CMPSC 465 Data Structures & Algorithms
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Worksheet 1

Monday, Sep 8, 2025

1. **Compare Growth Rates.** Order the following functions by asymptotic growth:

   (i) $f_1(n) = 3^n$
   (ii) $f_2(n) = n^{\frac{1}{3}}$
   (iii) $f_3(n) = 12$
   (iv) $f_4(n) = 2^{\log_2 n}$
   (v) $f_5(n) = \sqrt{n}$
   (vi) $f_6(n) = 2^n$
   (vii) $f_7(n) = \log_2 n$
   (viii) $f_8(n) = 2^{\sqrt{n}}$
   (ix) $f_9(n) = n^3$

2. **Running Time Analysis.** For each pseudo-code below, give the asymptotic running time in Θ notation.

   (a)
   ```
   for i := 1 to n do
       j := i;
       while j < n do
           j := j + 5;
       end
   end
   ```

   (b)
   ```
   for i := 1 to n do
       for j := 4i to n do
           s := s + 2;
       end
   end
   ```

   (c)
   ```
   for i := 1 to n do
       j := 2;
       while j < i do
           j := j^4;
       end
   end
   ```

3. **Identities.** Show that the following statements hold true.

(a) $\sum_{k=1}^{n} k^j = \Theta(n^{j+1})$ for any constant $j > 0$. Note that $k \leq n$ for all terms and $k \geq \frac{n}{2}$ for many terms.

(b) $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} ij = \Theta(n^4)$.

4. Recurrence Relations. Solve the following recurrence relations and give the tightest correct upper bound for each of them in O notation. Assume that $T(1) = O(1)$. Show all work.

(a) $T(n) = 16 \cdot T(n/2) + 15n^4 + 3n^3$

(b) $T(n) = 49 \cdot T(n/25) + n^{3/2} \log n$

(c) $T(n) = n^{\frac{1}{3}} T(n^{\frac{1}{3}}) + 5n^{\frac{2}{3}}$