CMPSC 465: LECTURE VII

More Examples of Divide-and-Conquer

Ke Chen

September 12, 2025

Median of medians as pivot for selection

Select(A, k):

1. Divide the input A into groups of 5.
2. Find the median of each group.
3. Recursively find the median m of all these $n/5$ medians.
4. Partition A with pivot m.
5. Do recursive call as in RandomizedSelect.

Time complexity?

- Worst-case: $T(n) = c \cdot n/5 + T(n/5) + \Theta(n) + T(\,?\,)$
$$= T(n/5) + T(\,?\,) + \Theta(n).$$

How good is MoM?

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19	7	15	42	5	88	91	4	29	21
13	67	54	18	20	73	8	36	49	2
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- m $\leq (n/5)/2 = n/10$ medians; also $\geq n/10$ medians.

$n = 30$, $m = 20$

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- m $\leq (n/5)/2 = n/10$ medians; also $\geq n/10$ medians.
- For each median $\leq m$, there are at least 2 more in its group $\leq m$.

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- For each median $\leq m$, there are at least 2 more in its group $\leq m$.
- For each median $\geq m$, there are at least 2 more in its group $\geq m$.

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- For each median $\leq m$, there are at least 2 more in its group $\leq m$.
- For each median $\geq m$, there are at least 2 more in its group $\geq m$.
- In total, we can guarantee $3(n/5)/2 = 3n/10$ numbers $\leq m$; also $3n/10$ numbers $\geq m$.

$n = 30$, $m = 20$

19	7	15	42	5	88	91	4	29	21
13	67	54	18	20	73	8	36	49	2
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$n = 30$, $m = 20$, 15 numbers $\leq m$, 15 numbers $\geq m$

19	7	15	5	4	13	18	3	8	2
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- (Exercise) Prove that $T(n) = O(n)$.
- (Exercise) What happens if we group by 3, or 7?

Matrix multiplication

Input: Two $n \times n$ matrices A and B

Output: $C = A \cdot B$

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Matrix multiplication by divide-and-conquer

Idea: Divide matrices into four $n/2 \times n/2$ blocks and perform multiplication block-wise.

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Same as direct computation!

Strassen's idea (1969)

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Let

$$\begin{array}{ll}
P_1 = A_{11}(B_{12} - B_{22}) & P_2 = (A_{11} + A_{12})B_{22} \\
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Then $A \cdot B = C = \begin{bmatrix} P_5 + P_4 - P_2 + P_6 & P_1 + P_2 \\ P_3 + P_4 & P_1 + P_5 - P_3 - P_7 \end{bmatrix}$

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Verify: $$\begin{aligned} P_1 + P_2 &= A_{11}(B_{12} - B_{22}) + (A_{11} + A_{12})B_{22} \\ &= A_{11}B_{12} - A_{11}B_{22} + A_{11}B_{22} + A_{12}B_{22} \\ &= A_{11}B_{12} + A_{12}B_{22} \end{aligned}$$

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$P_1, \ldots, P_7$ requires 7 $n/2 \times n/2$ matrix multiplications, therefore
$T(n) = 7T(n/2) + O(n^2) \rightsquigarrow$.

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$P_1, \ldots, P_7$ requires 7 $n/2 \times n/2$ matrix multiplications, therefore
$T(n) = 7T(n/2) + O(n^2) \rightsquigarrow O(n^{\log 7}) = O(n^{2.81})$.

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Can we do better?

Matrix multiplication

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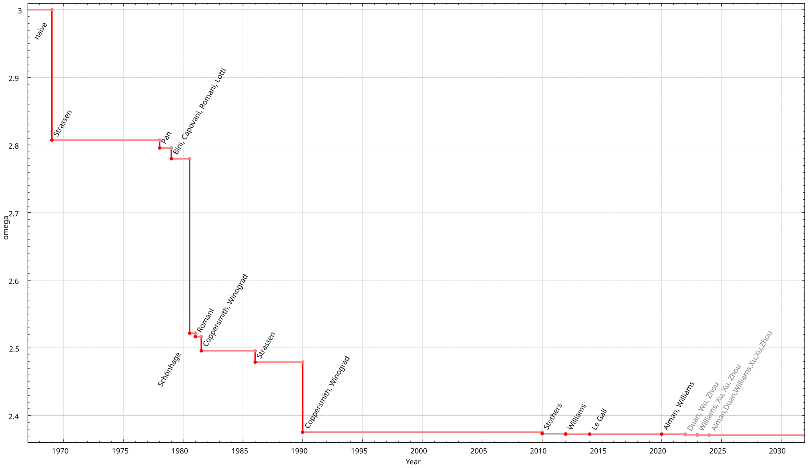
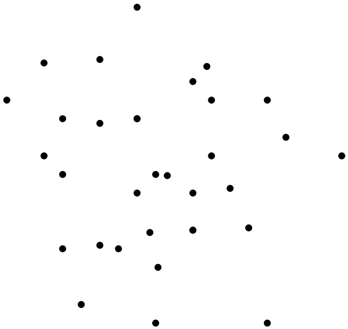


Figure by Jochen Burghardt

Closest pair of points

Input: A set of n points in the plane.

Output: A pair of points with the smallest Euclidean distance.



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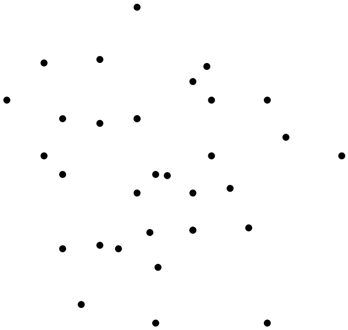

Brute-force: $O(n^2)$.

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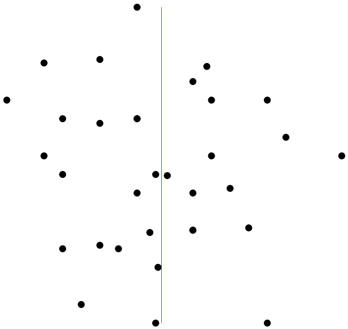

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Divide-and-conquer:

1. Divide the points into two parts.

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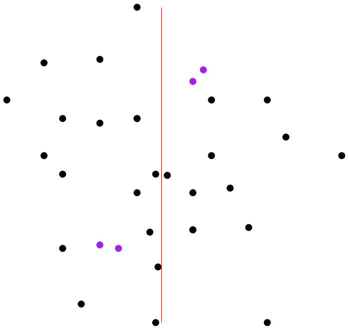

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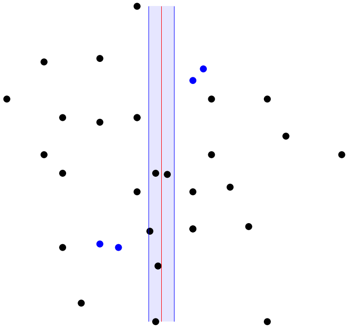

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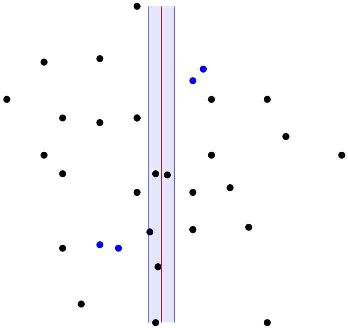

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Divide-and-conquer:

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Can be done in $O(n)$ time, therefore $T(n) = 2T(n/2) + O(n) \rightsquigarrow$

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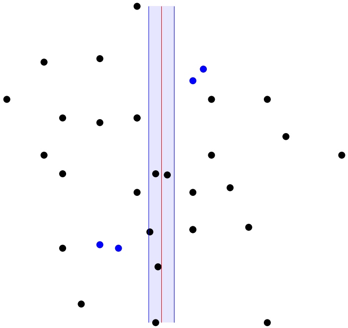
Input: A set of n points in the plane.

Output: A pair of points with the smallest Euclidean distance.



Brute-force: $O(n^2)$.

Divide-and-conquer:

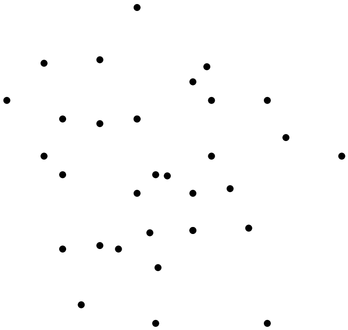
1. Divide the points into two parts.
2. Find a closest pair in each.
3. Find if any cross-boundary pair is closer.

Can be done in $O(n)$ time, therefore $T(n) = 2T(n/2) + O(n) \rightsquigarrow$ $\boxed{O(n \log n)}$.

Convex hull

Input: A set of n points in the plane.

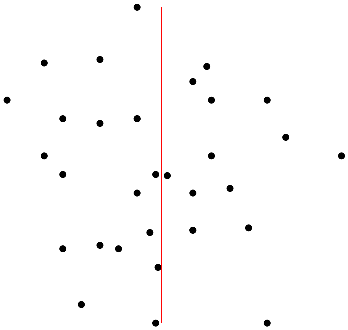
Output: The convex hull, i.e., smallest convex shape that contains all the points.



Convex hull

Input: A set of n points in the plane.

Output: The convex hull, i.e., smallest convex shape that contains all the points.



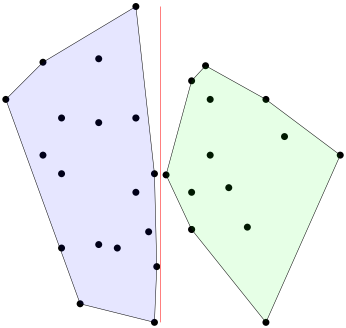
Divide-and-conquer:

1 Divide the points into two parts.

Convex hull

Input: A set of n points in the plane.

Output: The convex hull, i.e., smallest convex shape that contains all the points.



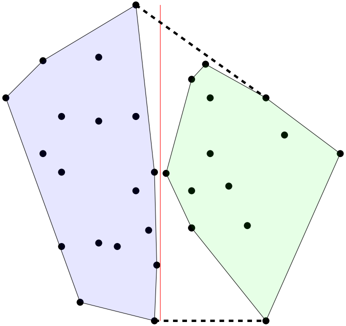
Divide-and-conquer:

1. Divide the points into two parts.
2. Compute the convex hull of each.

Convex hull

Input: A set of n points in the plane.

Output: The convex hull, i.e., smallest convex shape that contains all the points.



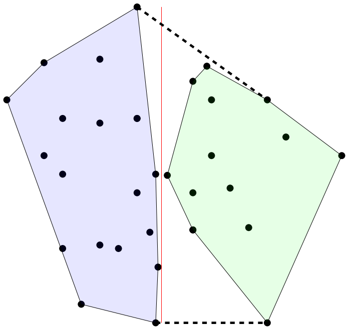
Divide-and-conquer:

1. Divide the points into two parts.
2. Compute the convex hull of each.
3. Merge the two convex polygons.

Convex hull

Input: A set of n points in the plane.

Output: The convex hull, i.e., smallest convex shape that contains all the points.



Divide-and-conquer:

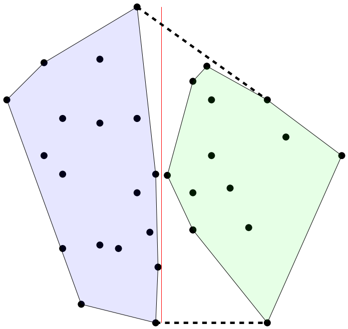
1. Divide the points into two parts.
2. Compute the convex hull of each.
3. Merge the two convex polygons.

Can be done in $O(n)$ time, therefore $T(n) = 2T(n/2) + O(n) \rightsquigarrow$

Convex hull

Input: A set of n points in the plane.

Output: The convex hull, i.e., smallest convex shape that contains all the points.



Divide-and-conquer:

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Can be done in $O(n)$ time, therefore $T(n) = 2T(n/2) + O(n) \rightsquigarrow O(n \log n)$.