Type System

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Simply Typed Lambda Calculus

PennState

- Type system: takes program as input and returns either the type of the program or rejects the program
- Types: int | bool | string | $\tau 1$ -> $\tau 2$

Examples:

- 3 — All the numbers are int and valid program. Well typed.
- λx:int.x — Identity function. Well typed.
- λx:int.(x+1) — Well typed. int-> int
- (true 3) — Not well typed. true can not be a function.
- λx:int.(x x) — Not well typed. x is int and int can not be a function.

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$$\Gamma \vdash n : \texttt{int}\ \text{(T-Num)} \qquad \Gamma \vdash \texttt{true} : \texttt{bool}\ \text{(T-True)}$$

$$\Gamma \vdash \texttt{false} : \texttt{bool}\ \text{(T-False)} \qquad \Gamma, x : \tau \vdash x : \tau\ \text{(T-Var)}$$

$$\Gamma \vdash s : \mathit{string} \quad \text{(T-String)}$$

$$\frac{\Gamma \vdash e_1 : \mathit{string} \qquad \Gamma \vdash e_2 : \mathit{string}}{\Gamma \vdash (e_1 :: e_2) : \mathit{string}}\ \text{(T-Con)}$$

$$\frac{\Gamma \vdash e_1 : \texttt{bool} \quad \Gamma \vdash e_2 : \texttt{bool}}{\Gamma \vdash (e_1 \wedge e_2) : \texttt{bool}}\ \text{(T-And)}$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash (e_1 + e_2) : \texttt{int}}\ \text{(T-Add)}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau\ .\ e) : \tau \to \tau'}\ \text{(T-Abs)}$$

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}\ \text{(T-App)}$$

Examples

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$$\Gamma \vdash n : \mathbf{int} \ \text{(T-NUM)}$$

$$\Gamma, x : \tau \vdash x : \tau \ \text{(T-VAR)}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau \,.\, e) : \tau \to \tau'} \ \text{(T-ABS)}$$

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \quad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash (e_1 + e_2) : \mathbf{int}} \ \text{(T-ADD)}$$

Return either the type of the program or reject the program

$\lambda x{:}\mathbf{int}.(x+1)$

Examples

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$$\Gamma \vdash n : \mathbf{int} \ \text{(T-NUM)}$$

$$\Gamma, x : \tau \vdash x : \tau \ \text{(T-VAR)}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau \,.\, e) : \tau \to \tau'} \ \text{(T-ABS)}$$

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$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 \; e_2 : \tau'} \ \text{(T-APP)}$$

Return either the type of the program or reject the program

$(\lambda x{:}\mathrm{int}.(x{+}1))\ 2$

Examples

PennState

Return either the type of the program or reject the program

$$\Gamma \vdash n : \texttt{int}\ \text{(T-Num)} \qquad \Gamma \vdash \texttt{true} : \texttt{bool}\ \text{(T-True)}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau\ .\ e) : \tau \to \tau'}\ \text{(T-Abs)}$$

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}\ \text{(T-App)}$$

true 3

Examples

PennState

$$\Gamma, x : \tau \vdash x : \tau \text{ (T-VAR)}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash (\lambda x : \tau \,.\, e) : \tau \to \tau'} \text{ (T-ABS)}$$

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'} \text{ (T-APP)}$$

Return either the type of the program or reject the program

$\lambda x{:}\text{int}.(x\ x)$