CMPSC 360 | Discrete Mathematics for Computer Science
Fall 2024 | Mahfuza Farooque

Worksheet 6

1. Determine if the following relations are reflexive, symmetric, antisymmetric, transitive, equivalence relation, or poset (i.e., partially ordered set) on the respective sets:
   (a) $R_0 = \{(a,a),(b,b),(a,c),(c,a)\}$ on $A_0 = \{a,b,c,d\}$
   (b) $R_1 = \{(a,a),(c,c),(b,b),(a,d),(d,b),(a,b),(a,c)\}$ on $A_1 = \{a,b,c,d,e\}$.
   (c) Also, determine the relational properties that R_1 satisfies on A_0 from above.
   (d) $R_2 = \{(x,y) \in P \times P \mid x \subseteq y\}$ defined on P such that P is the power set of an arbitrary set X

2. Prove that the following relation R is an equivalence relation on $\mathbb{Z}$:

$$R = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid \frac{x^2 - y^2}{4} \in \mathbb{Z}\}$$

3. Let $A = a,b,c,d,e$. Suppose R is an equivalence relation on A. Suppose R has three equivalence classes. Also aRd and bRc. Write out R as a set.

4. Define a relation R on $\mathbb{Z}$ as xRy if and only if $3x - 5y$ is even. Prove R is an equivalence relation. Describe its equivalence classes.

5. Determine if the following relations are functions:
   (a) $f_0 : \{1,2,3\} \to \{1,2,3\}$ and $f_0 = \{(1,1),(2,1),(3,2)\}$
   (b) $f_1 : \{1,2,3,4\} \to \{1,2,3\}$ and $f_1 = \{(1,1),(2,1),(3,2)\}$
   (c) $f_2 : \{a,b,c\} \to \{a,b\}$ and $f_2 : \{(a,a),(b,a),(b,c)\}$

6. Determine which functional properties the following functions satisfy and whether they are bijective:
   (a) $f : \mathbb{Z} \to \mathbb{Q}$ and $f(x) = \frac{2x-3}{5}$
   (b) $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = 3x^2 + 11x$