

Introduction to logistic Regression

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Logistic Regression is a type of regression that is used when the output is binary, that is Yes/No

probabilities > 0.5 result is likely to 1
" ≤ 0.5 " " " " 0

logistic regression aims to do the following

① independent variable ② selected observation ③ set of given independent variable

→ will model the probabilities of a particular event happening or not.

⇒ independent variable can be categorical or non categorical

Logistic Regression Equation

$$z = b + w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$

where x_1, x_2, x_3, \dots are independent variables

logistic Regression is defined by a sigmoid function

output

$$y = \frac{1}{(1 + e^{-z})}$$

$$1 + e^{-z} = \frac{1}{y}$$

$$e^{-z} = \frac{1}{y} - 1$$

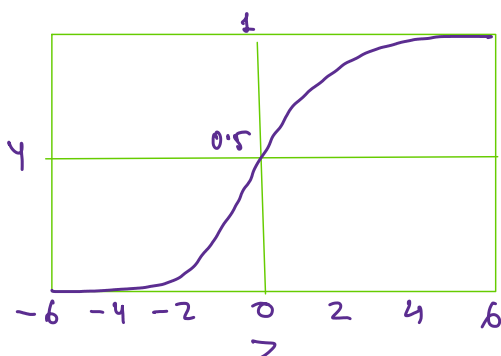
$$e^{-z} = \frac{1 - y}{y}$$

$$e^z = \frac{y}{1 - y}$$

Taking \ln both side

$$\ln e^z = \ln \frac{y}{1 - y}$$

$$z = \ln \left(\frac{y}{1 - y} \right)$$



Gradient descent Example

① $Y = mX + c$

$$(0.5, 1.4)$$

$$(2.3, 1.9)$$

$$(2.9, 3.2)$$

Step 1: Sum of Square Residual

$$\begin{aligned} RSS = & (1.4 - (c + m * 0.5))^2 \\ & + (1.9 - (c + m * 2.3))^2 \\ & + (3.2 - (c + m * 2.9))^2 \end{aligned}$$

Step 2: Derivatives of RSS with Respect to Intercept (c)

$$\frac{d}{dc} = 2 * (c + m * 0.5 - 1.4) + 2 * (c + m * 2.3 - 1.9) + 2 * (c + m * 2.9 - 3.2)$$

$$\frac{d}{dm} = 2 * (c + m * 0.5 - 1.4) * 0.5 + 2 * (c + m * 2.3 - 1.9) * 2.3 + 2 * (c + m * 2.9 - 3.2) * 2.9$$

$$\begin{aligned} \frac{d}{dc} &= -1.6 \\ \frac{d}{dm} &= 0.8 \end{aligned}$$

assume $m = 1$ $c = 0$

$$\begin{aligned} \text{new_m} &= m - \alpha \cdot \frac{d}{dm} = 1 + 0.01 * 0.8 = 1.008 \\ \text{new_c} &= c - \alpha \cdot \frac{d}{dc} = 0 + 0.01 * 1.6 = 0.016 \end{aligned}$$

$$\text{new_m} = ?$$

$$\text{new_c} = ?$$

1000 iteration

correct m & c

$$c = 0.94$$

$$m = 0.64$$

V.V. Imp Confusion matrix

	Actual Value	
	Positive	Negative
Predicted Value	Positive TP	Negative FP
	Negative FN	TN

- ① Accuracy is a metric that measures how often a machine learning model correctly predicts the outcome

$$\text{Accuracy} = \frac{TP + TN}{TP + FP + FN + TN}$$

- ② Precision refers to the number of true positives divided by the total number of positive prediction

- It helps to visualize the reliability of the machine learning model in classifying the model as positive

$$\text{Precision} = \frac{TP}{TP + FP}$$

- ③ Recall: The Recall measures the model's ability to detect positive samples

* higher Recall more positive samples are detected

$$\text{Recall} = \frac{TP}{TP + FN}$$

F1 Score F1-score is the evaluation, matrix that combines two matrices: Precision and Recall, into a single metric by taking their harmonic mean.

$$\text{F1 Score} = \frac{2 * \text{Recall} * \text{precision}}{\text{Recall} + \text{precision}}$$

$$\text{Harmonic Mean} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

$$= \frac{2}{\frac{1}{\text{Precision}} + \frac{1}{\text{Recall}}} \Rightarrow \frac{2 * (\text{Precision} * \text{Recall})}{\text{Precision} + \text{Recall}}$$

