CMPSC 360 Discrete Mathematics for Computer Science
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Worksheet 12 Solutions

1. Suppose a company has two factories, Factory A and Factory B, that produce 60% and 40% of its products, respectively. Factory A produces 2% defective products while Factory B produces 3% defective products. If a detective item is drawn at random, what is the probability that it was produced by factory B?
   Answer:
   Let D be the event that a product is defective, and let B be the event that the product was produced by Factory B. We want to find $P(D|B)$.

$$P(B) = 0.4, P(A) = 0.6, P(D|B) = 0.03, P(D|A) = 0.02$$

$$P(B|D) = \frac{P(D|B) \times P(B)}{P(D|B) \times P(B) + P(D|A) \times P(A)} = \frac{0.4 \times 0.03}{0.6 \times 0.02 + 0.4 \times 0.03} \approx 0.444$$

2. Two dice are rolled. Let:

   - A = "Sum of two dice equals 3."
   - B = "Sum of two dice equals 7."
   - C = "At least one of the dice shows a 1."

   (a) What is $P(A|C)$?
   (b) What is $P(B|C)$?

   Answer:

   (a) $P(A|C) = \frac{P(A \cap C)}{P(C)}$. There are only 2 outcome favorable to both A and C, which are (1,2), (2,1). Thus:

$$P(A \cap C) = \frac{2}{36}, \quad P(C) = \frac{11}{36}, \quad P(A|C) = \frac{\frac{2}{36}}{\frac{11}{36}} = \frac{2}{11}.$$

   (b) $P(B|C) = \frac{P(B \cap C)}{P(C)}$. There are 2 outcomes favorable to both B and C, which are (1,6) and (6,1). Thus:

$$P(B \cap C) = \frac{2}{36}, \quad P(C) = \frac{11}{36}, \quad P(B|C) = \frac{\frac{2}{36}}{\frac{11}{36}} = \frac{2}{11}.$$

3. A student is given a multiple choice exam with 10 questions, each question with five possible answers. This student guesses randomly for each question.

   (a) What's the probability this student gets exactly 6 questions correctly?

   (b) What's the probability this student gets at least 6 questions correctly?

   Answer:

   (a) The probability that the student guesses exactly 6 questions correctly out of 10 is calculated using the binomial probability formula:

   $$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

   where $n = 10$, $k = 6$, and $p = 0.2$. Therefore:

   $$P(X = 6) = \binom{10}{6} (0.2)^6 (0.8)^4 \approx 0.0055$$

   (b) The probability that the student guesses at least 6 questions correctly is the sum of the probabilities of guessing 6, 7, 8, 9, and 10 questions correctly:

   $$P(X \geq 6) = \sum_{k=6}^{10} \binom{10}{k} (0.2)^k (0.8)^{10-k} \approx 0.0063$$