CMPSC 465 Fall 2025 — Data Structures & Algorithms — Ke Chen and Yana Safonova

Quiz 2

Lecture Section: **Monday, Sep 15, 2025**

Student Name: **PSU Email ID:**

1. (2 pts.) We can apply the Master Theorem to recurrences of the form $T(n) = a \cdot T(\frac{n}{b}) + O(2^{n \cdot d})$ (where $a > 0$, $b > 1$ and $d \geq 0$) and conclude that:

$$T(n) = \begin{cases} \Theta(n^d) & \text{if } d > \log_b a \\ \Theta(n^d \log n) & \text{if } d = \log_b a \\ \Theta(n^{\log_b a}) & \text{if } d < \log_b a \end{cases}$$

(a) True
(b) False

Answer (b) False
Master Theorem applies to recurrences of the form $T(n) = a \cdot T(\frac{n}{b}) + \Theta(n^d)$. Everything besides the given $T(n)$ is correct.

2. (2 pts.) The recurrence relation for the median-of-medians algorithm can be approximated as $T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)$. What is the overall time complexity derived from this recurrence?

(a) $O(n^2)$
(b) $O(n \log n)$
(c) $O(n)$
(d) $O(\log n)$

Answer (c) Solving this recurrence shows that the total work is linear, as the subproblem sizes decrease geometrically, leading to a total time complexity of $O(n)$.

3. (2 pts.) Given two arrays of numbers $x = [2,4,12]$ and $y = [3,4,5]$. What would be the result of $Merge(x,y)$ in the merge-sort algorithm?

(a) $[2,3,4,5,4,12]$
(b) $[2,5,4,4,3,12]$
(c) $[3,4,4,2,5,12]$
(d) $[2,3,4,4,5,12]$

Answer (d) $[2,3,4,4,5,12]$
When Merge runs on two sorted arrays, it always returns a sorted array.

4. (2 pts.) What is the role of the pivot in the QuickSort algorithm?

(a) To sort the entire array in one step.
(b) To partition the array into two subarrays for recursive sorting.
(c) To compute the median of the array elements.
(d) To merge two sorted subarrays.

Answer (b) To partition the array into two subarrays for recursive sorting.

5. (2 pts.) What is the time complexity of the standard divide-and-conquer algorithm for matrix multiplication?

(a) $O(n^4)$
(b) $O(n^3)$
(c) $O(n^2)$
(d) $O(n \log \log n)$

Answer (b) $O(n^3)$