CMPSC 465 Fall 2025 — Data Structures & Algorithms — Ke Chen and Yana Safonova

Quiz 5

Lecture Section: **Monday, Oct 06, 2025**

Student Name: **PSU Email ID:**

1. (2 pts.) Suppose A and B are two strongly connected components in a directed graph, and there is an edge from a vertex in A to a vertex in B. Consider an arbitrary run of DFS on the graph that assigns pre and post numbers to each node. Which of the following must be true?

(a) $max_{w \in A}\ post(w) > max_{v \in B}\ post(v)$

(b) $max_{w \in A}\ pre(w) < max_{v \in B}\ pre(v)$

(c) $max_{w \in A}\ post(w) < max_{v \in B}\ post(v)$

(d) None of the above

Answer (a) $max_{w \in A}\ post(w) > max_{v \in B}\ post(v)$
When running DFS, if A is visited first, the vertex with an edge to B will have a higher post number than every vertex in B. If B is visited first, A cannot be reachable, or they would not be separate SCCs. DFS will assign post numbers to B without visiting A, and will have to visit A later.

2. (2 pts.) In an unweighted, connected, undirected graph, which of the following is true about using BFS to find shortest paths from a given node S?

(a) BFS may fail if the graph has cycles.

(b) BFS works only on trees.

(c) BFS always finds the shortest path.

(d) BFS doesn't work on undirected graphs.

Answer: (c) BFS always finds the shortest path.
As long as the graph is unweighted, BFS always finds the shortest path.

3. (2 pts.) Dijkstra's Algorithm **does not work correctly** on:

(a) Directed weighted graphs

(b) Graphs with negative-weight edges

(c) Undirected unweighted graphs

Answer: (b) Graphs with negative-weight edges
Dijkstra's algorithm assumes that once a node is finalized, its shortest path is known — which fails if a shorter path appears later via a negative edge.

4. (2 pts.) In a weighted graph where the weights of all edges are distinct, there is always a unique shortest path between two connected vertices.

(a) True

(b) False

Answer: (b) False
A counterexample is a graph where one edge of weight 3 connects the source and destination, and another path through a third node has weights 1 and 2.

5. (2 pts.) In Dijkstra's algorithm, after a vertex u is added to the visited set R, which of the following is always true?

(a) $dist(u)$ may still decrease later due to a shorter path.

(b) $dist(u)$ is equal to the true shortest-path distance from the source.

(c) $dist(u)$ is the minimum among all vertices in the graph.

(d) u has no outgoing edges.

Answer: (b) $dist(u)$ is equal to the true shortest-path distance from the source. Because Dijkstra's algorithm always selects the vertex with the smallest tentative distance, ensuring the shortest path to it is already found when it's added to R.