CMPSC 360 Discrete Mathematics for Computer Science
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Worksheet 7

1. Determine which functional properties the following functions satisfy and whether they are bijective:
   (a) $f : \mathbb{Z} \to \mathbb{Q}$ and $f(x) = \frac{2x-3}{5}$
   (b) $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = 3x^2 + 11x$

2. Suppose $f(x) = \sqrt{2x-5}$, $g(x) = 5x^2 - 3$. What is the domain of f? What is the domain of g? Find the composite functions below. For each composite function, state the domain.
   (a) $f \circ g(x)$
   (b) $g \circ f(x)$
   (c) $f \circ f(x)$
   (d) $g \circ g(x)$

3. Consider the function $f : \mathbb{R} \setminus \{-4\} \to \mathbb{R} \setminus \{2\}$ defined as $f(x) = \frac{2x-1}{x+4}$.
   (a) Determine whether the function f has an inverse, and if so, find the expression for $f^{-1}(x)$.
   (b) Would f have an inverse if we change the function's domain and codomain to $f : \mathbb{Z} \setminus \{-4\} \to \mathbb{Z} \setminus \{2\}$? Explain your reasoning.

4. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = x + 1$ and let $g : \mathbb{Z} \to \mathbb{Z}$ be defined by $g(x) = -x$.
   Find the composition $g \circ f$ and determine if it is a bijection over the set of integers, $\mathbb{Z}$.

5. Given the functions $h : M \to N$ and $k : N \to P$ where

$$M = \{a, b, c, d, e\}$$

$$N = \{j, k, l, m, n\}$$

$$P = \{s, t, u, v, w\}$$

   and the functions are represented by the rosters:

$$h = \{(a, j), (b, k), (c, l), (d, m), (e, n)\}$$

$$k = \{(j, t), (k, s), (l, u), (m, v), (n, w)\}$$

   Determine $k \circ h$ and $h \circ k$.