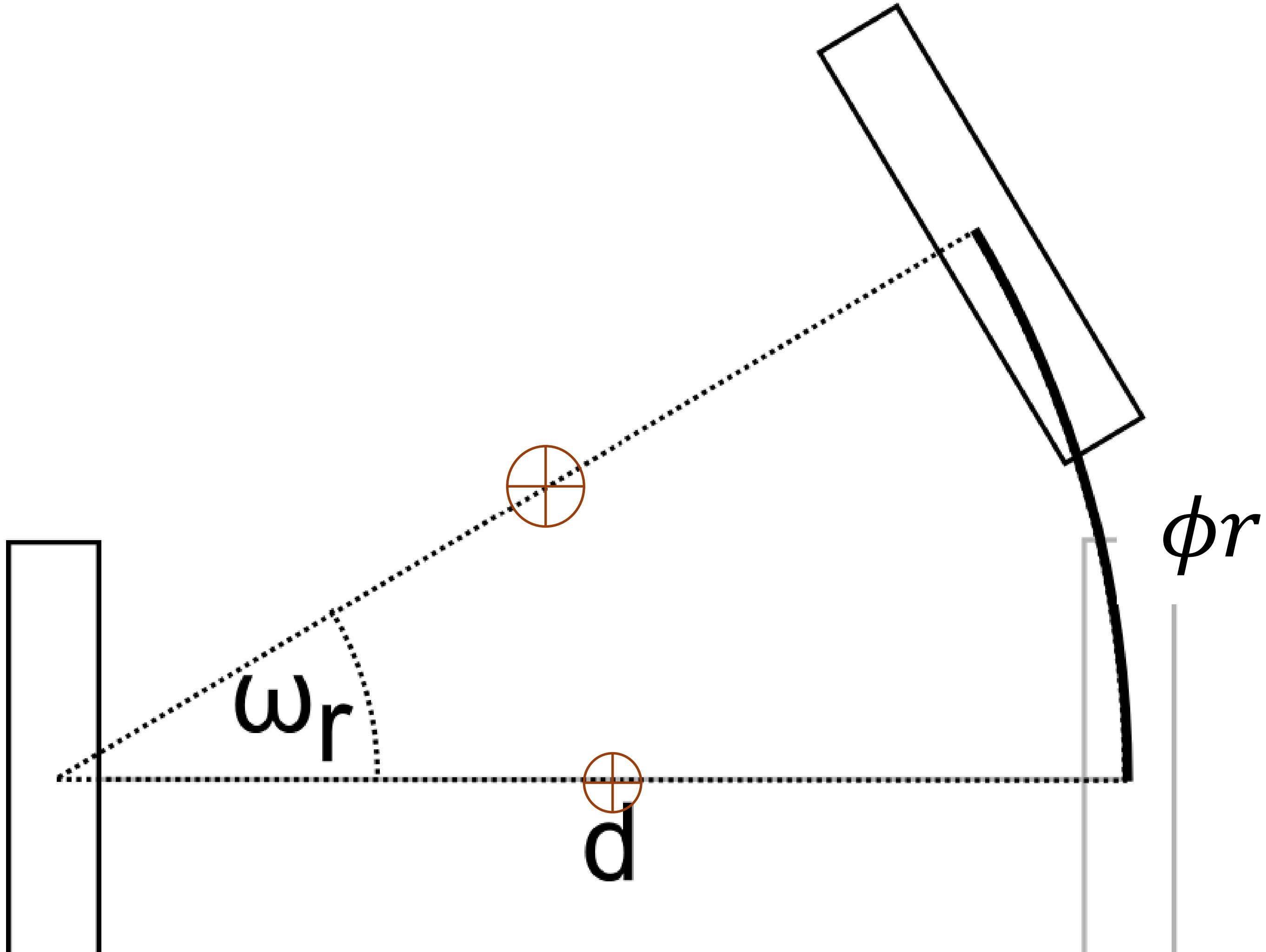


Distance traveled is angle of rotation times wheel radius:

$$x = r\phi$$

$$\dot{x} = r\dot{\phi}$$

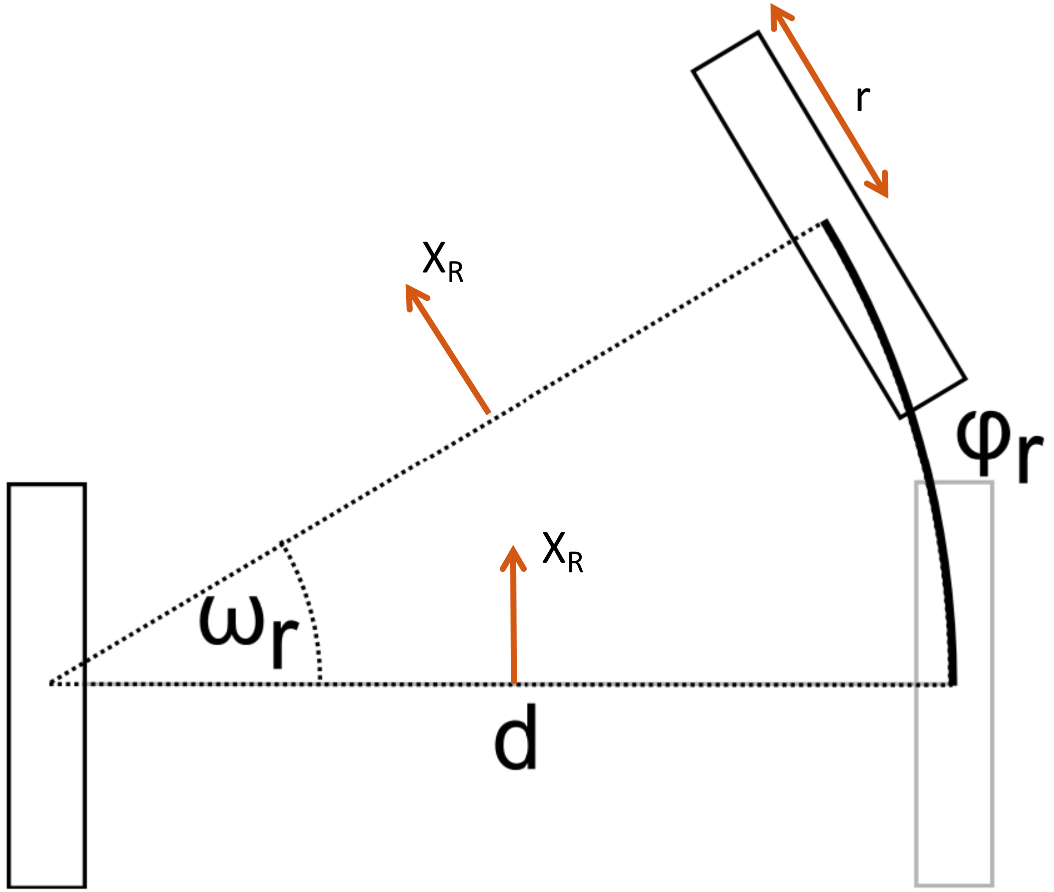
2. Wheel motion → Position Updates



What about the case where only one wheel is moving?

Its center of mass will move by $\frac{1}{2}\dot{\phi}r$!

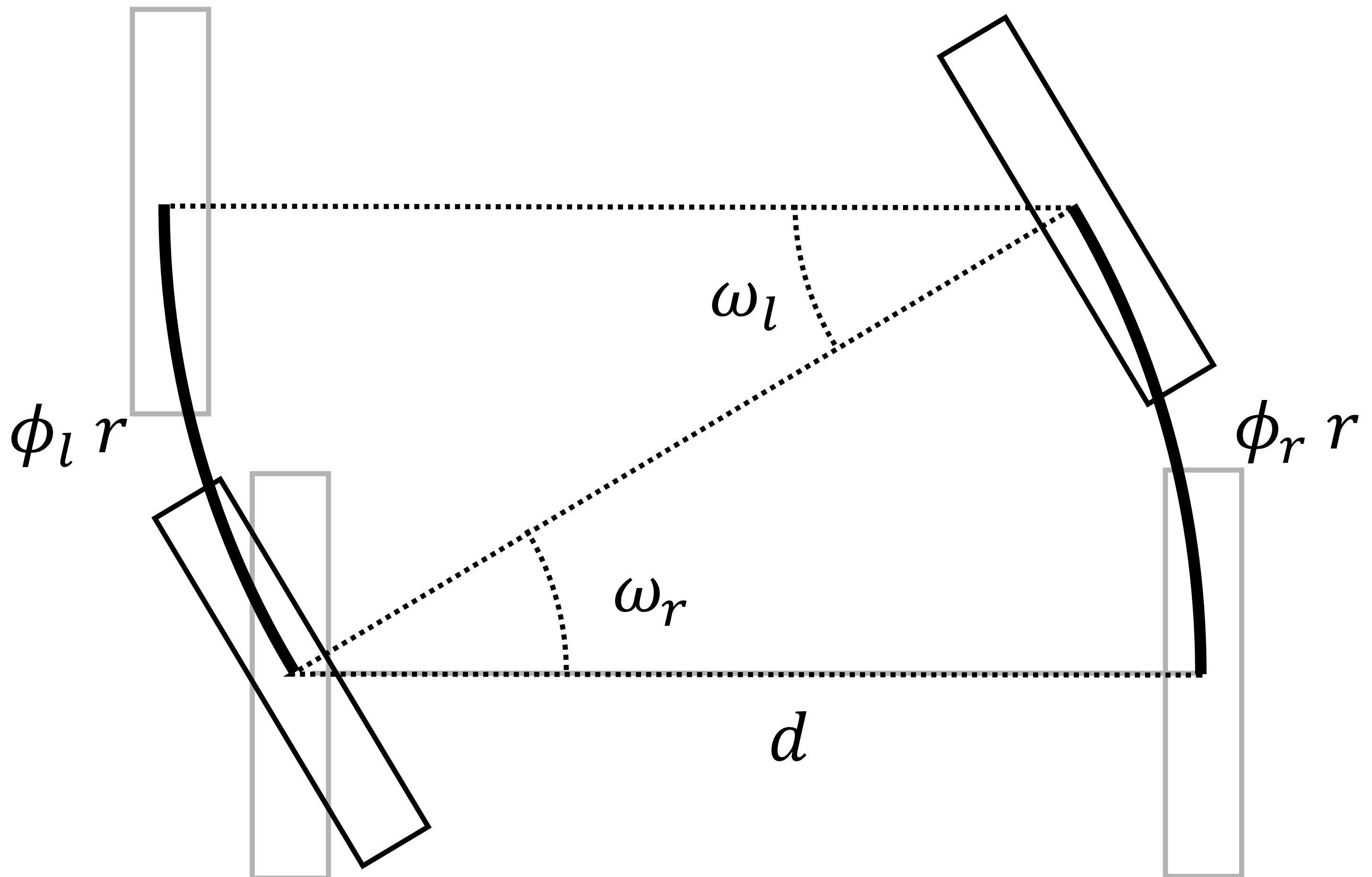
2. Wheel motion → Position Updates



$$\dot{x}_r = \frac{1}{2}\dot{\phi} r$$

$$\omega_r d = \phi_r r \;\rightarrow \dot{\omega}_r = \frac{\dot{\phi}_r r}{d}$$

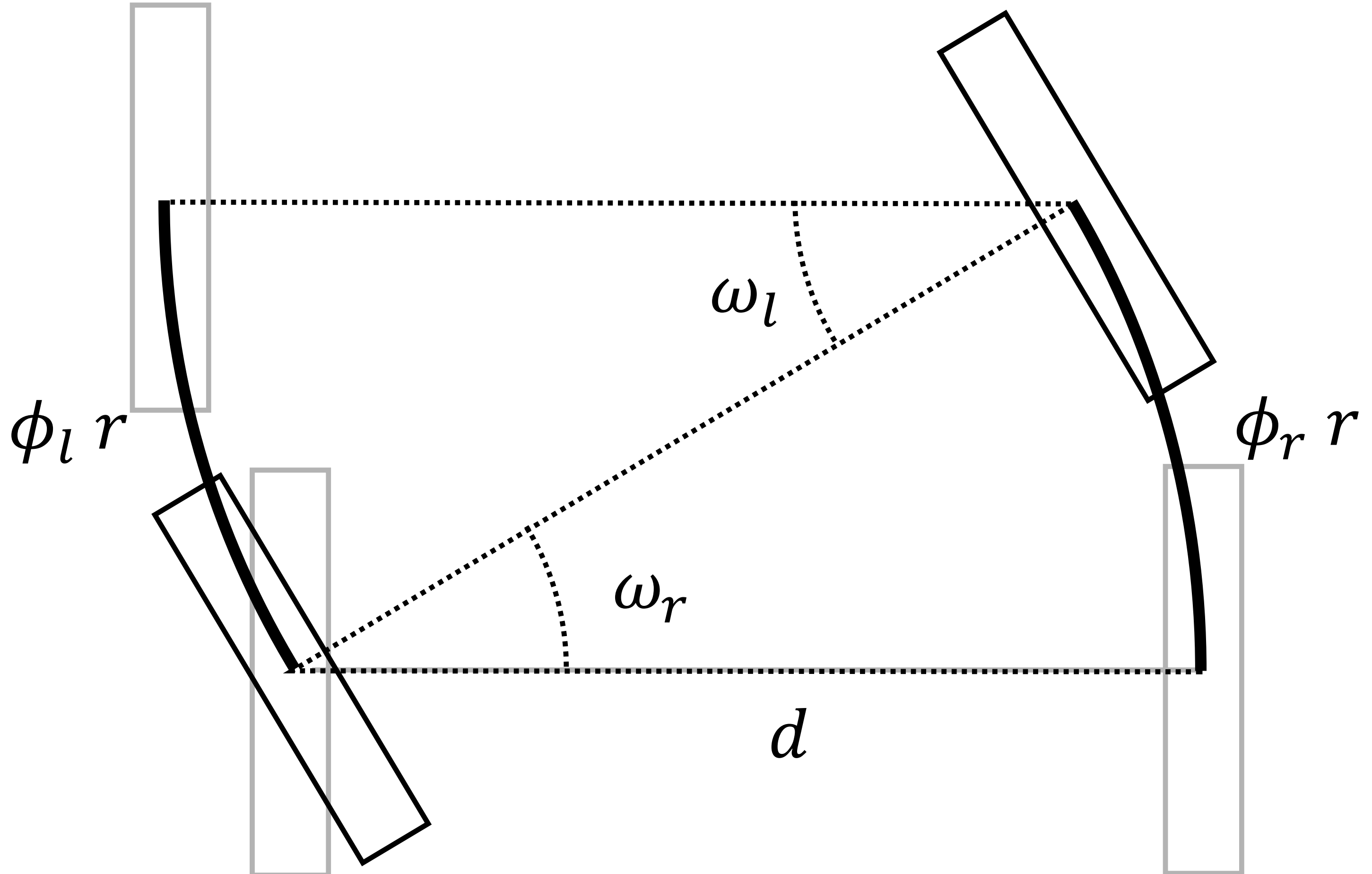
2. Wheel motions → Position Updates



What about the case where both wheels are moving at different speeds $\dot{\phi}_l$ and $\dot{\phi}_r$?

We can decouple the instantaneous contributions of $\dot{\phi}_l$ and $\dot{\phi}_r$!!

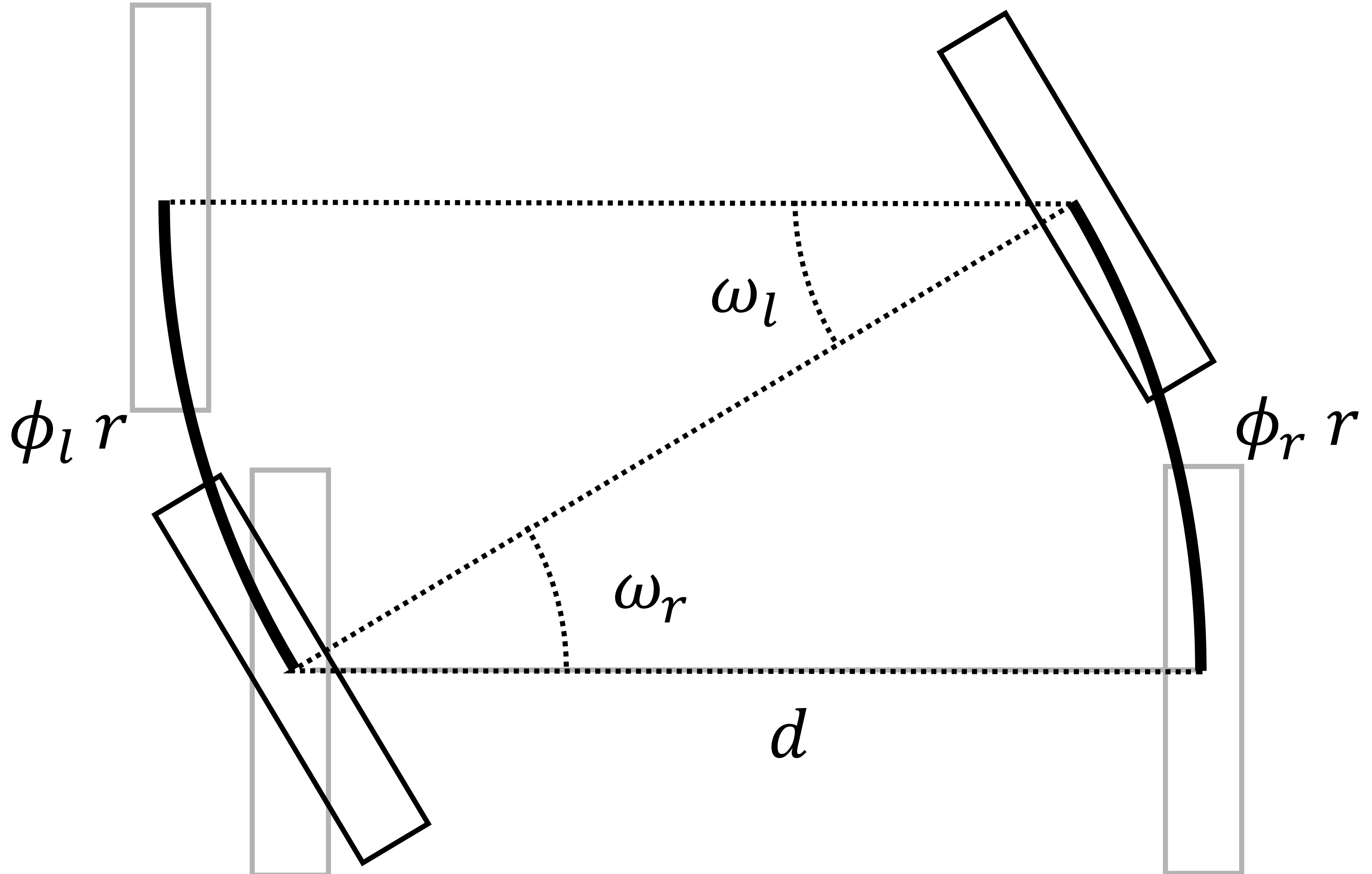
2. Wheel motions → Position Updates



What about the case where both wheels are moving at different speeds $\dot{\phi}_l$ and $\dot{\phi}_r$?

$$\dot{x}_r = \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2}$$

2. Wheel motions → Position Updates

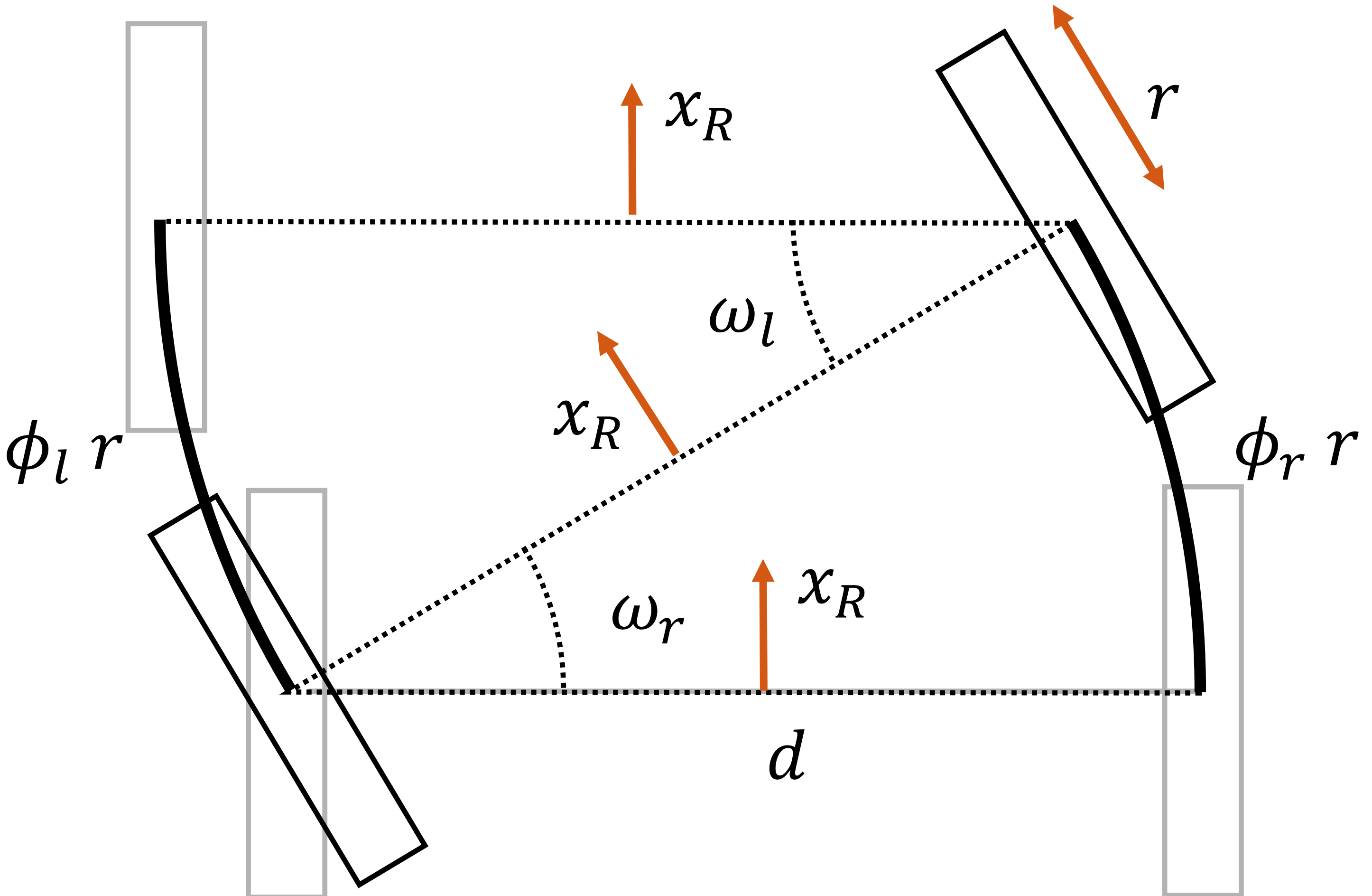


$$\dot{\omega}_r = \frac{\dot{\phi}_r r}{d}$$

$$\dot{\omega}_l = \frac{\dot{\phi}_l r}{d}$$

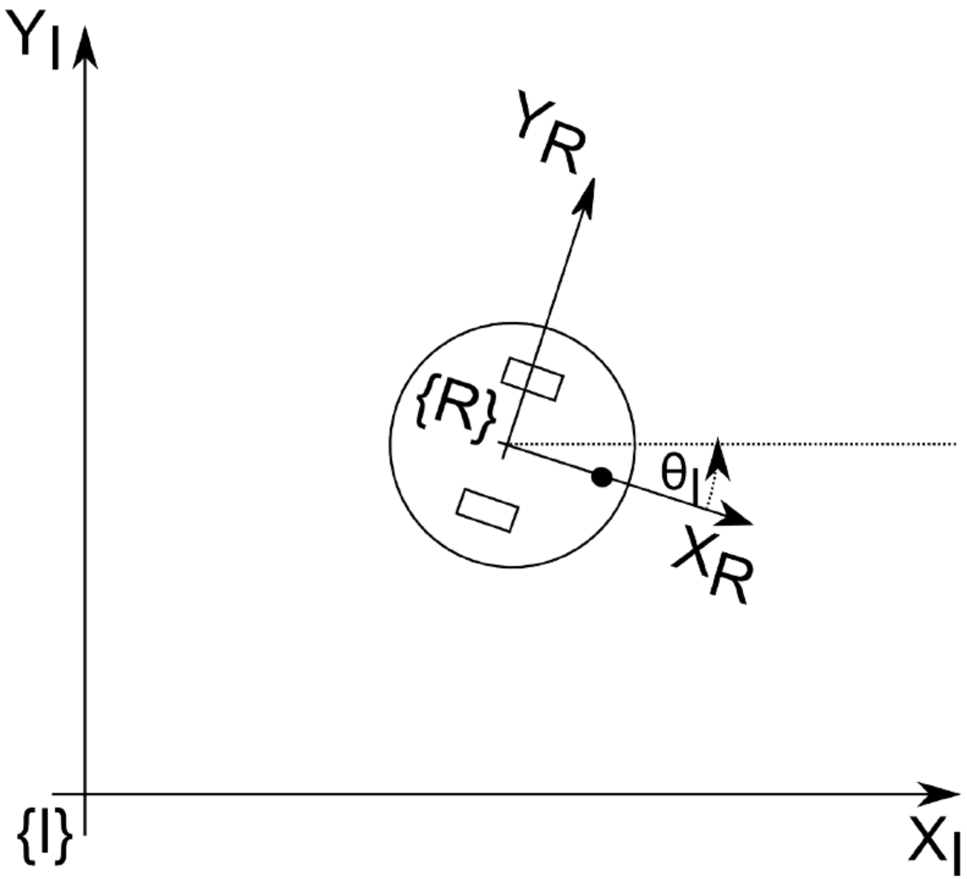
$$\dot{\theta} = \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d}$$

2. Forward Kinematics of mobile robot



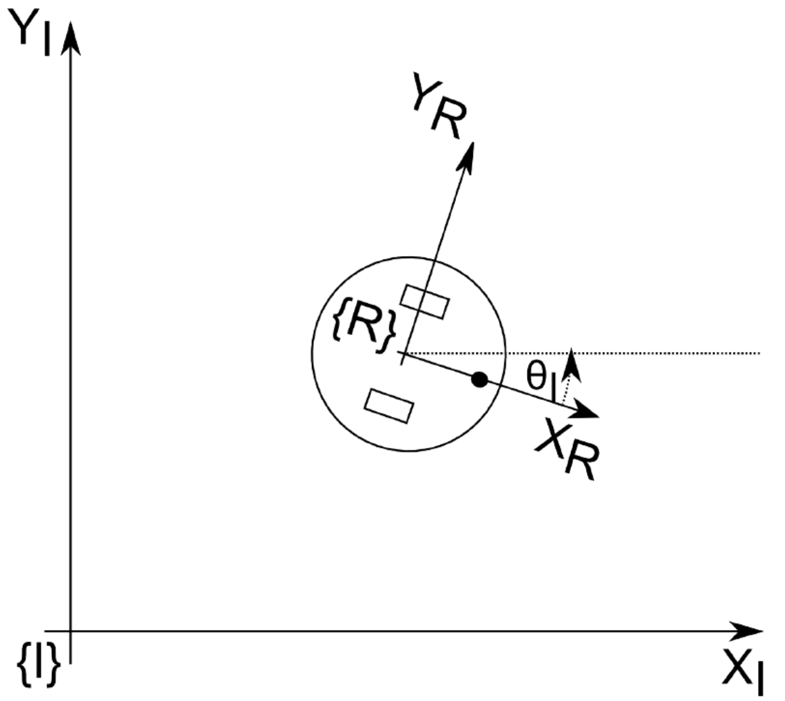
$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{bmatrix}$$

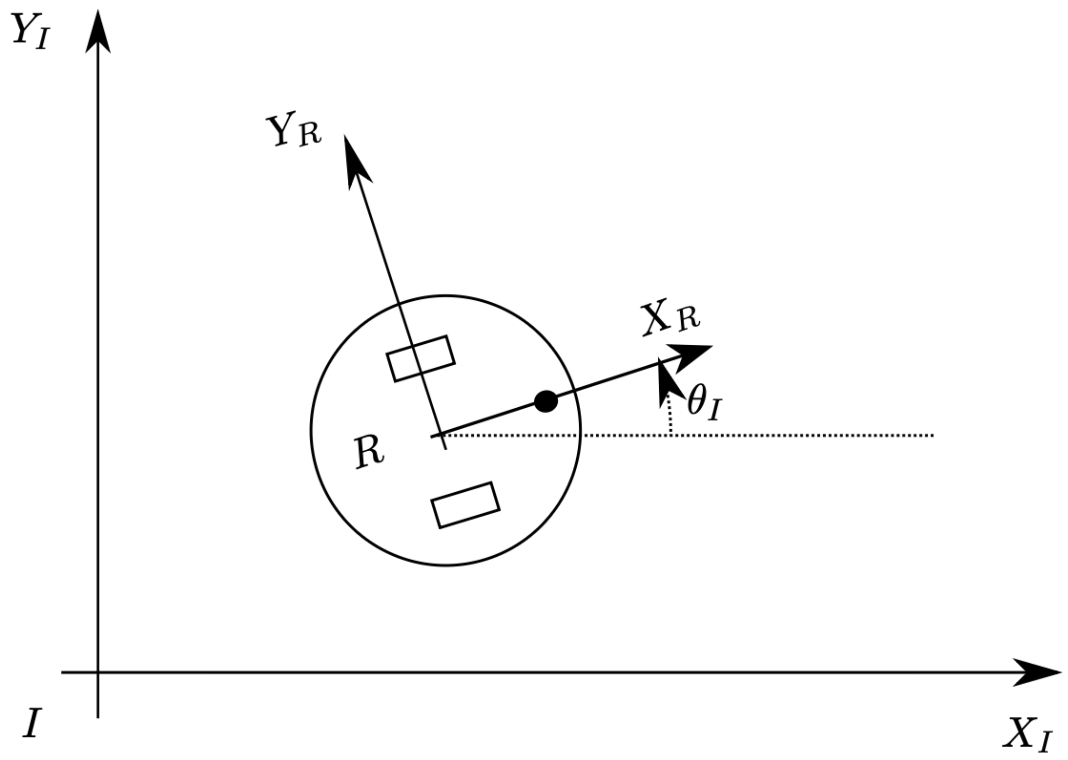
3. Forward Kinematics + Odometry



$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{bmatrix}$$

3. Forward Kinematics + Odometry

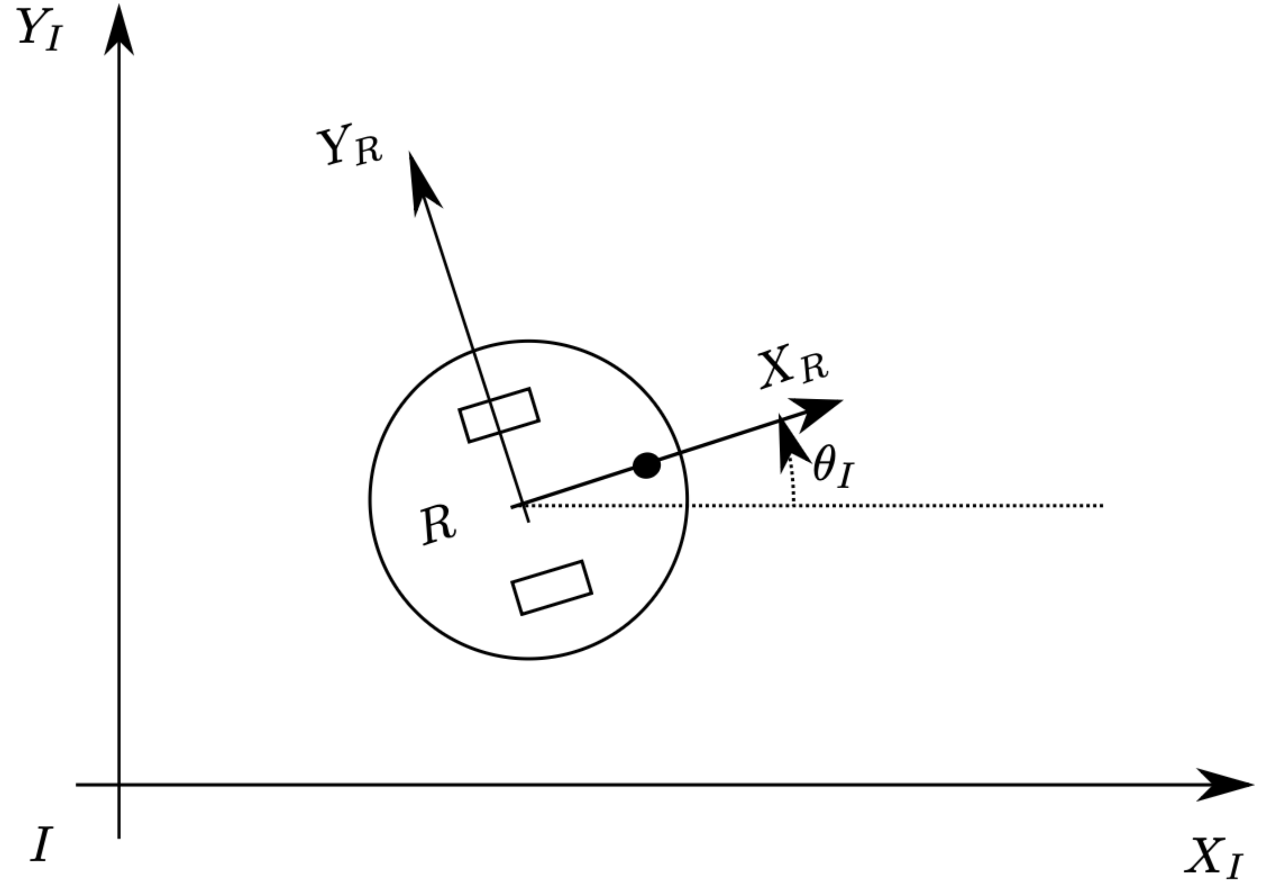




There was a mistake in lab slides! θ_I should have been negative according to the graph (starting from R into I).

In the book it was correct, but for simplicity we updated the book as well to have a positive θ_I. Equations were correct in both book and presentation though!

3. Forward Kinematics + Odometry



$$\dot{x}_{I,x} = cos(\theta)\dot{x}_R$$

$$\dot{x}_{I,y} = -sin(\theta)\dot{y}_R$$

$$\dot{x}_I = cos(\theta)\dot{x}_R - sin(\theta)\dot{y}_R$$

$$\dot{y}_I = sin(\theta)\dot{x}_R + cos(\theta)\dot{y}_R$$

$$\dot{\theta}_I = \dot{\theta}_R$$

$$\begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$

4. Speeds → How can we compute positions?

$$\begin{pmatrix} x_I(T) \\ y_I(T) \\ \theta(T) \end{pmatrix} =$$