A piece-wise harmonic Langevin model of EEG dynamics: Theory and application to EEG seizure detection

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Abstract

Electroencephalogram (EEG) dynamics results from the motion of charged particles in the brain. By invoking the laws of classical physics, we show that EEG dynamics obeys, at short enough times, a piece-wise quadratic Langevin equation characterized by an effective mass matrix, a piece-wise quadratic potential energy surface, a friction and random force matrix. In analyzing a sample seizure, we find with seizure onset a marked increase in the frequency of encounters with inflection points on the potential energy surface.

Keywords: EEG; seizure detection; potential energy; particle dynamics.

1 Introduction

Current automated EEG seizure detection techniques largely depend on pattern-recognition algorithms of high sophistication [7]. Newer techniques include various methods of measuring complexity as adapted from chaos theory [5,8,9], and the energy measure of Litt and coworkers [10]. The measures of complexity used in chaos theory include physically suggestive parameters such as the correlation dimension, similarity, synchrony and Lyapunov exponent. These measures all show that seizure onset is associated with a decrease in complexity, sometimes occurring 20 minutes or more before a clinical seizure.

The power of the chaos methods is that they are based on universal properties of dynamical systems and are independent of the details of the underlying dynamics. If at some point, however, one wishes to explore the details of the underlying dynamics, one must then turn to more explicit, physically based models. Of physically based models, the most established ones are based on an analogy between electrical circuits and neural or neuronal circuits. Nunez [12] emphasizes finding "global" solutions in his cable equations that represent long wavelength EEG standing waves. Neural networks is a vast field of research and is based on a computational paradigm [1]. Neural networks may be built using realistic neuronal models, as epitomized by the work of De Schutter and coworkers [4], applied to a study of cerebellar dynamics [11].

Here we offer another physically based model, drawing on an analogy between EEG tracings and the trajectory of classical particles, with each EEG channel output describing the trajectory of a 1-dimensional particle. We justify this analogy using techniques developed in the theoretical chemistry literature and analyze a sample seizure.

2 Formal theory

EEG data are measured as the difference in electric potential between one electrode and one or more other electrodes. The electric potential at position $\vec{r}_k$ due to n charges Q_i within the brain at coordinates $\vec{r}_i(t)$ ($i = 1$ to n) is given by

$$X_k(t) = \sum_{i=1}^{n} \frac{Q_i}{|\vec{r}_k - \vec{r}_i(t)|}. \tag{1}$$

The charge Q_i moves around in space as given by the coordinate $\vec{r}_i(t)$ but the coordinate $\vec{r}_k$ is fixed in space. The number of charged particles n is on the order of 10^{23}. The number of spatial coordinates $\vec{r}_k$ is the number of EEG electrodes used, typically on the order of $N = 19$. The dynamics of a few macroscopic variables that interact with an enormous number of microscopic particles has been intensively studied in the theoretical chemistry literature [2,14]. If the microscopic potential energy surface is continuous and differentiable, then it can be piecewise expanded, for consecutive short time intervals τ_o, as a quadratic function in the X_k's [3,6]. Within the Markov limit the equation of motion then takes the form:

$$\mathbf{M}\ddot{\mathbf{X}} = \mathbf{A} - \mathbf{K}\mathbf{X} - \mathbf{G}\dot{\mathbf{X}} + \mathbf{F_R}. \tag{2}$$

Here $\mathbf{X}$ is a vector whose components are the X_k's, and $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$ are the first and second time derivatives of $\mathbf{X}$. $\mathbf{M}$, $\mathbf{A}$, $\mathbf{K}$, $\mathbf{G}$ and $\mathbf{F_R}$ depend in a complicated way on the microscopic particle coordinates, charges and masses. For our purposes, we shall not need to know these dependences. Instead, we regard $\mathbf{M}$, $\mathbf{A}$, $\mathbf{K}$ and $\mathbf{G}$ as constants within each short time interval τ_o to be extracted from the EEG data (see below).

One can regard Eq 2 either as the equation of motion of an effective particle of mass $\mathbf{M}$ with an N-dimensional spatial coordinate $\mathbf{X}$, or equivalently as that of N coupled one-dimensional particles. We find the single particle in N-dimensional space representation more convenient. $\mathbf{A}$ and $\mathbf{K}$ are the linear and quadratic expansion coefficients of the potential energy surface; $\mathbf{G}$ is a friction matrix;

and $\mathbf{F_R}$ is the random force. Direct interactions of the effective particle with other degrees of freedom appear through the potential energy surface, while indirect interactions appear through the friction and random force [2,14]. The mass matrix varies across short time intervals τ_o because charged particles can move in and out of the EEG electrode field of detection. The potential energy surface expansion coefficients $\mathbf{A}$ and $\mathbf{K}$ also vary in time across time intervals τ_o because the true potential energy surface is in general not globally quadratic. We emphasize that the piecewise quadratic expansion of the potential energy surface in no way limits our analysis. Any continuous and differentiable potential energy function can be treated in this way.

To extract $\mathbf{M}$, $\mathbf{A}$, $\mathbf{K}$ and $\mathbf{G}$ from EEG data, define $\delta\mathbf{X} = \mathbf{X} - \langle\mathbf{X}\rangle_c$, where $\langle...\rangle_c$ denotes a time average over t_o to $t_o+\tau_c$. Here τ_c is a correlation interval $\tau_c > \tau_o$ over which a correlation function is calculated. The correlation interval is taken longer than the fitting interval τ_o because the statistics near the end of the correlation interval is less reliable. Define an N by N time-correlation matrix for the time interval t_o to $t_o+\tau_c$ by $\mathbf{C}(t) = \int_{t_o}^{t_o+\tau_c} dt'\delta\mathbf{X}(t+t')\delta\mathbf{X}(t')^T$, where superscript T denotes a transpose. Note that we do not normalize this correlation function. Then it can be shown that

$$\mathbf{M}\ddot{\mathbf{C}}(t) = -\mathbf{K}\mathbf{C}(t) - \mathbf{G}\dot{\mathbf{C}}(t) + \mathbf{C_{RX}}(t) \qquad (3)$$

where $\mathbf{C_{RX}}(t) = \int_{t_o}^{t_o+\tau_c} dt'\mathbf{F_R}(t+t')\delta\mathbf{X}(t')^T$. The parameters $\mathbf{A}$, $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{G}$ are extracted from EEG data by making $\mathbf{C_{RX}}$ as small as possible in a least squares sense. This optimization goal can be regarded as a way of defining the indirect degrees of freedom (i.e., the random force) such that they are as little correlated as possible with the explicit variable $\mathbf{X}$ [cf 2].

To proceed, for each time interval define an error measure by $E = \frac{1}{2}Tr[\langle\mathbf{C_{RX}}\mathbf{C_{RX}^T}\rangle_o]$ where $Tr[...]$ indicates a matrix trace, and $< ... >_o$ denotes a time average over the fitting interval τ_o. We use standard numerical methods for minimizing E [13], obtaining values for $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{G}$ so that E is as small as possible. The vector $\mathbf{A}$ is given by $\mathbf{A} = \mathbf{M}\langle\ddot{\mathbf{X}}\rangle_c + \mathbf{K}\langle\mathbf{X}\rangle_c + \mathbf{G}\langle\dot{\mathbf{X}}\rangle_c$. The eigenvalues of the mass matrix $\mathbf{M}$ are constrained to be positive so that every effective relative mass is positive, and the sum of the masses are normalized so that the average relative mass is one in dimensionless units. Absolute masses cannot be determined because there is a free multiplicative factor in Eq (3). Because the potential energy surface is assumed to be differentiable, the matrix $\mathbf{K}$ must be symmetric. Because of time-reversal symmetry, the matrix $\mathbf{G}$ must also be symmetric. Its eigenvalues must also all be positive. If it were not for the physical constraints on $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{G}$, the minimization procedure could be done analytically. However, these solutions would be unphysical. Once $\mathbf{M}$, $\mathbf{A}$, $\mathbf{K}$ and $\mathbf{G}$ are found, the random force $\mathbf{F_R}$ can be found using Eq 2.

3 Application to seizure detection

A 3000-second sample of a left central seizure was analyzed (Fig 1). EEG data were acquired digitally at 400 Hz with a notch filter of 60 Hz. Eyeblink artifact was removed by excluding the anterior electrodes, retaining N = 12 channels (T3-T5, T5-O1, F3-C3, C3-P3, P3-O1, T4-T6, T6-O2, F4-C4, C4-P4, P4-O2, C3-Cz, Cz-C4). Onset of seizure is labeled by $t = 0$. The end of the seizure occurs at about $t = 20$ seconds. No other seizure occurred in the sample. Time derivatives were taken in the frequency domain using Fast Fourier Transforms. Hamming windows were applied in frequency space to exclude frequencies below 20 Hz and above 50 Hz. The time correlation interval τ_c was taken to be 75 milliseconds but only the first $\tau_o = 50$ milliseconds were used for the least squares fit.

For each fitting window, the nearest extremum of the potential energy surface is given by $\mathbf{X_o} = \mathbf{K}^{-1}\mathbf{A}$. The numerical error in the inversion of $\mathbf{K}$ is never greater than 1 part in 10^6. Each extremum can be a maximum (a peak on the potential energy surface), a minimum (a potential well) or a saddle point. An extremum displacement series is constructed by tracking $|\mathbf{X_o}|$ in time. This series shows occasional peaks reaching hundreds to thousands of microvolts that are one to two orders of magnitude larger than the maximal amplitudes of the trajectories $\mathbf{X}$. These peaks likely represent encounters with inflection points on the potential energy surface, where the inverse of $\mathbf{K}$ becomes singular and the extrapolation to the extremum $\mathbf{X_o} = \mathbf{K}^{-1}\mathbf{A}$ diverges. In practice, $\mathbf{X_o}$ is always finite because $\mathbf{K}$ is extracted from a block of data covering a period of 50 milliseconds, during which time $\mathbf{X}$ may sample an inflection point but is not fixed to that point.

The peaks in the extremum displacement series are more frequent with onset of seizure. This increase is brought out by taking the *median* of $|\mathbf{X_o}|$ over each preceding 5 second interval, shown in Fig 2. For comparison, the rectified root mean square amplitude $|\mathbf{X}|$ is also shown in Fig 2. The relative magnitude $|\mathbf{C_{RX}}|/|\ddot{\mathbf{C}}|$ is a measure of the relative error of the fit. This error is on the order of 1% for each fitting window with standard deviation of 0.4%.

4 Discussion

A particle dynamics approach to EEG analysis allows us to think of EEG dynamics in terms of the motion of an effective particle on a potential energy surface. Most of the time, the particle moves inside a potential energy well (its motion is bounded). However, the encounters with inflection points on this potential energy surface tell us the surface is not globally quadratic, because a globally quadratic potential energy surface contains no inflection points. The non-quadratic terms in the potential energy surface give rise to nonlinear forces. Thus the increase in encounters with inflection points with seizure onset suggests a simultaneous increase in nonlinear forces. Note that we can detect nonlinearity in the dynamics even though the potential energy surface is taken to be piece-wise quadratic.

Encounters with inflection points on the potential energy surface may reflect an attempt to cross an energy barrier (an inflection point exists between every potential energy well and barrier) but our current analysis does not tell us whether an energy barrier is subsequently crossed. One would need to reconstruct the potential energy surface, piece by piece, across multiple time intervals τ_o, in order to demonstrate actual barrier crossing. We do not know if this reconstruction is technically feasible.

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Figure captions

1. A 30-second EEG segment containing a 20-second left central seizure. The arrow marks approximate time of seizure onset.

2. The upper curve gives the root mean square amplitude of $\mathbf{X}$ in microvolts. The lower curve gives the median of the extremum displacement series $|\mathbf{X}_o|$. Note the logarithmic scale. Onset of seizure is at $t = 0$. Seizure ends at approximately $t = 20$ seconds.

Fig 1: Sample EEG with left central seizure

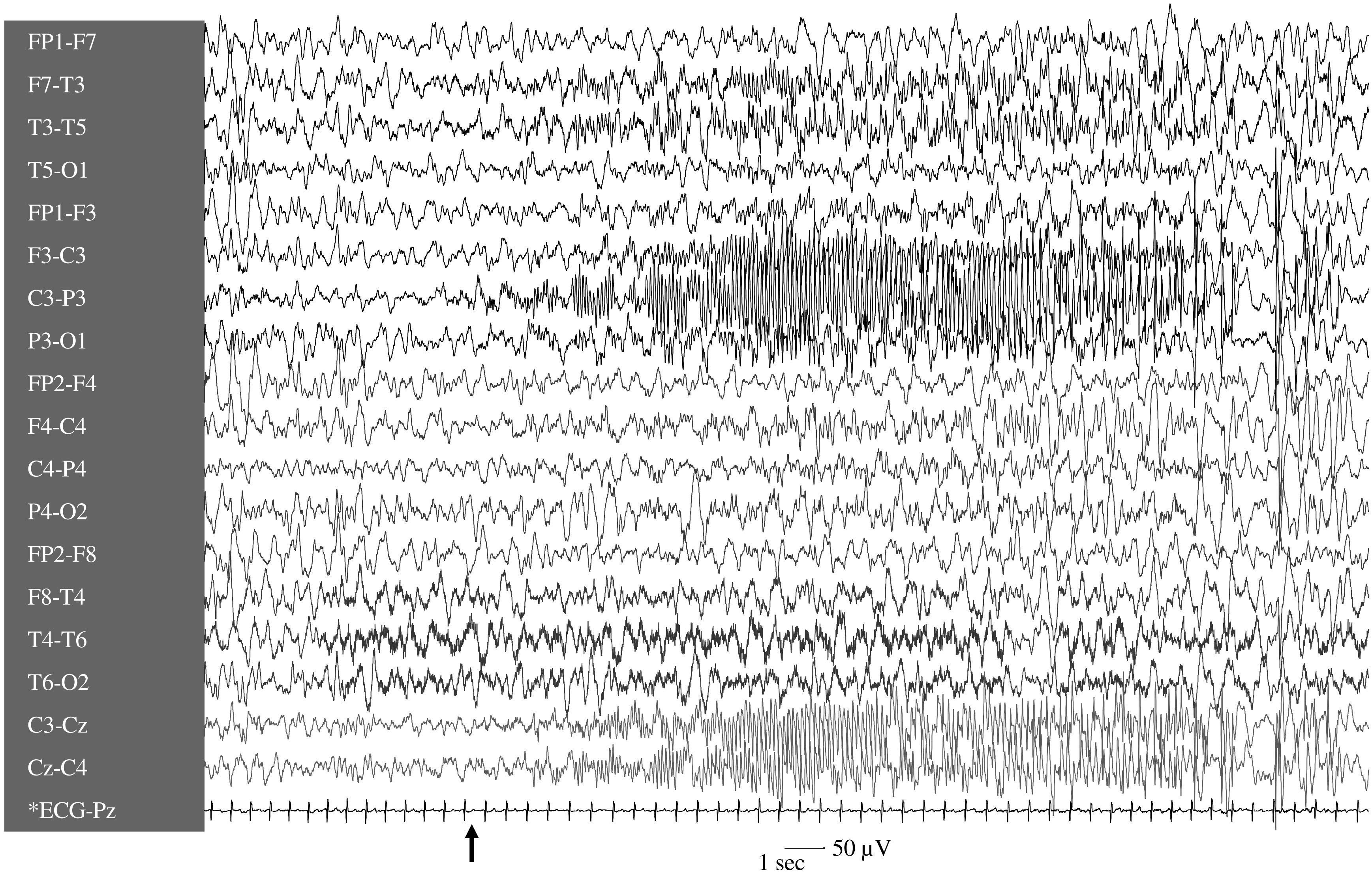
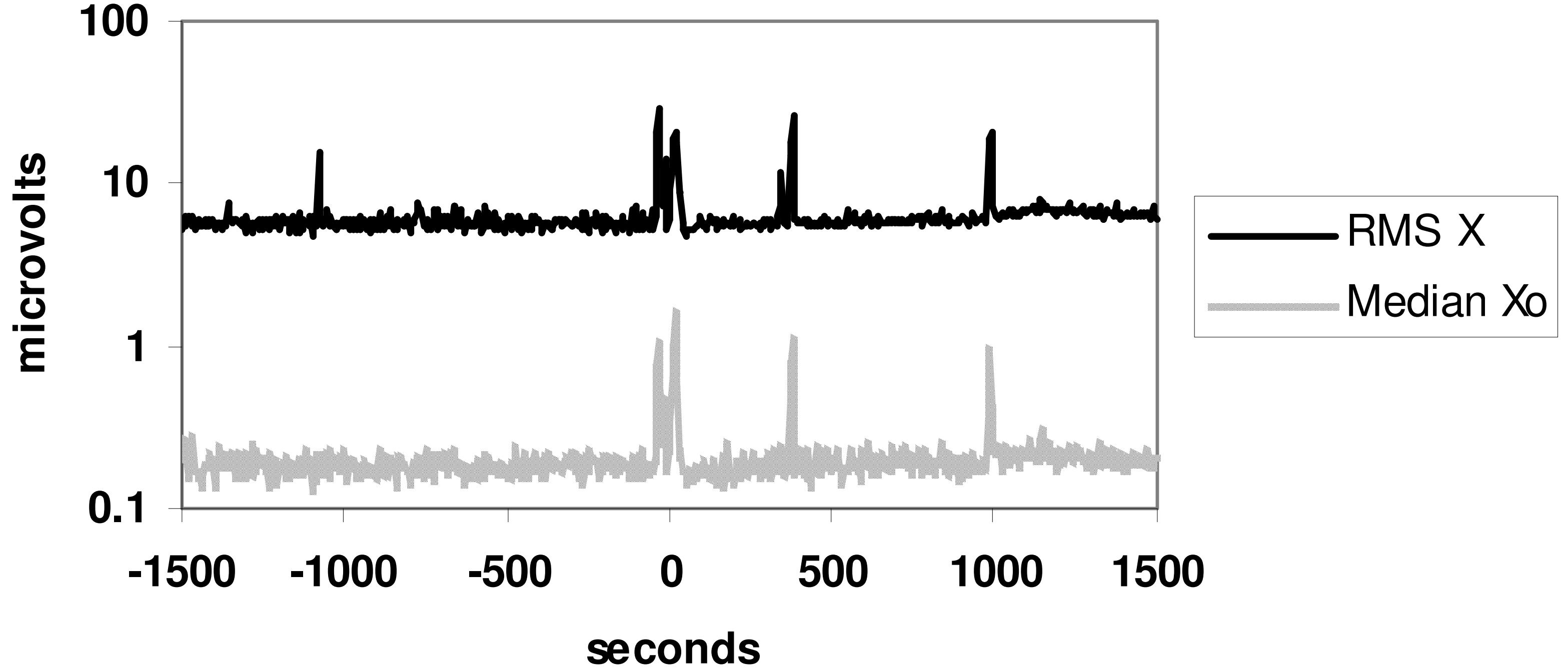


Fig 2



100
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