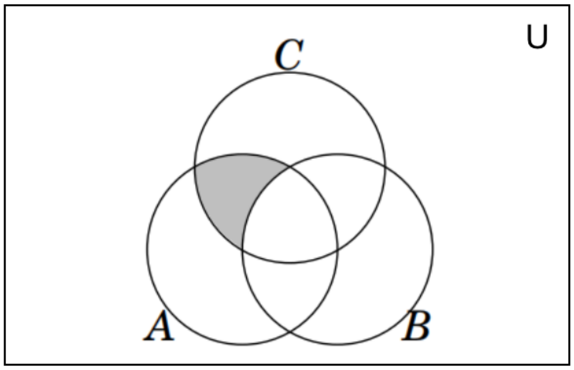
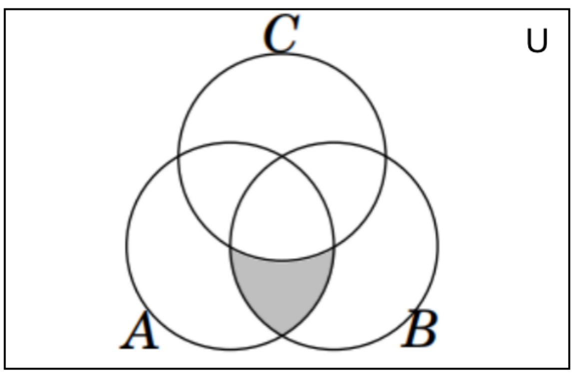
CMPSC 360 Discrete Mathematics for Computer Science
Fall 2024 Mahfuza Farooque

Worksheet 1 Solutions

1. Venn Diagrams: Consider three sets A, B, and C. Based on the questions below either draw a Venn diagram representing the set operation or name the set operation based on the Venn diagram.

1. $(A - B) \cap C$



2. 

$(A \cap B) - C$

2. Roster Method: Describe the sets given below using the Roster method.

(i) The set of all even prime numbers.
Answer: $\{2\}$

(ii) The set of all real-valued solutions for the equation $x^5 - x^4 + x - 1 = 0$.
Answer: $x^5 - x^4 + x - 1 = (x-1)(x^4+1) = 0 \implies x = 1 \implies \{1\}$

(iii) The set of all letters in the word "MATHEMATICS" that are consonants.
Answer:
{M, T, H, C, S}

(iv) The set of all integers x such that x is a perfect square between 10 and 50.
Answer:
$\{16, 25, 36, 49\}$

3. Set Builder method: Describe the sets given below using the Set Builder method.

(i) $\{-\sqrt{3}, \sqrt{3}\}$

(ii) $\{2, 4, 8, 16, 32, 64, \ldots\}$

(iii) $\{\ldots, -11, -6, -1, 4, 9, 14, 19, 24, 29, \ldots\}$

(iv) $\{1, 3, 6, 10, 15, \ldots\}$

(v) $\{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$

Answer:

(i) $\{x \in \mathbb{R} \,|\, x^2 = 3\}$

(ii) $\{2^x \,|\, x \in \mathbb{N}\}$

(iii) $\{5x - 1 \,|\, x \in \mathbb{Z}\}$

(iv) $\{x \,|\, x = \frac{n(n+1)}{2}, n \in \mathbb{N}\}$

(v) $\{2^n \,|\, n \in \mathbb{Z}\}$

4. Power set: Write the power set of each of the following sets in roster form:

(i) $\{1, (0,3), \{1\}\}$

(ii) $\{\#, \{n, m\}, \varnothing\}$

Answer:

(i) For the set $\{1, (0,3), \{1\}\}$: The power set is the set of all possible subsets.

$$
\begin{aligned}
&\{\}, \\
&\{1\}, \\
&\{(0,3)\}, \\
&\{\{1\}\}, \\
&\{1, (0,3)\}, \\
&\{1, \{1\}\}, \\
&\{(0,3), \{1\}\}, \\
&\{1, (0,3), \{1\}\}.
\end{aligned}
$$

(ii) For the set $\{\#, \{n, m\}, \varnothing\}$: The power set is the set of all possible subsets.

$$\begin{aligned}
&\{\},\\
&\{\#\},\\
&\{\{n,m\}\},\\
&\{\varnothing\},\\
&\{\#,\{n,m\}\},\\
&\{\#,\varnothing\},\\
&\{\{n,m\},\varnothing\},\\
&\{\#,\{n,m\},\varnothing\}.
\end{aligned}$$

5. Cartesian Products:

1. Suppose $A = \{0,1\}$ and $B = \{1,2\}$. Find out $(\wp(A)\cap\wp(B))\times(\wp(A)-\wp(B))$.
   First, we evaluate:

$$\wp(A)\cap\wp(B) = \big\{\emptyset,\{1\}\big\}.$$

$$\wp(A)-\wp(B) = \big\{\{0\},\{0,1\}\big\}.$$

   Finally,

$$\begin{aligned}
(\wp(A)\cap\wp(B))\times(\wp(A)-\wp(B)) =& \big\{\emptyset,\{1\}\big\}\times\big\{\{0\},\{0,1\}\big\}\\
=& \big\{(\emptyset,\{0\}),(\emptyset,\{0,1\}),(\{1\},\{0\}),(\{1\},\{0,1\})\big\}.
\end{aligned}$$

2. (i) How many elements are in $\{\}\times\{1,2\}$? (ii) Find out $\{\emptyset\}\times\{0,\emptyset\}\times\{0,1\}$
   (i):

$$\{\}\times\{1,2\} = \{(a,b)\mid a\in\emptyset, b\in\{1,2\}\} = \{\}.$$

   There are no ordered pairs (a,b) with $a\in\emptyset$.
   (ii)

$$\{\emptyset\}\times\{0,\emptyset\}\times\{0,1\} = \{(\emptyset,0,0),(\emptyset,0,1),(\emptyset,\emptyset,0),(\emptyset,\emptyset,1)\}$$

6. Sets Identity Laws: Use set identities for the following subproblems. Let A, B, and C be sets:

(i) Show that $(A-B)-C = A-(B\cup C)$

(ii) Show that $(B-A)\cup(C-A) = (B\cup C)-A$

(iii) Show that $A-(B\cup C) = (A-B)\cap(A-C)$

Answer:

(i)

$$\begin{aligned}
(A-B)-C &= \{x\mid x\in A\wedge x\notin B\}-C \text{ (By definition of Set Difference)}\\
&= \{x\mid x\in A\wedge x\notin B\wedge x\notin C\} \text{ (By definition of Set Difference)}\\
&= \{x\mid x\in A\wedge x\notin(B\cup C)\} \text{ (By De Morgan Law)}\\
&= A-(B\cup C) \text{ (By definition of Set Difference)}
\end{aligned}$$

(ii)

$$\begin{aligned}
(B-A)\cup(C-A) &= \{x \mid x\in B \wedge x\notin A\}\cup\{x \mid x\in C\wedge x\notin A\} \text{ (By definition of Set Difference)}\\
&= \{x \mid (x\in B\wedge x\notin A) \text{ OR } (x\in C\wedge x\notin A)\} \text{ (By definition of Set Union)}\\
&= \{x \mid (x\in B \text{ OR } x\in C)\wedge x\notin A\} \text{ (By Distributive Law)}\\
&= \{x \mid x\in (B\cup C)\wedge x\notin A\} \text{ (By definition of Set Union)}\\
&= (B\cup C)-A \text{ (By definition of Set Difference)}
\end{aligned}$$

(iii)

$$\begin{aligned}
A-(B\cup C) &= \{x \mid x\in A\wedge x\notin (B\cup C)\} \text{ (By definition of Set Difference)}\\
&= \{x \mid x\in A\wedge \neg(x\in B \text{ OR } x\in C)\} \text{ (By definition of Set Complement)}\\
&= \{x \mid x\in A\wedge (x\notin B) \text{ AND } (x\notin C)\} \text{ (By De Morgan Law)}\\
&= \{x \mid x\in A\wedge x\notin B\}\cap\{x \mid x\in A\wedge x\notin C\} \text{ (By definition of Set Intersection)}\\
&= (A-B)\cap(A-C) \text{ (By definition of Set Difference)}
\end{aligned}$$