

Comparison between networks of conductance- and current-driven neurons: stationary spike rates and subthreshold depolarization

Giancarlo La Camera, Walter Senn, Stefano Fusi¹

Institute of Physiology, University of Bern, B hlplatz 5, 3012 Bern, Switzerland

Abstract

The problem of an equivalence between conductance-driven and current-driven neurons in terms of mean stationary output rates is investigated. We show that it is possible to study a network of conductance-driven neurons by means of a mean field analysis of an equivalent network of current-driven neurons. The current drive is Gauss distributed and not voltage dependent. The equivalent network is composed by the same neurons and exhibits the same stable firing rates at the only price of having different connectivity. In addition, the differences in the subthreshold depolarization and the interspike-interval distribution can be studied at parity of output rates, providing a method to study those effects of the conductance drive which do not arise in a network of current-driven neurons.

Key words: conductance drive, current drive, integrate-and-fire neurons, neural networks, mean field, delay activity, coefficient of variability

1 Introduction

The problem of how conductance-driven neurons differ from current-driven neurons has become increasingly popular recently, to the point to question well established results obtained studying networks of current-driven spiking neurons [7]. To give a meaningful answer to this question, it is necessary to specify the observables with respect to which differently-driven neurons ought to behave equivalently. Here we study the problem of mean rates of asynchronous activity in large networks of spiking neurons. Population density approach provides a synthetic and reliable description of cortical phenomena like spontaneous or delay activity observed for example in many areas of behaving monkeys (see e.g. [1] and refs. therein). Briefly, neurons of the

¹ *E-mail addresses:* {lacamera,wsenn,fusi}@cns.unibe.ch

same kind are grouped in homogeneous populations which can be fully described by the population spike rate and whose neurons all share the same input *statistics*. The *stationary* patterns of activity in the network are given by the self-consistent solutions of *mean field equations* like $f_{out}^i = \Phi^i(\underline{f}_{in})$, where i labels the population and the frequencies f^i describe the state of the network. In such a framework, conductance- and current-driven neurons are to be considered equivalent if they are described by the same self-consistent equations. Given a network of conductance-driven neurons, we show how to build an equivalent network of current-driven neurons which exhibit the same stationary spike rates.

2 The network of conductance-driven neurons

For the sake of simplicity, we consider only two homogeneous populations, excitatory (e) and inhibitory (i). In the cortex, thousands of synaptic inputs delivered to the target cells by means of local conductance changes (independent or only weakly correlated), can be described in terms of Gauss distributed conductances $g_{e,i}(t)$ (see e.g. [3]), whose average $\mu_{e,i}$ and variance $\sigma_{e,i}^2$ are given by

$$\begin{aligned}\mu_{e,i} &= \bar{g}_{e,i} N_{e,i} c_{e,i} \nu_{e,i} \tau_{e,i} \\ \sigma_{e,i}^2 &= \frac{1}{2} \bar{g}_{e,i}^2 N_{e,i} c_{e,i} \nu_{e,i} \tau_{e,i}\end{aligned}$$

where N is the total number of neurons of a population; c is the fraction of neurons of this population which are directly connected; ν is the mean frequency of synaptic releases; \bar{g} is the mean conductance change induced by a single event; τ is the time constant of the exponential decay of a single post-synaptic change in conductance, which allows ionic current to flow into the cell. A good choice to match experimental data is $\tau \sim 5$ ms [3], therefore here we use $\tau_e = \tau_i = 5$ ms.

Note how the statistics of the input is completely specified by the *quadruplet* $\Omega = \{\nu_e, \bar{g}_e, \nu_i, \bar{g}_i\}$. The input current is obtained by multiplying the total synaptic conductances $g_{e,i}(\Omega, t)$ by the corresponding voltage dependent driving forces

$$I_{e,i}(\Omega, t) = g_{e,i}(\Omega, t)(V_{e,i} - V(t))$$

where $V_i = V_{rest} - 10$ and $V_e = V_{rest} + 70$ mV are the reversal potentials, and V_{rest} is the membrane potential of the post-synaptic cell. Its total input current is $I_{cond} \equiv I_e + I_i$.

3 The equivalent Gaussian current to get the same mean firing rates

If one injects a Gaussian current characterized by the same average and variance of the total current I_{cond} arising in conductance drive, in general one would *not* get the same spike rate as one would in the full simulation with the

conductance drive. This is because the driving force can skew the Gaussian distribution of the conductances and can introduce complex correlations in the resulting current I_{cond} , not taken into account by a Gaussian current with the same average and variance as I_{cond} .

The problem, then, is to find the *effective* or *rescaled* Gauss distributed, voltage-independent current which, given a large pool of inputs $\{\Omega\}$, gives for each Ω the same output frequency f_Ω which would be obtained under conductance drive. This would *map* a network of conductance-driven neurons in a network of current-driven neurons *which exhibit the same stationary spike rates*. The two networks will then be equivalent in the sense that their stationary mean field activities are the same.

The most natural candidate for an equivalent Gaussian current is probably that coming from the attempt to linearize the dynamical equations for the couple (V, I) around an holding potential $V^* = V(\Omega^*)$, where Ω^* represents a typical input in cortical conditions, as done e.g. in [2]. Such procedure amounts to looking for effective 'average' potentials $V_J^{e,i}$ (playing the role of 'average' driving forces $V_{e,i} - \langle V \rangle$, see [2]) such that the Gaussian current with average and variance given by

$$\hat{m}_I = \bar{g}_e V_J^e N_e c_e \nu_e \tau_e - \bar{g}_i V_J^i N_i c_i \nu_i \tau_i$$

$$\hat{s}_I^2 = \frac{1}{2} (\bar{g}_e V_J^e)^2 N_e c_e \nu_e \tau_e + \frac{1}{2} (\bar{g}_i V_J^i)^2 N_i c_i \nu_i \tau_i$$

gives the same output frequency at parity of input Ω . Although this may work for a single Ω , it does not work *for all* Ω s in a large pool, as we will show below.

Note that in the above procedure, the map is given in terms of effective synaptic efficacies $J_{e,i} \equiv \bar{g}_{e,i} V_J^{e,i}$. It is natural then to increase the number of degrees of freedom and look for a solution in terms, for example, of properly rescaled connectivities in addition to synaptic efficacies. In formal terms, for each quadruplet $\Omega = \{\nu_e, \bar{g}_e, \nu_i, \bar{g}_i\}$ characterizing the statistics of the input, we build a Gaussian current $I_{cur}(\Omega)$ with mean $m_I(\Omega)$ and variance $s_I^2(\Omega)$ according to:

$$m_I = J_e N_e \tilde{c}_e \nu_e \tau_e - J_i N_i \tilde{c}_i \nu_i \tau_i$$

$$s_I^2 = \frac{1}{2} J_e^2 N_e \tilde{c}_e \nu_e \tau_e + \frac{1}{2} J_i^2 N_i \tilde{c}_i \nu_i \tau_i$$

where $J_{e,i} = \bar{g}_{e,i} V_J^{e,i}$ are the peak currents per single pre-synaptic spike (units of current), and $\tilde{c}_{e,i} = \beta_{e,i} c_{e,i}$ are the rescaled connectivities in the network of current driven neurons. Two networks (of conductance- and current-driven neurons) will then be equivalent if values for $V_J^{e,i}$, $\beta_{e,i}$, *independent of* $\nu_{e,i}$, can be found such that the neuron's mean rate f_{cond} under conductance injection I_{cond} is the same as the mean rate f_{cur} under current injection I_{cur} . In such a case, in fact, the mean field activities for the two networks are the same. Note that the scaling parameters $\{V_J^{e,i}, \beta_{e,i}\}$ may still depend on the other

parameters defining Ω , i.e. $\bar{g}_{e,i}$, and yet the equivalence be achieved. We found that, for a network of integrate-and-fire neurons, a remarkable match can be found with unique (i.e. Ω -independent) scaling factors (Fig 1, top left). These scaling factors depend only on the parameters of the neurons (capacitance, membrane time constant, reset potential, threshold for spike emission, etc.)

Note now that two procedures outlined in this section are equivalent if and only if $\beta_{e,i} = \beta_{e,i}^2$, i.e. $\beta_{e,i} \equiv 1$, in which case we were not able to find any reasonable match between f_{cur} and f_{cond} for the same pool of inputs.

4 Beyond the spike rates

The scheme outlined above is not enough to get a complete match in the response as quantified by the entire spike train, but it allows to study conductance vs current driven neurons *at parity of output spike rates* for all inputs, in particular enables us to study the statistics of the depolarization and of the interspike intervals (ISIs). For a given quantity, higher moments like the variance, or equivalently the *coefficient of variability* (CV), are usually different even if the mean spike rates are the same (and often they *have* to be different in order to achieve the same spike rates, as shown in the previous sections).

In general, the rescaled Gaussian current is in a complex relationship with the input current in conductance drive, but for the subthreshold depolarization a clear feature emerges: its standard deviation is always larger in current drive, while its average is smaller. As a consequence the coefficient of variability of the depolarization (CV_V) is smaller in conductance drive (Fig 1, top right); at the same time, the CV_{ISI} is larger in conductance drive (Fig 1, bottom left).

The different statistics of the depolarization can be due to at least three different factors: a) the correlations between current and depolarization; b) the presence of an effective reflective barrier for the depolarization located at the inhibitory reversal potential V_i (below V_i , inhibitory inputs become excitatory and as a result excursions below V_i are forbidden in conductance drive); c) the correlation length τ_s of the current which naturally arises in conductance drive, here equal to 5 ms (the CV_{ISI} is always increased by a larger synaptic time [6], both in conductance and current drive).

To investigate the effect of the correlation length, we found the unique scaling parameters which give the frequency match with a δ -correlated Gaussian current ($\tau_s = 0$). The effect of τ_s on the CV_V is shown in Fig 1 (bottom right), in which one can see that there is still a residual difference in the CV_V between conductance and current drive, which cannot be accounted for by the correlation length only. We also checked that the presence of a reflecting barrier at V_i for the depolarization in current drive has a negligible effect (not shown). As a result, the cross-correlations between depolarization and current in conductance drive is likely to be an important factor responsible for the mismatch in

the depolarization and in the ISI statistics, at parity of output rates. These correlations may in fact also change the effective correlation length of the current, so that a comparison at parity of τ_s would be unfair. As an overall result, the dynamics of the two (equivalent) networks may be very different although the steady states of asynchronous activity are the same (but see [4]).

5 Discussion

In this work current- versus conductance-driven neurons were studied in the framework of the mean field theory of large recurrent networks of spiking neurons. For such networks, equivalence means having the same patterns of stable firing rates in homogeneous populations, regardless of the modality in which the input is delivered to the target cells, i.e. either in conductance or in current drive. Our main result is that it is possible to obtain the same mean firing rates of a network of conductance-driven neurons by studying an *equivalent* network of current-driven neurons, whose inputs are Gauss distributed and *not* voltage dependent. Compared to the original network, the companion network has the same number of neurons and the same stable firing rates at the only price of different connectivities and suitable synaptic efficacies. This result was obtained with a method that allows comparison, at parity of output rates, of the statistics of important quantities like the subthreshold depolarization and the ISIs distribution. Finally, our results also provide a bridge between *in vitro* experiments in which conductance injection is emulated *via* the *dynamic clamp* technique (see e.g. [5]), and *in vitro* experiments with current injection. To this purpose, the problem of an equivalent formulation in terms of current-driven neurons must be approached carefully, and its solution relies upon a correct choice of the relevant variables which are to be investigated. This work also represents an example of how such a procedure may be carried out.

References

- [1] Amit D. J. and Brunel, N., *Cerebral Cortex* **7**:237-252 (1997); Amit D.J. and Tsodyks M.V., *NETWORK* **2**:259 (1991)
- [2] Amit D.J. and Tsodyks M.V., *NETWORK* **3**:121-137 (1992)
- [3] Destexhe A. *et al*, *Neuroscience* **107**:13-24 (2001)
- [4] Fourcaud N. and Brunel N., *Neural Computation* **14**:2057-2110 (2002); Brunel N. and Sergi S., *J. Theor. Biol.* **195**:87-95 (1998)
- [5] Harsch A. and Robinson H.P.C., *J. Neurosci.* **16**:6181-6192 (2000)
- [6] La Camera *et al*, *Proceedings of ICANN2002, LNCS 2415, pp. 235-240, Springer-Verlag* (2002); Svirskis G. and Rinzel J., *Biophys J.* **5**:629-637 (2000)
- [7] Tiesinga P.H.E., Josè J.V. and Sejnowski T.J., *Physical Review E* **62**(6):8413-8419 (2000)

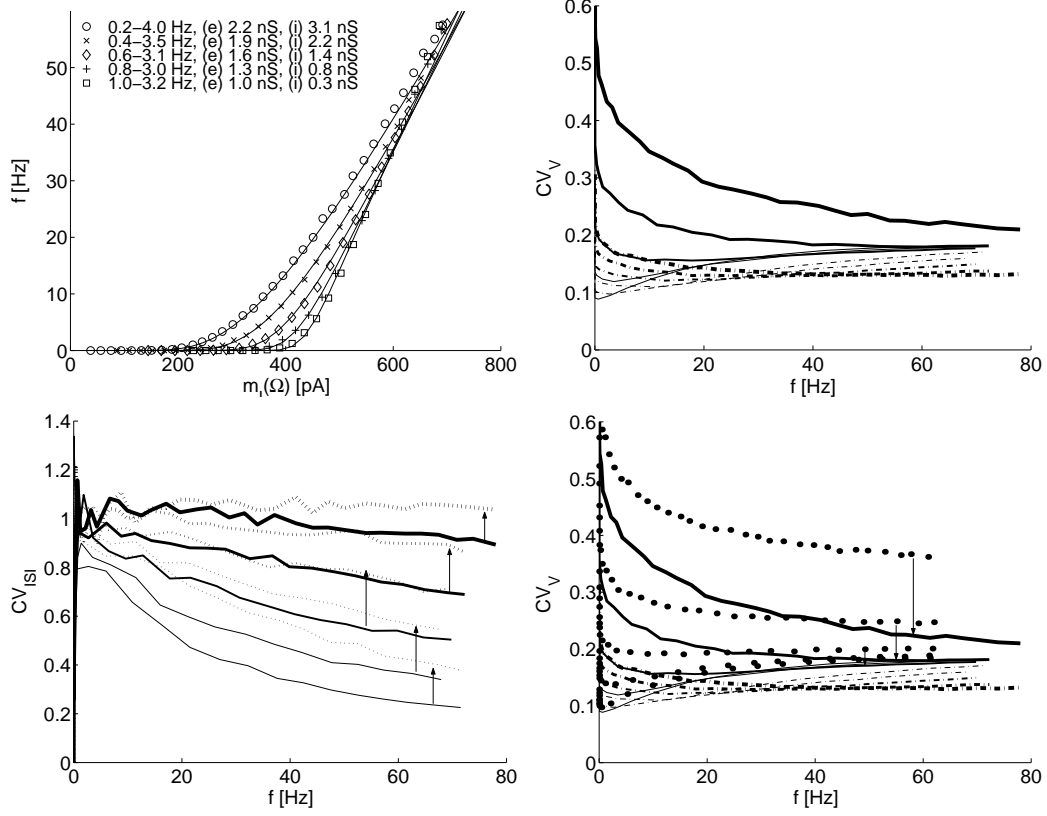


Fig. 1. **Top Left:** Comparison between a conductance-driven (symbols) and a current-driven (full curves) IF neuron after rescaling (simulations). The dynamics of the subthreshold membrane potential V evolve according to the Eq: $dV/dt = -V/\tau_m + I/C$, with $I = I_{cond}(\Omega)$, and thresholded at $\theta = 20$ mV to emulate a spike emission, with reset at $V_r = 10$ mV and no refractory period ($C = 500$ pF, $\tau_m = 20$ ms, $V_{rest} = 0$). The frequency match was obtained through a Montecarlo minimization of the squared difference between the observed frequencies and of the output mean rates of a current-driven neuron driven by the same inputs (whose stationary spike rate is given in [4] as a function of m_I, s_I). The best-fit parameters values were $V_f^e \approx 24.7$ mV, $V_f^i \approx 59.1$ mV, $\beta_e = \tilde{c}_e/c_e \approx 2.2$ and $\beta_i = \tilde{c}_i/c_i \approx 0.5$. Each curve was generated by setting $\bar{g}_{e,i}$ equal to constant values (reported in the top left corner of the figure), and then by sweeping along approximately the diagonal of the $\{\nu_e, \nu_i\}$ plane (i.e. $\nu_e \approx \nu_i$ for each point; ν_e ranges reported in the plot. $N_{e,i}c_{e,i} = 1000$). This procedure was chosen to explore different quadruplets Ω arising from realistic parameters and giving reasonable input currents and output frequencies. **Top Right:** Coefficient of variability of the depolarization in conductance (dot-dashed) and current (full) drive. A *thickness code* replaces the symbol code of the top left panel, with the thickest line corresponding to the circles. **Bottom Left:** CV_{ISI} as a function of the output frequency for both drives (same convention as for top right panel). Arrows help link corresponding curves. **Bottom Right:** Same as top right panel plus the CV_V for the case of an equivalent Gaussian, delta-correlated current (dots; see text). The arrows show the effect (in current drive) due to the time correlation length of the input (τ_s). A residual difference with the CV_V in conductance drive remains and cannot be accounted for by τ_s .