Grammar

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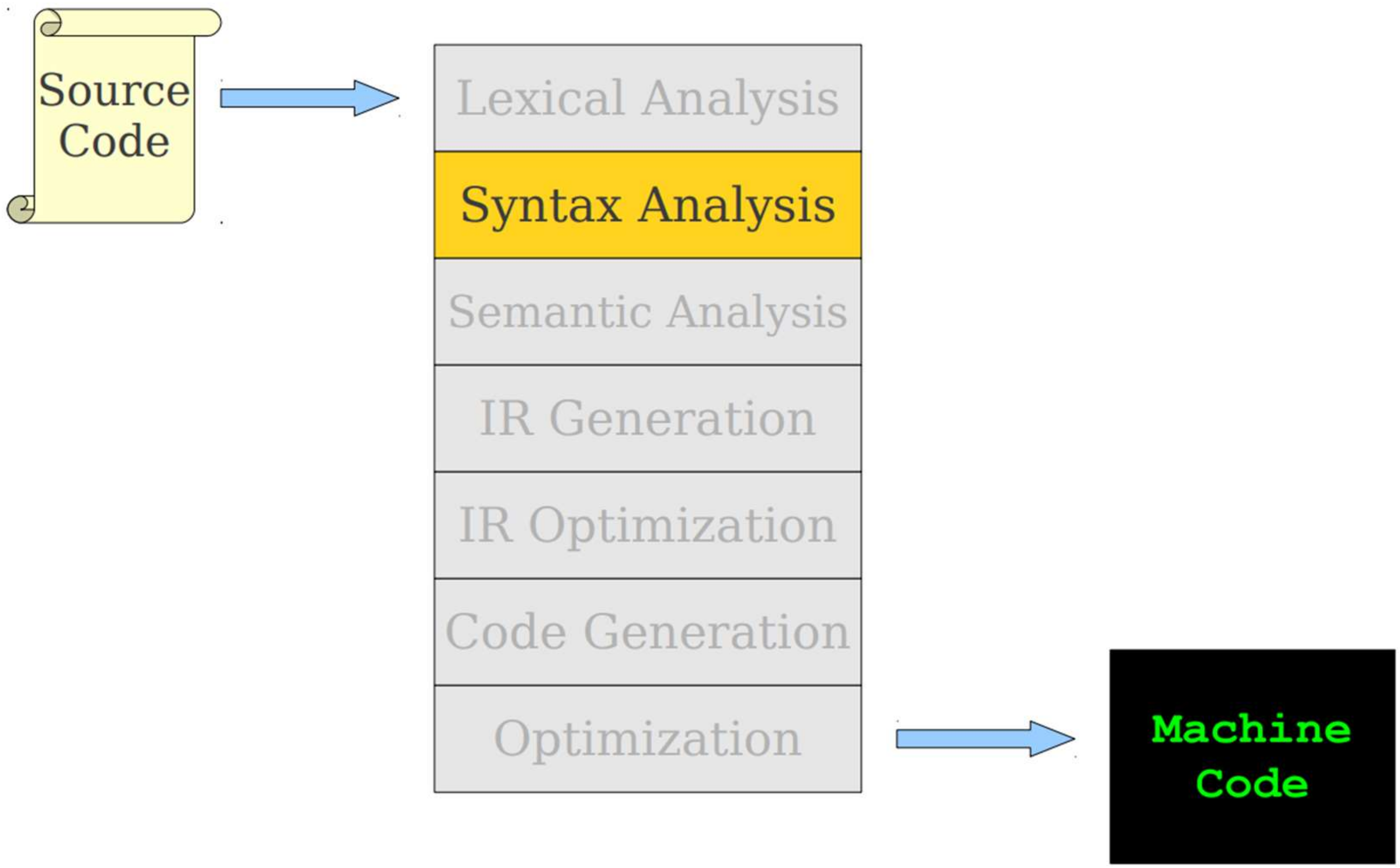
Where we are?



Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

What is Syntax Analysis?

- After lexical analysis (scanning), we have a series of tokens.
- In Syntax analysis (or parsing), we want to interpret what those tokens mean.
- Goal: Recover the *structure* describe by that series of tokens.
- Goal: Report errors if those tokens do not properly encode a structure.

Formal Languages

- An alphabet is a set Σ of symbols that act as letters.
- A language over Σ is a set of strings made from symbols in Σ.
- When scanning, our alphabet was ASCII or Unicode characters. We produced tokens.
- When parsing, our alphabet is the set of tokens produced by the scanner.

The Limits of Regular Languages

- When scanning, we used regular expressions to define each token.
- Unfortunately, regular expressions are (usually) too weak to define programming languages.
  - Cannot define a regular expression matching all expressions with properly balanced parentheses.
  - Cannot define a regular expression matching all functions with properly nested block structure.
- We need a more powerful formalism.

Grammars

- It is written in a metalanguage
- It defines all the legal strings of characters that can form a syntactically valid program

Context-Free Grammars

- Context-Free Grammars
  - Developed by Noam Chomsky in the mid-1950s
  - Describe the syntax of natural languages
  - Define a class of languages called context-free languages
  - Was originally designed for natural languages

Context-Free Grammars

- Using the notation of Backus-Naur Form (BNF) to describe CFG
- A grammar G <N, T, P, S> consists of the following
  - A finite set N of non-terminal symbols
  - A finite set T of terminal symbols, that is disjoint from N
  - A finite set P of production rules of the form

$$A \rightarrow \omega$$

  where ω is a string of nonterminal and terminal

  - Start symbol

Backus-Naur Form (BNF) Grammars

- A rule has a left-hand side (LHS), one or more right-hand side (RHS), and consists of terminal and nonterminal symbols
  - For instance
    - <binaryDigit> $\rightarrow$ 0
    - <binaryDigit> $\rightarrow$ 1
  - We can write <binaryDigit> $\rightarrow$ 0 | 1

Extended BNF Grammar

- Extended BNF simplifies writing a grammar by introducing metasymbols for iteration; option, and choice
- BNF

<expr> := <expr> + <term>
| <expr> - <term> | <term>

- EBNF

<expr> := <expr> {(+ | -) <term>} | <term>

Extended BNF Grammar

PennState

- BNF

<ifStmt> := if (<expr>) <stmt>
| if (<expr>) <stmt> else <stmt>

- EBNF

<ifStmt> := if (<expr>) <stmt> [else <stmt>]

Extended BNF Grammar

- However, EBNF is any more powerful than BNF for formally describing language syntax

$$A \rightarrow x\ \{y\}\ z$$

- Equivalent to

$$A \rightarrow x\ A'\ z$$
$$A' \rightarrow \varepsilon \mid y\ A'$$

Derivation

- To determine that the given string of symbols belongs to grammar
- A sequence of steps where nonterminals are replaced by the right-hand side of a production is called a derivation.
  - Leftmost derivation
  - Rightmost derivation

- Sentential form vs Sentence
  - A *sentential form* is any string derivable from the start symbol.
  - A *sentence* is a sentential form consisting only of terminals

- If string α derives string ω, we write $\alpha \Rightarrow^* \omega$.

Leftmost Derivation

- Say, we have grammar

$$\text{Integer} \rightarrow \text{Digit} \mid \text{Integer Digit}$$

$$\text{Digit} \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

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- 352 is an Integer?

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$$\begin{aligned}\text{Integer} &\rightarrow \text{Integer Digit}\\ &\rightarrow \text{Integer Digit Digit}\\ &\rightarrow \text{Digit Digit Digit}\\ &\rightarrow 3 \text{ Digit Digit}\end{aligned}$$

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What if I choose

$$\text{Integer} \rightarrow \text{Digit}$$

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$$\text{Integer} \Rightarrow^* 352$$

Rightmost Derivation

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$$\begin{aligned} \text{Integer} \quad & \rightarrow \text{Integer } {\color{red}\text{Digit}} \\ & \rightarrow \text{Integer } {\color{cyan}2} \end{aligned}$$

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$$\text{Digit} \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

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$$\begin{aligned} \text{Integer} \quad & \rightarrow \text{Integer Digit} \\ & \rightarrow \color{red}{\text{Integer}}\ \color{black}{2} \\ & \rightarrow \color{cyan}{\text{Integer Digit}}\ \color{black}{2} \end{aligned}$$

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& \rightarrow \text{Digit } 5\ 2 \\
& \rightarrow 3\ 5\ 2
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&\rightarrow \text{Integer } 5\ 2 \\
&\rightarrow \text{Digit } 5\ 2 \\
&\rightarrow 3\ 5\ 2
\end{aligned}$$

$$\text{Integer} \Rightarrow^* 352$$

The Language of a Grammar

- If *G* is a CFG with alphabet Σ and start symbol S, then the *language of G* is the set

$$L(G) = \{\omega \in \Sigma^* \mid S \Rightarrow^* \omega\}$$

- That is, L(G) is the set of strings derivable from the start symbol.

- Note: ω must be in Σ^*, the set of strings made from terminals. String involving nonterminals aren't in the language.

Context-Free Languages

- A language L is called a context-free language (or CFL) if there is a CFG *G* such that L = L(G).

From Regexes to CFGs

- CFGs consist purely of production rules of the form $A \longrightarrow \omega$. They do not have the regular expression operators * or ∪.
- However, we can convert regular expressions to CFGs as follows:

$$S \longrightarrow a^*b$$

From Regexes to CFGs

- CFGs consist purely of production rules of the form $A \rightarrow \omega$. They do not have the regular expression operators * or ∪.
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$$S \rightarrow Ab$$
$$A \rightarrow Aa \mid \varepsilon$$

From Regexes to CFGs

- CFGs consist purely of production rules of the form $A \rightarrow \omega$. They do not have the regular expression operators * or ∪.
- However, we can convert regular expressions to CFGs as follows:

$$S \rightarrow a(b \cup c^*)$$

From Regexes to CFGs

- CFGs consist purely of production rules of the form $A \rightarrow \omega$. They do not have the regular expression operators * or ∪.
- However, we can convert regular expressions to CFGs as follows:

$$S \rightarrow aX$$
$$X \rightarrow b \mid C$$
$$C \rightarrow Cc \mid \varepsilon$$

Regular Languages and CFLs

- Theorem: Every regular language is context-free.
- Proof Idea: Use the construction from the previous slides to convert a regular expression for L into a CFG for L.
- Problem Set Exercise: Instead, show how to convert a DFA/NFA into a CFG

The Language of a Grammar

- Consider the following CFG G:

$$S \rightarrow aSb \mid \varepsilon$$

- What strings can this generate?

a	a	a	a	b	b	b	b

$$L(G) = \{\, a^n b^n \mid n \in \mathbb{N} \,\}$$

Designing CFGs

- Like designing DFAs, NFAs, and regular expressions, designing CFGs is a craft.
- When thinking about CFGs:
  - Think recursively: Build up bigger structures from smaller ones.
  - Have a construction plan: Know in what order you will build up the string.
  - Store information in nonterminals: Have each nonterminal correspond to some useful piece of information.

Designing CFGs

- Let $\Sigma = \{a, b\}$ and let $L = \{w \in \Sigma^* \mid w \text{ is a palindrome}\}$
- We can design a CFG for L by thinking inductively:
- Base case: ε, a, and b are palindromes.
- If ω is a palindrome, then $a\omega a$ and $b\omega b$ are palindromes.

$$S \rightarrow \varepsilon \mid a \mid b \mid aSa \mid bSb$$

Designing CFGs

- Let $\Sigma = \{\, (, \,) \,\}$ and let $L = \{w \in \Sigma^* \mid w$ is a string of balanced parentheses $\}$
- Some sample string in L

(((())))

(())()

(()())(()())

((((()))(()))))

ε

()()

Designing CFGs

- Let $\Sigma = \{ (,) \}$ and let $L = \{w \in \Sigma^* \mid w \text{ is a string of balanced parentheses} \}$
- Let's think about this recursively.
  - Base case: the empty string is a string of balanced parentheses.
  - Recursive step: Look at the closing parenthesis that matches the first open parenthesis.

((()(()))(()))(())((()))

Designing CFGs

- Let $\Sigma = \{ (,) \}$ and let $L = \{w \in \Sigma^* \mid \text{w is a string of balanced parentheses} \}$
- Let's think about this recursively.
  - Base case: the empty string is a string of balanced parentheses.
  - Recursive step: Look at the closing parenthesis that matches the first open parenthesis. Removing the first parenthesis and the matching parenthesis forms two new strings of balanced parentheses.

$$S \rightarrow (S)S \mid \varepsilon$$

Designing CFGs: A Caveat

- Let $\Sigma = \{ a, b \}$ and let $L = \{w \in \Sigma^* \mid w$ has the same number of a's and b's $\}$
- Is this a CFG for L?

$$S \rightarrow aSb \mid bSa \mid \varepsilon$$

- Can you derive the string $abba$?

Designing CFGs: A Caveat

- When designing a CFG for a language, make sure that it
  - generates all the strings in the language and
  - never generates a string outside the language.
- The first of these can be tricky – make sure to test your grammars!

CFG Caveats II

- Is the following grammar a CFG for the language $\{\ a^n b^n \mid n \in \mathbb{N}\ \}$?

$$S \rightarrow aSb$$

- What strings can you derive?
  - Answer: None!
- What is the language of the grammar?
  - Answer: Ø
- When designing CFGs, make sure your recursion terminates!

CFG Caveats III

- When designing CFGs, remember that each nonterminal can be expanded out independently of the others.
- Let Σ = $\{a, \stackrel{?}{=}\}$ and let $L = \{a^n \stackrel{?}{=} a^n \mid n \in \mathbb{N}\}$.
- Is the following a CFG for L?

$$S \to X \stackrel{?}{=} X$$
$$X \to aX \mid \varepsilon$$

$$\begin{aligned} & S \\ \Rightarrow\ & X \stackrel{?}{=} X \\ \Rightarrow\ & aX \stackrel{?}{=} X \\ \Rightarrow\ & aaX \stackrel{?}{=} X \\ \Rightarrow\ & aa \stackrel{?}{=} X \\ \Rightarrow\ & aa \stackrel{?}{=} aX \\ \Rightarrow\ & aa \stackrel{?}{=} a \end{aligned}$$

Finding a Build Order

- Let Σ = {a, $\stackrel{?}{=}$} and let $L = \{a^n \stackrel{?}{=} a^n \mid n \in \mathbb{N}\}$.
- To build a CFG for L, we need to be more clever with how we construct the string.
  - If we build the strings of a's independently of one another, then we can't enforce that they have the same length.
  - Idea: Build both strings of a's at the same time.
- Here's one possible grammar based on that idea:

$$S \rightarrow \stackrel{?}{=} \mid aSa$$

$$\begin{aligned} & S \\ \Rightarrow\ & aSa \\ \Rightarrow\ & aaSaa \\ \Rightarrow\ & aaaSaaa \\ \Rightarrow\ & aaa\stackrel{?}{=}aaa \end{aligned}$$

Function Prototypes

- Let $\Sigma = \{$void, int, double, name, (,), ,, ;$\}$.
- Let's write a CFG for C-style function prototypes!
- Examples:
  - void name(int name, double name);
  - int name();
  - int name(double name);
  - int name(int, int name, int);
  - void name(void);

Function Prototypes

- Here's one possible grammar:
  - S → Ret name (Args);
  - Ret → Type | void
  - Type → int | double
  - Args → ε | void | ArgList
  - ArgList → OneArg | ArgList, OneArg
  - OneArg → Type | Type name

- Fun question to think about: what changes would you need to make to support pointer types?

CFGs for Programming Languages

BLOCK → STMT
| { STMTS }
STMTS → ε
| STMT STMTS
STMT → EXPR;
| if (EXPR) BLOCK
| while (EXPR) BLOCK
| do BLOCK while (EXPR);
| BLOCK
| …

EXPR → identifier
| constant
| EXPR + EXPR
| EXPR – EXPR
| EXPR * EXPR
| …

Reading and Exercises

Reading

- Chapter: 2.2 (Michael Scott Book)

References

Lecture Materials of CS 103, Stanford University