Value at Risk

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Brief

Assignment Task Option 2: Risk Measurement You are a risk analyst for an investment bank and have been tasked with analyzing the risk of an investment in options on Procter and Gamble (PG) stock. Compute the VaR at a confidence level of 75% for a position in a call option on PG stock with a maturity that is as close as possible to three months, and for a strike price that is about 90% of the current stock price. The VaR horizon is the maturity of the option. You should also estimate the corresponding VaR for a position in the underlying stock. You should incorporate the expected return of the stock, and any dividends that it pays, in the simulation.

Introduction

Risk is inherent in any business operation, including those across investment bank functions involving uncertain cash flows. Ignoring risk associated with investment uncertainty can have disastrous implications. Multiple sources of risk are associated with investment banking, including market risk, credit risk, exchange rate risk and operational risk. Banks and other financial institutions must engage in Risk management to mitigate the implications for the broader economy that can arise due to neglecting the risk of investments.

Before banks and financial institutions can manage risk, they must measure it. Value at risk (henceforth VaR) is the most crucial framework to measure risk. Although it can be scaled to measure a whole business, in its general form, VaR measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence interval[Damodaran, 2008]. In this report, VaR is used by a risk analyst to capture the potential loss in value in an investment into Proctor and Gamble option arising from adverse market movements over the option's maturity. In practice, this can then be compared with the investment bank's available capital and cash reserves to ensure the firm can remain operational should such losses be incurred.

Following the great depression, the establishment of the Securities Exchange Commission required banks to discontinue borrowing above 2000% of the available equity capital [Damodaran, 2008]. Since then, capital requirements have continued to develop to address the increased risk complexities from the growing derivatives markets. In 1980, VaR was first formally tied to capital requirements, and VaR began to be used widely to measure the risks of trading portfolios [Linsmeier and Pearson, 1996].

There are three ways to calculate VaR. Firstly, the historical method, which is the simplest, orders the historical returns for an asset or portfolio's returns for a given period and then calculates the appropriate value for a percentile from this distribution.

The second method is the Variance Covariance approach, also known as the parametric approach, since it assumes parameters of the return distribution. By assuming the distribution for the returns of an asset or portfolio, the maximum loss at the 75% confidence level can be calculated with mean and standard deviation.

Finally, as to be explained in further detail later in this report, there is the Monte Carlo method. Monte Carlo is advantageous when evaluating risk for portfolios with complex securities that contain non-linear relationships and correlations.

This report aims to illustrate, with an option contract for Proctor and Gamble Stock, the general methodology and framework required to compute VaR for derivatives. By doing this, we find quite a substantial difference between the VaR for an investment in the option and an investment in the stock. This illustrates the increased risk associated with increasingly complex instruments and derivatives, reinforcing the requirement for investment banks and other financial institutions to monitor their risks actively.

Method

Monte Carlo (MC) simulation relies heavily on probability theory to drive the simulation process [Cheung and Powell, 2012]. By generating simulated price paths of the underlying asset, we value an option portfolio at the VaR horizon for each price path. With the distribution of these portfolio values we estimate the portfolio VaR, the investment risk of the option portfolio. The MC approach is suitable for pricing all options, including a call on proctor and gamble stock, which is dependent on a single underlying asset [Kyrtsos, nd].

Firstly, once we have obtained the option price, underlying stock price, time to expiry and strike (exercise) price, we use the maturity to calculate the horizon of VaR. Then, we calculated daily log returns and estimated daily volatility from daily data of the adjusted close price for the underlying stock. We measured this by finding the standard deviation of daily log returns from a sample of historical data, dating back 1 year, to establish our initial input parameters needed for the model.

In the parametric method, we would define the probability distribution of the risk factor (the return of the option); however, using the MC simulation, we derive the distribution of the option returns using a stochastic process. Let P_t be the stock price on day t, P_{t-1} be the stock price on the previous day, and r_t be the daily return for day t. The relationship between these variables can be expressed using the natural logarithm as follows. In our model, we assume that asset prices follow a stochastic process described by the following equation:

$$\ln P_t = \ln P_{t-1} + r_t \tag{1}$$

If we assume $r_t \sim N(0, \sigma^2)$, that daily log returns are normally distributed with a zero mean and variance of σ^2, we are able to transform (1) as follows:

$$\ln P_t = \ln P_{t-1} + \sigma \varepsilon_t \tag{2}$$

Where σ is the standard deviation of daily log returns calculated from the data, and $\varepsilon_t \sim N(0, 1)$ is a standard random normal variable that represents a random shock. This is our model of the underlying stock price for the MC simulation.

After defining the process, we simulate price paths for all variables of interest. At the 45-day horizon, we consider in our example; our portfolio is marked-to-market using the full valuation, comprising 45 different daily stock prices. Although there are 46 trading days until expiration, we do not forecast today's price ($t = 0$). The MC simulation attempts to represent all outcomes given our assumptions by repeating the simulation 1,000 times. For each iteration, our standard normal variable takes on a different random value for each day, and the stock price for any day is given by (2). Once we have 1,000 completed simulations, we analyse the distribution of the outcomes. In our case, when option pricing, the appropriate outcome is the underlying stock's final future price, P_{final}, in a natural logarithmic format. We then raised the exponent to this logarithmic value to get an absolute value of price as in (3).

$$P_{final} = e^{\ln P_{final}} \tag{3}$$

Each of these 'Pseudo' realizations of the share price at the expiry date is used to calculate the option's value at expiry. We compute simple returns on the stock and options over the option's maturity for each realization with equations (4) and (5) respectively.

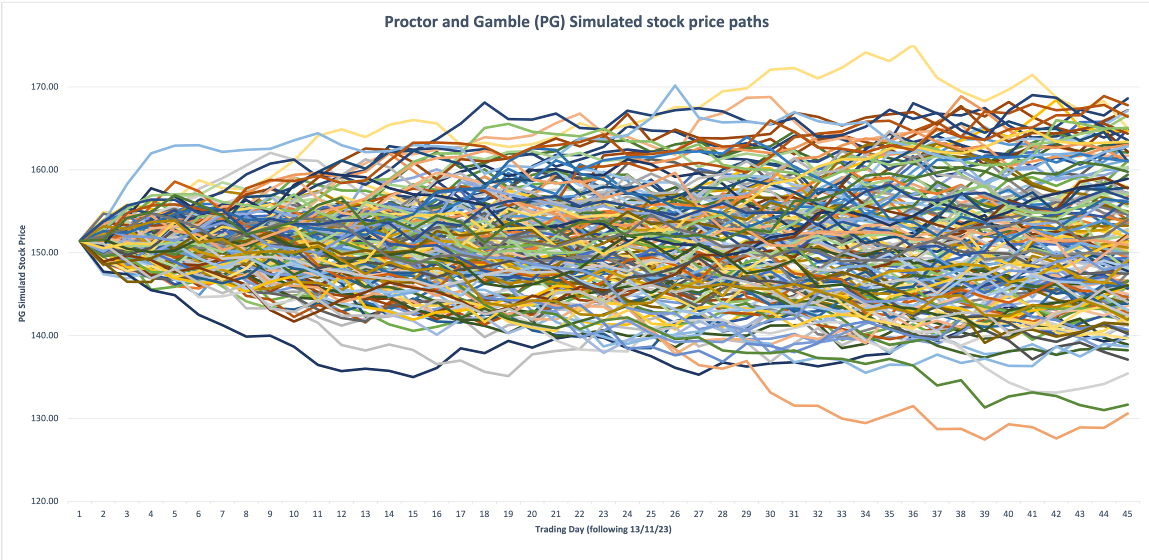


Figure 1: 156 Simulated Price Paths for PG stock over 45 day maturity of the Call Option

$$R_{stock} = \frac{P_{final} - P_{0,stock}}{P_{0,stock}} = \frac{P_{final} - \$151.54}{\$151.54} \tag{4}$$

$$R_{option} = \frac{P_{final,option} - P_{0,option}}{P_{0,option}} = \frac{P_{final,option} - \$17.8}{\$17.8} \tag{5}$$

Where R_{stock} and R_{option} are the simple returns for an investment in the PG stock and option respectively. $P_{0,stock}$ is the initial stock price, which we can substitute with \$151.54 and $P_{0,option}$ is the initial price of the option which we can substitute with \$17.8. P_{final} is the stocks final price for each simulation when $(t = 45)$. $P_{final,option}$ is the price of the option based on the final price of the stock. To ensure positive cash-flows from this investment the option contract would not be exercised if the strike price is greater than the price of the contract. Equation (6) shows how the final value of the option is computed with this condition. In our case P_E is \$135.

$$P_{final,option} = \begin{cases} P_{final} - P_E, & \text{if } P_{final} > P_E \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

We can then compile and plot a distribution of stock and option returns, from which a VaR figure can be measured for both the underlying stock and the option. We do this by calculating the appropriate percentile of the distribution of returns that we are interested in; for our case, it is the 75% level. This provides us with a table of VaR values using historical data in the MC simulation case. We also calculated VaR using the Variance-Covariance approach, whereby we used the standard deviation and mean value of returns for both the stock and option. This allowed us to calculate VaR with (7) whereby q^s_α is the α percent quantile of the standard empirical distribution and σ and μ are standard deviation and mean respectively.

$$VaR = -\left(q^s_\alpha \sigma + \mu\right) \tag{7}$$

Data

PG Stock Historical Adjusted Close Data

To compute the assumed volatility of the stock return. We sourced publicly available historical data on the daily adjusted close price of PG stock.

The data is available here.A CSV like the one used to build our model that includes the daily Adjusted Close ($USD) along with the daily Open, High, Low, Close, and Volume can be downloaded by setting the period ranging from 12/11/2022 -12/11/2023 and the frequency as daily [Yahoo Finance, 2023]. Although we accessed and downloaded the data on 13/11/2023, the first trading day with data available at the time of download in our range was 14/11/2022, and the last trading day in our range was 10/11/2023.

The adjusted close is the close price adjusted for splits and dividend and capital gain distributions. Using the adjusted close price, we incorporate the stock's expected return and the dividends paid on specific dates into the simulation. In our historical data, dividends were paid on the following dates: 19/11/23, 20/07/2023, 20/04/2023 & 19/01/2023.

Table 1: Summary Statistics of Adjusted Close Price Data

Statistic	Value
Mean	$146.65
Minimum	$134.03
Maximum	$156.10
Standard Deviation	$5.65

Since the daily log returns are data points directly transformed from the adjusted close price data, it is also worth providing the following summary statistics as in Table 2.

Table 2: Summary Statistics of Daily Log Returns

Statistic	Value
Mean	0.04%
Minimum	-2.72%
Maximum	3.40%
Standard Deviation	0.90%

Option Contract Data

To compute the VaR with the MC method, we require the initial option price, associated initial underlying stock price, and time to expiry date for an specified option contract. The closest maturity date to 3 months from 13/11/2023 for proctor and gamble options is 19/01/2024. Data for several option contracts with that expiry date can be publicly accessed here. By configuring the filters as the following: Volume: All, Expiration Type: All, Options Range: All, Expiration: January 2024, one can download data on several option contracts with a maturity date 19/01/2024 [CBOE, 2023].

When we accessed the Data on 13 November 2023 at 07:01 GMT-5, the current stock price was $151.41. We sought to identify a call option with a strike (exercise) price of $135 to use in our model. We selected this strike price as it was the closest to 90% of the current stock price ($136.27). We were not able to identify a contract with a price history in February 2024 which would have been closer to 3 months, so opted to use an option maturing in January.

Market data provided by Cboe

PG240119C00135000 Last: **17.8** Change: **+0.975 (+5.79494%)**

Chart

Price Data Table

2023-11-10 14:11:52 ET (Delayed)

Last Sale	17.8	Tick	up
Time of Last Sale	2023-11-10 14:11:52	Symbol	PG240119C00135000
Net Change	+0.975	Percent Change	+5.79494
Previous Close	17.875	Open	17.8
High	17.8	Low	17.8
Bid	17.9	Ask	18.35
Bid Size	39	Ask Size	14
Volume	1	Security Type	option
Open Interest	849	Expiration Date	Fri Jan 19 2024
Strike Price	135	IV	0.1946
Theta	-0.0077	Theo	18.0874
Rho	0.2304	Delta	0.9404
Vega	0.079	Gamma	0.0095

Figure 2: Screen Clipping of our Chosen Option Contract

Based on this option's last sale on 10/11/2023, the price was \$17.8. The time to expiry (maturity) was 45 trading days as of 13/11/2023, as there are five holidays where there is no trading on the NYSE in our period.

Results

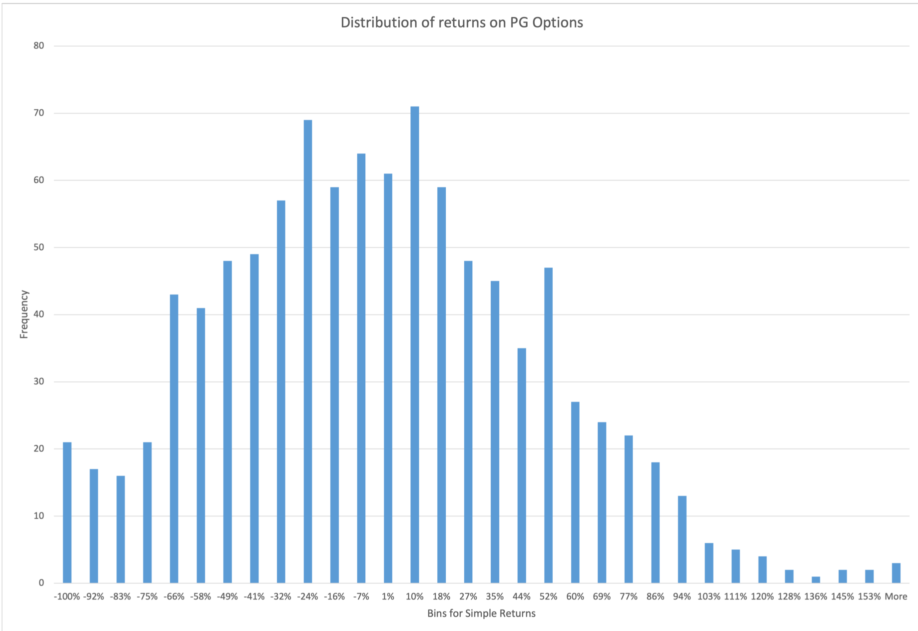
PG Option Returns

The distribution of the 1,000 simulated option contract returns has a mean of -5.69% and a standard deviation of 49.95%. Using Excel we can sort the returns into frequency bins as in Table 3. The row highlighted shows that the bin with the 25.6 cumulative percentile has a return of -40.90%, we can expect our VaR to be just below this since it is the 25th percentile. With the table we can visualise the distribution as a histogram in Figure 3.

Table 3: Option Return Frequencies

Bin	Freq	Cum-%
-100.00%	21	2.10%
-91.56%	17	3.80%
-83.11%	16	5.40%
-74.67%	21	7.50%
-66.23%	43	11.80%
-57.78%	41	15.90%
-49.34%	48	20.70%
-40.90%	49	25.60%
-32.45%	57	31.30%
-24.01%	69	38.20%
-15.57%	59	44.10%
-7.12%	64	50.50%
1.32%	61	56.60%
9.76%	71	63.70%
18.21%	59	69.60%
26.65%	48	74.40%
35.09%	45	78.90%
43.53%	35	82.40%
51.98%	47	87.10%
60.42%	27	89.80%
68.86%	24	92.20%
77.31%	22	94.40%
85.75%	18	96.20%
94.19%	13	97.50%
102.64%	6	98.10%
111.08%	5	98.60%
119.52%	4	99.00%
127.97%	2	99.20%
136.41%	1	99.30%
144.85%	2	99.50%
153.30%	2	99.70%
> 153%	3	100.00%

Figure 3: Distribution of PG Option Returns Based on an investment at a price of $17.8 on 13/11/23 Maturing on 19/01/24



For a 75% confidence level, we can compute the VaR through the return on the option contract at the exact 25th percentile of this distribution. Using Excel's percentile function, we can report that in our sample of 1,000 simulations, 25% of the returns on the option contract were below -42.52%. The VaR of the call option into PG stock at the 75% confidence level over an investment held over 49 days till expiration is 43.51%.

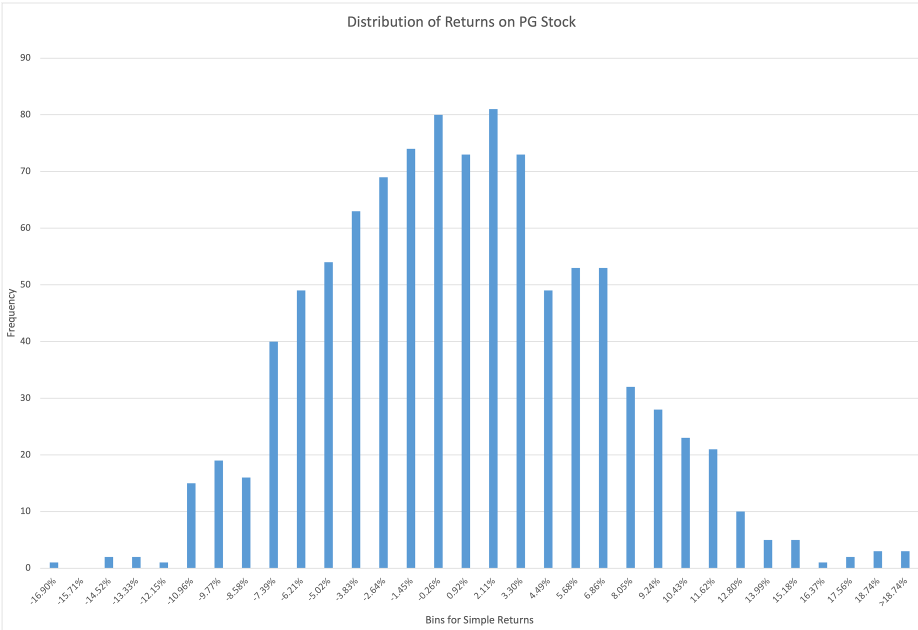
PG Stock Returns

The distribution of the 1,000 simulated PG stock returns has a mean of 0.22% and a standard deviation of 5.94%. Using Excel we can sort the returns into frequency bins as in Table 4. The row highlighted shows that the bin with the 26.2 cumulative percentile has a return of -3.83%, we can expect our VaR to be just below this since it is the 25th percentile. With the table we can visualise the distribution as a histogram in Figure 4.

Table 4: Stock Return Frequencies

Bin	Freq	Cum-%
-16.90%	1	0.10%
-15.71%	0	0.10%
-14.52%	2	0.30%
-13.33%	2	0.50%
-12.15%	1	0.60%
-10.96%	15	2.10%
-9.77%	19	4.00%
-8.58%	16	5.60%
-7.39%	40	9.60%
-6.21%	49	14.50%
-5.02%	54	19.90%
-3.83%	63	26.20%
-2.64%	69	33.10%
-1.45%	74	40.50%
-0.26%	80	48.50%
0.92%	73	55.80%
2.11%	81	63.90%
3.30%	73	71.20%
4.49%	49	76.10%
5.68%	53	81.40%
6.86%	53	86.70%
8.05%	32	89.90%
9.24%	28	92.70%
10.43%	23	95.00%
11.62%	21	97.10%
12.80%	10	98.10%
13.99%	5	98.60%
15.18%	5	99.10%
16.37%	1	99.20%
17.56%	2	99.40%
18.74%	3	99.70%
>18.74%	3	100.00%

Figure 4: Distribution of PG Stock Returns Based on an investment at a price of $151.41 on 13/11/23 held until 19/01/24



For a 75% confidence level, we can compute the VaR through the return on the proctor and gamble stock at the 25th percentile of this distribution. Using Excel's percentile function, we can report that in our sample of 1,000 simulations, 25% of the returns on the PG stock were below -4.08%. The VaR of position in underlying PG stock at the 75% confidence level for an investment held over 49 days till the options expiration date is 4.08%. This means there is a 75% probability that the loss in the investment will be at most 4.08% of the initial investment.

Sensitivity Analysis

The pricing model that evaluates the stock price at each date along the maturity of the option is largely dependent on the standard deviation of the log returns in the data. We can conduct a sensitivity analysis to determine how different values for the volatility of the returns can affect the 75% VaR estimates for both securities.

Table 5: VaR Return Volatility Sensitivity Analysis

Return Volatility(σ)	75%VaR:Option	75%VaR Stock
0.40%	23.35%	1.83%
0.50%	27.20%	2.28%
0.60%	31.02%	2.73%
0.70%	34.83%	3.18%
0.80%	38.62%	3.62%
0.90%	42.39%	4.07%
1.00%	46.14%	4.51%
1.10%	49.88%	4.95%
1.20%	53.60%	5.38%
1.30%	57.30%	5.82%
1.40%	60.99%	6.25%

Variations in the constant return Volatility we assumed based on the historical data can have significant variations in the VaR. With a one percentage point increase in return volatility from 0.4% to 1.4%, the option VaR computed varies from 23.35% to 60.99%. This large variance highlights a potential flaw in our computation.

Conclusion

In producing this report, we have acted as a risk analyst at an investment bank in order to evaluate the risk of an investment in options for Proctor and Gamble stock. While the process we have used yields two explainable and interpretable values that represent the worst-case scenario for both potential investments 75% of the time. There are a couple of shortcomings in our method and VaR in general that if addressed in the future will yield more robust and reliable figures.

The main hindrance in the reliability of our estimates is derived from the simplicity in the stochastic model we use to derive our logs of the prices of the underlying security during the period. The model described in (1) and (2) satisfies the standard finance assumption that stock prices are log-normally distributed or equivalently that stock returns are normally distributed [Goshu, 2014]. However, a common property incorporated into a models assumptions is that the average return from holding a stock tends to increase over time.

While our model and process adheres to most of these properties, it does not consider the fourth, that the average return from holding a stock tends to increase over time as (1) and (2) do not incorporate the expected return of the PG stock.

If we were to use a more complex model, such as the widely used Geometric Brownian Motion we would be able to account for the expected drift as well as the random shock in the stock price. The equation for geometric Brownian motion is given in (8)[Harper, 2022].

$$\frac{\triangle P}{P} = \mu\triangle t + \sigma\epsilon\sqrt{\triangle t} \tag{8}$$

Where P is the stock price, $\triangle P$ is the change in stock price, μ is the annualized expected return of the stock, σ is the standard deviation of the returns, ϵ is same standard normal random variable as in (2), and $\triangle t$ is the elapsed time period.

To present it in line with our methodology we can alter notation and simplify:

$$\frac{P_t - P_{t-1}}{P_t} = \mu\triangle t + \sigma\epsilon\triangle t \tag{9}$$

$$P_t - P_{t-1} = P_t(\mu\triangle t + \sigma\epsilon\triangle t) \tag{10}$$

Equation (10) suggests the change in price of the stock over a continuous interval is a function of the initial price is multiplied by two terms. The first term represents a "drift" calculated with expected return, the second term is the "random shock" as in equation 2. A model that includes a drift is more in line with our data as the mean return was positive as well as PG stock in general. On the other hand our model that only includes the stochastic part representing the random fluctuations may not capture the long-term trends in the underlying stock.

There are also widely accepted limitations preventing practitioners from using VaR as a comprehensive measure of risk. Firstly, VaR f, figures produced internally can easily be erroneous, misrepresenting levels of risk. Firstly, all three approaches are dependent in some form on historical data. The biggest culprit is the historical method, which bases VaR entirely on historical data. However, the variance-covariance approach uses historical data to build a covariance matrix, and the MC very often derives distribution from historical data [Damodaran, 2008]. VaR that tends to be a function of the period of the data collected. Our VaR might underestimate the risk as the year up to 13/11/2023 should have less volatility than the following year, particularly for US consumer stocks with the expected market volatility associated with the oncoming US presidential election. Furthermore our VaR estimate may be false if we incorrectly assumed the return distributions for the underlying stock.In addition to being potentially erroneous, our use of VaR is atypical of industry practice. In most real world applications VaR is computed over shorter periods than 45 days, typically a day, week or 10 days. The reason for this a trading desk in an investment bank would prioritize being able to hedge the shorter term, more immediate risks and regulatory bodies tend to demand shorter risk exposure profiles.

With these limitations acknowledged, if we were to repeat this project and fully satisfy the objective of measuring the risk of the position, we would seek to use VaR as one of many measures of risk. The easiest additional metric to produce would be stressed VaR which entails producing VaR estimates via any chosen method but running them under an environment that assumes extreme market volatility. [Ross, 2023] There is also the Cash flow at risk (CFaR) method. Whilst VaR focuses on changes in the overall value of a portfolio as market risks vary, C-FaR is has a focus on the operating cash flow in a specified period and the market-induced variances within it. As a result, it assesses the likelihood that operating cash flows will drop below a prespecified level and is computed over longer time horizons than VaR. C-FaR may be a better choice for non-financial institutions which are primarily concerned with managing the risks inherent in operating cash flows, as finding VaR's focus on mark-to-market profit or loss over a holding period may not suit their perspective [Linsmeier and Pearson, 1996].

In demonstrating the steps taken to produce a VaR estimate for an investment into Proctor and Gamble stock options with the Monte Carlo simulation method, we have been able to asses the suitability of VaR in the greater risk measurement framework, emphasizing the need for additional measurements by highlighting its vulnerabilities. These vulnerabilities will only be greater exposed as we progress into 2024 with several geo-political events such as developing global conflicts and significant elections both impacting market volatility [Board, 2023].

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