CMPEN362 — Preliminaries

Ting He

Probability

Problem 1

Given: n i.i.d. Bernoulli(p) random variables $X_1, \ldots, X_n$

Q1: probability that $X_1, \ldots, X_k$ are 1, rest are 0?
A1: $p^k(1-p)^{n-k}$

Q2: probability that (any) k variables are 1, rest are 0?
A2: $\binom{n}{k} p^k(1-p)^{n-k}$

Q3: probability that at least k variables are 1?
A3: $\sum_{i=k}^{n} \binom{n}{i} p^i(1-p)^{n-i}$

Problem 2

Given: n independent processes $X_1(t), \ldots, X_n(t)$, each a sequence of i.i.d. Bernoulli(p) random variables

Q1: probability that at $t=1$, only $X_1(t) = 1$ (rest are 0)?
A1: $p(1-p)^{n-1}$

Q2: probability that at $t=1$, only one variable is 1 (rest are 0)?
A2: $q := \binom{n}{1} p(1-p)^{n-1}$

Q3: probability that at $t=1$, at least two variables are 1?
A3: $\sum_{k=2}^{n} \binom{n}{k} p^k(1-p)^{n-k} = 1-(1-p)^n - np(1-p)^{n-1}$

Q4: probability that the first time of having only one variable being 1 is t ($t \geq 1$)?
A4: $q(1-q)^{t-1}$

Definitions

Given a discrete random variable $X \in \mathcal{X}$ with distribution $(p_x)_{x \in \mathcal{X}}$

Mean: $\mathrm{E}[X] := \sum_{x \in \mathcal{X}} p_x \cdot x$
Empirical mean: Given samples $x_1, \ldots, x_n$ from n trials, $\overline{X} := \frac{1}{n} \sum_{i=1}^{n} x_i$

Variance: $\mathrm{Var}[X] := \mathrm{E}[(X - \mathrm{E}[X])^2] = \sum_{x \in \mathcal{X}} p_x \cdot (x - \mathrm{E}[X])^2 = \mathrm{E}[X^2] - (\mathrm{E}[X])^2$
Standard deviation: $\sqrt{\mathrm{Var}[X]}$

Series

Arithmetic series $\sum_{i=1}^{n} (a_0 + (i-1)d) = \frac{n}{2}(2a_0 + (n-1)d)$
Geometric series $\sum_{i=1}^{n} a_0 r^{i-1} = \frac{a_0(1-r^n)}{1-r}$