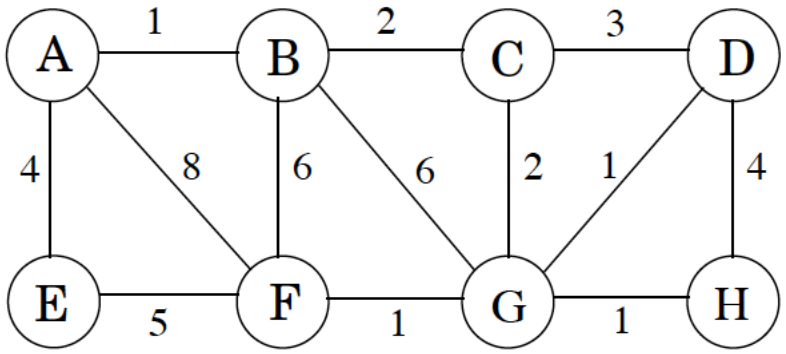
CMPSC 465 Data Structures & Algorithms
Fall 2025 Ke Chen and Yana Safonova

Worksheet 8

Monday, Oct 27, 2025

1. **Minimum Spanning Trees.** Run Prim's Algorithm to find a minimum spanning tree for the following graph. Whenever there is a choice of nodes, always use alphabetic ordering (e.g. start from node A). Show the order edges are added and the weight of the partial MST at each step.



2. **Minimum Spanning Trees and Subgraphs.** Let T be an MST of graph G. Given a connected subgraph H of G, show that $T \cap H$ is contained in some MST of H.

3. **Edge Weight Incrementing.** Consider an undirected graph $G = (V, E)$ with nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all nodes from a particular node $s \in V$.
Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$.

   (a) Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

   (b) Do the shortest paths change? Give an example where they change or prove they cannot change.

4. **Minimum Spanning Trees**

   (a) Given an undirected graph $G = (V, E)$ and a set $E' \subset E$, briefly describe how to update Kruskal's algorithm to find the minimum spanning tree that includes all edges from E'.

   (b) Assume you are given a graph $G = (V, E)$ with positive and negative edge weights and an algorithm that can return a minimum spanning tree when given a graph with only positive edges. Describe a way to transform G into a new graph G' containing only positive edge weights so that the minimum spanning tree of G can be easily found from the minimum spanning tree of G'.