CMPSC 360 Discrete Mathematics for Computer Science
Fall 2024 Mahfuza Farooque **Worksheet 11 Solutions**

1. In an art exhibition, there are three galleries, each displaying 5 pieces of artwork. An art enthusiast wishes to study a total of 6 artworks, making sure to observe at least one piece from each gallery. In how many ways can the art enthusiast do this?

   Answer:

   Case 1: Selecting 4 artworks from one gallery, and 1 artwork from each of the other two galleries.

   Case 2: Selecting 3 artworks from one gallery, 2 artworks from the second gallery, and 1 artwork from the third gallery.

   Case 3: Selecting 2 artworks from each gallery.

   The total number of unique combinations can be calculated by summing the outcomes of these scenarios.

   - For Case 1: $3 \times \left(\binom{5}{4} \times \binom{5}{1} \times \binom{5}{1}\right) = 3 \times (5 \times 5 \times 5) = 375$.
   - For Case 2: $3! \times \left(\binom{5}{3} \times \binom{5}{2} \times \binom{5}{1}\right) = 6 \times (10 \times 10 \times 5) = 3000$.
   - For Case 3: $\left(\binom{5}{2} \times \binom{5}{2} \times \binom{5}{2}\right) = (10 \times 10 \times 10) = 1000$.

   There is no priority in selection for case 3, so we do not permute the selections.

   Adding these together, the total number of ways is:

   $$375 + 3000 + 1000 = 4375$$

2. What is the total number of six-digit numbers in which only and all the five odd digits appear?

   Answer:

   After selecting the 5 odd numbers 1,3,5,7 and 9 for 5 places in 1 way, 1,3,5,7 and 9 can be selected in $\binom{5}{1}$ ways for the 6th digit place. The permutations of the 6 digits are $\frac{6!}{2!}$

   Therefore, the total number of 6-digit numbers in which only and all 5 digits appear is $5 \cdot \frac{6!}{2}$

3. Answer the following about a standard deck of cards:

   (a) In how many ways can you choose four cards that are of the same suit?

   (b) In how many ways can you choose four cards such that each one is of a different suit?

   (c) In how many ways can you choose three face cards?

   Answer:

   (a) The number of ways to choose four cards that are of the same suit is calculated using the combination formula for choosing 4 out of 13 cards, and then multiplying by the number of suits:

   $$4 \times \binom{13}{4} = 4 \times \frac{13!}{4!(13-4)!} = 2860$$

(b) The number of ways to choose four cards such that each one is of a different suit, where there are 13 choices per suit and one card is chosen from each of the four suits:

$$\binom{13}{1} \times \binom{13}{1} \times \binom{13}{1} \times \binom{13}{1} = 13^4 = 28561$$

(c) The number of ways to choose three face cards from the 12 available face cards in the deck:

$$\binom{12}{3} = \frac{12!}{3!(12-3)!} = 220$$

4. How many nine-character vehicle license plates can be made using the 26 letters of the English alphabet, where each letter can be either uppercase or lowercase, and the 10 digits,

(a) with repetition?

(b) without repetition?

How many of these license plates can be created if the plate must start and end with a letter and must include exactly three digits (with no repeated digits), and the remaining characters must be letters? Repetitions are allowed for letters.

Answer:

(a) Each character has 52 possibilities (26 uppercase + 26 lowercase) plus 10 digits, making a total of $52+10=62$ possible characters. For a straightforward nine-character license plate with repetitions allowed:

$$62^9$$

(b) If no character is to be repeated, the first character can be any of the 62 characters, the second can be any of the remaining 61 characters, and so on, down to the 54th character for the ninth position:

$$P(62,9) = 62 \times 61 \times 60 \times 59 \times 58 \times 57 \times 56 \times 55 \times 54$$

License Plates with Specific Conditions:

License plates that must:

- Start and end with a letter (52 choices each).
- Include exactly three digits in the remaining seven characters. These digits are non-repeating and chosen from 10 digits.
- The remaining four characters must be letters (repetitions allowed).

Calculation:

- Choose 3 positions for the digits among the 7 available (excluding first and last positions): $\binom{7}{3}$ ways.
- Arrange 3 non-repeating digits in these positions: $P(10,3) = 10 \times 9 \times 8$ ways.
- Fill the remaining 4 spots with letters: 52^4 ways.

- Multiply by the choices for the first and last positions: 52×52.

The total number of such license plates is calculated by:

$$52 \times 52 \times \binom{7}{3} \times (10 \times 9 \times 8) \times 52^4$$