

Monday, Oct 20, 2025

1. **Max Flow Min Cut** Consider the following network (Figure 1, the numbers are edge capacities).

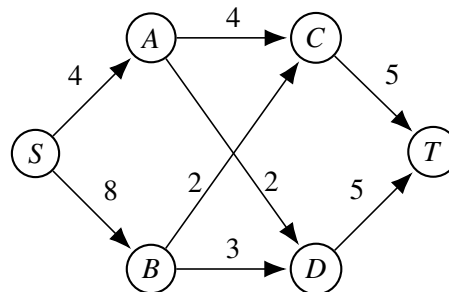


Figure 1: Flow Graph

- Find the maximum flow f and a minimum cut using Ford-Fulkerson algorithm.
 - Draw the residual graph G_f (along with its edge capacities). In this residual network, mark the vertices reachable from S and the vertices from which T is reachable.
2. **Max-Flow Extended** Suppose, there are many sources and many sinks, and we wish to maximize the total flow from all sources to all sinks. This problem can be solved efficiently. Show this by reducing it to the original `max-flow` problem.
3. **Verifying a max-flow** Suppose someone presents you with a solution to a max-flow problem on some network. Give a *linear* time algorithm to determine whether the solution does indeed give a maximum flow.
4. **Assigning Projects.** A company has n employees and m projects. Each employee is qualified to work on a subset of the projects, and each project requires exactly one employee. However, each employee can be assigned to at most one project. We need to determine whether it is possible to assign employees to projects so that every project is staffed, and if so, find such an assignment. Reduce this problem to max-flow problem and analyze why it is correct.