Discrete Mathematics for Computer Science
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Worksheet 8

1. a) Given two functions f and g such that $g:\mathbb{R}\to\mathbb{R}$ defined by $g(x)=\frac{a}{x^4+2}$ and $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=bx-1$. Determine the values of constants a and b such that:
$\begin{cases} g\circ Id_{\mathbb{R}}(0)=f\circ Id_{\mathbb{R}}(0) \\ g\circ Id_{\mathbb{R}}(1)=f\circ Id_{\mathbb{R}}(1) \end{cases}$, where $Id_{\mathbb{R}}(x)=x$ for all $x\in\mathbb{R}$.

   b) Given two functions f and g such that $g:\mathbb{R}\to\mathbb{R}$ defined by $g(x)=ae^x+b$ and $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=cx^2+|x|+1$. Determine the values of constants a, b and c such that:
$\begin{cases} g\circ Id_{\mathbb{R}}(0)=f\circ Id_{\mathbb{R}}(0) \\ g\circ Id_{\mathbb{R}}(1)=f\circ Id_{\mathbb{R}}(1) \\ g\circ Id_{\mathbb{R}}(2)=f\circ Id_{\mathbb{R}}(2) \end{cases}$, where $Id_{\mathbb{R}}(x)=x$ for all $x\in\mathbb{R}$.

2. Evaluate the following:

   (a) $(\sum_{x=1}^{12}\frac{1}{x+6})(\prod_{y=1}^{17}-4y+y^2-21)$

   (b) $\left(\prod_{m=1}^{5}m^8\right)^{\frac{1}{4}}$

   (c) $\sum_{x=1}^{17}(x+3)-\sum_{x=1}^{19}(x+9)$

3. Use $\sum$ notation and/or $\prod$ notation to rewrite the following sums and/or products.

   (a) $x_1y_1^4+x_2(y_1^4-y_2^4)+x_3(y_1^4-y_2^4+y_3^4)$

   (b) $\frac{2}{2}+\frac{2^2}{2(2+1)}+\frac{2^3}{2(2+1)(2+2)}+\ldots+\frac{2^n}{2(2+1)(2+2)\ldots(2+(n-1))}$

   (c) $\left(1+\frac{1}{1^2}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{3^2}\right)\ldots\left(1+\frac{1}{n^2}\right)$

   (d) $1-\left(\frac{1}{2}\cdot\frac{3}{2}\right)+\left(\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\right)-\ldots+(-1)^{n+1}\left(\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\ldots\frac{(2n-1)}{2}\right)$

   (e) $1-\frac{2}{1!}+\frac{3^2}{2!}-\frac{4^3}{3!}+\cdots+(-1)^{n+1}\frac{n^{n-1}}{(n-1)!}$

4. Prove using induction that $n!>3^n$ for all natural numbers $n\geq 7$.

5. Using induction, prove that for any $n\in\mathbb{N}$, $1^3+2^3+3^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$

6. Prove by induction that 6^n+4 is divisible by 5, for all $n\in N$.