CMPSC 465 Data Structures & Algorithms
Fall 2025 Ke Chen and Yana Safonova **HW 1**

HW 1

Due September 15, 10:00 pm

Instructions: You are encouraged to solve the problem sets on your own, or in groups of three to five people, but you must write your solutions strictly by yourself. You must explicitly acknowledge in your write-up all your collaborators, as well as any books, papers, web pages, etc. you got ideas from.

Formatting: Each problem should begin on a new page. Each page should be clearly labeled with the problem number. The pages of your homework submissions must be in order. You risk receiving no credit for it if you do not adhere to these guidelines.

Late homework will not be accepted. Please, do not ask for extensions since we will provide solutions shortly after the due date. Remember that we will drop your lowest three scores.

This homework is due Monday, September 15, at 10:00 pm electronically. You need to submit it via Gradescope (Course ID: 1087979). Please ask on Campuswire about any details concerning Gradescope and formatting.

1. (5 pts.) **Getting started**. Please read the course policies on the syllabus, especially the course policies on collaboration. If you have any questions, contact the instructors. Once you have done this, please write "I understand the course policies." on your homework to get credit for this problem

2. (36 pts.) **Comparing growth rates.** In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). Justify your answers.

	$f(n)$	$g(n)$
a)	$4n \cdot 3^n + n^{50}$	4^n
b)	$\log(5n)$	$\log(4n)$
c)	$e^n + n^{10}$	2^n
d)	7^n	8^n
e)	$\log(n^8 + n^2)$	$\log(6n)$
f)	$2^n + n^{100}$	$3 \cdot 2^n + \log n$
g)	$\sqrt{n} + \log n$	$4\sqrt{n} + n^{1/3}$
h)	$(\log_3 n)^{\log_3 n}$	$3^{(\log_3 n)^2}$
i)	$\frac{\log n}{n}$	$n^{\frac{1}{n}}$

3. (24 pts.) **Proofs.**

(a) Prove that $3n^2 + 4n + 5 = \Theta(n^2)$.

(b) Prove that if $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$, then $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$.

(c) Prove that if $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$, then $f(n) = \Theta(h(n))$ i.e., transitivity of theta notation.

4. (14 pts.) **Useful Identities.** Show the following statements hold true.

(a) Show that

$$\log(n!) = \Theta(n \log n).$$

(b) Show that

$$\sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n).$$

(*Hint:* To show an upper bound, compare $\frac{1}{i}$ with $\frac{1}{t}$, where t is a power of two just smaller than i. To show a lower bound, compare $\frac{1}{i}$ with $\frac{1}{t}$, where t is a power of two just greater than i.)

5. (21 pts.) **Recurrence Relations**. Solve the following recurrence relations and give the tightest correct upper bound for each of them in O notation. Assume that $T(O(1)) = O(1)$. Show all work.

(a) $T(n) = 4T(n/2) + 5n^3$

(b) $T(n) = 3T(n/4) + n^{0.6}$

(c) $T(n) = 7T(n/3) + n^2$

(d) $T(n) = 8T(n/2) + 3n^3$

(e) $T(n) = T(n-1) + d^n$, where $d \geq 1$ is a constant

(f) $T(n) = T(n/3) + 0.75^n$

(g) $T(n) = T(4n/7) + T(3n/7) + \Theta(n)$. (*Hint: Assume $T(k) \leq ck \log k$ for some constant $c > 0$ for all $k < n$. Prove that $T(n) \leq c_1 n \log n$ for some constant $c_1 > 0$ by induction.*)