CMPSC 360 Fall 2024 | Discrete Mathematics for Computer Science | Mahfuza Farooque

Worksheet 3

1. Simplify the logical expression $q \vee \neg(\neg(p \wedge r) \rightarrow (p \vee \neg q))$ using logical equivalence rules.

2. Use the logical equivalence properties below to verify the logical equivalence.

$$(p \rightarrow \neg q) \wedge (p \rightarrow \neg r) \equiv \neg(p \wedge (q \vee r))$$

3. Using truth tables, determine whether $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

4. (A) Express the following using predicates, quantifiers, logical connectives, and mathematical operators if necessary.

(a) Every positive integer is the sum of the squares of four integers. (The universe of discourse contains all integers)

(b) Every user has access to exactly one mailbox. (Assume that the domain consists of all users and all mailboxes)

(B) Let G(x, y) mean that child x has played video-game y, where the domain for x consists of all the children in your school and the domain for y consists of all video-games. Express these statements as an English sentence.

(a) $\exists a \forall b (a \neq (child_1) \wedge (G(child_1, b) \rightarrow G(a, b)))$

(b) $\exists x \exists y \forall z ((x \neq y) \wedge (G(x, z) \leftrightarrow G(y, z)))$

5. Prove or disprove that the following compound proposition is a contingency.

$$((p \vee r) \vee ((q \wedge p) \vee (q \wedge r))) \wedge \overline{r} \wedge \overline{p}$$