CMPSC 360 Discrete Mathematics for Computer Science
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Worksheet 4

1. Simplify the following expressions.

   (a) $\neg\forall x \exists y (P(x,y) \land \exists z R(x,y,z))$

   (b) $\neg\forall x (\exists y \forall z P(x,y,z) \land \exists z \forall y P(x,y,z))$

   Answer:

2. Use the rules of inference to show that the following arguments are valid:

   (a) $q \leftrightarrow \overline{p},\ \overline{q} \land r,\ p \to s,\ s \to t\ \vdash t \lor r$

   (b) $u \to s,\ \overline{p} \land (\overline{p} \lor \overline{q}) \to \overline{s},\ u\ \vdash p$

   Answer:

3. **Inference rules for predicate logics:**
A plant is considered endangered if it has not been found in at least three different locations. The Blueleaf plant has been found in only one location this year. According to environmental regulations, an endangered plant cannot be commercially traded. We have a specific Blueleaf plant in question.

This scenario can be formalized using the following predicates, assuming the universe of discourse is the set of all plants:

- $E(x)$: x is endangered.
- $F(x,n)$: x is found in n different locations.
- $T(x)$: x can be traded.

The term "Blueleaf" refers to a particular instance of a plant.

Given:

(a) $\forall x(\neg F(x,3) \rightarrow E(x))$ - A plant not found in at least three locations is endangered.
(b) $\forall x(E(x) \rightarrow \neg T(x))$ - An endangered plant cannot be traded.
(c) $F(\text{Blueleaf}, 1)$ - The Blueleaf plant has been found in only one location.

To Prove:

- $E(\text{Blueleaf}) \wedge \neg T(\text{Blueleaf})$ - The Blueleaf plant is endangered and cannot be traded.

Answer:

4. Let n be a positive integer. Prove that if n is an even, then $4n^2+48n+144$ is a perfect square.

   (a) What is the given statement and your assumptions?

   (b) What is your conclusion?

   (c) Construct the proof:

   Answer:

5. Using the direct proof method, prove that if n is an odd perfect square, then n is of the form 4k+1. Answer the questions as directed.

   (a) What are the assumptions?

   (b) What are you trying to conclude?

   (c) Show your work.

   Answer:

6. Consider the following theorem:
   ”For an integer n, if n is odd, then n^3 is odd.”

   Verify that the theorem is true using direct proof.
   Answer: