CMPSC 360 — Discrete Mathematics for Computer Science
Fall 2024 — Mahfuza Farooque

Worksheet 3

1. Simplify the logical expression $q \vee \neg(\neg(p \wedge r) \rightarrow (p \vee \neg q))$ using logical equivalence rules.

Answer:

$$\begin{aligned}
q \vee \neg(\neg(p \wedge r) \rightarrow (p \vee \neg q)) &\equiv q \vee \neg((p \wedge r) \vee (p \vee \neg q)) && \text{(Implication Equivalence)}\\
&\equiv q \vee ((\neg p \vee \neg r) \wedge (\neg p \wedge q)) && \text{(De Morgan's Law)}\\
&\equiv q \vee [(\neg p \wedge \neg p \wedge q) \vee (\neg r \wedge \neg p \wedge q)] && \text{(Distributive Law)}\\
&\equiv q \vee [(\neg p \wedge q) \vee (\neg r \wedge \neg p \wedge q)] && \text{(Idempotent Law)}\\
&\equiv q \vee (\neg p \wedge q) && \text{(Absorption Law)}\\
&\equiv q && \text{(Absorption Law)}
\end{aligned}$$

2. Use the logical equivalence properties below to verify the logical equivalence.

$$(p \rightarrow \neg q) \wedge (p \rightarrow \neg r) \equiv \neg(p \wedge (q \vee r))$$

Answer:

$$\begin{aligned}
(p \rightarrow \neg q) \wedge (p \rightarrow \neg r) &\equiv (\neg p \vee \neg q) \wedge (\neg p \vee \neg r) && \text{(Implication Equivalence)}\\
&\equiv \neg p \vee (\neg q \wedge \neg r) && \text{(Distributive Law)}\\
&\equiv \neg p \vee \neg(q \vee r) && \text{(De Morgan's Law)}\\
&\equiv \neg(p \wedge (q \vee r)) && \text{(De Morgan's Law)}
\end{aligned}$$

3. Using truth tables, determine whether $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

Answer:

p	q	r	$(p \wedge q)$	$(p \wedge q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The two propositions are not logically equivalent.

4. (A) Express the following using predicates, quantifiers, logical connectives, and mathematical operators if necessary.

(a) Every positive integer is the sum of the squares of four integers. (The universe of discourse contains all integers)

(b) Every user has access to exactly one mailbox. (Assume that the domain consists of all users and all mailboxes)

(B) Let G(x, y) mean that child x has played video-game y, where the domain for x consists of all the children in your school and the domain for y consists of all video-games. Express these statements as an English sentence.

(a) $\exists a \forall b (a \neq (child_1) \land (G(child_1, b) \rightarrow G(a, b)))$

(b) $\exists x \exists y \forall z ((x \neq y) \land (G(x, z) \leftrightarrow G(y, z)))$

Answer:

(A) (a) $\forall x \exists a \exists b \exists c \exists d ((x > 0) \rightarrow x = a^2 + b^2 + c^2 + d^2)$

(b) Let A(x, y) mean that user x has access to mailbox y.
$\forall x \exists y (A(x, y) \land \forall z (z \neq y \rightarrow \neg A(x, z))$

(B) (a) There is a child other than $child_1$ who has played all the video-games that $child_1$ has played. (This does not mean that this child has only played the video-games played by $child_1$. The child could have also played other games.)

(b) There exist two different children who have played exactly the same video-games.

5. Prove or disprove that the following compound proposition is a contingency.

$$((p \lor r) \lor ((q \land p) \lor (q \land r))) \land \overline{r} \land \overline{p}$$

Answer:

$$\begin{aligned}
&((p \lor r) \lor ((q \land p) \lor (q \land r))) \land \overline{r} \land \overline{p} \\
&\equiv ((p \lor r) \lor (q \land (p \lor r))) \land \overline{r} \land \overline{p} \text{ (Distributive Law)} \\
&\equiv (p \lor r) \land \overline{r} \land \overline{p} \text{ (Absorption Law)} \\
&\equiv (p \lor r) \land \overline{(p \lor r)} \text{ (DeMorgan's Law)} \\
&\equiv F \text{ (Complement Law)}
\end{aligned}$$