CMPSC 465: LECTURE II

Asymptotic Notations

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August 29, 2025

For two functions $f, g : \mathbb{N} \to \mathbb{R}^+$, we say $f(n) = O(g(n))$ if there exist $c > 0$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$

$$f(n) \leq c \cdot g(n).$$

$f(n) = O(g(n))$ means:

- asymptotically, $f(n)$ grows no faster than $g(n)$.
- in terms of growth rate, $f(n) \leq g(n)$.
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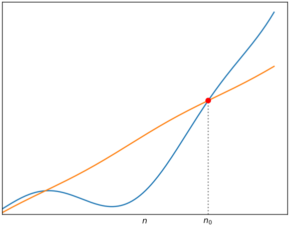

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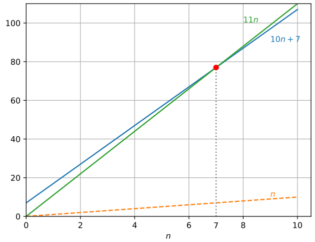

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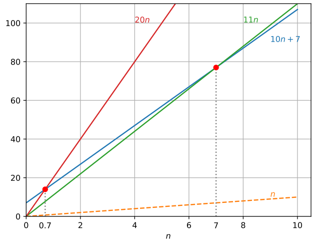

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Therefore, we say $10n + 7 = O(n)$.

However, comparing $f(n) = 0.01n^2$ and $g(n) = n$, no matter how large we pick c, for $n > 100c$, we have

$$f(n) = 0.01n^2 = 0.01n \cdot n > 0.01 \cdot 100c \cdot n = c \cdot n = c \cdot g(n).$$

So, $0.01n^2 \neq O(n)$.

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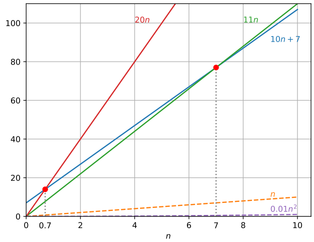

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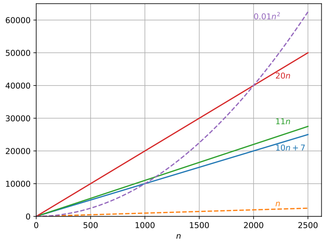

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These are all valid upper bounds, but we always choose the lowest possible upper bound because it provides the most accurate analysis.

Big-O: useful facts

1. Multiplicative constants can be omitted: $a \cdot f(n) = O(f(n))$.
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3. If $f(n) = O(h(n))$ and $g(n) = O(h(n))$, then
   $f(n) + g(n) = O(h(n))$.
   Proof: There exist c, c', n_0, n'_0 such that
   $f(n) \leq c \cdot h(n)$ for all $n \geq n_0$ and
   $g(n) \leq c' \cdot h(n)$ for all $n \geq n'_0$.
   So, for $n \geq \max(n_0, n'_0)$, $f(n) + g(n) \leq (c + c')h(n)$.

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4. Polynomials are easy:
   $a_d \cdot n^d + a_{d-1} \cdot n^{d-1} + \cdots + a_1 \cdot n + a_0 = O(n^d)$.
   Proof: Follows from 1, 2, and 3.

Big-O: useful facts

5. Any polynomial dominates any logarithm: $(\log n)^a = O\left(n^b\right)$ for all constants $a, b > 0$.

Examples: $(\log n)^3 = O(n)$, $(\log n)^{100} = O(n^{0.01})$.

Proof.

By the property of logarithms, $\log n = \frac{a}{b} \log n^{\frac{b}{a}}$.

Since $\log x \leq x$ for all $x \geq 1$, we get $\frac{a}{b} \log n^{\frac{b}{a}} \leq \frac{a}{b} n^{\frac{b}{a}}$ for all $n \geq 1$.

Combined, we have $\log n \leq \frac{a}{b} n^{\frac{b}{a}}$, for all $n \geq 1$.

Raise to the power a: $(\log n)^a \leq \underbrace{\left(\frac{a}{b}\right)^a}_{c} n^b$, for all $n \geq 1 = n_0$. $\square$

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6. The base of the logarithms are irrelevant: $\log_a n = O\left(\log_b n\right)$ for all constants $a, b > 1$.

Proof. $\log_a n = \underbrace{\log_a b}_{c} \cdot \log_b n$.

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6. The base of the logarithms are irrelevant: $\log_a n = O\left(\log_b n\right)$ for all constants $a, b > 1$.

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7. Any exponential dominates any polynomial: $n^a = O\left(b^n\right)$ for all constants $a > 0, b > 1$.

Examples: $n^{99} = O(2^n)$, $n^{1000} = O(1.1^n)$.

Proof. Exercise.

Big-O: useful facts

8. Transitivity: If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.

Proof.

By definition, $\exists c > 0, n_0 \geq 1$ such that $\forall n \geq n_0$, $f(n) \leq c \cdot g(n)$.

Similarly, $\exists c' > 0, n_0' \geq 1$ such that $\forall n \geq n_0'$, $g(n) \leq c' \cdot h(n)$.

Combined, for all $n \geq \max\{n_0, n_0'\}$, $f(n) \leq c \cdot c' \cdot h(n)$. $\square$

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9. If $f(n) = O(h_1(n))$ and $g(n) = O(h_2(n))$, then $f(n) \cdot g(n) = O(h_1(n) \cdot h_2(n))$.

Examples: $n \log n = O(n^2)$.

Proof. Exercise.

Big Omega describes lower bounds

For two functions $f, g : \mathbb{N} \to \mathbb{R}^+$, we say $f(n) = \Omega(g(n))$ if there exist $c > 0$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$,

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Examples: true or false?

- ◇ $3n + 1 = \Omega(n)$
- ◇ $0.01n^2 = \Omega(n)$
- ◇ $2^{2n} = \Omega(2^n)$
- ◇ $n^5 + 1888n^3 + n \log n = \Omega(n^6)$

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Theta describes tight bounds

Observe that $f(n)$ can be both $O(g(n))$ and $\Omega(g(n))$, in this case, we say

$$f(n) = \Theta(g(n)).$$

In words, functions $f(n)$ and $g(n)$ are asymptotically of the same order.

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- $\diamond$ $n^5 + 1888n^3 + n \log n = \Theta(n^5)$
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- $\diamond$ $5n \log n + 1000n - 6 = \Theta(n \log n)$

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Useful facts using limits

For two functions $f, g : \mathbb{N} \to \mathbb{R}^+$,

- if $\lim\limits_{n\to\infty} \frac{f(n)}{g(n)} = a > 0$, then $f(n) = \Theta(g(n))$.
- if $\lim\limits_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty$, then $f(n) = O(g(n))$.
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Proof: Use the "(ε, δ)-definition of limits", we have

for any $\varepsilon > 0$, there exists n_0 such that for all $n \geq n_0$,

$$\left| \frac{f(n)}{g(n)} - a \right| \leq \varepsilon.$$

So $|f(n) - a \cdot g(n)| \leq \varepsilon \cdot g(n)$, or

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Watch out, limits may not exist!

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Notes

When using asymptotic notation, you need to bring your expressions to the most concise form, where only the main function remains. For example, the following are *not acceptable* final answers; they need to be worked out as shown:

- $O(n + n\sqrt{n})$

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- $\Theta(\sum_{i=1}^{n} \log i)$

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- $\Theta(\sum_{i=1}^{n} \log i) = \Theta(n \log n)$
- $\Omega(n + n\sqrt{n}) = \Omega(n\sqrt{n})$
- $o(n + n\sqrt{n})$

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- $o(n! + 2^n) = o(n!)$
- $o(n^2 2^n + 3^n/n^4)$

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- $\omega(n + n\sqrt{n}) = \omega(n\sqrt{n})$
- $o(n! + 2^n) = o(n!)$
- $o(n^2 2^n + 3^n/n^4) = o(3^n/n^4)$

Exercises

1. Let $f(n)$ and $g(n)$ be continuous positive functions, where $f(n)$ and $g(n)$ are integers for each integer n. Prove or disprove:
   - $f(n+1) = \Theta(f(n))$.
   - $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

   (*Hint.* Look for fast growing functions.)
2. Order the following list of functions by the big-Oh notation. Group together those functions that are big-Theta of one another. Justify your ordering briefly, in each case.

$$\begin{array}{ccccc} \sqrt{n} & \log(n!) & 2^{13\log n} & \log^5 n & 2^{\log n} \\ \lceil \sqrt{n\log n} \rceil & n^{0.51} & n(8/3)^n & 75n^{\sqrt{2}} & 18n^{1.41} \end{array}$$