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## 1 基本

### 1.1 型態大小

- int:
  - -2,147,483,648 to 2,147,483,647 (10 digits)
  - $-2^{31}$  to  $2^{31} - 1$
  - $-10^9$  to  $10^9$
- unsigned long long int:
  - Begins with 9, and has a total of 19 digits
  - $2^{63} - 1$
  - $10^{18}$
- array:
  - Do not declare with a size larger than 30,005.

## 2 語法

### 2.1 c++

```
1 #include<bits/stdc++.h>
2 using namespace std;
3 typedef unsigned long long int ll;
4 int main(){
5     std::ios::sync_with_stdio(false);
6     return 0;
7 }
```

## 2.2 c++ 函式庫

```
1 // <string>
2
3 // 查找substr第一次出現的位置
4 str.find(substr);
5 // 返回substr最后一次出現的位置
6 str.rfind(substr);
7
8 s1 = s1+s2 // 連接兩個字串
9 s1 + s2 // 跟上面一樣
10 s1.append(s2) // s2插在s1的屁股
11 if(s1 == s2) // 比較兩字串
12
13 // <ctype>
14
15 // 檢查系列
16 isalpha(c) // 字母
17 isdigit(c) // 數字
18 isalnum(c) // 字母or數字
19 isspace(c) // 空格or換行
20 isupper(c) // 大寫
21 islower(c) // 小寫
22 ispunct(c) // 標點符號
23 toupper(c) // to大寫
24 tolower(c) // to小寫
25
26
27 // <algorithm>
28
29 // 酷東西
30 reverse(v, v+n)
31 find(v, v+n, 3) //查找3是否在v中
32 count(v, v+n, 3) // 算3在v裡出現幾次(只能算字元or數字)
33
34 // sort
35 sort(v.begin(), v.end())
36 sort(v, v+n)
37 sort(v, v+n, greater<int>())
38
39 sort(v, v+n, cmp)
40 bool cmp(型態 a, 型態 b){
41     return a > b; // 大到小
42 }
43
44 // <numeric>
45
46 // 返回鄰近數值的差
47 int arr[10]={1,2,3,4,5,6,7,8,9,10};
48 int a[9] = {0};
49 adjacent_difference(arr, arr+10, a);
50 for(int i= 0; i < 9; i++){
51     cout<<a[i]<<' ';
52 }
53
54 // <cmath>
55 round(x) // 返回最接近x的整數
```

### 2.3 宣告法

```
1 // <vector>
2 vector<int> v;
3 vector<int> v = {1,2,3,4};
4 vector<int> v(5); // v={0,0,0,0,0}
5 vector<int> v(5,1); // v={1,1,1,1,1}
6 vector<vector<int>> v; //二維
7 // v[2][3] v[樓][層]
8 vector<vector<int>> v(2, vector<int>(3));
9 // v = {(1,1),(1,1),(1,1)}
10 vector<vector<int>> v(2, vector<int>(3, 1));
11
12 v.push_back(1) // 推入數字
```

```

13 v.pop_back() // 拔出尾端數字
14
15 // 在二維陣列中插入元素
16 vector<vector<int>> > arr(5, vector<int>(3, 1));
17 arr[1].push_back(2);
18 for(size_t i= 0; i < arr.size();i++){
19     for(size_t j= 0; j < arr[i].size();j++){
20         cout<<arr[i][j]<< ' ';
21     }
22     cout<<endl;
23 }
24 /*
25 Output
26 1 1 1
27 1 1 1 2
28 1 1 1
29 1 1 1
30 1 1 1
31 */
32
33 // struct
34 struct s{
35     int x[100];
36     int y[100];
37 };
38 s num; //一組s
39 num.x[1]=1; num.y[1]=2;
40
41 // set
42 set<int> s;
43
44 s.insert(x)
45 s.count(x) // x是否存在於set中
46 s.erase(x)
47
48 s.clear()
49 s.empty()
50 s.size()
51
52 // stack
53 stack<int> s;
54
55 s.push(1); // 把1推到尾巴
56 s.pop(); // 刪掉尾巴
57 s.top(); // 返回尾巴
58
59 // queue
60 queue<int> q;
61 q.pop(); // 刪掉前
62 q.front(); // 返回前
63 q.back(); // 返回尾
64 q.push(1); // 把1推到前

```

## 2.4 強制轉型

```

1 // 數字 to 字元(串)
2 str = to_string(num)
3 c = num + '0';
4
5 // 字符串流轉型法
6 stringstream ss;
7 ss << num; // 把num塞進字符串流
8 string str;
9 ss >> str; // 把字符串留丟進str
10
11 // 字串 to 數字
12 int num = stoi(str) //整數
13 double num = stod(str) //小數

```

## 2.5 python

```

1 # EOF
2 while True:
3     try:
4         '''
5             你要執行的程式碼
6             '''
7         except EOFError:
8             break
9
10 # 有規定終止條件
11 while True:
12     if a==0:
13         break
14
15 # 數學符號
16 a//=10 # 整除
17 a**b # a^b
18
19 # 邏輯
20 a=True
21 b=False
22 print(a and b) #False
23 print(a or b) #True
24
25 # scan
26 a = int(input())
27 n=list(input().split(' ')) #
28     連續輸入一串用空格隔開的數字
29
30 for i in range(a):
31     c, d = map(int, input().split()) # 連續輸入兩個數
32
33 # print
34 print('for is not a multiple of 11.'.format(a))
35 print(a+" and "+b+" sitting in the tree")
36 print('The parity of ',a,' is ',count,' (mod 2).')
37
38 # 標頭檔math
39 import math
40 math.gcd(a, b, c, d, e) # 最大公約數
41 math.lcm(a, b, c, d, e) # 最小公倍數
42 math.fabs(x) # 絕對值
43 math.isqrt(n) # 整數平方根
44 math.sqrt(x) # 平方根
45 math.pow(x, y) # x^y
46
47 # count
48 c+=b.count("商店") # 用在要計算好幾個字串時
49 c=b.count('1') # 一次算出一串字串有幾個 '1'
50
51 # 進制轉換
52 a = bin(a)[2:] # 10 to 2
53 a = hex(a)[2:] # 10 to 16
54 a = oct(a)[2:] # 10 to 8
55
56 # 大小寫轉換
57 a.lower()
58 a.upper()
59
60 # 取長度
61 a.len()

```

## 3 字串

### 3.1 KMP

```

1 #include <iostream>
2 using namespace std;
3
4 void KMP(string text, string pattern)
5 {
6     int m = text.length();

```

```

7   int n = pattern.length();
8
9   // 如果模組沒東東
10  if (n == 0)
11  {
12      cout << "The pattern occurs with shift 0";
13      return;
14  }
15
16  // 如果文本的長度小於模組的長度
17  if (m < n)
18  {
19      cout << "Pattern not found";
20      return;
21  }
22
23  // next[i] 存儲下一個最佳部分匹配的索引
24  int next[n + 1];
25
26  for (int i = 0; i < n + 1; i++) {
27      next[i] = 0;
28  }
29
30  for (int i = 1; i < n; i++)
31  {
32      int j = next[i];
33
34      while (j > 0 && pattern[j] != pattern[i]) {
35          j = next[j];
36      }
37
38      if (j > 0 || pattern[j] == pattern[i]) {
39          next[i + 1] = j + 1;
40      }
41  }
42
43  for (int i = 0, j = 0; i < m; i++)
44  {
45      if (text[i] ==
46          pattern[j])//一樣如果+1j下一個檢查
47      {
48          if (++j == n) {
49              cout << "The pattern occurs with
50                  shift " << i - j + 1 << endl;
51          }
52          else if (j > 0)
53          {
54              j = next[j];//把她休崩變回來
55              i--;      // 還要回去啾啾
56          }
57      }
58  }
59  int main()
60  {
61      string text = "ABCABAABCABAC";
62      string pattern = "CAB";
63
64      KMP(text, pattern);
65
66      return 0;
67  }

```

## 4 數論

### 4.1 快速幕

```

1  long long binpow(long long a, long long b){
2      if(b==0) return 1;
3      int res = binpow(a, b/2);
4      if(b%2==0) return res*res;
5      else return res*res*a;
6  }

```

### 4.2 窮舉 (選 or 不選)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3
4  int k, all = 0;
5  int Min = 9999999;
6  int arr[100] = {0};
7
8  void Find(int sum, int now){
9      if(now == k) return;
10     Min = min(abs(all-sum-sum), Min);
11
12     Find(sum, now+1);
13     Find(sum+arr[now], now+1);
14     return;
15 }
16 int main(){
17     cin >> k;
18     for(int i = 0; i < k; i++){
19         cin >> arr[i];
20         all+=arr[i];
21     }
22     Find(0, 0);
23     cout << Min;
24 }

```

### 4.3 喵

```

1  #include <iostream>
2  using namespace std;
3  int a[20];
4
5  int main() {
6      int cases, target, sticks, num, tmp, i;
7      bool flag;
8      while (cin >> cases){
9          while (cases--){
10             cin >> target;
11             cin >> sticks;
12             for (int i = 0; i < sticks; i++){
13                 cin >> a[i];
14             }
15             num = 1;
16             for (int i = 1; i < sticks; i++){
17                 num <= 1;
18                 num++;
19             }
20             flag = false;
21             for (int _i = 0; _i <= num; _i++){
22                 tmp = 0;
23                 i = _i;
24                 for (int j = 0; j < sticks; j++){
25                     if (i & 1) tmp += a[j];
26                     i >>= 1;
27                 }
28                 if (tmp == target){
29                     flag = true;
30                     break;
31                 }
32             }
33             if (flag) cout << "YES\n";
34             else cout << "NO\n";
35         }
36     }
37 }

```

### 4.4 小費馬實踐

```

1  ll s(ll b, ll p, ll m){
2      if(p == 0) return 1;
3      else if(p%2!=0) return ((b%m)*(s(b, p-1, m)%m))%m;
4      else{

```

```

5 |     ll t = s(b, p/2, m);
6 |     return ((t%m)*(t%m))%m;
7 | }
8 | }

```

## 4.5 Fibonaccimal

```

1 | #include<bits/stdc++.h>
2 | using namespace std;
3 |
4 | int main(){
5 |
6 |     int N;
7 |     int Fibonacci[40] = {0, 1}; //開始的兩個數
8 |     int i;
9 |
10 |    for(i = 2; i < 40; i++){
11 |        Fibonacci[i] = Fibonacci[i - 1] + Fibonacci[i
        - 2];
12 |    }
13 |
14 |    scanf("%d", &N);
15 |
16 |    while(N--){
17 |
18 |        int num;
19 |        bool flag = false;
20 |
21 |        scanf("%d", &num);
22 |        printf("%d = ", num);
23 |
24 |        for(i = 39; i >= 2; i--){
25 |            if(num >= Fibonacci[i]){
26 |                num = num - Fibonacci[i];
27 |                flag = true;
28 |                printf("1");
29 |            }
30 |            else if(flag){
31 |                printf("0");
32 |            }
33 |        }
34 |
35 |        printf(" (fib)\n");
36 |    }
37 |
38 |    return 0;
39 | }

```

## 4.6 LCM

```

1 | int GCD(int num1, int num2)
2 | {
3 |     if(num2==0)
4 |     {
5 |         return num1;
6 |     }
7 |
8 |     return GCD(num2, num1%num2);
9 | }
10 |
11 | int LCM(int num1, int num2) //2個最小公倍數
12 | {
13 |     return((num1*num2)/GCD(num1, num2));
14 | }
15 |
16 | int LCM2(int num1, int num2, int num3) //3個最小公倍數
17 | {
18 |     return((num1*num2)/GCD((num1, num2), num3));
19 | }
20 |
21 | int main()
22 | {
23 |     cout<<GCD(6,3);

```

```

24 |     cout<<LCM(6,3);
25 |     cout<<LCM2(6,3,3);
26 |
27 |     return 0;
28 | }

```

## 4.7 LCS

```

1 | #include<bits/stdc++.h>
2 | using namespace std;
3 | int main()
4 | {
5 |     string str1, str2;
6 |     short lcs[2][1000];
7 |     while (cin >> str1 >> str2)
8 |     {
9 |         lcs[0][0] = str1[0] == str2[0];
10 |        for (int j=1; j<str2.length(); j++) lcs[0][j]
            = max(lcs[0][j-1],
            short{str1[0]==str2[j]});
11 |        for (int i=1; i<str1.length(); i++)
12 |        {
13 |            bool r = i & 1;
14 |            lcs[r][0] = max(lcs[r^1][0],
            short{str1[i]==str2[0]});
15 |            for (int j=1; j<str2.length(); j++)
16 |            lcs[r][j] = str1[i]==str2[j] ?
            lcs[r^1][j-1] + 1 :
            max(lcs[r^1][j], lcs[r][j-1]);
17 |        }
18 |        cout <<
            lcs[(str1.length()-1)&1][str2.length()-1]
            << '\n';
19 |    }
20 |    return 0;
21 | }

```

## 4.8 LPS

```

1 | #include<bits/stdc++.h>
2 | #include<iostream>
3 | using namespace std;
4 |
5 | int main(){
6 |     string s;
7 |     cin >> s;
8 |     int maxlen=0, l, r;
9 |     for(int i=0; i<s.length(); i++){
10 |        //奇
11 |        int x = 0;
12 |        while((s[i-x]==s[i+x]) && (i-x>=0) &&
            (i+x<s.length())){ //當找到一個中心點以其為中間然後左
            x++;
13 |        }
14 |        x--;
15 |        if(2*x+1 >
            maxlen){ //只有第一次會max==後面就追不到那女孩
            maxlen = 2*x+1; //最大的
16 |            l = i-x; //計算頭頭
            r = i+x; //計算尾巴
17 |        }
18 |        //偶
19 |        x = 0;
20 |        while( (s[i-x]==s[i+1+x]) && (i-x>=0) &&
            (i+1+x<s.length())) {
            x++;
21 |        }
22 |        if(2*x > maxlen){
23 |            maxlen = 2*x;
24 |            l = i-x+1;
25 |            r = i+x;
26 |        }
27 |    }
28 | }
29 |
30 | }
31 | }

```

```

32     cout << maxlen << '\n';
33     cout << l+1 << ' ' << r+1 << '\n';
34 }
35 }

```

## 4.9 Pairty

```

1 #include <iostream>
2 #include <algorithm>
3 using namespace std;
4
5 int main() {
6     int I, n;
7     while (cin >> I) {
8         if (I == 0) break;
9         string B = "";
10        n = I;
11        int cnt = 0;
12        while (n){
13            cnt += (n & 1);
14            B += '0' + (n & 1);
15            n >>= 1;
16        }
17        reverse(B.begin(), B.end());
18        cout << "The parity of " << B << " is " <<
19            cnt << " (mod 2).\n";
20    }
21    return 0;
22 }

```

## 5 圖論

### 5.1 最短路徑 dijkstra

```

1 // 邊權重皆為正數時使用
2 // 1.
3 // 輸入有總點、總邊，接著輸入點,點,距離(權重)時使用
4 #include <iostream>
5 #include <vector>
6 #include <limits>
7 using namespace std;
8
9 // 定義城市數量的上限
10 #define MAX_CITIES 100
11
12 // 定義無限大的距離
13 #define INF INT_MAX
14
15 // 城市數量、道路數量
16 int numCities, numRoads;
17
18 // 圖的鄰接矩陣表示法
19 vector<vector<int>> graph(MAX_CITIES,
20     vector<int>(MAX_CITIES, INF));
21 //
22 // Dijkstra演算法，計算從指定城市出發到其他城市的最短路徑
23 void dijkstra(int startCity) {
24     vector<int> dist(numCities, INF);
25     vector<bool> visited(numCities, false);
26
27     dist[startCity] = 0;
28
29     for (int i = 0; i < numCities - 1; i++) {
30         int u = -1;
31         for (int j = 0; j < numCities; j++) {
32             if (!visited[j] && (u == -1 || dist[j] <
33                 dist[u])) {
34                 u = j;
35             }
36         }
37     }
38 }

```

```

34     }
35
36     visited[u] = true;
37
38     for (int v = 0; v < numCities; v++) {
39         if (!visited[v] && graph[u][v] != INF) {
40             dist[v] = min(dist[v], dist[u] +
41                 graph[u][v]);
42         }
43     }
44 }
45
46 // 輸出最短路徑結果
47 cout << "從城市 " << startCity << "
48     出發到其他城市的最短路徑如下：" << endl;
49 for (int i = 0; i < numCities; i++) {
50     if (i != startCity) {
51         cout << "到城市 " << i << " 的最短距離為
52             " << dist[i] << endl;
53     }
54 }
55
56 int main() {
57     // 讀取城市數量和道路數量
58     cin >> numCities >> numRoads;
59
60     // 初始化圖的鄰接矩陣
61     for (int i = 0; i < numRoads; i++) {
62         int city1, city2, distance;
63         cin >> city1 >> city2 >> distance;
64         graph[city1][city2] = distance;
65         graph[city2][city1] = distance; //
66         因為是雙向道路
67     }
68
69     // 選擇起始城市，這裡以城市0為例
70     int startCity = 0;
71
72     // 執行Dijkstra演算法
73     dijkstra(startCity);
74
75     return 0;
76 }

```

### 5.2 DFS

```

1 // 印出最快路徑(座標)
2 #include <bits/stdc++.h>
3 #define N 100
4 using namespace std;
5
6 int map[N][N], visited[N][N]={0};
7 typedef pair<int, int> p;
8 int n,m,found=0;
9 deque<p> path;
10
11 void dfs(int x, int y){
12     if (found==1) return;
13     visited[x][y]=1;
14     path.push_back(make_pair(x,y));
15     if (x==n-1 && y==m-1){
16         found=1;
17         cout<<"Path: ";
18         while(!path.empty()){
19             cout<<"(" << path.front().first << ", " << path.front().second <<
20                 " ";
21             path.pop_front();
22         }
23         cout<<endl;
24         return;
25     }
26     if (x+1<n && visited[x+1][y]==0 && map[x+1][y]==0){
27         dfs(x+1,y);
28     }
29 }

```

```

28     path.pop_back();
29 }
30 if (y+1<m && visited[x][y+1]==0 && map[x][y+1]==0){
31     dfs(x,y+1);
32     path.pop_back();
33 }
34 if (x-1>=0 && visited[x-1][y]==0 && map[x-1][y]==0){
35     dfs(x-1,y);
36     path.pop_back();
37 }
38 if (y-1>=0 && visited[x][y-1]==0 && map[x][y-1]==0){
39     dfs(x,y-1);
40     path.pop_back();
41 }
42 }
43
44 int main(){
45     cin>>n>>m;
46     for (int i=0; i<n; i++)
47         for (int j=0; j<m; j++)
48             cin>>map[i][j];
49     dfs(0,0);
50     if (found==0){
51         cout<<"No routes accessible.\n";
52     }
53     return 0;
54 }
55 // 顯示最短距離
56 #include <iostream>
57 #include <utility>
58 #include <deque>
59 #define N 100
60 using namespace std;
61
62 int map[N][N], visited[N][N]={0};
63 typedef pair<int, int> p;
64 int n,m,dis=-2;
65 deque<p> path;
66
67 void dfs(int x, int y){
68     visited[x][y]=1;
69     path.push_back(make_pair(x,y));
70     if (x==n-1 && y==m-1){
71         if (dis==-1){
72             dis=path.size()-1;
73         }
74         else {
75             if (path.size()-1<dis) dis=path.size()-1;
76         }
77     }
78     if (x+1<n && visited[x+1][y]==0 && map[x+1][y]==0){
79         dfs(x+1,y);
80         visited[x+1][y]=0;
81         path.pop_back();
82     }
83     if (y+1<m && visited[x][y+1]==0 && map[x][y+1]==0){
84         dfs(x,y+1);
85         visited[x][y+1]=0;
86         path.pop_back();
87     }
88     if (x-1>=0 && visited[x-1][y]==0 && map[x-1][y]==0){
89         dfs(x-1,y);
90         visited[x-1][y]=0;
91         path.pop_back();
92     }
93     if (y-1>=0 && visited[x][y-1]==0 && map[x][y-1]==0){
94         dfs(x,y-1);
95         visited[x][y-1]=0;
96         path.pop_back();
97     }
98 }
99
100 int main(){
101     cin>>n>>m;
102     for (int i=0; i<n; i++)
103         for (int j=0; j<m; j++)
104             cin>>map[i][j];

```

```

105     dfs(0,0);
106     if (dis==-2)
107         cout<<"No routes accessible.\n";
108     else
109         cout<<"Shortest distance: "<<dis<<endl;
110     return 0;
111 }

```

### 5.3 merge sort

```

1 #include <iostream>
2 using namespace std;
3
4 //做比較大小的部分
5 void merge(int arr[], int l, int m, int r, int size)
6 {
7     int i = l;
8     int j = m + 1;
9     int k = l;
10
11     /* create temp array */
12     int temp[size];
13
14     while (i <= m && j <= r) {
15         if (arr[i] <= arr[j]) {
16             temp[k] = arr[i];
17             i++;
18             k++;
19         }
20         else {
21             temp[k] = arr[j];
22             j++;
23             k++;
24         }
25     }
26     / Copy the remaining elements of first half, if
       there are any /
27     while (i <= m) {
28         temp[k] = arr[i];
29         i++;
30         k++;
31     }
32     / Copy the remaining elements of second half, if
       there are any /
33     while (j <= r) {
34         temp[k] = arr[j];
35         j++;
36         k++;
37     }
38
39     / Copy the temp array to original array /
40     for (int p = l; p <= r; p++) {
41         arr[p] = temp[p];
42     }
43 }
44
45 //做分開陣列的部分
46 void mergeSort(int arr[], int l, int r, int size)
47 {
48     if (l < r) {
49         // 找中間點 ex:陣列五個元素0-4 2是中間點
50         //陣列分成兩組 0-2/3-4兩個部分
51         //舉0-2陣列來說 中間點是1
52         //陣列再分成 0-1/2兩個部分
53         int m = (l + r) / 2;
54
55         / 遞迴第一和第二部分*/
56         //(也就是不斷的分)
57         mergeSort(arr, l, m, size);
58         mergeSort(arr, m + 1, r, size);
59
60         // merge
61         //當我分到不能再分 比較陣列內數值 小的放前面
62         merge(arr, l, m, r, size);
63     }

```

```

64     }
65 }
66
67 int main()
68 {
69     cout << "Enter size of array: " << endl;
70     int size;
71     cin >> size;
72     int myarray[size];
73
74     cout << "Enter " << size << " integers in any
75         order: " << endl;
76     for (int i = 0; i < size; i++) {
77         cin >> myarray[i];
78     }
79     cout << "Before Sorting" << endl;
80     for (int i = 0; i < size; i++) {
81         cout << myarray[i] << " ";
82     }
83     cout << endl;
84     mergeSort(myarray, 0, (size - 1), size); //
85         mergesort(arr, left, right) called
86
87     cout << "After Sorting" << endl;
88     for (int i = 0; i < size; i++) {
89         cout << myarray[i] << " ";
90     }
91     return 0;

```

## 5.4 quick sort

```

1  include <iostream>
2  using namespace std;
3  // quick sort sorting algorithm
4  int Partition(int arr[], int s, int e)
5  {
6      int pivot = arr[e];
7      int pIndex = s;
8
9      for(int i = s; i < e; i++)
10     {
11         if(arr[i] < pivot)
12         {
13             int temp = arr[i];
14             arr[i] = arr[pIndex];
15             arr[pIndex] = temp;
16             //swapping 也就是說如果當前數值比指標小
17             //他就移到最前面
18             //也就是陣列0的位置
19             pIndex++;
20             //下一個比指標小的數值放進陣列i的位置
21         }
22     }
23
24     int temp = arr[e];
25     arr[e] = arr[pIndex];
26     arr[pIndex] = temp;
27     //比指標數值小的都去前面了
28     //將指標放到目前計數到的陣列位置
29     //那指標前都比她小 指標後都比他大
30     return pIndex; //回傳給p值
31 }
32
33 void QuickSort(int arr[], int s, int e)
34 //s stand for start index
35 //e stand for end index also (size-1)
36 {
37     if(s < e)
38     {
39         int p = Partition(arr, s, e);
40         QuickSort(arr, s, (p-1));
41         // recursive QS call for left partition
42         QuickSort(arr, (p+1), e);
43         // recursive QS call for right partition
44     }
45 }

```

```

42 QuickSort(arr, (p+1), e);
43 // recursive QS call for right partition
44 }
45 }
46
47 int main()
48 {
49     int size=0;
50     cout<<"Enter Size of array: "<<endl;
51     cin>>size;
52     int myarray[size];
53
54     cout<<"Enter "<<size<<" integers in any order:
55         "<<endl;
56     for(int i=0;i<size;i++)
57     {
58         cin>>myarray[i];
59     }
60     cout<<"Before Sorting"<<endl;
61     for(int i=0;i<size;i++)
62     {
63         cout<<myarray[i]<<" ";
64     }
65     cout<<endl;
66
67     QuickSort(myarray,0,(size-1)); // quick sort called
68
69     cout<<"After Sorting"<<endl;
70     for(int i=0;i<size;i++)
71     {
72         cout<<myarray[i]<<" ";
73     }
74
75     return 0;
76 }

```

## 5.5 二分搜

```

1  #include <iostream>
2  using namespace std;
3
4  int binarySearch(int arr[], int left, int right, int
5      x) {
6      while (left <= right) {
7          int mid = left + (right - left) / 2;
8
9          if (arr[mid] == x) {
10             return mid;
11         }
12         else if (arr[mid] < x) {
13             left = mid + 1;
14         }
15         else {
16             right = mid - 1;
17         }
18     }
19     return -1;
20 }
21
22 int main() {
23     int myarr[10];
24     int num;
25     int output;
26
27     cout << "Please enter 10 elements ASCENDING order"
28         << endl;
29     for (int i = 0; i < 10; i++) {
30         cin >> myarr[i];
31     }
32     cout << "Please enter an element to search" << endl;
33     cin >> num;
34
35     output = binarySearch(myarr, 0, 9, num);

```

```

36 if (output == -1) {
37     cout << "No Match Found" << endl;
38 } else {
39     cout << "Match found at position: " << output <<
        endl;
40 }
41
42 return 0;
43 }

```

如果我們超過25頁我還可以再縮減程式區 這是比較完整的 Floyd

```

44 void floyd(){
45     for(int k=0; k<n; k++){ //中間點
46         for(int i=0; i<n; i++){
47             for(int j=0; j<n; j++){
48                 dp[i][j]=min(dp[i][j], dp[i][k]+dp[k][j]);
49                 //經過中間點k的路徑是否小於原始路徑
50                 //小於則更新 不小於則不變動
51                 //窮舉所有鬆弛的可能
52             }
53         }
54     }
55 }
56
57 }

```

## 6 dp

### 6.1 階乘 1

```

1  '''
2  !注意! long long 存到21!就會爆掉
3
4  1. 要你輸出階乘
5  好懶，請你直接用python
6  '''
7  a = 0
8  while True:
9      try:
10         a = int(input())
11         sum = 1
12         for i in range(1,a+1):
13             sum*=i
14         a = str(a)
15         print(a+'!')
16         print(sum)
17     except EOFError:
18         break

```

### 6.2 階乘 2

```

1  /*
2  2. 要你輸出階乘最後一個非0的數字
3  用dp表格先存1-10000數字的階乘，
4  同時因為我們只關心最後一個非0的數字，
5  所以可以每次乘完一階就讓他進while迴圈裡%10，
6  把0都去掉，到while迴圈外後再把arr[i]%=10000，
7  只留下剩下可能會影響結果的數值部分。
8  */
9  typedef long long ll;
10 ll arr[10000];
11 void s(){
12     arr[0]=1;
13     for(ll i = 1; i <= 10000; i++){
14         arr[i] = arr[i-1]*(i+1);
15         while (arr[i] % 10 == 0) {
16             arr[i] /= 10;
17         }
18         arr[i] %= 1000000;
19     }
20 }

```

## 6.3 階梯問題

```

1  /*
2  1. 問從左上角走到右下角有幾種解法
3  - 此問題可分為(1)往下(2)往右，兩個走法。
4  */
5  const int H = 8, W = 8;
6  int f[2][W]; //
    兩條陣列，儲存最近算出來的問題答案。
7
8  void staircase_walk()
9  {
10     // [Initial States]
11     for (int j=0; j<W; ++j) f[0][j] = 1;
12
13     // [Computation]
14     for (int i=1; i<H; i++)
15         for (int j=1; j<W; j++)
16             // 只是多了 mod 2，
17             // 外觀看起來就像兩條陣列輪替使用。
18             f[i % 2][j] = f[(i-1) % 2][j] + f[i %
                2][j-1];
19
20     // 輸出結果
21     cout << "由(0,0)走到(7,7)有 " << f[7 % 2][7] <<
        "種走法";
22     // cout << "由(0,0)走到(7,7)有 " << f[(H-1) %
        2][W-1] << "種走法";
23 }

```

## 6.4 極值問題 (格子有權重)

```

1  const int H = 8, W = 8;
2  int a[H][W];
3  int f[H][W];
4
5  void staircase_walk()
6  {
7      // [Initial States]
8      f[0][0] = a[0][0];
9      for (int i=1; i<H; i++)
10         f[i][0] = f[i-1][0] + a[i][0];
11      for (int j=1; j<W; j++)
12         f[0][j] = f[0][j-1] + a[0][j];
13
14      // [Computation]
15      for (int i=1; i<H; i++)
16         for (int j=1; j<W; j++)
17             f[i][j] = max(f[i-1][j], f[i][j-1]) + a[i][j];
18
19      // 輸出結果
20      cout << "由(0,0)走到(7,7)的最小總和 " << f[7][7];
21      // cout << "由(0,0)走到(7,7)的最小總和 " <<
        f[H-1][W-1];
22
23      int h, w;
24      while (cin >> h >> w)
25         cout << "由(0,0)走到(h,w)的最小總和 " << f[h][w];
26 }

```

## 7 數學

### 7.1 理論

#### • 三角形邊長定理

- $a + b > c$
- 三角形形狀判定：
- 直角  $a^2 + b^2 = c^2$
- 銳角  $a^2 + b^2 > c^2$
- 鈍角  $a^2 + b^2 < c^2$



## 7.2 公式

### • 積

$$\begin{aligned}
 - \sum_{i=1}^n i &= \frac{n(n+1)}{2} \\
 - \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} \\
 - \sum_{i=1}^n i^3 &= \frac{(n^2(n+1)^2)}{4} \\
 - \sum_{i=1}^n i^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \\
 - \sum_{i=1}^n i^5 &= \frac{n^2(n+1)^2(2n^2+2n-1)}{12} \\
 - \sum_{i=1}^n i^6 &= \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42} \\
 - \sum_{k=1}^n (k-1)(k-1)! &= n! - 1
 \end{aligned}$$

### • log

$$\begin{aligned}
 - \log \frac{a}{b} &= \log a - \log b \\
 - \log_a b &= \frac{\log a}{\log b} \\
 - \log ab &= \log a + \log b \\
 - \log_a 1 &= 0
 \end{aligned}$$

### • 三角形面積

$$- \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2}$$

### • 圓形面積

$$\begin{aligned}
 - \text{面積：} \\
 - \pi * r^2 \\
 - \text{周長：} \\
 - 2\pi * r
 \end{aligned}$$

### • 圓形方程

$$\begin{aligned}
 - (x-h)^2 + (y-k)^2 &= r^2, \quad (h, k) = \text{point} \\
 - x^2 + y^2 + Dx + Ey + F &= 0 \\
 - \text{point} &= \left(-\frac{D}{2}, -\frac{E}{2}\right) \\
 - r &= \sqrt{\left(\frac{D^2}{4} + \frac{E^2}{4} - F\right)}
 \end{aligned}$$

### • 座標幾何

$$\begin{aligned}
 - \text{兩點距離：} \\
 - \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\
 - \text{斜率：} \\
 - \frac{y_2 - y_1}{x_2 - x_1} \\
 - \text{中點：} \\
 - \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\
 - \text{平行：} \\
 - m_1 = m_2 \\
 - \text{垂直：} \\
 - m_1 * m_2 = -1
 \end{aligned}$$

### • 二元方程式

$$\begin{aligned}
 - x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 - \text{判別式：} \\
 - x &= b^2 - 4ac \\
 - x > 0: \text{相異根}, x = 0: \text{重根}, x < 0: \text{無實根} \\
 - \text{根和：} \\
 - r_1 + r_2 &= -\frac{b}{a} \\
 - \text{根積：} \\
 - r_1 * r_2 &= \frac{c}{a} \\
 - \text{頂點：} \\
 - \left(-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a}\right)
 \end{aligned}$$

### • 等差級數

$$\begin{aligned}
 - S_n &= \frac{(a_1 + a_n)n}{2} = a_1 n + \frac{(n-1)nd}{2} \\
 - a_n &= a_1 + (n-1)d \\
 - \sum_{i=1}^n a_n &= \frac{(2a_1 + (n-1)d)n}{2}
 \end{aligned}$$

### • 等比級數

$$\begin{aligned}
 - S_n &= \frac{a_1(1-r^n)}{1-r} = \frac{a_1(r^n-1)}{r-1}, (r \neq 1) \\
 - \frac{a_n}{a_{n-1}} &= r
 \end{aligned}$$

### • 恆等式

$$\begin{aligned}
 - a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\
 - a^3 - b^3 &= (a-b)(a^2 + ab + b^2) \\
 - (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 - (a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
 - (a+b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \\
 - 1 \\
 - 1 \quad 1 \\
 - 1 \quad 2 \quad 1 \\
 - 1 \quad 3 \quad 3 \quad 1 \\
 - 1 \quad 4 \quad 6 \quad 4 \quad 1 \\
 - 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
 - 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
 - 1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
 - 1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\
 - 1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1
 \end{aligned}$$

### • 離散

$$\begin{aligned}
 - (1+x)^n &= \binom{n}{0} + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n \\
 - \frac{1}{(1+x)^n} &= \sum_{r=0}^{\infty} \binom{n+r-1}{r}(-1)^r(x)^r \\
 - \frac{1}{(1+x)^n} &= \sum_{r=0}^{\infty} \binom{n+r-1}{r}(x)^r \\
 - \frac{1}{1-x} &= 1 + x + x^2 + x^3 + \dots \\
 - \frac{1}{(1-x)^2} &= 1 + 2x + 3x^2 + 4x^3 + \dots + nx^{n-1}
 \end{aligned}$$

### • 線代

$$\begin{aligned}
 - A_x &= [a_1 a_2 \dots a_n][\begin{pmatrix} x_1 \\ x_n \end{pmatrix}] = x_1 * a_1 + x_2 * a_2 + \dots + x_n * a_n \\
 - u &= [], v = [], A(u+v) = A_u + A_v
 \end{aligned}$$

### • 指數

$$\begin{aligned}
 - a^n a^m &= a^{n+m} \\
 - \frac{a^n}{a^m} &= a^{n-m} \\
 - (a^n)^m &= a^{n*m}
 \end{aligned}$$

### • 小費馬

$$\begin{aligned}
 - (a+b)\%n &= (a\%n + b\%n)\%n \\
 - (a*b)\%n &= (a\%n * b\%n)\%n \\
 - a * (b\%m) &= (a * b)\%m
 \end{aligned}$$