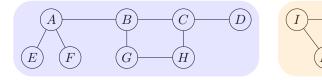
CMPSC 465: LECTURE XI

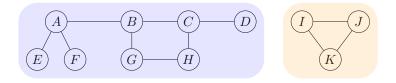
DFS on Undirected Graphs

Ke Chen

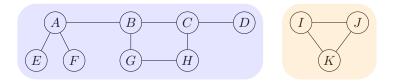
September 24, 2025



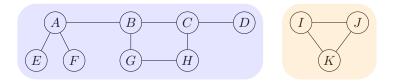
Recall that the connected component of an undirected graph is defined as a maximal set of connected vertices.



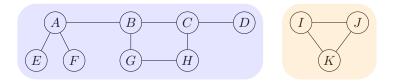
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- ► How?



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- Finding the connected components helps answer queries like "is node v connected to node w?".
- ► How? Just explore!

Intuition Explore a maze with a chalk and a string.



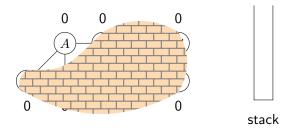
The Fastest Maze-Solving Competition On Earth by Veritasium

DFS on a graph follows the same idea:

- ▶ The graph is the maze, with vertices corresponding to intersections.
- Integer array (one int per vertex) as the colored cyber-chalk.
- ► String can be modeled by a stack, to backtrack we pop from the stack.

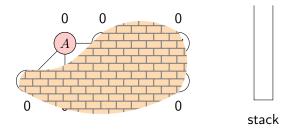
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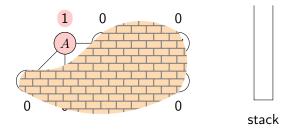
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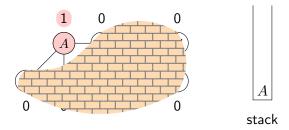
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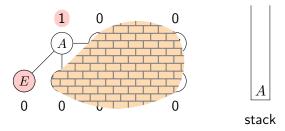
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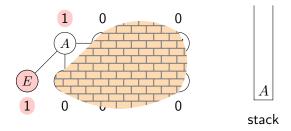
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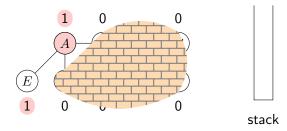
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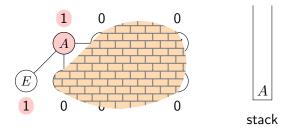
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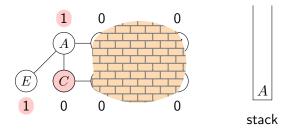
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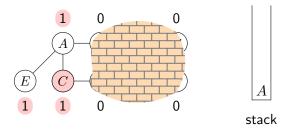
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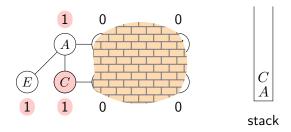
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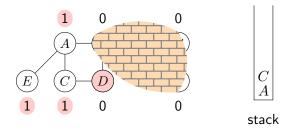
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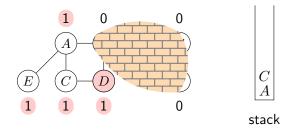
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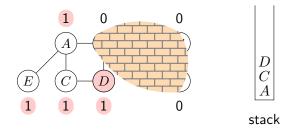
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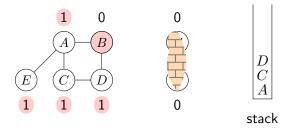
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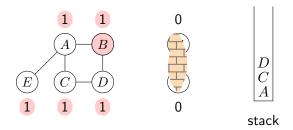
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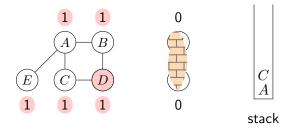
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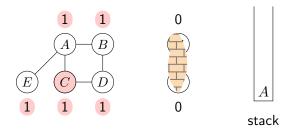
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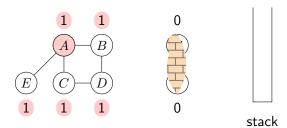
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Input: Graph G=(V,E), starting vertex s, chalk color **Output:** Mark all nodes reachable from s with color

```
\frac{\mathsf{Explore}(G, \, s, \, color)}{\mid}
```

```
Input: Graph G=(V,E), starting vertex s, chalk color Output: Mark all nodes reachable from s with color // visited is an array of length |V|, filled with 0's Explore(G, s, color)
```

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 \begin{array}{l} \textbf{Input: Graph } G = (V, E) \text{, starting vertex } s \text{, chalk } color \\ \textbf{Output: Mark all nodes reachable from } s \text{ with } color \\ \textit{// visited is an array of length } |V| \text{, filled with 0's } \\ \hline \hline & x \text{ explore}(G, s, color) \\ \hline & x \text{ visited}[s] = color \\ \hline \end{array}
```

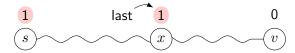
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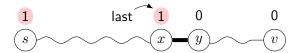
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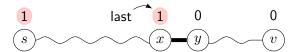






Explore by Depth First Search (DFS)

Correctness?



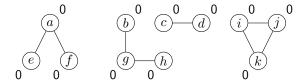
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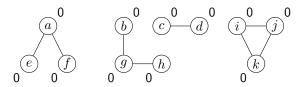
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Example



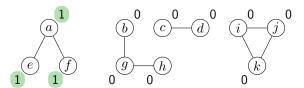
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Example Explore(G, a, 1)



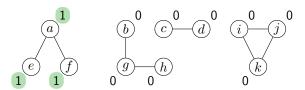
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Example Explore(G, a, 1)



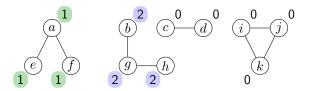
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Example Explore(G, b, 2)



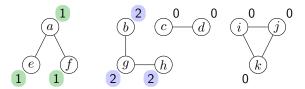
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Example Explore(G, b, 2)



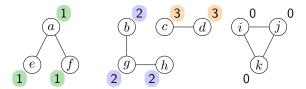
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Example Explore(G, c, 3)



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Example Explore(G, c, 3)

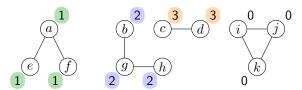


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$\frac{\mathsf{DFS}(G = (V, E))}{\bot \text{ exist ad is an a}}$

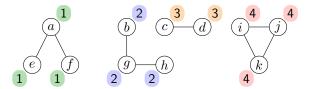
$$\begin{array}{c|c} \textbf{if} \ visited[s] == 0 \ \textbf{then} \\ & \text{Explore}(G, \ s, \ color) \\ & color = color + 1 \end{array}$$

Example Explore(G, i, 4)



- The Explore procedure reveals one connected component.
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Time complexity?

- The Explore procedure reveals one connected component.
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Time complexity?

Explore is called on each vertex in V.

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G in adjacency matrix representation:

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Time complexity?

Explore is called on each vertex in V.

G in adjacency matrix representation:

Explore checks all neighbors of a vertex.

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Time complexity?

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G in adjacency matrix representation:

Explore checks all neighbors of a vertex.

 $O(|V|^2)$

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Time complexity?

Explore is called on each vertex in V.

G in adjacency list representation:

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Time complexity?

Explore is called on each vertex in V.

G in adjacency list representation:

Explore checks each edge twice.

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Time complexity?

Explore is called on each vertex in V.

G in adjacency list representation:

Explore checks each edge twice.

$$O\left(|V| + |E|\right)$$

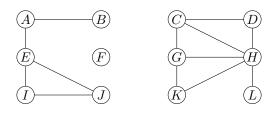
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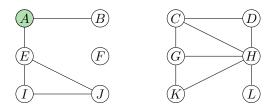
Can we do better?

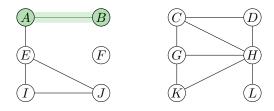
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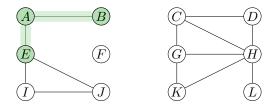
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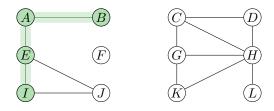
O(|V| + |E|) is the best possible, since we need to read the graph.

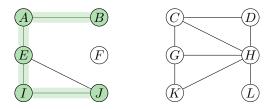


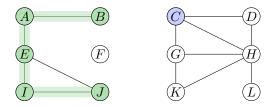


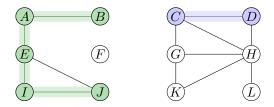


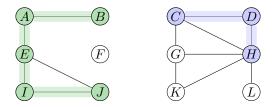


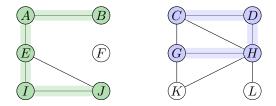


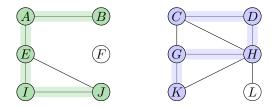


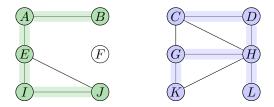




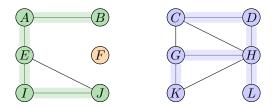




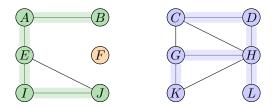




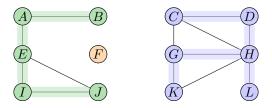
Example Suppose nodes are visited in lexicographical



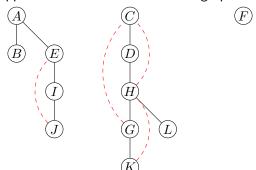
► This is a DFS forest.



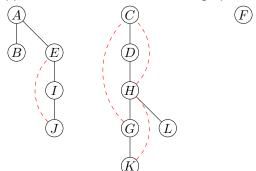
- ► This is a DFS forest.
- ► Colored edges are called tree edges.



- ► This is a DFS forest.
- ► Colored edges are called tree edges.
- ▶ Unused edges are called back edges .



- ► This is a DFS forest.
- ► Solid edges are called tree edges .
- ► Dashed edges are called back edges.



- ► This is a DFS forest.
- ► Solid edges are called tree edges .
- ▶ Dashed edges are called back edges.
- ► Back edges correspond to cycles.

Cycle detection (undirected)

To find cycles, it is sufficient to find back edges, which can be done with a simple modification to the Explore procedure.

We can collect more information during Explore by keeping a global clock that ticks every time we:

- visit a node for the first time, and
- leave a node for good.

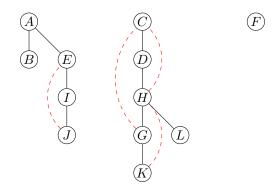
We can collect more information during Explore by keeping a global clock that ticks every time we:

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```
// pre and post are integer arrays of size |V|
// clock is an integer counter starting at 1
Explore(G, s, color)
   visited[s] = color
   pre[s] = clock
    clock = clock + 1
   foreach edge \{s,v\} \in E do
      if visited[v] == 0 then
          \mathsf{Explore}(G, v, color)
    post[s] = clock
    clock = clock + 1
```

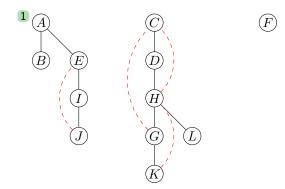
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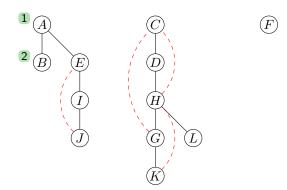
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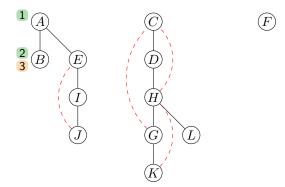
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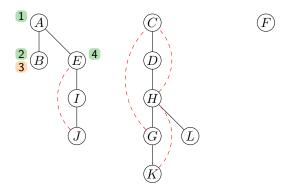
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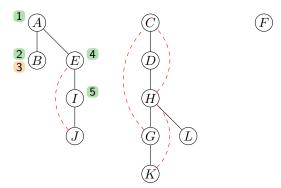
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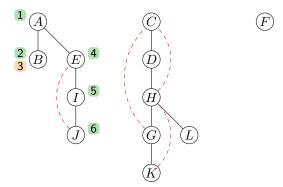
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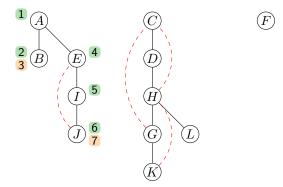
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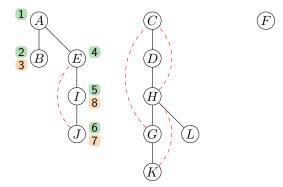
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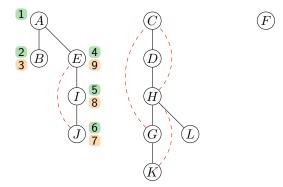
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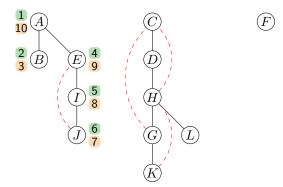
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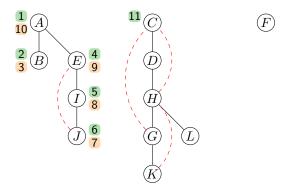
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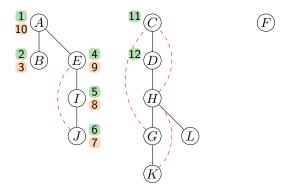
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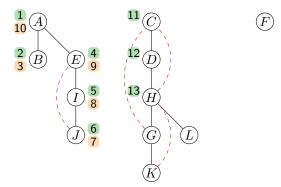
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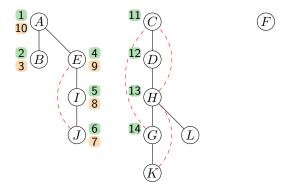
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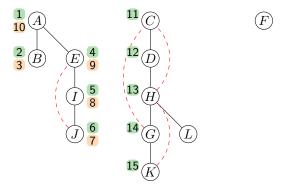
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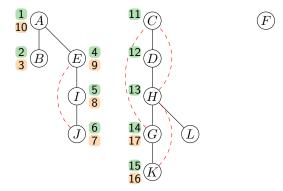
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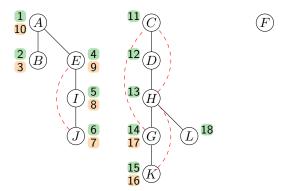
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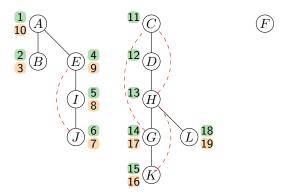
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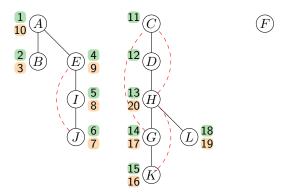
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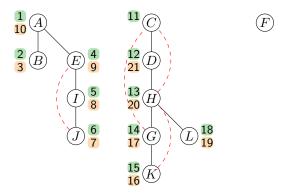
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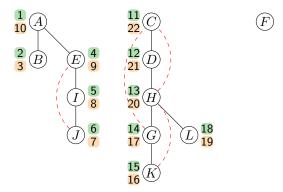
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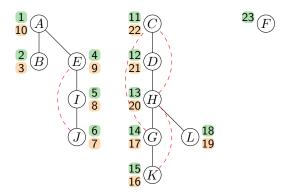
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