1. Set Identities: Use set identities to show that the following statements are true:

(i) $\overline{(A-B)} = \bar{A} \cup B$

(ii) $\overline{\overline{(A \cup B) \cap C} \cup \bar{B}} = B \cap C.$

Answer:

(i) $\overline{A-B} = \bar{A} \cup B$

From the definition of sets operations, $A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap \bar{B}$. Therefore,

$$
\begin{aligned}
\overline{A-B} &= \overline{A \cap \bar{B}} && \text{Definition of Set Difference}\\
&= \bar{A} \cup \bar{\bar{B}} && \text{DeMorgan's Law}\\
&= \bar{A} \cup B && \text{Law of Double Complement}
\end{aligned}
$$

(ii) $\overline{\overline{(A \cup B) \cap C} \cup \bar{B}} = B \cap C.$

$$
\begin{aligned}
&\overline{\overline{(A \cup B) \cap C} \cup \bar{B}}\\
&= \overline{\overline{((A \cup B) \cap C)}} \cap \bar{\bar{B}} && \text{DeMorgan's Law}\\
&= ((A \cup B) \cap C) \cap B && \text{Law of Double Complement}\\
&= (A \cup B) \cap (C \cap B) && \text{Associative Law}\\
&= (A \cup B) \cap (B \cap C) && \text{Commutative Law}\\
&= [(A \cup B) \cap B] \cap C && \text{Associative Law}\\
&= B \cap C && \text{Absorption Law}
\end{aligned}
$$

2. Propositions: Determine if the following sentences are propositions. If the sentence is a proposition, determine its truth value.

(i) There are a finite number of even integers.

(ii) If x is an integer number, $x+1$ is odd.

(iii) $x = x$

(iv) For all integer values of x, is $2x+1$ always odd?

Answer:

(i) The sentence is a proposition. Truth value: **False**

(ii) Sentence is not a proposition. Truth value changes depending on the value of x.

(iii) This sentence (or expression) is not a proposition. x is not well-defined and may not even be a numerical value. Similarly, the equals operator may not be well-defined for such cases. Thus, it is not possible to determine if the statement is true or false.

(iv) This is a question. Therefore, this is not a proposition.

3. **Truth Table:** Given the logical expression $\neg(p \wedge q) \oplus [(q \leftrightarrow p) \wedge (\neg q \to (p \wedge q))]$, construct a truth table to determine whether the expression is a tautology, contradiction or contingency.

Answer:

p	q	$\neg(p \wedge q)$	$q \leftrightarrow p$	$\neg q$	$p \wedge q$	$\neg q \to (p \wedge q)$	$(q \leftrightarrow p) \wedge (\neg q \to (p \wedge q))$	Entire Expression
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	F	F	T
F	T	T	F	F	F	T	F	T
F	F	T	T	T	F	F	F	T

4. **Converse, Inverse and Contrapositive:** Find the converse, inverse and contrapositive of the following statement.

If a car is fuel-efficient, then it is both economical and environmentally friendly.

Answer:

Converse: If a car is economical and environmentally friendly, then it is fuel-efficient.

Inverse: If a car is not fuel-efficient, then it is not economical or not environmentally friendly.

Contrapositive: If a car is not economical or not environmentally friendly, then it is not fuel-efficient.

5. **Propositional Logic:**

Let p be the proposition "Birds are reptiles.",
q be the proposition "All rivers flow uphill.",
r be "Squares have four sides," and
s be "Not all reptiles lay eggs."
Express the sentence "If all rivers flow uphill and not all reptiles lay eggs, then birds are reptiles or squares do not have four sides." using compound logic.

Answer:

$(q \wedge s) \to (p \vee \neg r)$

6. **Translation of Sentences:** Translate the following English statements using propositional logic

1. I will not pass this class unless I go to class every day and do all of the homework exercises.

2. Eating a balanced diet and exercising regularly is necessary for maintaining good health.

3. Reading books or attending seminars is sufficient for gaining knowledge.

4. Being a member of the club and having attended at least three meetings is necessary and sufficient for obtaining voting rights.

Answer:

1. **Answer:** p: I pass this class. g: I go to class everyday. h: I do all the homework exercises. Then the statement translates as: $(\neg g \vee \neg h) \to \neg p$ or $p \to (g \wedge h)$

2. **Answer:** b: eating a balanced diet. e: exercising regularly. h: maintaining good health. Then the statement translates as: $h \to (b \wedge e)$

3. **Answer:** r: reading books. s: attending seminars. k: gaining knowledge. The statement translates as: $(r \vee s) \to k$

4. **Answer:** m: member of the club. a: attended at least three meetings. v: obtain voting rights. The statement translates as: $(m \wedge a) \leftrightarrow v$