The 1D Ising Model on a Quantum Computer

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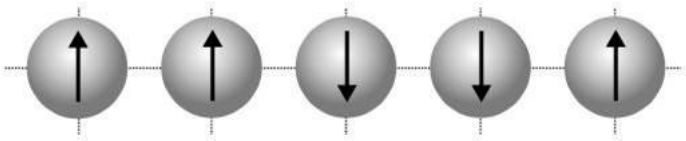
Background: Metropolis sampling of the Ising model

- Prepare initial system configuration with energy Econf
- Generate test configuration with energy Etest
- If Etest < Econf the test configuration becomes the new system configuration
- If not, the test configuration becomes the new configuration with probability p = exp(-(Etest-Econf)/T)
- Repeat



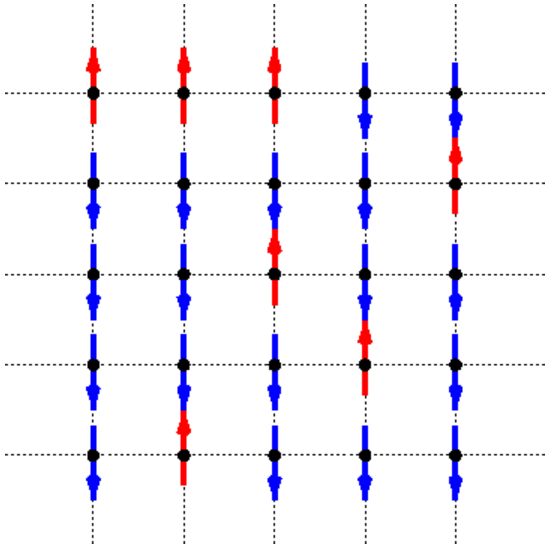


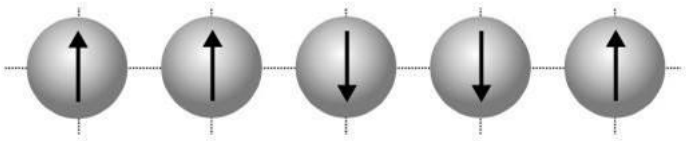
Image from https://personal.math.ubc.ca/~andrewr/research/intro_html/node14.html

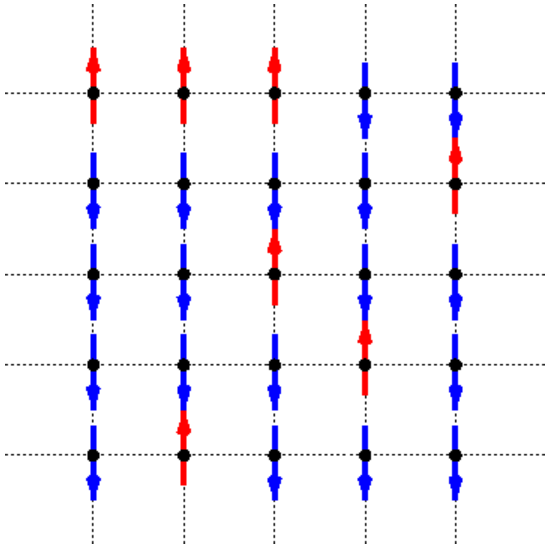
Background: Metropolis sampling of the Ising model

S

$$P = \exp(-\beta \cdot \Delta E)$$

S'





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1D Ising Model at H=0: the algorithm

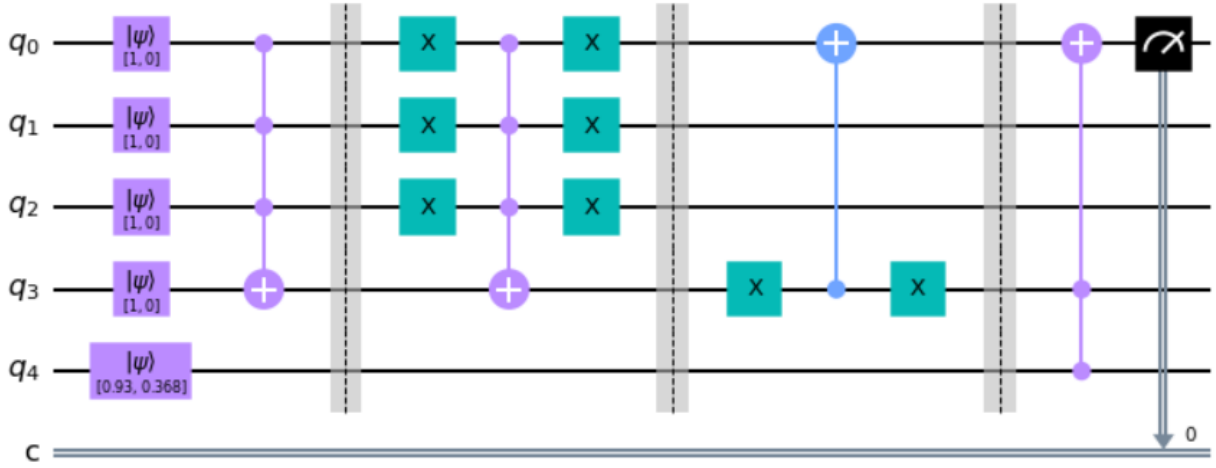
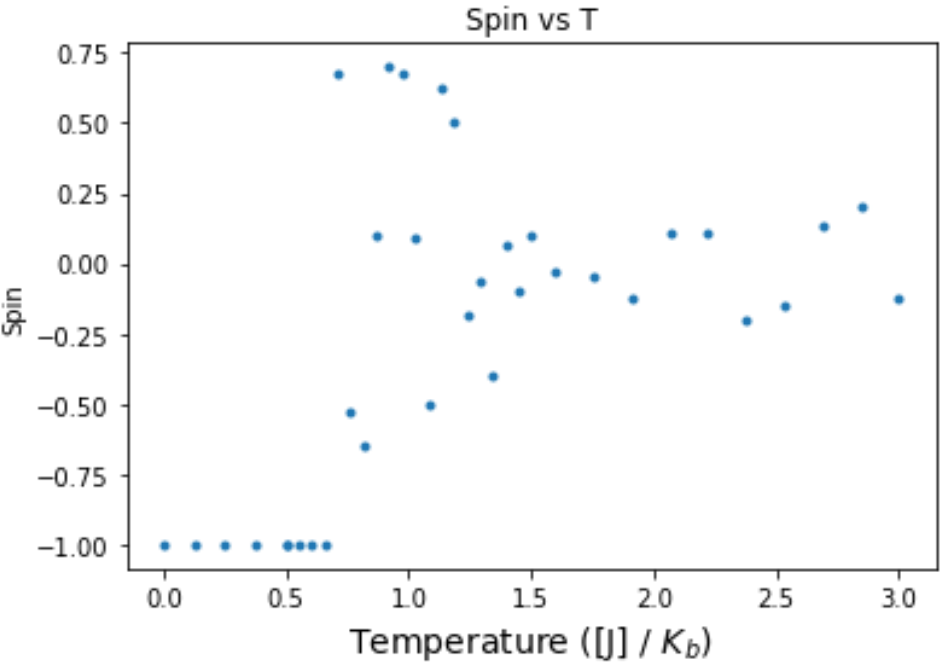


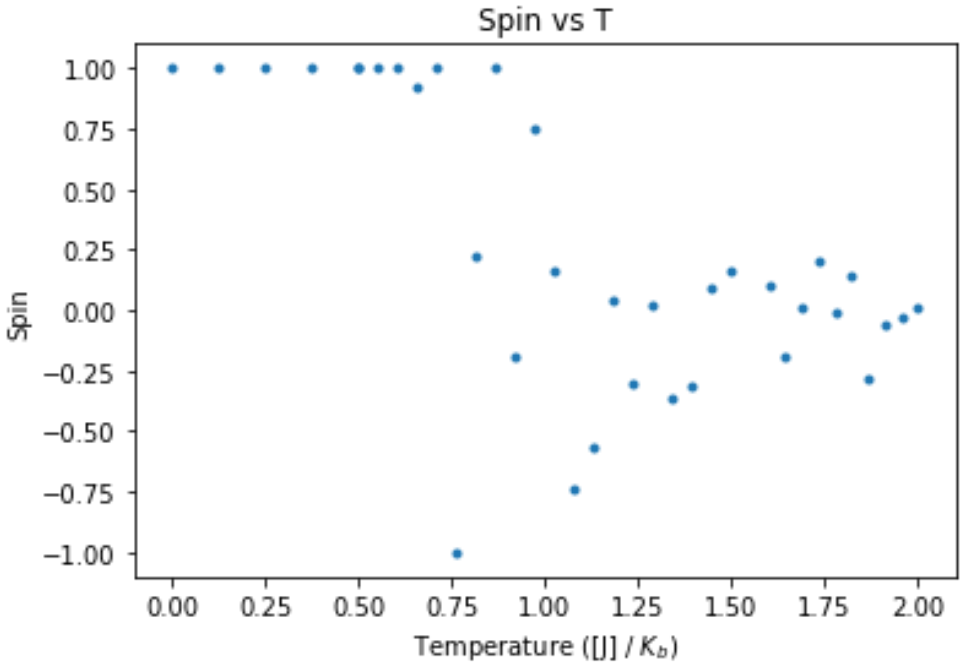
Table from: https://arxiv.org/abs/quant-ph/0404143

ASB	$\|S'\rangle$	S'_{cl}	p_{cl}
$\downarrow\downarrow\downarrow$	$\sqrt{P}\|\uparrow\rangle + \sqrt{1-P}\|\downarrow\rangle$	$\downarrow$	$1-P$
		$\uparrow$	P
$\downarrow\downarrow\uparrow$	$\|\uparrow\rangle$	$\uparrow$	1
$\downarrow\uparrow\downarrow$	$\|\downarrow\rangle$	$\downarrow$	1
$\downarrow\uparrow\uparrow$	$\|\downarrow\rangle$	$\downarrow$	1
$\uparrow\downarrow\downarrow$	$\|\uparrow\rangle$	$\uparrow$	1
$\uparrow\downarrow\uparrow$	$\|\uparrow\rangle$	$\uparrow$	1
$\uparrow\uparrow\downarrow$	$\|\downarrow\rangle$	$\downarrow$	1
$\uparrow\uparrow\uparrow$	$\sqrt{P}\|\downarrow\rangle + \sqrt{1-P}\|\uparrow\rangle$	$\uparrow$	$1-P$
		$\downarrow$	P

Note: $P = e^{-\frac{4J}{T}}$

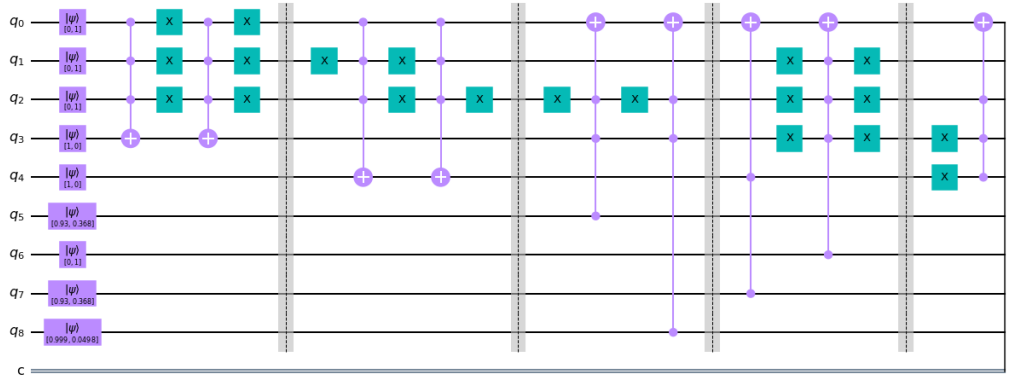
1D Ising Model at H=0: magnetization vs. temperature

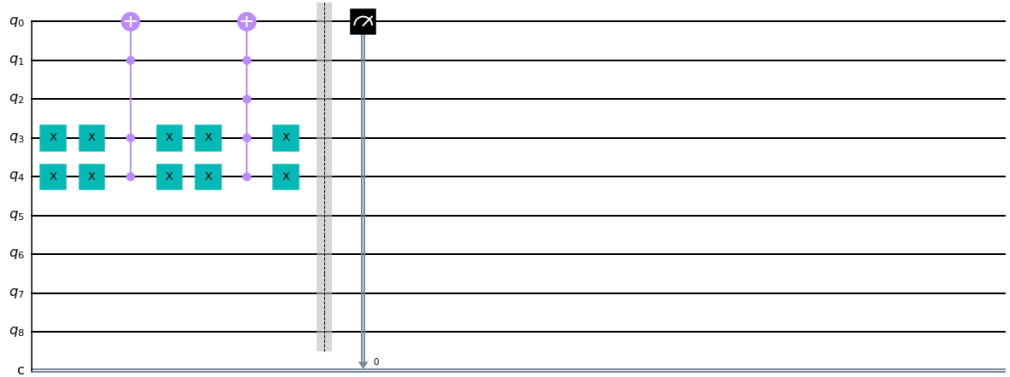




$|\uparrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow>$ Initial states $|\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,>$

Extending 1D Ising Model for H≠0: the algorithm





A S B	$\|S'\rangle$
↓ ↓ ↓	$\sqrt{P_1}\|\uparrow\rangle + \sqrt{1-P_1}\|\downarrow\rangle$
↓ ↓ ↑	$\|\uparrow\rangle$
↓ ↑ ↓	$\sqrt{P_2}\|\downarrow\rangle + \sqrt{1-P_2}\|\uparrow\rangle$
↓ ↑ ↑	$\sqrt{P_3}\|\downarrow\rangle + \sqrt{1-P_3}\|\uparrow\rangle$
↑ ↓ ↓	$\|\uparrow\rangle$
↑ ↓ ↑	$\|\uparrow\rangle$
↑ ↑ ↓	$\sqrt{P_3}\|\downarrow\rangle + \sqrt{1-P_3}\|\uparrow\rangle$
↑ ↑ ↑	$\sqrt{P_4}\|\downarrow\rangle + \sqrt{1-P_4}\|\uparrow\rangle$

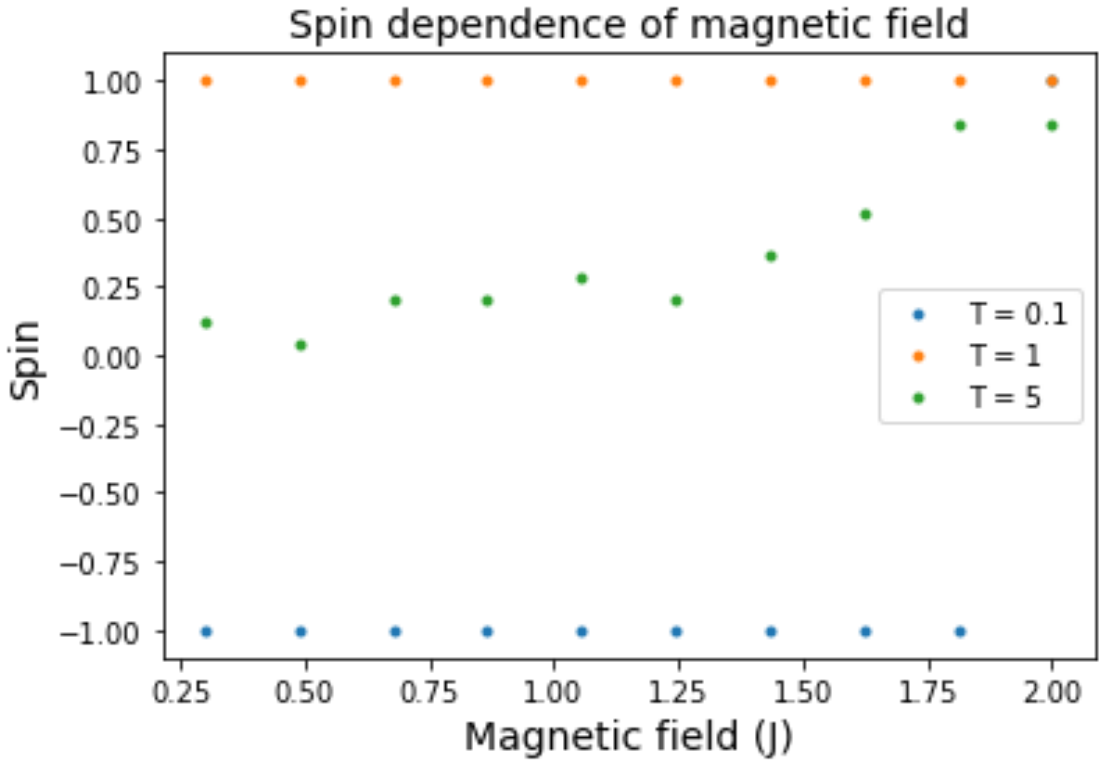
$$P_1 = \min\{1, \exp((-4J + 2h)/T)\}$$

$$P_2 = \min\{1, \exp((+4J - 2h)/T)\}$$

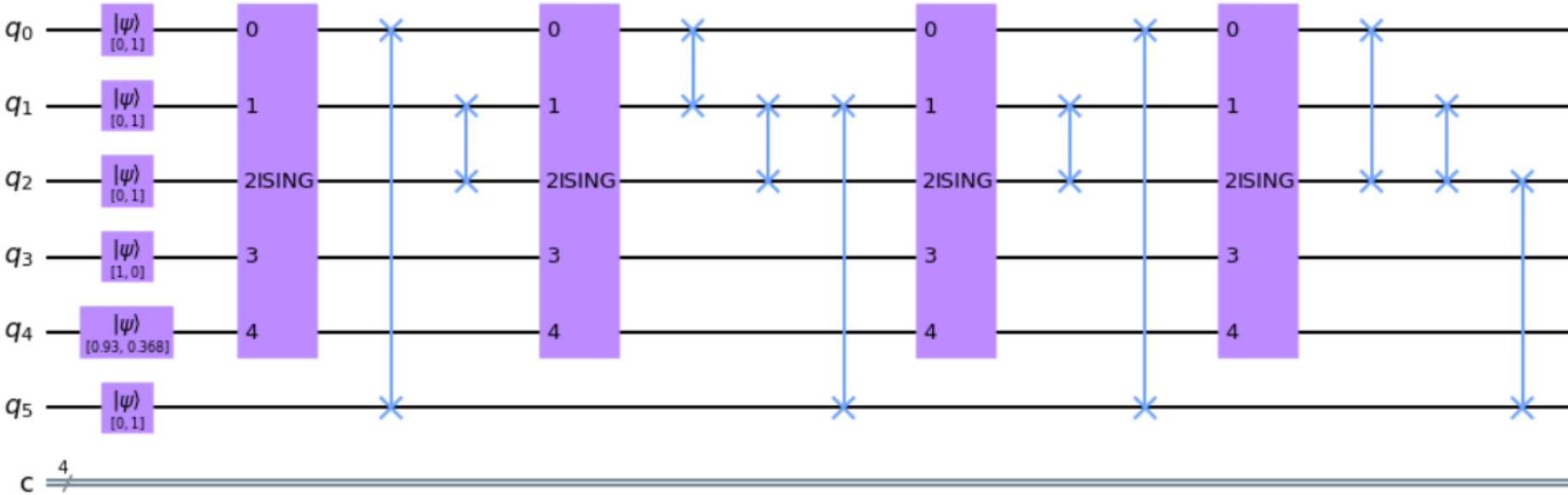
$$P_3 = \exp(-2h/T)$$

$$P_4 = \exp(-4J - 2h)/T)$$

Extending 1D Ising Model for H≠0: the results



Introducing entanglement: the algorithm



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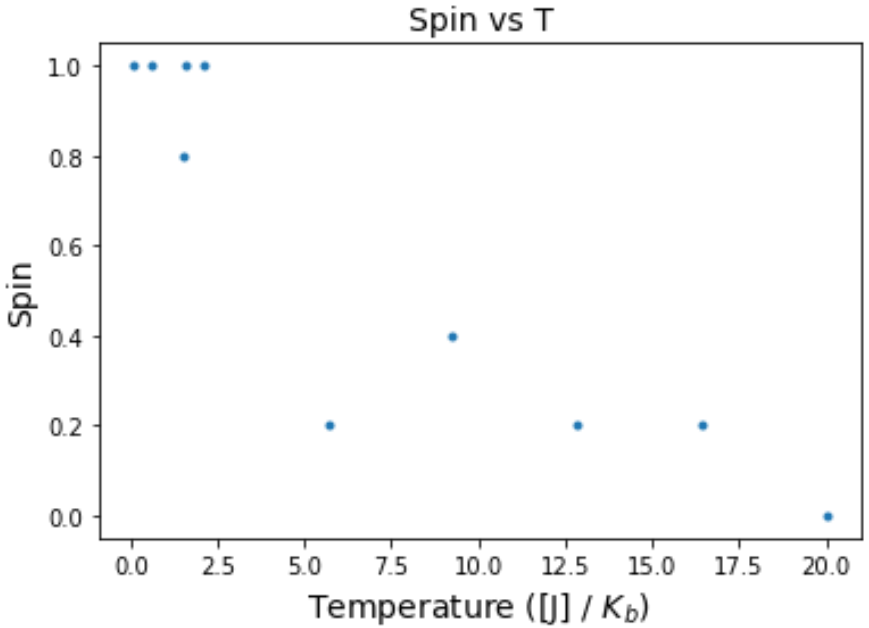
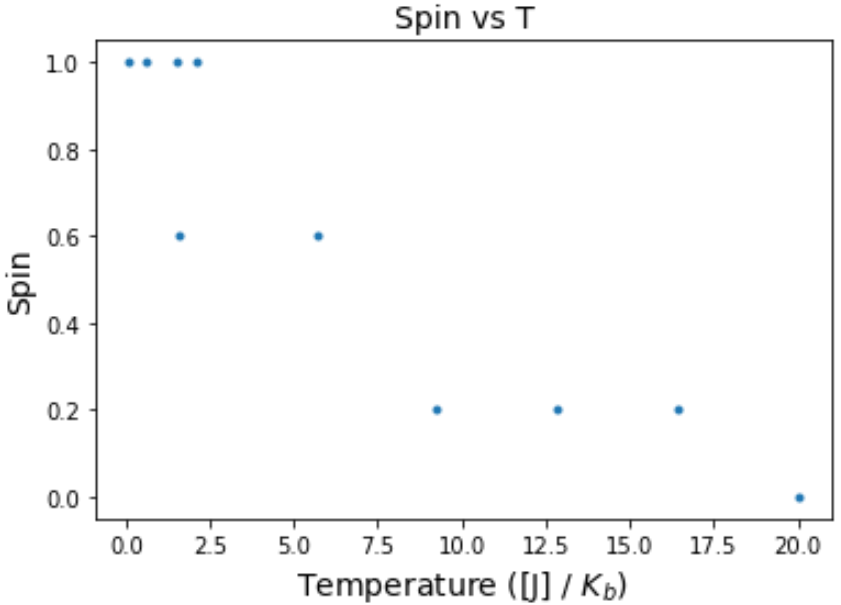
Introducing entanglement: the results



(1 iteration) $|\downarrow,\uparrow,\downarrow,\uparrow>$



(4 iterations) $|\downarrow,\uparrow,\downarrow,\uparrow>$