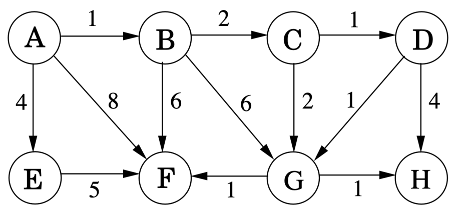
CMPSC 465 | Data Structures & Algorithms |
Fall 2025 | Ke Chen and Yana Safonova | Worksheet 6

Monday, Oct 13, 2025

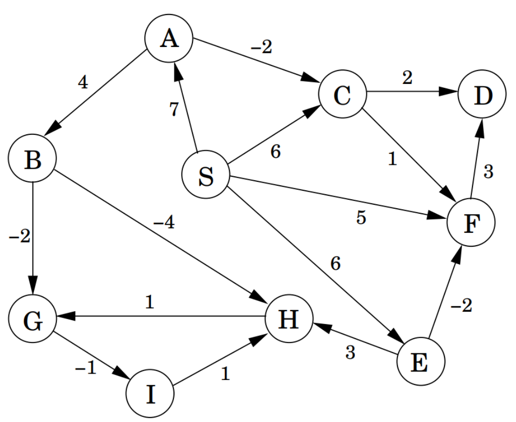
1. **Dijkstra's.** Suppose Dijkstra's Algorithm is run on the following graph, starting at node A.



   (i) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

   (ii) Show the final shortest-path tree.

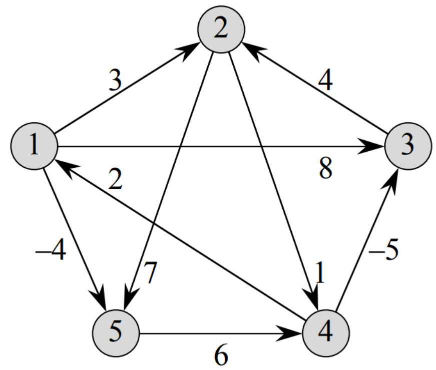
2. **Bellman-Ford.** Suppose Bellman-Ford is used to find all the shortest paths from node S.



   (i) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

   (ii) Show the final shortest-path tree.

3. **Floyd-Warshall.** Run Floyd-Warshall to find all pairs of shortest paths in the following graph. Show the distance matrix for each step of the algorithm, including the initial and final matrices.



4. Dijkstra's with Negative Edges. Professor F. Lake suggests the following algorithm for finding the shortest path from node s to node t in a directed graph with some negative edges: add a large constant to each edge weight so that all the weights become positive, then run Dijkstra's algorithm starting at node s, and return the shortest path found to node t. Is this a valid method? Either prove that it works correctly or give a counterexample.

5. Network Flow. Answer the following questions on the given flow network $G = (V, E)$:

(a) Consider a function $f : E \to \mathbb{N}$ defined by $f(s,a) = 6$, $f(s,b) = 5$, $f(a,c) = 5$, $f(a,d) = 2$, $f(b,c) = 2$, $f(b,d) = 3$, $f(c,t) = 8$, and $f(d,t) = 5$. Is it a valid flow on G? If not, list all the violations and fix them.

(b) It turns out the above (after your fixes) flow f is a maximum flow in this network. We call an edge in the network a *bottleneck edge* if increasing its capacity results in an increase in the maximum flow, namely, f can improved to have a larger value. List all bottleneck edges in the network G.

(c) Give an example of a network that has no bottleneck edges.