

**1. Fermat's Little Theorem:** Simplify the following using Fermat's Little Theorem:

(a)  $5^{300} \bmod 13$

(b)  $3^{47} \bmod 7$

**2. Linear Congruence:**

(a) Find all solutions, if possible to  $6x \equiv 4 \pmod{17}$

(b) Find all solutions, if possible to  $2x \equiv 5 \pmod{8}$

(c) Find all solutions, if possible to  $3x \equiv 6 \pmod{12}$

**3. Chinese Remainder Theorem:** Consider the following system of linear congruences:

$$x \equiv 2 \pmod{4}$$

$$6x \equiv 3 \pmod{15}$$

Solve for all solutions using CRT. (Hint: There are 3 unique solutions modulo 60)

**4. RSA:**

(a) Given  $p = 23$  and  $q = 19$ . Compute the public and the private keys.

(b) Daniel wants to send the message  $M = 13$  to Alice. Using Alice's public and private keys, calculate the ciphertext  $C$ , and the value for  $R$  when Alice recovers the message.