CMPSC 461, Fall 2025
Programming Language Concepts
Instructor: Dr. Suman Saha

Midterm-I Practice Exam Solutions

General Instructions about Midterm-I:

Exam Date: September 29, 2025
Exam Time: 8:10 PM - 10:00 PM

1. If you have any conflict, you are requested to contact the instructor **at least 72 hours**[1] before the exam and inform them about it.
2. If you are someone who requires any special accommodations, please reach out to the instructor **at least 72 hours** before the exam and inform them of your requirements, in order to accommodate well in advance.
3. If you are sick or unfit and are prescribed rest or quarantine by the medical authorities during the exam hours, we request you to kindly inform the instructor with the relevant paperwork.
4. Kindly arrive at the exam hall **10-15 minutes before** the scheduled time to settle yourself in and follow the instructions for seating (if any). You will have the seating plan **informed to you 36 hours** before the exam schedule.
5. It is **mandatory** to carry **Penn State Student ID** (Physical/ Mobile ID+) to the exam hall. No other ID(s) will be accepted for verifying your submission.
6. This exam will be a closed book and closed note with **no cheat sheets permitted**.
7. **No electronic devices** are allowed on the desk while taking the exam. You should only have your writing instruments and the exam paper on your desk while working on the exam.
8. Make sure your **writing instruments are high contrast** (this is required for better scanning of your submissions on Gradescope).
9. If you have any questions during the exam, kindly raise your hand and one of the proctors will reach out to you.
10. Any violations will lead to direct consequences as per the course's academic integrity policies.
11. As per Penn State policy, most campus areas no longer require masks, with a few exceptions. For safety amidst rising COVID cases, if you choose to wear a mask, we support you but remember to verify your identity (without the mask) when submitting your exam copy.

All the very best for Midterm :)

[1] unless it's an emergency- in that case, an exception will be made based on the severity of the situation.

Problem 1: Regular Expressions I

1. Construct a regular expression for URLs that:

   - Must begin with `http://` or `https://`.
   - Domain must be letters only, 2–10 in length.
   - Must end with one of the extensions: `.com`, `.org`, or `.net`.

   Valid:
   `http://psu.com`
   `https://example.org`
   `https://abc.net`

   Invalid:
   `http://psu.edu` – extension not allowed
   `https://1abc.com` – domain contains a digit

2. Construct a regular expression for time in 12-hour format `HH:MMAM` or `HH:MMPM` where:

   - `HH` ranges from 01 to 12.
   - `MM` ranges from 00 to 59.
   - The string must end with either `AM` or `PM`.

   Valid:
   `01:00AM`
   `12:45PM`
   `09:59PM`

   Invalid:
   `00:15AM` – hour cannot be 00
   `13:00PM` – hour cannot exceed 12
   `7:30AM` – hour must be two digits

3. Construct a regular expression over $\Sigma = \{a, b, c\}$ for strings where the first and last symbols are different, and the string length is at least 3.

   Valid:
   `abc`
   `bac`
   `cbbba`

   Invalid:
   `aa` – too short and same start/end
   `cb` – too short
   `cacac` – starts and ends with same symbol

Solution

1. `https?://[a-zA-Z]{2,10}\.(com|org|net)`

2. `(0[1-9]|1[0-2]):[0-5][0-9](AM|PM)`

3. `a[abc]+(b|c)|b[abc]+(a|c)|c[abc]+(a|b)`

Problem 2: Finite Automata I

1. Design a DFA recognizing the string over the alphabet $\{1, 2, 3\}$ where the string starts with a single 3 followed by 1 or 2.

   For example, accepted strings: 31, 32, 31231, 313323; rejected strings: ϵ, 3, 33, 123.

   Draw the graph clearly with states and transitions. Mark the start state and all accepting state(s). No explanation required. No transition table required.

2. Design a NFA recognizing the string over the alphabet $\{a, b, c\}$ where the string has consecutive as.

   For example, accepted strings: *aab*, *baaac*, *aabb*, *abccaaa*; rejected strings: ϵ, *a*, *aba*, *caba*.

   Draw the graph clearly with states and transitions. Mark the start state and all accepting state(s). No explanation required. No transition table required.

3. Convert the following NFA transition table into equivalent DFA transition table using subset construction.

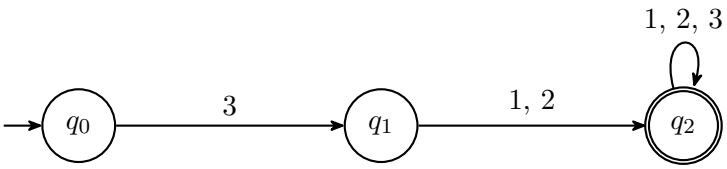
   Only DFA transition table is required. No Automata graph required. No explanation required. Also, mention the start state and all accepting state(s) in the derived DFA. You can omit the row if no transitions going out from the state or it cannot be reached from the start state.

NFA State	x	y	z
q_0	$\{q_1, q_2\}$	$\{q_0\}$	$\{q_2\}$
q_1	$\{q_2\}$	$\{q_1\}$	ϕ
q_2	ϕ	$\{q_0, q_2\}$	$\{q_1\}$

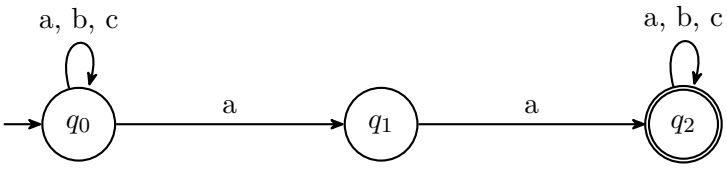
   The start state is q_0, and the accepting state is q_2.

Solution

1. One correct DFA is shown below:



2. One correct NFA is shown below:



3. The transition table is shown below:

DFA State	x	y	z
q_0	q_1q_2	q_0	q_2
q_1q_2	q_2	$q_0q_1q_2$	q_1
q_2	ϵ	q_0q_2	q_1
$q_0q_1q_2$	q_1q_2	$q_0q_1q_2$	q_1q_2
q_1	q_2	q_1	ϵ
q_0q_2	q_1q_2	q_0q_2	q_1q_2

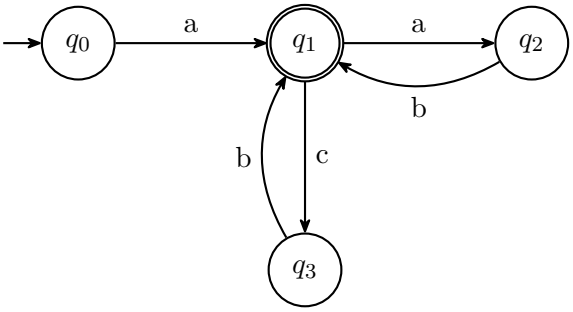
The start state is q_0, and the accepting states are q_1q_2, q_2, $q_0q_1q_2$, and q_0q_2.

Problem 3: Finite Automata II

1. Given the regular expression "$a(ab|cb)*$" over the alphabet $\Sigma = \{a, b, c\}$, build an equivalent DFA.

Note: Draw the graph clearly with states and transitions. Mark the start state and all accepting state(s). No explanation required. No transition table required.

Solution



Problem 4: Context Free Grammar - I

Consider the grammar:

1. $L = \{w \in \{0,1\}^* \mid w \text{ starts with 0 and ends with 1}\}$. Some valid strings are 01, 001, 0111, 000011111, etc.
   (a) Provide CFG for the language mentioned above.
   (b) Using grammar generated in part a), provide the left-most derivation for string 0011.
2. $L = \{a^n b^{2n} \mid n \geq 1\}$. Some valid strings in the language are ab, aabbbb, aaabbbbbb, etc.
   (a) Provide a CFG for the language.

Solution

1. (a)

$$\begin{aligned} S &\rightarrow 0A1 \\ A &\rightarrow 0A \mid 1A \mid \epsilon \end{aligned}$$

   (b) $S \Rightarrow 0A1 \Rightarrow 00A1 \Rightarrow 001A1 \Rightarrow 001\epsilon 1 = 0011$

2.

$$S \rightarrow aSbb \mid abb$$

Problem 5: Context Free Grammar - II

1. (True or False): All regular grammars are also context-free grammars, but not all context-free grammars are regular. Explain your answer clearly.

2. Briefly explain the four components of a context-free grammar (CFG). Give an example production rule for illustration.

Solution

1. True. Regular grammars are a strict subset of context-free grammars. While every regular grammar can be expressed as a CFG, many CFGs (such as those for balanced parentheses) cannot be represented by regular grammars.

2. A context-free grammar is defined by a 4-tuple $G = (V, \Sigma, R, S)$:

   - V (Variables / Non-terminals): Symbols that can be replaced (e.g., E, T).
   - Σ (Terminals): Actual symbols of the language (e.g., id, $+$, $*$).
   - R (Rules / Productions): Rewriting rules of the form $A \to \alpha$ where A is a non-terminal and α is a string of terminals/non-terminals (or ϵ).
   - S (Start symbol): A distinguished non-terminal from which derivations begin.

Problem 6: Ambiguity - I

Convert each of the following regular expressions into an equivalent CFG using BNF notation. Your grammar should be unambiguous and use a minimal number of production rules. To abbreviate ranges of consecutive characters, you can use the syntax $a \mid \cdots \mid z$.

1. `<[1-9][0-9]+(:[0-9][0-9]+)*>`

   Matches strings like `<10>` and `<1234:567>` and `<12:345:6789>` but not `<01>` or `<1234:56>` or `<9876:>`

2. `(0|[1-9][0-9]*)+(0|[1-9][0-9]*)(\.(0|[0-9]*[1-9]))?`

   Matches strings like `0+0` and `123+456.7` and `0.123+456.007` but not `1.2+456` or `01+123` or `123+456.700`

Solution

1.
$$
\begin{aligned}
S &\to \texttt{<}D_1\ D_0\ N_0\ N_1\texttt{>} \\
N_0 &\to D_0\ N_0 \mid \epsilon \\
N_1 &\to \texttt{:}\ D_0\ N_0\ N_1 \mid \epsilon \\
D_0 &\to \texttt{0} \mid \cdots \mid \texttt{9} \\
D_1 &\to \texttt{1} \mid \cdots \mid \texttt{9}
\end{aligned}
$$

2.
$$
\begin{aligned}
S &\to N_0\texttt{+}N_0N_1 \\
N_0 &\to \texttt{0} \mid D_1\ D_n \\
N_1 &\to \texttt{.}\ D_n\ D_1 \mid \epsilon \\
D_n &\to D_0\ D_n \mid \epsilon \\
D_0 &\to \texttt{0} \mid \cdots \mid \texttt{9} \\
D_1 &\to \texttt{1} \mid \cdots \mid \texttt{9}
\end{aligned}
$$

Problem 7: Ambiguity - II

In Python, functions are declared using the `def` keyword, a function name, and a pair of parentheses containing a *parameter list*. This list contains zero or more variable names followed by zero or more variables with default values. This parameter list can be expressed using the following CFG:

$$
\begin{aligned}
S &\to X, Y \mid X \mid Y \mid \epsilon \\
X &\to X, X \mid \langle var \rangle \\
Y &\to \ Y, Y \mid \langle var \rangle = \langle expr \rangle
\end{aligned}
$$

This grammar matches inputs like $var_1, var_2 = expr_2$ but rejects inputs like $var_1 = expr_1, var_2$.

1. Show that the grammar is ambiguous using *left-most derivation* and the smallest possible example. Clearly indicate which rule is being applied at each step.
2. The *Dangling Else Problem* showed that ambiguity can change the structure of a program. Does this grammar cause the same issue? Why or why not?

Solution

1. The grammar is ambiguous for three or more parameters of a single kind. $\langle var \rangle, \langle var \rangle, \langle var \rangle$ and $\langle var \rangle = \langle expr \rangle, \langle var \rangle = \langle expr \rangle, \langle var \rangle = \langle expr \rangle$ are functionally equivalent, so only the former is shown below:

$\underline{S}$	$S \to X$
$\underline{X}$	$X \to X, X$
$\underline{X}, X$	$X \to X, X$
$\underline{X}, X, X$	$X \to \langle var \rangle$
$\langle var \rangle, \underline{X}, X$	$X \to \langle var \rangle$
$\langle var \rangle, \langle var \rangle, \underline{X}$	$X \to \langle var \rangle$

(a) First Derivation

$\underline{S}$	$S \to X$
$\underline{X}$	$X \to X, X$
$\underline{X}, X$	$X \to \langle var \rangle$
$\langle var \rangle, \underline{X}$	$X \to X, X$
$\langle var \rangle, \underline{X}, X$	$X \to \langle var \rangle$
$\langle var \rangle, \langle var \rangle, \underline{X}$	$X \to \langle var \rangle$

(b) Second Derivation

2. **No**, the ambiguity would not change the structure of a program. The differences between the derivations would not cause the precedence of the parsed symbols to change.

Problem 8: Names, Scopes and Bindings I

Consider the following pseudo-code:

```
int x = 100;

void helper() {
    print(x);
}

void recur(int n) {
    if (n > 0) {
        int x = n;
        recur(n - 1);
    } else {
        helper();
    }
}

void caller() {
    int x = 50;
    recur(2);
}

void main() {
    caller();
}
```

(a) Draw a diagram of the **runtime stack** at the exact moment the `helper()` function is called. For each activation frame, clearly label its **static link** and **dynamic link**.

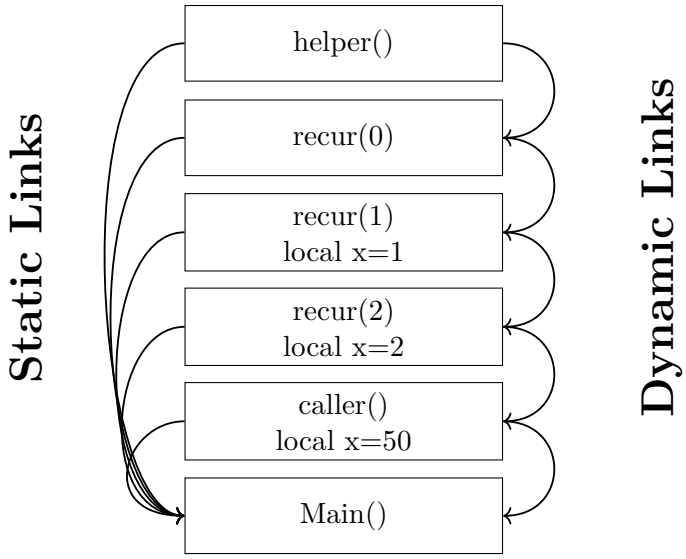
(b) What is the output of this program, assuming the language uses:

i. Static scoping?

ii. Dynamic scoping?

Solution

(a) **Runtime Stack at `helper()` call**



(b) **Program Outputs**

i. Static scoping: `100`

ii. Dynamic scoping: `1`

Problem 9: Name, Scope, and Binding II

Consider the following pseudo-code:

```
typedef void (*FuncPtr)();

void worker() {
    print(val);
}

FuncPtr setup() {
    int val = 50;
    return worker;
}

void recursive_executor(int n, FuncPtr F) {
    if (n > 0) {
        int val = n * 10;
        recursive_executor(n - 1, F);
    } else {
        F();
    }
}

void main() {
    FuncPtr my_func;
    my_func = setup();
    recursive_executor(2, my_func);
}
```

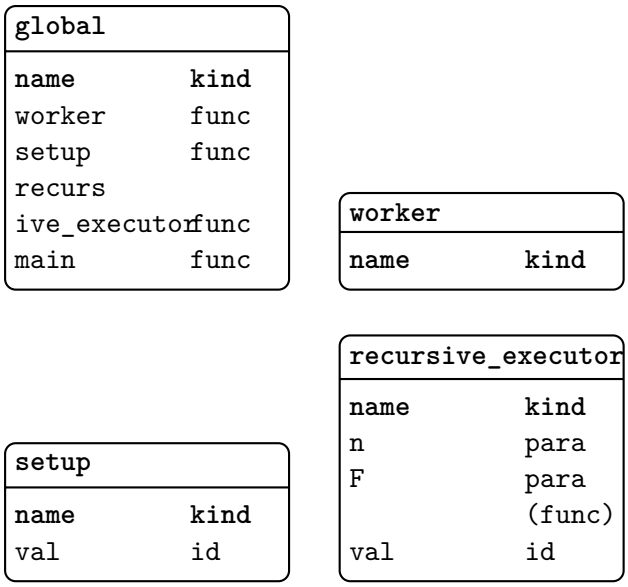
(A) Draw the **symbol tables** for the scopes `global`, `worker`, `setup`, and `recursive_executor`.

(B) What is the output if the language uses **shallow binding**? Justify by tracing variable resolution from the call site.

(C) What is the output if the language uses **deep binding**? Explain which referencing environment is captured when the function reference is created.

Solution

(A) Symbol Tables



(B) Shallow binding output: `10`.

Explanation: Shallow binding resolves variables using the environment of the **call site**. The call ‘F()‘ happens inside ‘recursive_executor(0)‘. The call stack is ‘main ->
recursive_executor(2) ->
recursive_executor(1) ->
recursive_executor(0) ->
worker‘.
The search for ‘val‘ begins up this dynamic chain:
it checks ‘worker‘ (none),
then its caller ‘recursive_executor(0)‘ (none),
and then its caller’s caller, ‘recursive_executor(1)‘, where it finds the local ‘val = 1 * 10 = 10‘.

(C) Deep binding output: `50`.

Explanation: Deep binding resolves variables using the environment from when the function reference was **created**. The reference to ‘worker‘ is returned by ‘setup()‘. At that moment, a closure is created, capturing the environment of ‘setup()‘, which includes the binding ‘val = 50‘. When ‘worker‘ is later called, it uses this captured environment to find ‘val‘, printing ‘50‘.

Problem 10: LL(1) Parsers

Consider the grammar:

```
S → A B
A → a A |
B → b B | c
```

(a) Compute the **FIRST** and **FOLLOW** sets for S, A, B.

(b) Construct the **LL(1) parsing table** and state whether the grammar is LL(1).

Solution

- FIRST sets: $\text{FIRST}(A) = \{a, \epsilon\},\ \text{FIRST}(B) = \{b, c\},\ \text{FIRST}(S) = \{a, b, c\}$.
- FOLLOW sets: $\text{FOLLOW}(S) = \{\$\},\ \text{FOLLOW}(A) = \{b, c\},\ \text{FOLLOW}(B) = \{\$\}$.
- Parsing table:

Non-terminal	a	b	c	$
S	$S \to AB$	$S \to AB$	$S \to AB$	
A	$A \to aA$	$A \to \epsilon$	$A \to \epsilon$	
B		$B \to bB$	$B \to c$	

No conflicts $\Rightarrow$ Grammar is LL(1).