1. Given the scores of 10 students in a mathematics test are as follows: 75, 58, 84, 59, 68, 72, 83, 60, 76, 65.

   1.1 Find the mean. (1 points) $\frac{75+58+84+59+68+72+83+60+76+65}{10} = \frac{700}{10} = 70$

   1.2 Find the median. (1 points) 58, 59, 60, 65, 68, 72, 75, 76, 83, 84 $= \frac{68+72}{2} = \frac{140}{2} = 70$

   1.3 Find the mode. (1 points) ไม่มี (mode คือตัวที่ซ้ำกันมากสุด)

   1.4 Find the variance. (3 points)

   1.5 Find the quartiles Q_1. (3 points)

   1.6 Find the deciles D_5. (3 points)

   1.7 Find the percentiles P_{75}. (3 points)

   (25 minutes)

2. Given the training data in the below table. (20 minutes)

Instance	Outlook	Temperature	Humidity	Wind	PlayTennis
1	sunny	hot	high	weak	no
2	sunny	hot	high	strong	no
3	overcast	hot	high	weak	yes
4	rain	mild	high	weak	yes
5	rain	cool	normal	weak	yes
6	rain	cool	normal	strong	no
7	overcast	cool	normal	strong	yes
8	sunny	mild	high	weak	no
9	sunny	cool	normal	weak	yes
10	rain	mild	normal	weak	yes
11	sunny	mild	normal	strong	yes
12	overcast	mild	high	strong	yes
13	overcast	hot	normal	weak	yes
14	rain	mild	high	strong	no

2.1 Find the prior probability P(PlayTennis = yes).

(2 points) 9/14

2.2 Find the conditional probability P(Wind = strong | PlayTennis = yes).

(2 points) 3/9

2.3 Find the conditional probability P(Outlook = rain | PlayTennis = no).

(2 points) 2/5

2.4 Find the conditional probability P(Temperature = mild | PlayTennis = yes).

(2 points) 4/9

2.5 Find the conditional probability P(Humidity = high | PlayTennis = no).

(2 points) 4/5

3. Find the equation $y = \beta_0 + \beta_1 x$ of the regression line that best fits the given data points (1,0), (2,1), (4,2) and (5,3).

(10 points, 20 minutes)

	x	y	x^2	xy
	1	0	1	0
	2	1	4	2
	4	2	16	8
	5	3	25	15
Σ	12	6	46	25

$$\hat{\beta}_1 = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

$$= \frac{4(25) - (12)(6)}{4(46) - (12)^2}$$

$$= \frac{100 - 72}{184 - 144} = \frac{28}{40} = 0.7$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= \frac{\cancel{6}^{3}}{\cancel{4}_{2}} - 0.7 \times \frac{\cancel{12}^{3}}{\cancel{4}_{1}}$$

$$= -0.6$$

สมการ $y = -0.6 + 0.7X$

$y = \beta_0 + \beta_1 X$

(ก) MSE $= \frac{1}{n} \sum_i (y_i - \hat{y}_i)^2$

$SSR = \Sigma (y_i - (\beta_0 + \beta_1 x_i))^2$

$= 0.1$

$\hat{y}_i \rightarrow y = -0.6 + 0.7X$

x	y	$\hat{y}$	$y - \hat{y}$	$(y-\hat{y})^2$
1	0	$-0.6 + 0.7(1) = 0.1$	-0.1	0.01
2	1	$-0.6 + 0.7(2) = 0.8$	0.2	0.04
4	2	$-0.6 + 0.7(4) = 2.2$	-0.2	0.04
5	3	$-0.6 + 0.7(5) = 2.9$	0.1	0.01
				$\Sigma = 0.1$

$= \frac{1}{4}(0.01 + 0.04 + 0.04 + 0.01)$

$= \frac{1}{4}(0.1)$

$= 0.025$ MSE #

(ข) RMSE $= \sqrt{MSE}$

$= \sqrt{0.025}$

$= 0.1581$ RMSE #

(ค) R^2

$$1 - \frac{\Sigma (y_i - \hat{y}_i)^2}{\Sigma (y_i - \bar{y})^2}$$

$$= 1 - \frac{0.1}{5}$$

$= 0.98$ R^2 #

$\bar{y} = \frac{0+1+2+3}{4} = \frac{\cancel{6}}{\cancel{4}} \; \frac{3}{2} = 1.5$

$y_i - \bar{y}$		$(y_i - \bar{y})^2$
$0 - 1.5$	$= -1.5$	2.25
$1 - 1.5$	$= -0.5$	0.25
$2 - 1.5$	$= 0.5$	0.25
$3 - 1.5$	$= 1.5$	2.25
		$\Sigma = 5$

4. Find the global maximum and global minimum values of the function $f(x) = x^3 - 6x^2 + 5$ on the interval [-3, 5].

(10 points, 20 minutes)

$f(x) = x^3 - 6x^2 + 5$

$f'(x) = 3x^2 - 2(6x) + \frac{d5}{dx} \rightarrow 0$

$= 3x^2 - 12x$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 4, 0$

Case 1 $f(x) = 0$

$f(0) = (0)^3 - 6(0)^2 + 5$

$= 0 - 0 + 5$

$= 5$

case 2 $f(x) = 4$

$f(4) = (4)^3 - 6(4)^2 + 5$

$= 64 - 96 + 5$

$= -27$

ใน global $[-3, 5]$

$f(x) = x^3 - 6x^2 + 5$

$f(-3) = (-3)^3 - 6(-3)^2 + 5$

$= -27 - 54 + 5$

$= -76$

$f(x) = x^3 - 6x^2 + 5$

$f(5) = (5)^3 - 6(5)^2 + 5$

$= -20$

$\therefore$ global maximum $= 5$

global minimum $= -76$

5. If $f(x,y) = \frac{xy}{(x+y)^2}$, find $f_x(1,2)$ and $f_y(1,2)$.

(10 points, 20 minutes)

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6. Find the local maximum and minimum values of the function
   $f(x, y) = x^3 + y^3 - 3x^2 - 3y^2 - 9x$.

   (15 points, 25 minutes)

7. Find the gradient of $f(x, y, z) = 5x^2 - 3xy + xyz$ at the point (3, 4, 5).

   (10 points, 20 minutes)

8. A farmer has 200 m of fencing and wants to fence off a rectangular field. What are the dimensions of the field that has the largest area (by using method of Lagrange Multipliers)?

   (20 points, 30 minutes)