Université de Montréal

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IFT-6155 Quantum Computing

Assignment 3

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Question 1

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Considérez l'état intriqué

$$|\Gamma\rangle = \tfrac{1}{\sqrt{3}}|01\rangle + \tfrac{1}{\sqrt{3}}|10\rangle + \tfrac{1}{\sqrt{3}}|11\rangle\,,$$

où l'on donne le premier qubit à Alice et le second à Bob. Au moment où ils reçoivent leur qubit, Alice et Bob décident indépendamment de soit le mesurer directement (D), soit lui appliquer la transformation de Hadamard (H) avant de le mesurer.

a) Afin de déterminer l'état des deux qubits avant la mesure pour chacune des éventualités, calculez $(\mathsf{H}\otimes\mathsf{I})|\Gamma\rangle$, $(\mathsf{I}\otimes\mathsf{H})|\Gamma\rangle$ et $(\mathsf{H}\otimes\mathsf{H})|\Gamma\rangle$, où I est l'opération identité qui envoie $|0\rangle$ sur $|0\rangle$ et $|1\rangle$ sur $|1\rangle$.
Note : Vous pouvez vous épargner quelques calculs si vous utilisez le fait que $(\mathsf{H}\otimes\mathsf{I})|\Gamma\rangle$ est symétrique à $(\mathsf{I}\otimes\mathsf{H})|\Gamma\rangle$ et que $(\mathsf{H}\otimes\mathsf{H})|\Gamma\rangle$ est égal à $(\mathsf{H}\otimes\mathsf{I})(\mathsf{I}\otimes\mathsf{H})|\Gamma\rangle$.

b) Remplissez un tableau des probabilités d'obtention des quatre résultats possibles (00, 01, 10 et 11 ; le premier bit étant pour Alice et le second pour Bob) en fonction des choix d'Alice et de Bob (DD, DH, HD et HH). Il s'agit donc d'un tableau 4×4.

c) (optionnel) Dites pourquoi ce tableau de probabilités est remarquable.

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a)

$(H \otimes I)|\Gamma\rangle$ $= (1/\sqrt{3})\,(H|0\rangle \otimes I|1\rangle) + (1/\sqrt{3})\,(H|1\rangle \otimes I|0\rangle) + (1/\sqrt{3})\,(H|1\rangle \otimes I|1\rangle)$

Knowing that: $(H|0\rangle = ((1/\sqrt{2})|0\rangle + ((1/\sqrt{2})|1\rangle)$, $(H|1\rangle = ((1/\sqrt{2})|0\rangle - ((1/\sqrt{2})|1\rangle)$, Identity Operation (I): $|0\rangle \rightarrow |0\rangle$ & $|1\rangle \rightarrow |1\rangle$

$= (1/\sqrt{3})\,(((1/\sqrt{2})|0\rangle + ((1/\sqrt{2})|1\rangle)\otimes I|1\rangle)$
$+ (1/\sqrt{3})\,(((1/\sqrt{2})|0\rangle - ((1/\sqrt{2})|1\rangle)\otimes I|0\rangle)$
$+ (1/\sqrt{3})\,(((1/\sqrt{2})|0\rangle - ((1/\sqrt{2})|1\rangle)\otimes I|1\rangle)$

$= (1/\sqrt{3}\sqrt{2})\,|01\rangle + (1/\sqrt{3}\sqrt{2})\,|11\rangle$
$+ (1/\sqrt{3}\sqrt{2})\,|00\rangle - (1/\sqrt{3}\sqrt{2})\,|10\rangle$
$+ (1/\sqrt{3}\sqrt{2})\,|01\rangle - (1/\sqrt{3}\sqrt{2})\,|11\rangle$

$= (1/\sqrt{6})\,|00\rangle + (2/\sqrt{6})\,|01\rangle - (1/\sqrt{6})\,|10\rangle$

$(I \otimes H)|\Gamma\rangle$ $= (1/\sqrt{3})\,(I|0\rangle \otimes H|1\rangle) + (1/\sqrt{3})\,(I|1\rangle \otimes H|0\rangle) + (1/\sqrt{3})\,(I|1\rangle \otimes H|1\rangle)$

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$= (1/\sqrt{3})\,(|0\rangle \otimes ((1/\sqrt{2})|0\rangle - ((1/\sqrt{2})|1\rangle))$
$+ (1/\sqrt{3})\,(|1\rangle \otimes ((1/\sqrt{2})|0\rangle + ((1/\sqrt{2})|1\rangle))$
$+ (1/\sqrt{3})\,(|1\rangle \otimes ((1/\sqrt{2})|0\rangle - ((1/\sqrt{2})|1\rangle))$

$= (1/\sqrt{6})\,|00\rangle - (1/\sqrt{6})\,|01\rangle + (2/\sqrt{6})\,|10\rangle$

$(H \otimes H)|\Gamma\rangle$ $= (H \otimes I)(I \otimes H)|\Gamma\rangle$

$= (H \otimes I)\,((1/\sqrt{3}\sqrt{2})\,|00\rangle - (1/\sqrt{3}\sqrt{2})\,|01\rangle + (2/\sqrt{3}\sqrt{2})\,|10\rangle)$

$= ((1/\sqrt{3}\sqrt{2})\,(H|0\rangle \otimes I|0\rangle)$
$- (1/\sqrt{3}\sqrt{2})\,(H|0\rangle \otimes I|1\rangle)$
$+ (2/\sqrt{3}\sqrt{2})\,(H|1\rangle \otimes I|0\rangle)$

$= ((1/\sqrt{3}\sqrt{2})\,(((1/\sqrt{2})|0\rangle + ((1/\sqrt{2})|1\rangle) \otimes |0\rangle)$
$- (1/\sqrt{3}\sqrt{2})\,(((1/\sqrt{2})|0\rangle + ((1/\sqrt{2})|1\rangle) \otimes |1\rangle)$
$+ (2/\sqrt{3}\sqrt{2})\,(((1/\sqrt{2})|0\rangle - ((1/\sqrt{2})|1\rangle) \otimes |0\rangle)$
$= (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|00\rangle + (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|10\rangle)$
$- (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|01\rangle + (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|11\rangle$
$+ (2/\sqrt{3}\sqrt{2}\sqrt{2})\,|00\rangle - (2/\sqrt{3}\sqrt{2}\sqrt{2})\,|10\rangle$

$= (3/\sqrt{3}\sqrt{2}\sqrt{2})\,|00\rangle - (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|01\rangle - (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|10\rangle) + (1/\sqrt{3}\sqrt{2}\sqrt{2})\,|11\rangle$

- Linear Algebra Way:

$$|\Gamma\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}0\\1\\1\\1\end{pmatrix}$$

$$(H\otimes I)|\Gamma\rangle = \frac{1}{\sqrt{2}\sqrt{3}}\begin{pmatrix}I & I\\ I & -I\end{pmatrix}\begin{pmatrix}0\\1\\1\\1\end{pmatrix} = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\-1\\0\end{pmatrix}$$

$$(I\otimes H)|\Gamma\rangle = \frac{1}{\sqrt{2}\sqrt{3}}\begin{pmatrix}H & 0\\ 0 & H\end{pmatrix}\begin{pmatrix}0\\1\\1\\1\end{pmatrix} = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\-1\\2\\0\end{pmatrix}$$

$$(H\otimes H)|\Gamma\rangle = \frac{1}{\sqrt{2}\sqrt{2}\sqrt{3}}\begin{pmatrix}H & H\\ H & -H\end{pmatrix}\begin{pmatrix}0\\1\\1\\1\end{pmatrix} = \frac{1}{2\sqrt{3}}\begin{pmatrix}3\\-1\\-1\\-1\end{pmatrix}$$

A \ B		D 0	D 1	H 0	H 1
D	0	0	1/3	1/6	1/6
	1	1/3	1/3	4/6	0
H	0	1/6	4/6	9/12	1/12
	1	1/6	0	1/12	1/12

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b)

	00	01	10	11
DD	0	0.333	0.333	0.333
DH	0.166	0.166	0.666	0
HD	0.166	0.666	0.166	0
HH	0.75	0.083	0.083	0.083

First bit --> Alice
Second bit --> Bob

In the context of the course's theory:

A_i' is H (The outcome of a rotated measurement)
A_i is D (The outcome of a direct measurement of the ith particle)

B_i' is H (The outcome of a rotated measurement)
B_i is D (The outcome of a direct measurement of the ith particle)

*Those sequences will never be available in their entirety to the experimenter: for any given i, either Ai or A'i is measured, and similarly for Bi or B'i. But local hidden variable theories **must** be **defined** in a way that the four binary sequences **exist** even in places where they have not been observed.*

l'état de Hardy n'est pas une classe d'état, c'est cet état là.

c)

First, athough those are not Bell States (maximally entangled), they are called Hardy states, which constitute a large class of entangled states that represents every entangled state that is not maximally entangled. Now the probability table is remarquable as that **obtaining such statistics is impossible with classical local hidden variable theories as it implies correlations that are much stronger than what can be explained classically**. Indeed, if we observe well, whenever both A and B make direct measurements, they never both get 0 (yellow). Also, it happens with probability 1/12 that they both get 1 when they both apply a Hadamard transform before their measurement, but they never both get 1 when one makes a direct measurement while the other applies a Hadamard transform before measurement (turquoise). Such statistics are impossible with classical local hidden variable theories as it may seem that the decision of applying a transformation on the state of an entanglement qubit has a non-local impact on the state of the other qubit that may be extremely far away. On the other hand, this match the predictions of quantum theory that can be validated experimentally.

c'est effectivement impossible de reproduire classiquement, mais vous n'en faites pas la preuve.

Question 2

5/8

Considérez l'état suivant :

$$|\Psi\rangle = \tfrac{1}{2}|00\rangle + \tfrac{\sqrt{3}}{2\sqrt{2}}|01\rangle - \tfrac{1}{2}|10\rangle + \tfrac{1}{2\sqrt{2}}|11\rangle\,.$$

Comme d'habitude, on donne le premier qubit à Alice et le second à Bob.

a) Exprimez $|\Psi\rangle$ explicitement sous la forme $|\Psi\rangle = \alpha|\Psi_0\rangle|0\rangle + \beta|\Psi_1\rangle|1\rangle$ où $|\alpha|^2 + |\beta|^2 = 1$ et $|\Psi_0\rangle$ et $|\Psi_1\rangle$ sont des états légitimes sur un qubit.

b) Donnez explicitement les probabilités p_0 et p_1 d'obtenir 0 ou 1, respectivement, si Bob mesure son qubit. **Attention**, c'est le *second* qubit qui est mesuré et non pas le premier tel que nous avions vu au cours de la semaine dernière.

c) Donnez explicitement les résidus quantiques laissés sur le qubit d'Alice suite à la mesure de Bob, selon qu'il ait obtenu 0 ou 1 à sa mesure.

d) Supposons maintenant qu'Alice mesure son qubit suite à la mesure de Bob. Calculez la probabilité qu'elle obtienne 0 et celle qu'elle obtienne 1. La réponse finale ne suffit pas : vous devez raisonner explicitement en fonction des deux possibilités du résultat classique obtenu par Bob à la mesure, de la probabilité de chacun d'entre eux, et de l'état quantique résiduel chez Alice dans chacun de ces deux cas avant qu'elle fasse sa mesure.

e) Quelle aurait été la probabilité qu'Alice obtienne 0 si elle avait mesuré son qubit en premier (voulant dire avant que Bob fasse sa mesure) ? Ici encore, la réponse finale ne suffit pas : vous devez montrer un calcul explicite, bien que très simple dans ce cas. Cette probabilité est-elle la même que celle obtenue en d) ? Si oui, est-ce rassurant et pourquoi ? Si non, qu'est-ce qui explique ce phénomène ?

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a)

|Ψ⟩ = (1/2) |00⟩ + (√3/2√2) |01⟩ - (1/2) |10⟩ + (1/2√2) |11⟩

- Second qubit measured: |Ψ⟩ = α |Ψ₀⟩ |0⟩ + β |Ψ₁⟩ |1⟩

- Instead of first qubit measured: |Ψ⟩ = α |0⟩ |Ψ₀⟩+ β |1⟩ |Ψ₁⟩

2/2

a) Soit $|\Psi\rangle = 1/2\,|00\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|01\rangle - \frac{1}{2}|10\rangle + \frac{1}{2\sqrt{2}}|11\rangle$.

Nous avons $p_0^B = (1/2)^2 + (-1/2)^2 = \frac{1}{2}$ et $p_1^B = \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{4}{8} = \frac{1}{2}$.

En posant $\begin{cases} x_0 = 1/\sqrt{p_0} \\ x_1 = 1/\sqrt{p_1} \end{cases}$ on normalise :

$$\begin{cases} \left|x_0 \times \frac{1}{2}\right|^2 + \left|x_0 \times \frac{-1}{2}\right|^2 = \left|\frac{1}{\sqrt{1/2}} \times 1/2\right|^2 + \left|\frac{1}{\sqrt{1/2}} \times \frac{-1}{2}\right|^2 = \frac{2}{4} + \frac{2}{4} = 1 \\ \left|x_1 \times \frac{\sqrt{3}}{2\sqrt{2}}\right|^2 + \left|x_1 \times \frac{1}{2\sqrt{2}}\right|^2 = \left|\frac{1}{\sqrt{1/2}} \times \frac{\sqrt{3}}{2\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{1/2}} \times \frac{1}{2\sqrt{2}}\right|^2 = \frac{3}{4} + \frac{1}{4} = 1 \end{cases}$$

On a alors :

$$|\Psi_0\rangle = \frac{1}{2} \times x_0 |0\rangle + \frac{-1}{2} \times x_0 |1\rangle = \frac{1}{2} \times \frac{1}{\sqrt{1/2}} |0\rangle + \frac{-1}{2} \times \frac{1}{\sqrt{1/2}} |1\rangle$$
$$= \frac{\sqrt{2}}{2}|0\rangle - \frac{\sqrt{2}}{2}|1\rangle$$

$$|\Psi_1\rangle = \frac{\sqrt{3}}{2\sqrt{2}} \times x_1 |0\rangle + \frac{1}{2\sqrt{2}} \times x_1 |1\rangle = \frac{\sqrt{3}}{2\sqrt{2}} \times \frac{1}{\sqrt{1/2}}|0\rangle + \frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{1/2}}$$
$$= \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle.$$

Et donc, $|\Psi\rangle = \sqrt{p_0}\,|\Psi_0\rangle|0\rangle + \sqrt{p_1}\,|\Psi_1\rangle|1\rangle$

$$= \sqrt{1/2}\left(\frac{\sqrt{2}}{2}|0\rangle - \frac{\sqrt{2}}{2}|1\rangle\right)|0\rangle + \sqrt{1/2}\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)|1\rangle$$

$$|\Psi\rangle = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}|0\rangle - \frac{\sqrt{2}}{2}|1\rangle\right)|0\rangle + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)|1\rangle$$

où $|\alpha|^2 + |\beta|^2 = |\sqrt{p_0}|^2 + |\sqrt{p_1}|^2 = 1/2 + 1/2 = 1$.

b)

- The probability p0 that Bob measurement yields 0 is the sum of the probabilities that measuring both qubits would give either 0**0** or 1**0**:

$p0 = |(1/2)|^2 + |-(1/2)|^2$ **= 0.5**

1/1

- The probability p1 that Bob measurement yields 1 is the sum of the probabilities that measuring both qubits would give either 0**1** or 1**1**:

$p1 = |(\sqrt{3}/2\sqrt{2})|^2 + |(1/2\sqrt{2})|^2$ **= 0.5**

c)

- Steps:
- Keep only classical states that have b as last qubit
- Normalize the resulting state
- Factorize according to collapsed qubits

In the case of Bob getting 0:

- His qubit collapsed to classical state $|0\rangle$
- And the qubit of Alice collapsed to $|\Psi_0\rangle$

- We need to consider only 0**0** or 1**0**
- Resulting state on both qubits after Bob's measurement is 0: $(1/2)\,|00\rangle - (1/2)\,|10\rangle$ up to renormalization
- The renormalization factor x must be such that $|x\alpha|^2 + |x\beta|^2 = 1$
- Solution is to take $x = 1/\sqrt{p0}$
- Knowing that:

$$|\Psi_0\rangle = ((1/2) / \sqrt{p0})\,|0\rangle - ((1/2)/ \sqrt{p0})\,|1\rangle$$
$$\mathbf{= (\sqrt{2}/2)\,|0\rangle - (\sqrt{2}/2)\,|1\rangle}$$

1/1

- Then: $\mathbf{= |\Psi_0\rangle \otimes |0\rangle}$
$\mathbf{= ((\sqrt{2}/2)\,|0\rangle - (\sqrt{2}/2)\,|1\rangle) \otimes |0\rangle}$

In the case of Bob getting 1:

- His qubit collapsed to classical state $|1\rangle$
- And the qubit of Alice collapsed to $|\Psi_1\rangle$

- We need to consider only 0**1** or 1**1**
- Resulting state on both qubits after Bob's measurement is 1: $(\sqrt{3}/2\sqrt{2})\,|01\rangle + (1/2\sqrt{2})\,|11\rangle$ up to renormalization
- Renormalization factor: $x = 1/\sqrt{p1}$
- Knowing that:

$$|\Psi_1\rangle = ((\sqrt{3}/2\sqrt{2})/\sqrt{p1})\,|0\rangle + ((1/2\sqrt{2})/\sqrt{p1})\,|1\rangle$$
$$\mathbf{= (\sqrt{3}/2)\,|0\rangle + (1/2)|1\rangle}$$

- Then: $\mathbf{= |\Psi_1\rangle \otimes |1\rangle}$
$\mathbf{= ((\sqrt{3}/2)\,|0\rangle + (1/2)\,|1\rangle) \otimes |1\rangle}$

d)

P (Alice measures=0) **=** [(P(Bob obtained 0) x P (Alice measures 0 | Bob obtained 0)) + P(Bob obtained 1) x P (Alice measures 0 | Bob obtained 1)]
$= [\ (0.5 \times |(\sqrt{2}/2)|^2) + \ (0.5 \times |-\ \cancel{(\sqrt{2}/2)}|^2)]$
= 0.5

0/2

P (Alice measures=1) **=** [P(Bob obtained 0) x P (Alice measures 1 | Bob obtained 0) + P(Bob obtained 1) x P (Alice measures 1 | Bob obtained 1)]
$= [\ (0.5 \times |-(\sqrt{2}/2)|^2) + \ (0.5 \times |\cancel{(\sqrt{2}/2)}|^2)]$
= 0.5

e)

- Probability of Alice getting 0 if she measured her qubit before Bob's measurement
*- We consider the sum of the probabilities for **0**0 or **0**1:*

1/2

$|\Psi\rangle = \underline{(1/2)\,|\mathbf{0}0\rangle + (\sqrt{3}/2\sqrt{2})\,|\mathbf{0}1\rangle} - (1/2)\,|10\rangle + (1/2\sqrt{2})\,|11\rangle$

P (Alice measures 0) $= |(1/2)|^2 + |(\sqrt{3}/2\sqrt{2})|^2$
= 0.625 or (5/8)

This probability **is not** the same as the one in (d) as **0.625 ≠ 0.5**. Therefore, the probability of Alice getting 0 if she measured her qubit before Bob's measurement is different than if she measured her qubit after Bob, which shows that the 2 events (Alice Measurement and Bob measurement) are not "independant" statistically. Indeed, if Alice was performing the measurement first, the probabilities will not be equal in this case and will depend on the different probability amplitudes of the various states of the multi-qubit system. On the other hand, if Alice was performing the measurement after Bob, the measure of Bob's qubit will have an influence on the wavefunction of the system, which will change the probabilities to be random for the case of Alice obtaining 0.

Si ceci était vrai, ils pourraient communiquer plus vite que la vitesse de la lumière, on aurait un GROS problème!!

Question 3

6/6

Considérez l'état intriqué

$$|\Psi\rangle = \frac{1}{2\sqrt{2}}\big(|000\rangle + |001\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle - |110\rangle - |111\rangle\big)$$

dont chacun des trois qubits est distribué à un joueur. Supposons maintenant que chaque joueur reçoive un bit classique en entrée et fasse une transformation de Hadamard H sur son qubit si et seulement si son bit d'entrée est 1.

a) Sous la promesse que le nombre de 1 en entrée est impair, calculez l'état résultant des trois qubits selon chacune des quatre possibilités. Dit autrement, calculez

$$(\mathsf{H}\otimes\mathsf{I}\otimes\mathsf{I})|\Psi\rangle \qquad (\mathsf{I}\otimes\mathsf{H}\otimes\mathsf{I})|\Psi\rangle$$
$$(\mathsf{I}\otimes\mathsf{I}\otimes\mathsf{H})|\Psi\rangle \quad \text{et} \quad (\mathsf{H}\otimes\mathsf{H}\otimes\mathsf{H})|\Psi\rangle\,,$$

où I est encore l'opération identité.

b) Supposons maintenant que les joueurs mesurent leur qubit dans la base de calcul (après avoir effectué H si et seulement si leur bit d'entrée était 1). Trouvez une relation simple entre le nombre de 1 parmi les sorties et le nombre de 1 parmi les entrées, toujours sous la promesse qu'il y a un nombre impair de 1 parmi les entrées.

c) (optionnel) Dites pourquoi cette relation est remarquable.

|Ψ⟩ = (1/2√2) (|000⟩ + |001⟩ + |010⟩ − |011⟩ + |100⟩ − |101⟩ − |110⟩ − |111⟩)

a)

- Scenario: Only 1 input bits is 1 4/4

(H ⊗ I ⊗ I)|Ψ⟩ = (1/2) (|000⟩ − |011⟩ + |101⟩ + |110⟩)

(I ⊗ H ⊗ I)|Ψ⟩ = (1/2) (|000⟩ + |011⟩ − |101⟩ + |110⟩)

(I ⊗ I ⊗ H)|Ψ⟩ = (1/2) (|000⟩ + |011⟩ + |101⟩ − |110⟩)

- Scenario: All three input bits are 1

(H ⊗ H ⊗ H)|Ψ⟩ = (1/2) (|001⟩ + |010⟩ + |100⟩ − |111⟩)

(Calculations to the right)

a) $(H\otimes I\otimes I)|\Psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} I & I \\ I & -I \end{pmatrix}\otimes I\,|\Psi\rangle$

$= \frac{1}{2\times\sqrt{2}\sqrt{2}}\begin{pmatrix} I0 & I0 \\ 0I & 0I \\ I0 & -I0 \\ 0I & 0-I \end{pmatrix}\begin{pmatrix}1\\1\\1\\-1\\1\\-1\\-1\\-1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}2\\0\\0\\-2\\0\\2\\2\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\\0\\-1\\0\\1\\1\\0\end{pmatrix} = \frac{1}{2}(|000\rangle - |011\rangle + |101\rangle + |110\rangle).$

b) $(I\otimes H\otimes I)|\Psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} H & 0 \\ 0 & H \end{pmatrix}\otimes I\,|\Psi\rangle$

$= \frac{1}{\sqrt{2}}\times\frac{1}{2\sqrt{2}}\begin{pmatrix} II & 00 \\ I-I & 00 \\ 00 & II \\ 00 & I-I \end{pmatrix}\begin{pmatrix}1\\1\\1\\-1\\1\\-1\\-1\\-1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}2\\0\\0\\2\\0\\-2\\2\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\\0\\1\\0\\-1\\1\\0\end{pmatrix} = \frac{1}{2}(|000\rangle + |011\rangle - |101\rangle + |110\rangle).$

c) $(I\otimes I\otimes H)|\Psi\rangle = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle - |110\rangle)$, par symétrie.

d) $(H\otimes H\otimes H)|\Psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} H & H \\ H & -H \end{pmatrix}\otimes H\,|\Psi\rangle$

$= \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\begin{pmatrix} HH & HH \\ H-H & H-H \\ HH & -H-H \\ H-H & -HH \end{pmatrix}\times\frac{1}{2\sqrt{2}}\begin{pmatrix}1\\1\\1\\-1\\1\\-1\\-1\\-1\end{pmatrix} = \frac{1}{4\sqrt{2}}\times\frac{1}{\sqrt{2}}\begin{pmatrix}0\\4\\4\\0\\4\\0\\0\\0\\-4\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\1\\1\\0\\1\\0\\0\\-1\end{pmatrix}$

$= \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle - |111\rangle).$

b)

As we can see from the results above, in the scenario where only 1 input bits is 1 (odd), which means that the participants are measuring their qubits after only one of them performed a H Transform (Hadamard), **an even number of 1s will results** (0 or 2 output 1s). Now, in the other case considered, where all three input bits are 1s (odd) and each participants performed a H Transform (Hadamard), we can see that their measured outputs will be an odd number of 1s (one or three 1s). Therefore, the relation is clearly visible in this context.

Number of 1s as input = 1 (odd) --> Number of outputs 1s = 0 or 2 **(even)**
Number of 1s as input = 3 (odd) --> Number of outputs 1s = 1 or 3 **(odd)**

2/2

c)

This relation is remarquable as it can be referred to the concept of "Pseudo-telepathy", as it allows seemingly impossible correlations to happens classically without some sort of communication between parties. A good definition of Pseudo-telepathy is given by [1]: *"We say that a bipartite game G exhibits pseudo-telepathy if bipartite measurements of an entangled quantum state can yield a winning strategy, whereas no classical strategy that does not involve communication is a winning strategy."*

This number presents "exactly" a game of this type, ~~often known as the Magic Square game~~. Now looking through a classical lens, it is possible to show that a table which allows a classical winning strategy is impossible. On the other hand, looking through a quantum mechanical lens, it is possible to show that correlations of a shared quantum state can give the required bits of information that allows a winning strategy without requiring instantaneous communication.

C'est effectivement de la pseudotélépathie! Mais ce n'est pas le magic square game et vous n'en faites pas la preuve!

[1] Methot, Andre Allan. *On local-hidden-variable no-go theorems*. DIRO. Université de Montréal. July 30, 2018. *https://arxiv.org/pdf/quant-ph/0507149.pdf*

[2] Berthelette, Sophie. *Complexité de Kolmogorov et corrélations quantiques ; étude du carré magique.* Université de Montréal. 2019. ";)"
https://papyrus.bib.umontreal.ca/xmlui/bitstream/handle/1866/23810/Berthelette_Sophie_2019_memoire.pdf?sequence=2&isAllowed=y