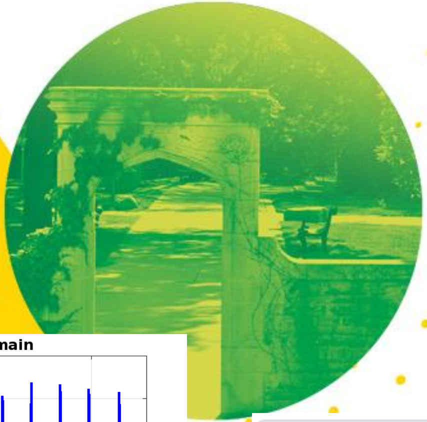
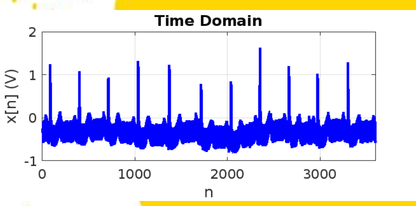
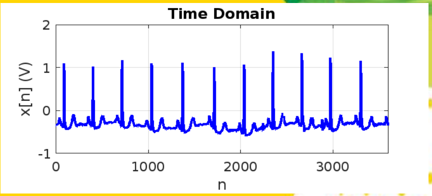
Fourier Analysis and Filtering of Biomedical Signals

Omar Boursalie, Ph.D.







IBEHS 3A03
Biomedical Signals and Systems

Lecture Slides, ECG data, and Matlab Code:
https://github.com/OBoursalie/MicroTutorial

McMaster University

Land Acknowledgement

We take this time to recognize that McMaster University is currently on the traditional territory shared between the Haudenosaunee confederacy and the Anishinabe nations, which was acknowledged in the Dish with One Spoon Wampum belt.

That wampum uses the symbolism of a **dish to represent the territory**, and **one spoon to represent that the people** are to **share the resources** of the land and only take what they need.

https://www.torontomu.ca/aec/land-acknowledgment/

https://healthsci.mcmaster.ca/docs/librariesprovider59/resources/mcmaster-university-land-acknowledgment-guide.pdf?sfvrsn=7318d517_2

My Biomedical Engineering Journey

Omar Boursalie, Ph.D.

You Are Here

- 1991: Born (at McMaster Hospital!)
- 2009-2014: **Undergraduate Electrical, Computer, and Biomedical Engineering** (McMaster)
  - 2012-2013: 12-Month co-op Instructional Assistant Intern (IAI) for 1C03 (McMaster)
- 2014-2016: M.A.Sc. Biomedical Engineering (McMaster)
- 2016-2021: Ph.D. Biomedical Engineering (McMaster)
  - Artificial intelligence in healthcare
- 2022-2023: Postdoctoral Fellow (Toronto Metropolitan University)
  - Sessional Instructor (Winter 2022): Electrical and Computer Engineering (McMaster)
- My Goal: Teaching Professor Position in Biomedical Engineering

boursao@mcmaster.ca if you want to chat!
Meet with students 2:15 pm- 2:45 pm MDCL 3515

“Fourier Analysis and Filtering of Biomedical Signals”
Lecture Objectives

1. **Motivation**
2. Time and Frequency Domain
3. Fourier Analysis
   - Time to Frequency Domain
   - Frequency to Time Domain
4. Filtering

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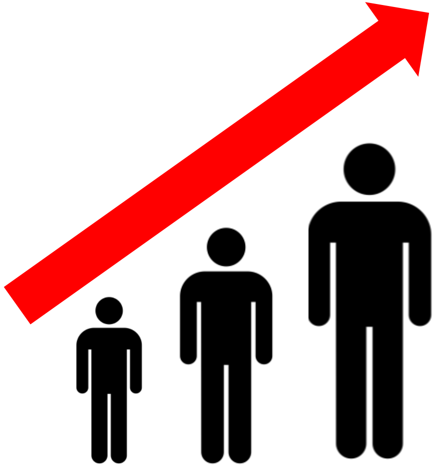
Textbook: Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd edition

Motivation

Challenges in Healthcare

Growing Population

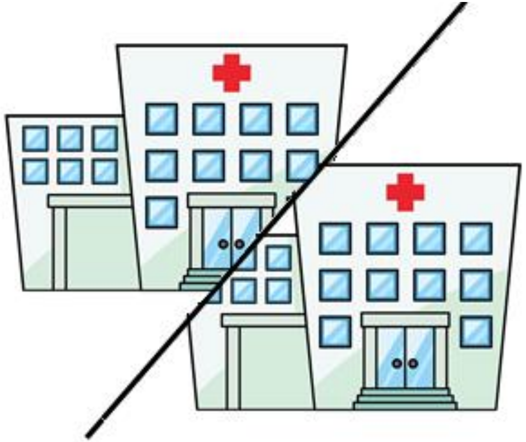


51 million by 2063
(Stats Canada, 2014)

Aging Population

25% of Population by 2036
(Stats Canada, 2016)

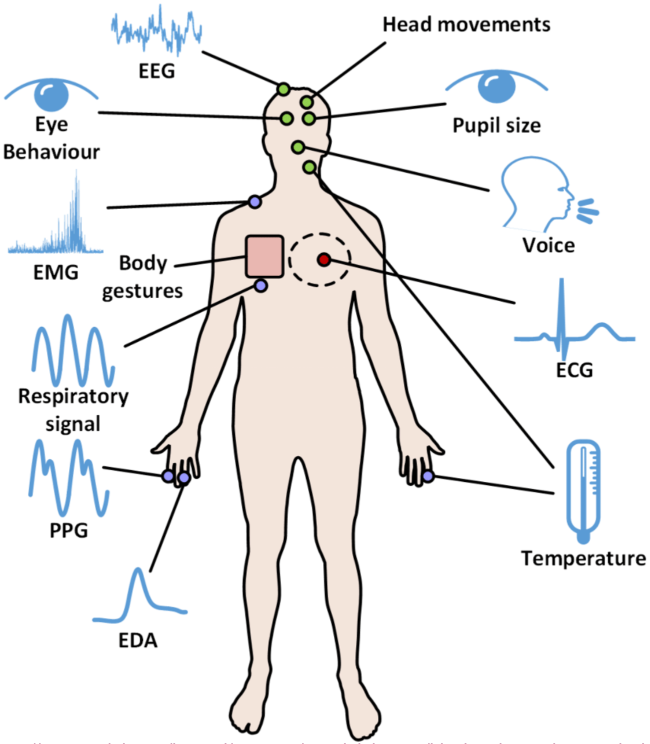
Medical resources not increasing fast enough



Length of stay for admitted patients was up 11% in 2018 (CIHI, 2018)

Motivation

Biomedical signals can be captured (mostly) non-invasively and used as indicators of overall health



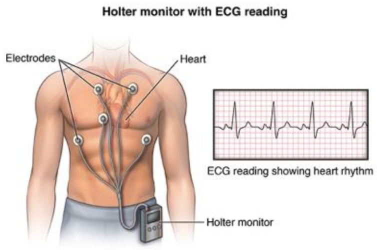
https://www.mdpi.com/journal/sensors/special_issues/biosignal_sensing_analysis

- Electroencephalogram (EEG)
- Electrooculogram (EOG)
- Electrocardiogram (ECG)
- Electromyogram (EMG)
- Photoplethysmogram (PPG)
- Electrodermal activity (EDA)

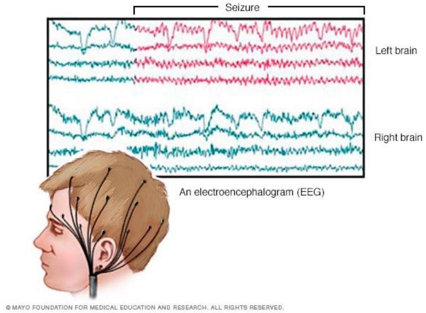
Motivation

Biomedical Signals Applications

Screening
(e.g., Depression Voice Analysis)



Diagnosis
(e.g., ECG Holter Monitor)
https://www.hopkinsmedicine.org/health



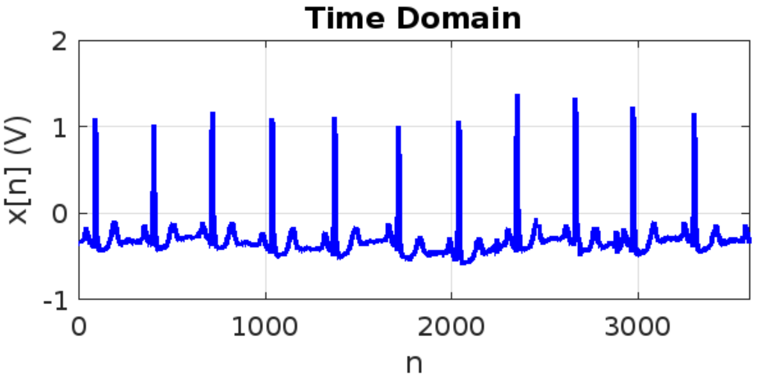
Treatment
(e.g., EEG Brain Control Interface)
https://www.mayoclinic.org/tests-procedures/eeg/about/pac-20393875

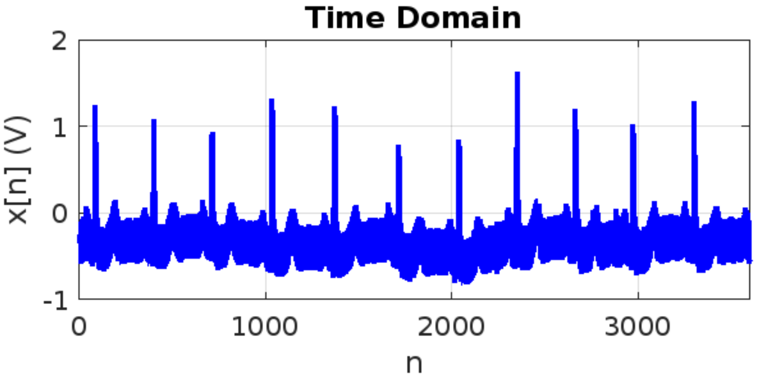
Monitoring
(e.g., Apple Watch)
https://www.apple.com/ca/shop/buy-watch

Disease Management
(e.g., Blood Glucose)
https://www.cbc.ca/news/canada/edmonton/glucose-monitoring

Motivation

Biomedical Signals are Noisy





- These are real ECG signals
  - Sample 101 from the MIT-BIH Arrhythmia Database
  - https://archive.physionet.org/physiobank/database/mitdb/
  - Lots of real data available (great resume builder!)
- Biomedical signals contain
  - Noise of different types e.g., movement, electricity interference)
  - Aggregated information from different concurrent sources (e.g., EOG, EEG, and EMG)
- Signal processing techniques are needed to extract clinically meaningful information from the biomedical signals
- Working with biomedical signals requires a collaboration between between health professionals, physicists and engineers
- Today we will discuss filtering of biomedical signals

Motivation

Opportunities in Biomedical Signals Processing

- Undergraduate Studies
  - 3A03 is prerequisite for
    - IBEHS 4A03 Biomedical Control Systems
    - IBEHS 4DO3 Medical Imaging
    - IBEHS 4F04 Biomedical Instrumentation and Measurement
  - IBEHS 3P04 Health Solutions Design Projects III: Analysis and Decision Making
  - IBEHS 3H03/ 3I06/ 3T03/ 4T06/ 4H03 Research Project Course
  - Biomedical Engineering Capstone Design Project

- Co-ops
  - Nomic R&D Assay Engineering Intern
  - https://www.glassdoor.ca/Job/signal-processing-intern-jobs-SRCH_KO0,24.htm
- Graduate Studies
  - Dr. Bruce Auditory Engineering Lab
  - https://www.ece.mcmaster.ca/~ibruce/ael.htm
- Careers for Signal Processing
  - Data Scientist / Biosignals Algorithms Engineer
  - BRAEBON Medical Corporation
  - https://ca.indeed.com/viewjob?cmp=BRAEBON-Medical&t=Algorithm+Engineer&jk=cca1153e3752c5f0&q=biomedical+Signal+Processing&vjs=3

“Fourier Analysis and Filtering of Biomedical Signals”
Lecture Objectives

1. Motivation
2. **Time and Frequency Domain**
3. Fourier Analysis
   - Time to Frequency Domain
   - Frequency to Time Domain
4. Filtering
5. EDI

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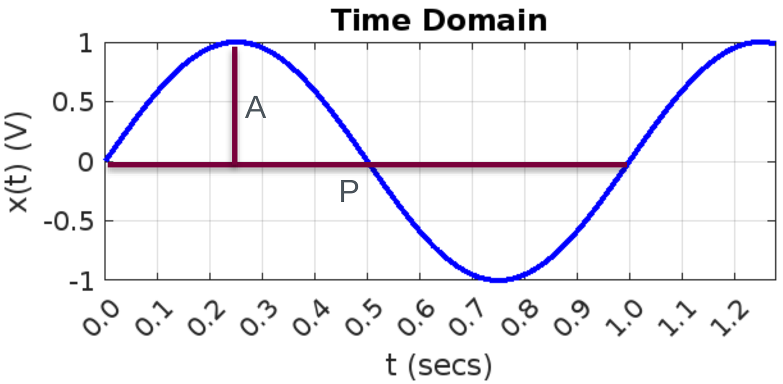
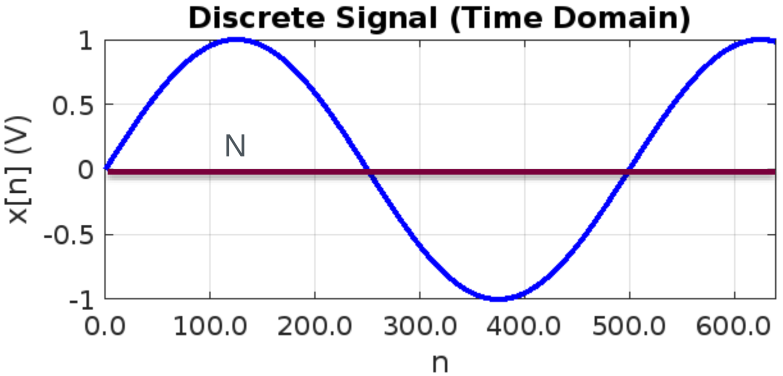
Continuous to Discrete-time Signal

Storing digital signals on a computer





We know our signal is a sine wave

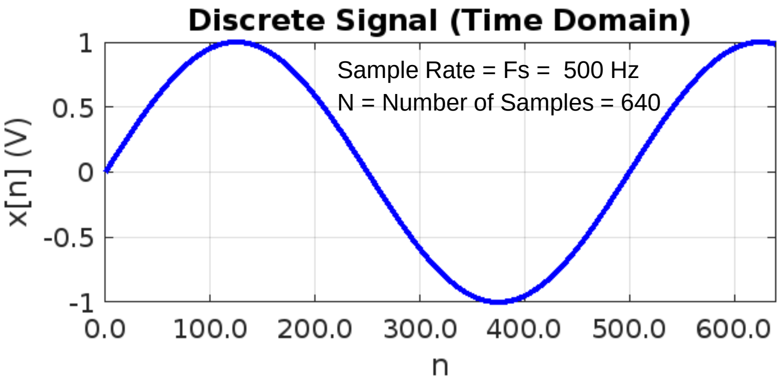
- Continuous-time function x(t) = Asin(2πft) (Not known usually)
- Amplitude = A = 1 V
- Period = P = 1 second (repeats every 1 second)
- Frequency = f = 1/P = 1 Hz (repeats every 1 second)
- X-axis = time (t) in secs
- Y-axis = x(t)

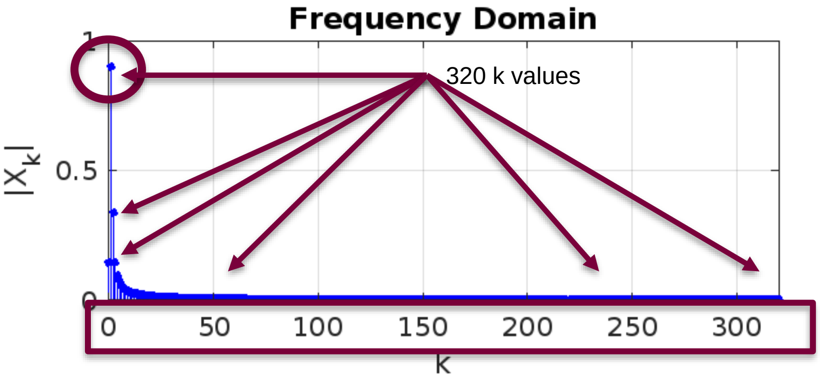
We sample x(t) at a fixed rate and record a vector of n discrete values x[0], x[1], …, x[N] (blue dots)

- X-axis = sample number n
- Y-axis = x[n]
- Sample Rate = F_s = 500 Hz (500 samples every 1 sec)
- N = Number of Samples = 640

Discrete-time to Frequency Domain

Discrete-time signals can be described in frequency domain

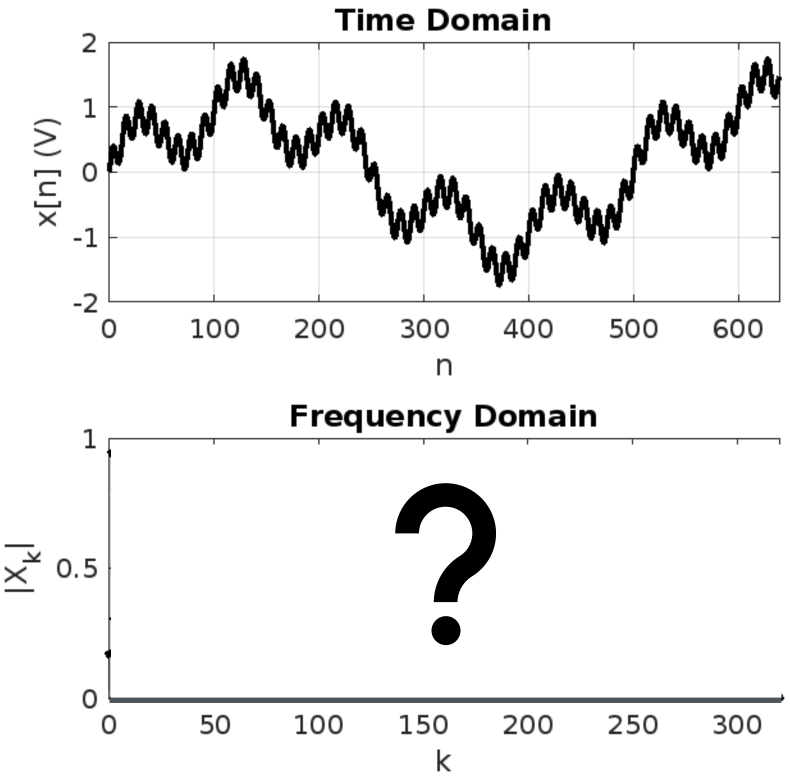




- X-axis = k = the current frequency we're considering
  - Signal can have multiple frequencies
  - Number of k frequencies we can look at is N/2
    - L_k= Number of k = N/2 = 640/2 = 320
    - k = 0 to 319 (we can look at 320 frequencies)
  - What frequencies can we look at?
    - R_k = frequency range is from 0 to Fs*(N/2)/N Hz (250 Hz)
    - Frequency we look at increments by Fs/ R_k = 0.7812 Hz
      - f_k=k* Fs/ R_k
      - f_k= 0 Hz, 0.7812 Hz, 1.56 Hz, …, 250 Hz

- Y-axis = $|X_k|$ = the magnitude of frequency *k* in the signal
  - In this example, we only have one frequency
  - $|X_k|$ is highest magnitude around 1 Hz (k = 1)
  - Right now, I can measure frequency directly (How do we calculate $|X_k|$?)

What if we don't know everything beforehand?



We **know** the overall amplitude of the signal

We **know** the overall frequency of the signal

We **don't know** the wave-form shape (sine? cosine?)

We **don't know** which frequencies are in the signal

We **don't know** the amount of each frequency in the signal

- Last example was a special case where what we can directly measure the above properties

Now what?

Fourier Analysis

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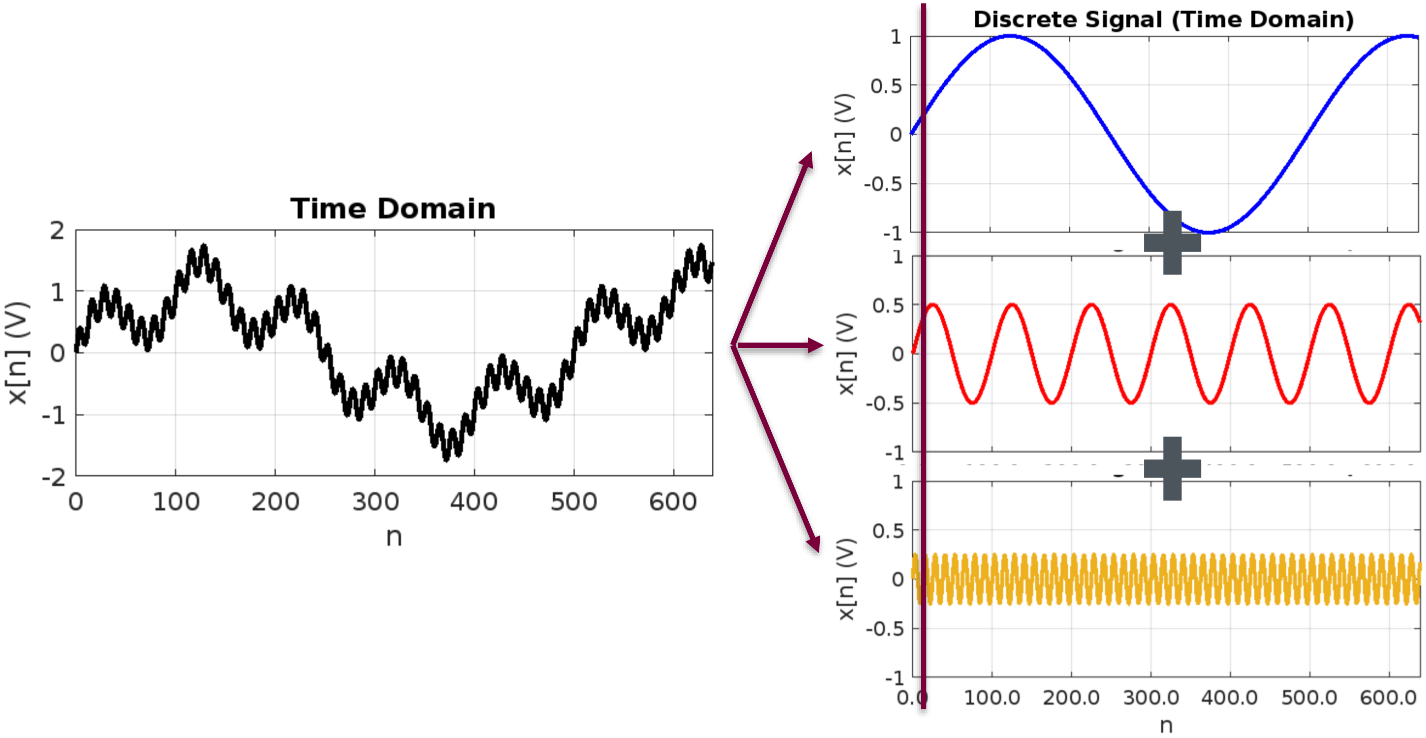
Fourier Series

Any signal can be approximated as a sum of sines and cosines of increasing frequencies



Sine wave
Amplitude = A = 1 V
Frequency = f = 1 Hz
Sample Rate = 500 Hz

Sine wave
Amplitude = A = 0.5 V
Frequency = f = 5 Hz
Sample Rate = 500 Hz

Sine wave
A = 0.25 V
Frequency = f = 40 Hz
Sample Rate = 500 Hz

Discrete Fourier Transform (DFT)

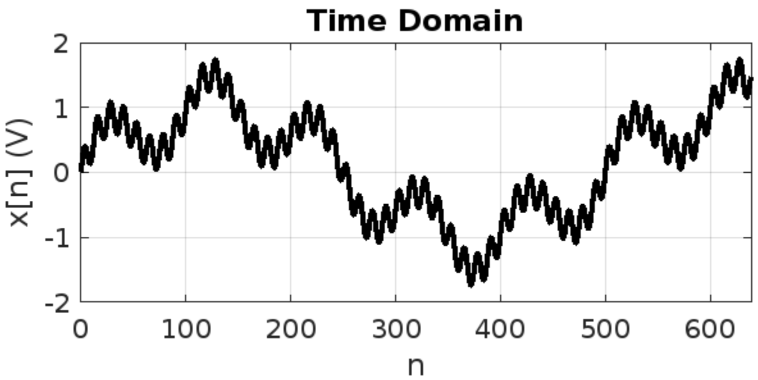
Any signal can be approximated as a sum of sines and cosines of increasing frequencies

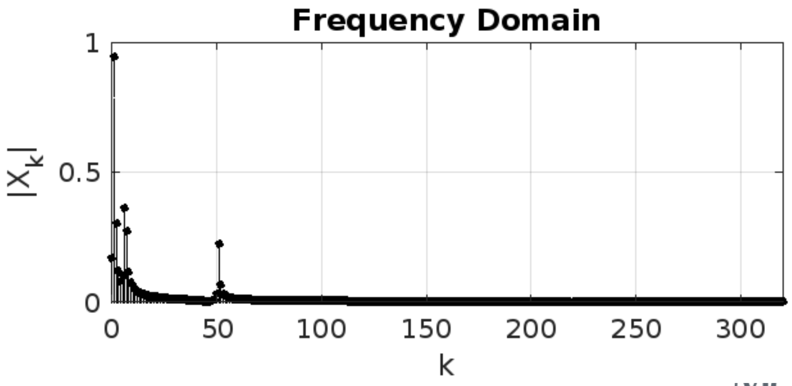
$$X_k = \sum_{n=0}^{N-1} x[n]\left(\cos\frac{2\pi kn}{N} - j\sin\frac{2\pi kn}{N}\right), k = 0,1,\ldots,N-1$$

Fast Fourier Transform (FFT) is a computationally efficient way of computing the DFT

$$= \sum_{n=0}^{N-1} x[n]e^{-\frac{j2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

k= target frequency
n= sample number
N= total number of samples
x[n] = value of nth sample





Discrete Fourier Transform (DFT)

k= target frequency
n= sample number
N= total number of samples
x[n] = value of nth sample

Calculate DFT at k = 1 (0.78 Hz), N = 640

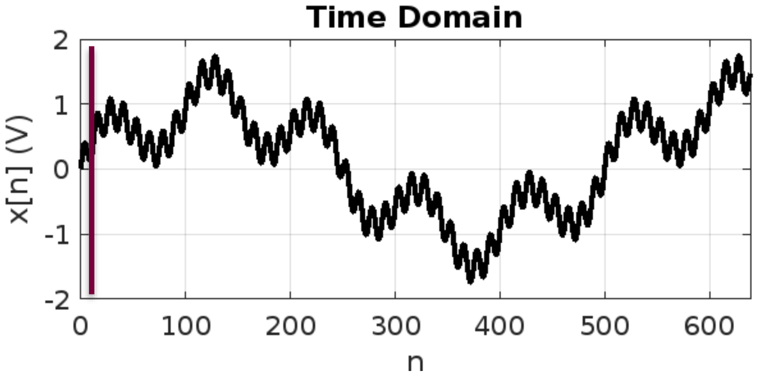
$$X_k = \sum_{n=0}^{N-1} x[n]\left(\cos\frac{2\pi kn}{N} - j\sin\frac{2\pi kn}{N}\right), k = 0,1,\ldots,N-1$$

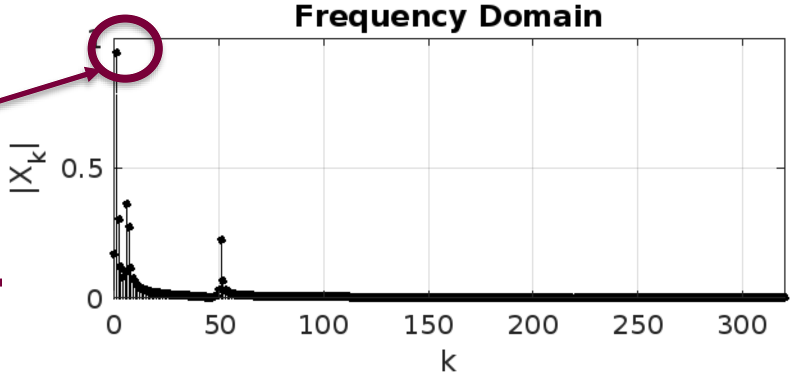
$$X_1 = \sum_{n=0}^{639} x[n]\left(\cos\frac{2\pi * 1 * n}{640} - j\sin\frac{2\pi * 1 * n}{640}\right)$$

$$= \sum_{n=0}^{639} x[n]\left(\cos(0.0098 * n) - j\sin(0.0098 * n)\right)$$

$$= 256.82 - j157.48$$

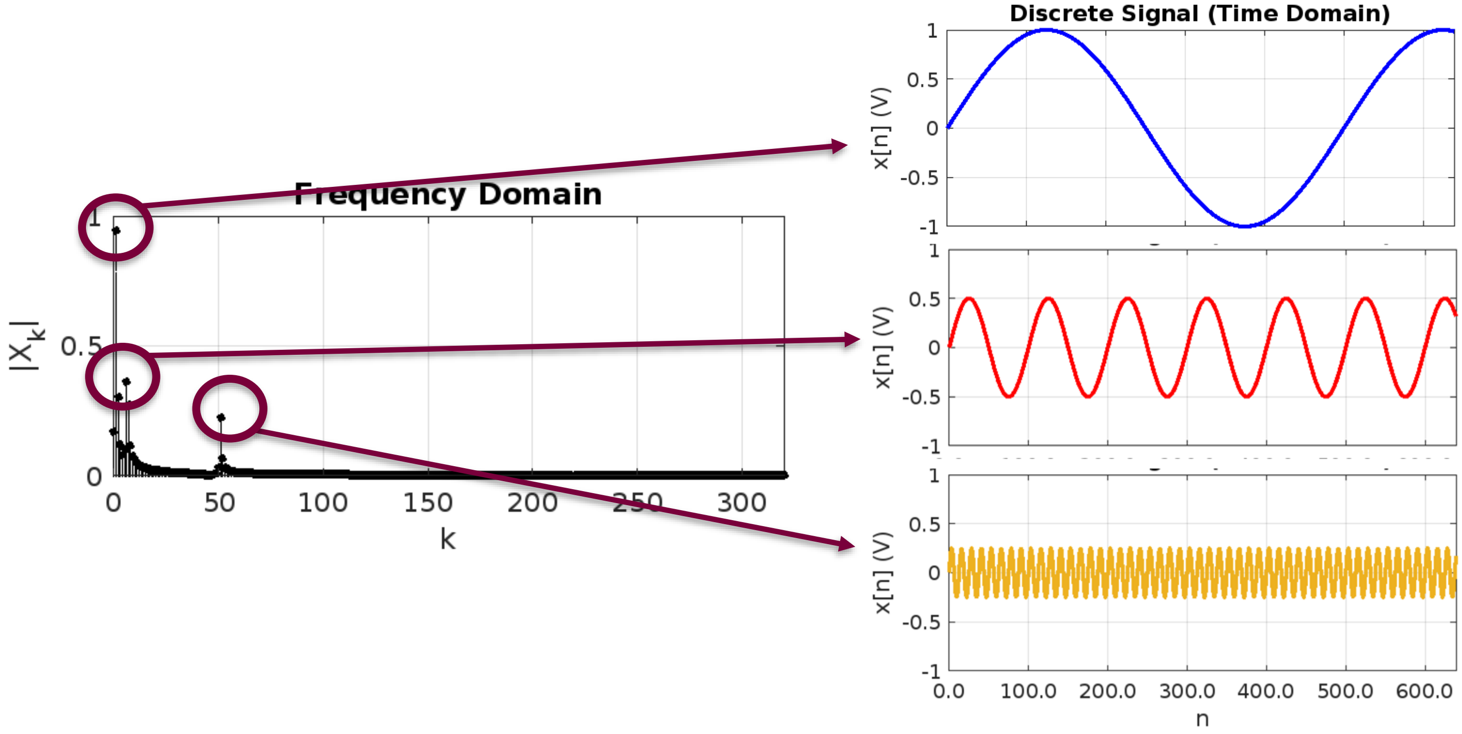
$$|X_1| = \frac{\sqrt{256.82^2+157.48^2}}{N/2} = 0.9414$$





Fourier Series

Any signal can be approximated as a sum of sines and cosines of increasing frequencies



Sine wave
Amplitude = A_{total} = 1 V
Frequency = B = 1 Hz
Sample Rate = 500 Hz

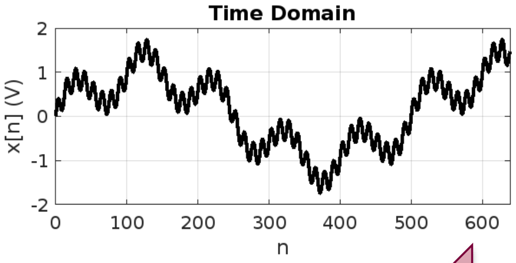
Sine wave
Amplitude = A_{total} = 0.5 V
Frequency = B = 5 Hz
Sample Rate = 500 Hz

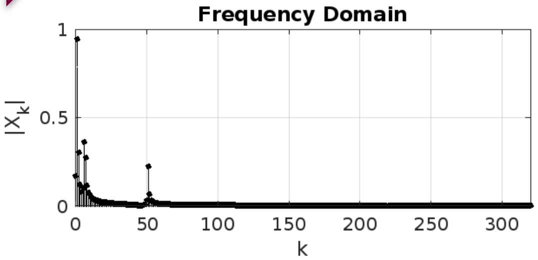
Sine wave
A_{total} = 0.25 V
Frequency = B = 40 Hz
Sample Rate = 500 Hz

Discrete Fourier Transform and Inverse Discrete Fourier Transform

$$X_k = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

Discrete Fourier Transform

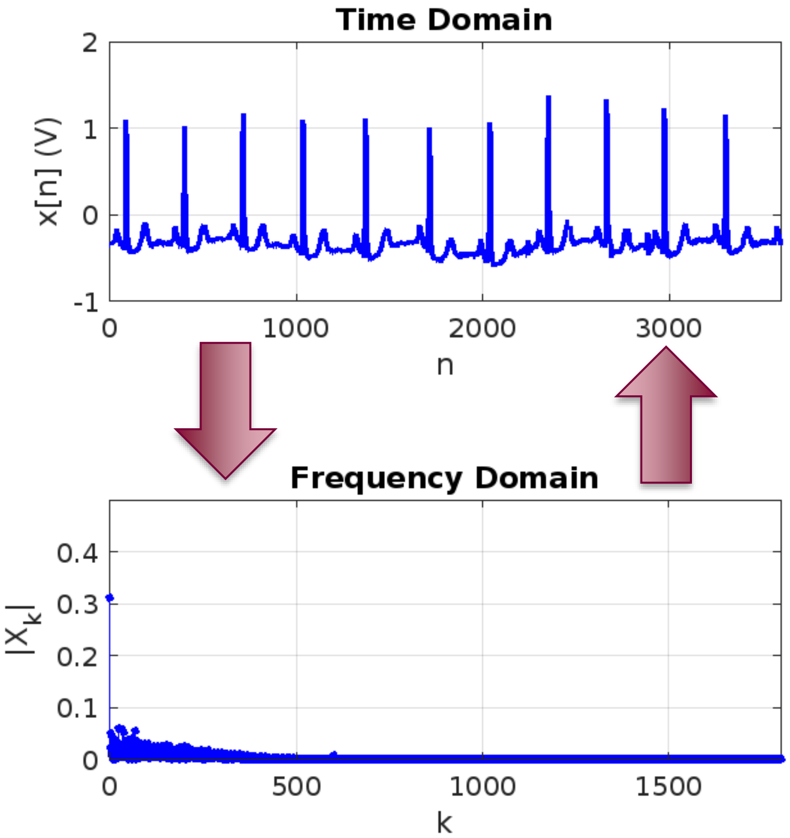




Inverse Discrete Fourier Transform

$$x[n] = \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, n = 0, 1, \ldots, N-1$$

Time and Frequency Domain of Electrocardiogram



- Frequency domain has no time component
- Frequencies that are fixed in time have clean frequency breakdowns
- ECG signal is full of frequencies that vary in time
  - We can not resolve it clearly in frequency domain

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4. **Filtering**

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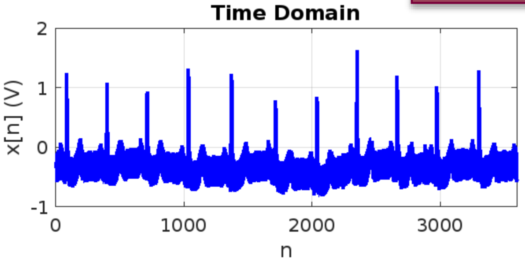
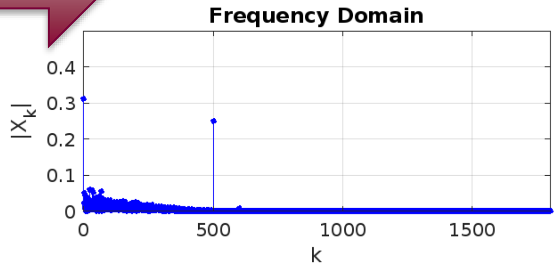
Filtering (Denoising) Step 1

Use FFT to convert ECG discrete-time domain to frequency domain

$$X_k = \sum_{n=0}^{N-1} x[n]e^{-\frac{j2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

Discrete Fourier Transform



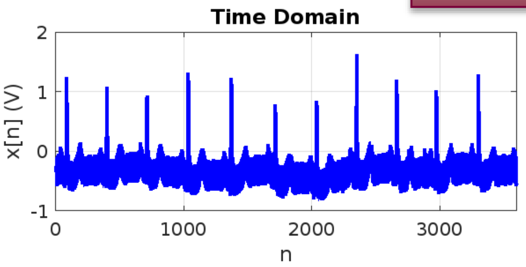


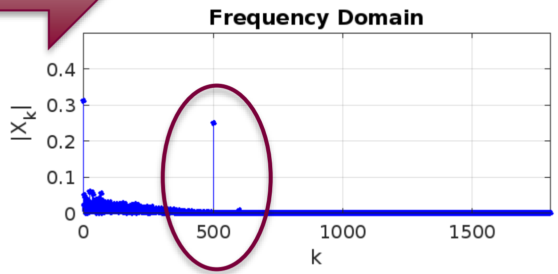
Filtering (Denoising) Step 2

Identify and isolate unwanted frequency components (e.g., 50 Hz)

$$X_k = \sum_{n=0}^{N-1} x[n]e^{-\frac{j2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

Discrete Fourier Transform





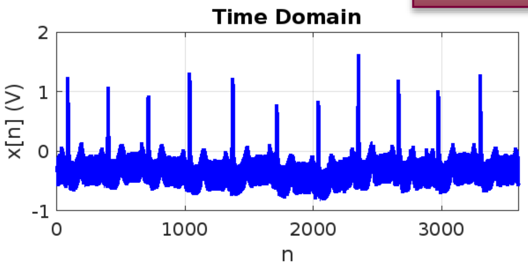
50 Hz noise occurs in ECG measurements because of magnetically induced interference, interference currents in the body, and interference currents in the electrode leads

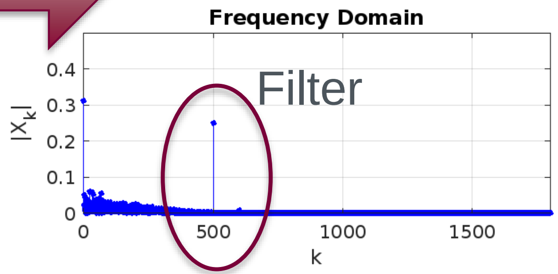
Filtering (Denoising) Step 3

Filter 50 Hz noise

$$X_k = \sum_{n=0}^{N-1} x[n]e^{-\frac{j2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

Discrete Fourier Transform





Filter Types

MATLAB has built in libraries for designing and testing filters

- Low-pass
  - Keep low frequencies
  - Rejects high
- High-pass
  - Keep high frequencies
  - Rejects low
- Band-pass
  - Keep frequencies in a certain range
  - Reject outside of range
- Band-stop (or notch)
  - Keep frequencies outside a certain range
  - Reject inside of range

Many Challenges Designing Filters

- Noise intermixed with target signal
- Filtering removes wanted signals

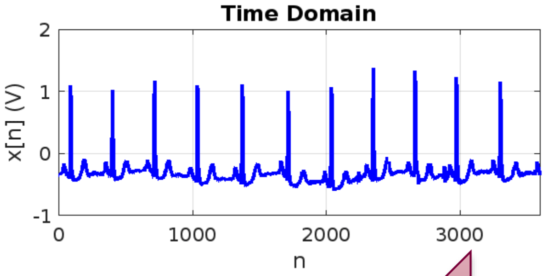
Use Fourier Transforms in voice and image analysis too!

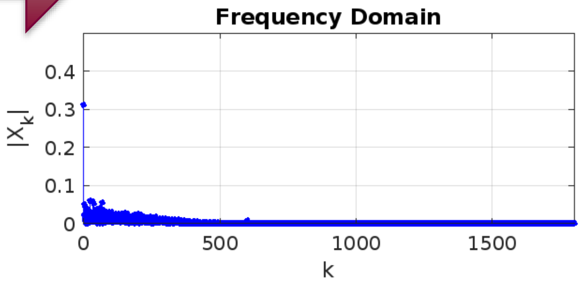
Filtering (Denoising) Step 4

Use IFT to convert clean frequency domain back to time domain

$$X_k = \sum_{n=0}^{N-1} x[n]e^{-\frac{j2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

Discrete Fourier Transform

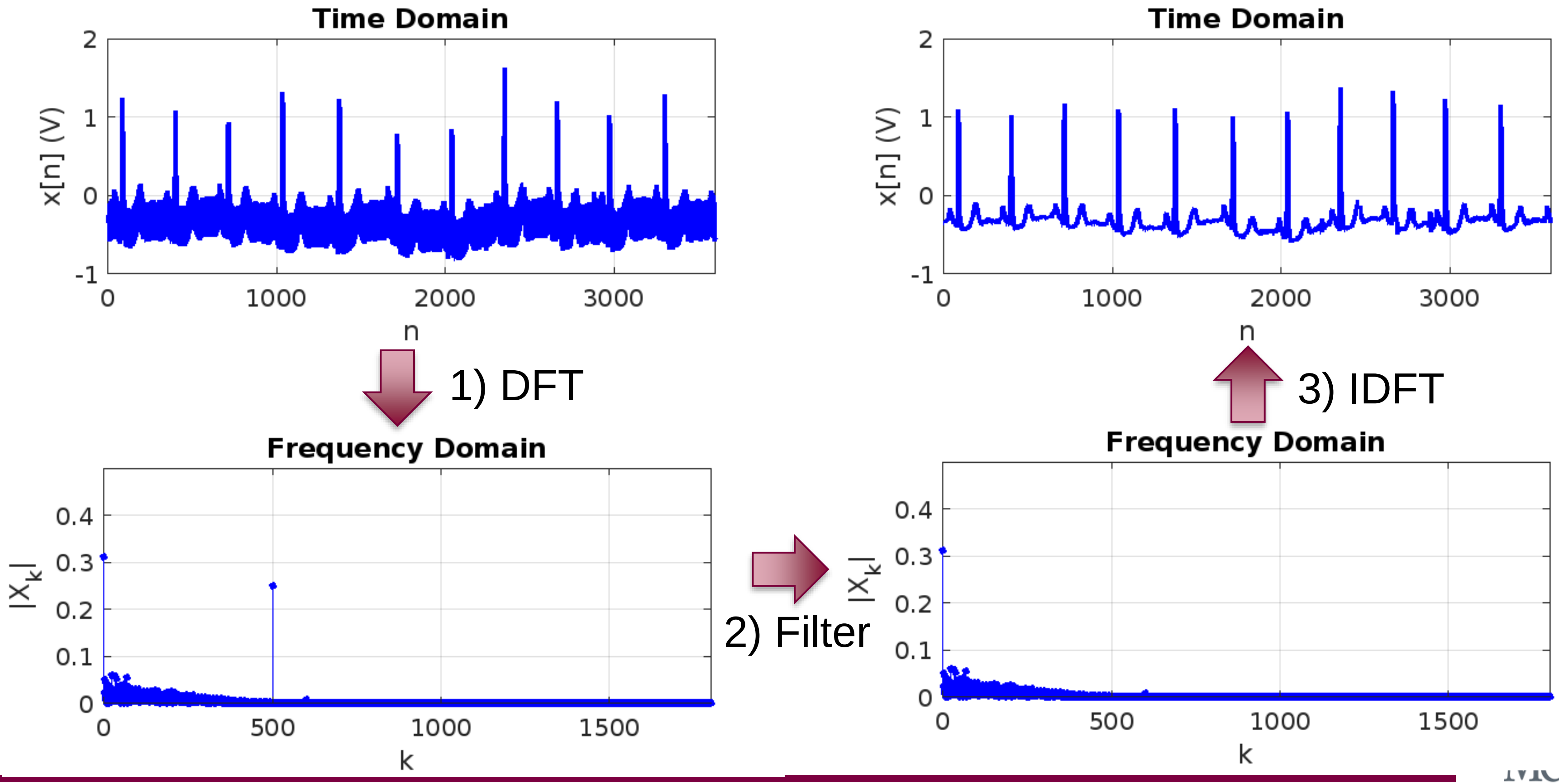




Inverse Discrete Fourier Transform

$$x[n] = \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, n = 0, 1, \ldots, N-1$$

From Time to Frequency Domain and Back Again



Summary

- Continuous-time signals are stored as discrete-time signals
- We can represent signals in the time and frequency domain
- The FFT and IFT are used to convert signals from time to frequency domain (and back)
- Filtering in the frequency domain allows us to remove noise and clean up our signal in the time domain

Lessons Learned

(Or what I wished I knew before I started)

- Co-op
  - Longer co-ops are easier to get then four month
  - Start looking early!
- Extracurricular engineering projects
  - Your undergraduate courses are a starting point
  - Pick your favorite courses and use them as a launching board for your own side projects (e.g., Raspberry Pi)
  - Usually what you discuss in interviews
- Writing (especially if you are interested in doing graduate school)
  - Academic writing is an important skill
  - University has lots of free resources you can take advantage on your own
  - Classes may not require it but you can take courses and apply it to your labs on your own
- Mental Health
  - Importance of weekends and breaks

References

- Kamen, Edward W., and Bonnie S. Heck. *Fundamentals of signals and systems using the web and matlab*. Prentice-Hall, Inc., 2006
- Brunton, Steven L., and J. Nathan Kutz. *Data-driven science and engineering: Machine learning, dynamical systems, and control*. Cambridge University Press, 2022.
- MATLAB Code
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  - Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PCh, Mark RG, Mietus JE, Moody GB, Peng C-K, Stanley HE. PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals. *Circulation* **101**(23):e215-e220 [Circulation Electronic Pages; http://circ.ahajournals.org/content/101/23/e215.full]; 2000 (June 13).

THANK YOU

Omar Boursalie

boursao@mcmaster.ca

Meet with students 2:15 pm- 2:45 pm MDCL 3515

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