

1. Proof or disprove the following statements.
  - (a) For  $a, b \in \mathbb{R}$ , if  $a^2 = b^2$ , then  $a = b$ .
  - (b)  $n^2 + n + 41$  is prime for  $n \in \mathbb{Z}$ .
2. Prove by contrapositive that if a number is divisible by 4, then its last two digits in base 10 is divisible by 4.
3. Suppose  $q \in \mathbb{Z}$ . Prove by contrapositive that if  $6q + 7$  is even, then  $q$  is odd.
4. For  $a, b, c \in \mathbb{R}^+$ , prove that if  $ab = c$ , then  $a \leq \sqrt{c}$  or  $b \leq \sqrt{c}$ .
5. Suppose  $n \in \mathbb{Z}$  and  $p \in \mathbb{P}$  (i.e.,  $p$  is a prime number). Prove the statement: if  $p \mid n$ , then  $p \nmid (n+1)$ .