

Stochastic Interaction in Associative Nets

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Abstract

Spatio-temporal correlations in spike trains of simultaneously recorded cortical neurons have been a matter of intensive research during the last years. Information theoretic measures for “spatial” and “temporal stochastic interaction” can characterize spike correlations on a mathematical base. In the present work we calculate these interaction measures for associative networks that have been proposed as models for cortical gamma-oscillations and precisely repetiting spike patterns (synfire chains).

Key words: Spike correlations; Stochastic interaction; Associative memory models

1 Spike-correlations and Associative Models

Simultaneous recordings of multiple cortical units revealed various types of correlation patterns on the time-scale of single spikes [1,2,7,8,11]. Fig. 1A, for instance, displays the correlation function of two (artificially generated) spike trains and reveals a dominant peak at time zero indicative for frequent synchronous firings. The side-peaks at $\pm 20, 40, \dots$ milliseconds further relate to a preference of the cells to fire periodically (here at 50Hz in the gamma-range). “Synchronized gamma-oscillations” of this type in real data have been proposed as expressing the binding of feature coding neurons into “coherent” object representations [7,11]. Figure 1B displays part of a correlation function for three units. The abscissa counts occurrences of spike-patterns in a long trial, where the first unit fires at some time t , the second unit fires τ_1 ms

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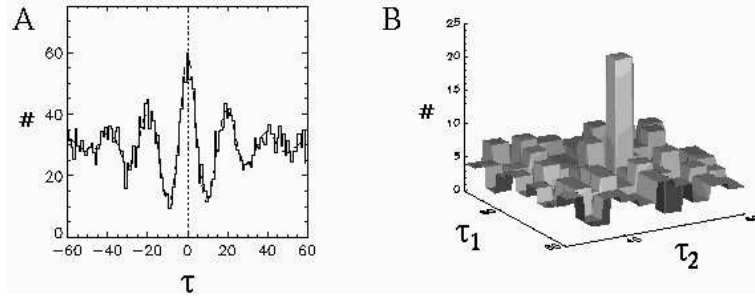


Fig. 1. Correlation patterns in spike-trains. Left: Synchronized oscillation of 2 neurons; Right: Synfire pattern comprising 3 units, i.e., 2 time-shifts.

later, and the third one τ_2 ms after the first. Most “triplets” defined by τ_1 and τ_2 occur at a low rate consistent with the assumption of independent firing. The prominent peak in Fig. 1B, however, defines a certain pattern that appears significantly more often than chance level. Recordings from monkey prefrontal cortex by Abeles [1] indicate that such “synfire patterns” occur in a behavior-dependent manner, but their functional role still is unknown.

Attempts have been made, to relate the above types of spatio-temporal correlations to tentative network architectures. These networks are instances of associative memories, which comprise a number of artificial neurons and store a set of binary patterns in the synaptic connectivity matrix using the correlation learning rule or Hebb-rule [10,12]. Figure 2 displays simulations of simple associative networks comprising leaky-integrate-and-fire neurons (cf., e.g., [6]). Figure 2A shows an (auto-)associative synaptic matrix storing six non-overlapping patterns in a network of 36 units. Figures 2B and C reveal two possible modes of persistent dynamic activity in the net: Each dot represents the firing of a cell at the respective time. In B all neurons of the first pattern fire periodically and synchronized. Although more detailed models for synchronized gamma-oscillations have been proposed, the example shown here is the most simple one that explains prototypically the type of correlations in Fig. 1A. In Fig. 2C, where the fifth pattern has been excited, the same model reveals a second mode of behavior, where neurons still fire periodically, but asynchronously. Thus, the type of spike patterns and correlations observable in simulations is not a unique function of the network connectivity.

Figure 2D shows a connectivity matrix, where six patterns have been stored in cyclic order. Correspondingly, the model dynamics consists in a sequence of firings of the whole patterns, see Fig. 2E. This model is basically the “synfire chain model” as proposed by Abeles [1] in order to explain precise repeating firing patterns as described in Fig. 1B. However, observe that also the active cells in Fig. 2C fire in a fixed temporal order although they are homogeneously coupled. Thus, repeating patterns can occur in different architectures.

These simple examples show that 1) the same architecture can lead to various types of firing correlations, and 2) different architectures can lead to spike pat-

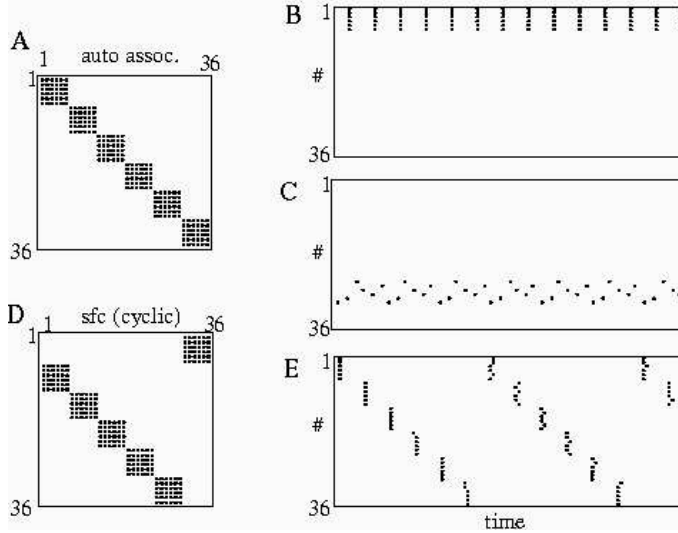


Fig. 2. A: Auto-accociative synaptic coupling matrix storing six non-overlapping patterns of six neurons each. B and C: Persistent synchronized (B) and asynchronous (C) activity as a raster plot over time in a network of spiking neurons. D: Hetero-associative cycle of six patterns. E: Ordered retrieval of the stored sequence.

terns with comparable properties. Clearly, the situation in real neural systems is much more complex than our simple examples, but this makes the problem even more severe (cf., e.g., [2,8]). Therefore, one should carefully distinguish between the correlation structure in spike data (also called “functional couplings” [2]) and the underlying physical connectivity.

2 Spatial and Temporal Stochastic Interaction

Recently, various attempts have been made to characterize spike correlations as well as more general stochastic interactions in spike data mathematically [3–5,8,9]. For a stationary probability distribution p on the joint state space Ω of N units, *spatial (stochastic) interaction* can be defined as

$$I(p) = \sum_{v=1}^N H(p_v) - H(p) , \quad (1)$$

where the p_v are the marginals of p . This is the well-known mutual information or transinformation of the units with respect to p expressed in terms of Shannon-entropies. $I(p)$ measures the Kullback-Leibler distance from independence, whence, the name “interaction”. Spatial interaction has been extended to stochastic processes by Ay [4]. Given a probability distribution p on the states of N units and a Markov kernel K for the probabilities of state transitions, *temporal (stochastic) interaction*, $I(p, K)$ can be represented as

$$I(p, K) = \sum_{v=1}^N H(p_v, K_v) - H(p, K) , \quad (2)$$

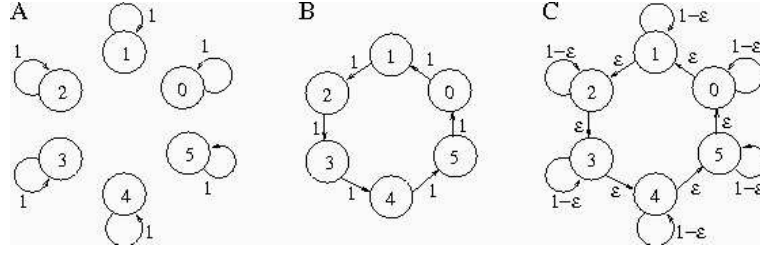


Fig. 3. Transition diagrams for (A) the auto-associative and (B) the synfire chain model and six stored patterns. Labels at edges denote one-step transition probabilities. Graph C contains A and B as limiting cases for $\epsilon = 0$ and 1.

where the *conditional entropy* of a Markov transition (p, K) is defined as

$$H(p, K) := - \sum_{\omega, \omega' \in \Omega} p(\omega) K(\omega' | \omega) \ln K(\omega' | \omega) .$$

In (2) the p_v and K_v are marginal probabilities and Markov kernels for unit v , respectively (for precise definitions see [4]).

3 Spatial and Temporal Interaction in Associative Networks

Because associative models have been proposed as explaining spatio-temporal correlations in spike train data we now study their stochastic interaction measures. In order to map the time-continuous dynamics in Fig. 2 to the setting of (time-discrete) Markov chains, we assume state-transitions between activated patterns as depicted in Fig. 3. This simplifies the dynamics somewhat by neglecting spike jitter (e.g., Fig. 2E) and the different firing modes in Fig. 2D and E), but still enables a comparison of the auto-associative and hetero-associative modes (attractor network and synfire chain, respectively). Formally, we consider M patterns $s^i = (s_1^i, \dots, s_N^i) \in \{0, 1\}^N$, $i = 0, 1, \dots, M-1$ such that for all units $v \in \{1, \dots, N\}$: $|\{i : s_v^i = 1\}| \leq 1$ (that is, each unit is in at most one pattern). We define the (support) size of the patterns s^i as $N_i := |\{v : s_v^i = 1\}|$, and their average size as $\bar{N} := \frac{1}{M} \sum_{i=0}^{M-1} N_i$. Corresponding with Fig. 3C we use a parametric family of transition kernels:

$$K^\epsilon(s^i | s^j) := \begin{cases} \epsilon & , \text{ if } i = (j+1) \bmod M \\ 1 - \epsilon & , \text{ if } i = j \\ 0 & , \text{ otherwise} \end{cases} , \quad \epsilon \in [0, 1] .$$

The probability distribution $p := \frac{1}{M} \sum_{i=0}^{M-1} \delta_{s^i}$ is stationary for all transition kernels K^ϵ . With these definitions, $I(p)$ results from (1) as

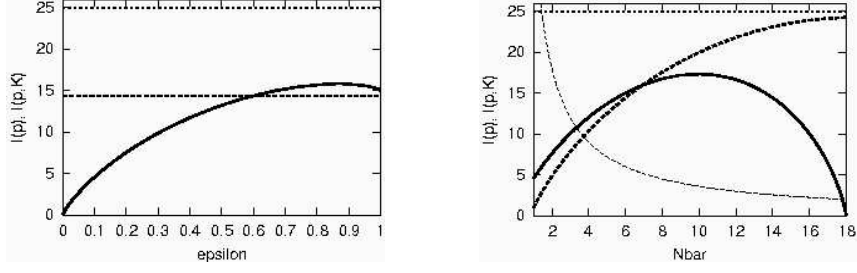


Fig. 4. Temporal (solid) and spatial interaction (dashed) in the Willshaw associative memory; dotted line: maximally possible value, $N \ln 2 \approx 24.95$. Left: Interaction as function of ε with $N = 36$, $\bar{N} = 6$, $M = 6$. Right: Interaction as a function of pattern size \bar{N} with $N = 36$, $M = N/\bar{N}$, $\varepsilon = 1$. The thin line represents the number of storable (non-overlapping, equal sized) patterns.

$$\begin{aligned}
 I(p) &= \left(\sum_{i=0}^{M-1} N_i \right) \cdot \left(-\frac{1}{M} \ln \frac{1}{M} - \frac{M-1}{M} \ln \frac{M-1}{M} \right) - \ln M \\
 &= (\bar{N}M - 1) \ln M - \bar{N}(M-1) \ln(M-1),
 \end{aligned}$$

which obviously is independent of ε . Now, we compute the temporal stochastic interaction. The marginal kernels K_v^ε are for all v

$$K_v^\varepsilon(0|0) = \frac{M-1-\varepsilon}{M-1}, K_v^\varepsilon(1|0) = \frac{\varepsilon}{M-1}, K_v^\varepsilon(0|1) = \varepsilon, K_v^\varepsilon(1|1) = 1-\varepsilon.$$

Using these kernels and (2) the temporal stochastic interaction gets

$$\begin{aligned}
 I(p, K^\varepsilon) &= \bar{N}(M-1) \ln(M-1) - \bar{N}(M-1-\varepsilon) \ln(M-1-\varepsilon) \\
 &\quad - (2\bar{N}-1) \varepsilon \ln \varepsilon - (\bar{N}-1)(1-\varepsilon) \ln(1-\varepsilon).
 \end{aligned}$$

Figure 4 (left) displays $I(p)$ and $I(p, K)$ for $M = 6$ and $N_i = \bar{N} = 6$, $i = 0, \dots, 5$ as in Fig. 2. Apparently, $I(p) = 14.4$ is constant at roughly 58 percent of the maximally possible value of $36 \ln 2 = 24.95$ for 36 independent binary units. So, the attractor and synfire chain dynamics have equal spatial interaction. Nonetheless, as Fig. 4 (left) shows, the attractor dynamics ($\varepsilon = 0$) has zero temporal interaction, whereas that of the synfire dynamics ($\varepsilon = 1$) is high. Figure 4 (right) displays interaction values as a function of pattern size for $\varepsilon = 1$, i.e., the synfire chain model. Increasing pattern size decreases the number of storable (non-overlapping) patterns (thin line). For $\bar{N} = N/2 = 18$ we can store just two patterns which must therefore alternate in each step ($\varepsilon = 1$). Then, the temporal interaction becomes zero. Conversely, for $\bar{N} = 1$, units are silent for $N-1$ steps after firing also corresponding with a low interaction. Only for intermediate pattern sizes the temporal interaction can be large. In contrast, spatial interaction increases monotonically and obtains its optimal value at $\bar{N} = N/2 = 18$, where $M = 2$, such that the single unit

entropies are maximal (each unit is 0 in one pattern and 1 in the other). These entropies decrease for smaller patterns such that $I(p)$ gets small for small \bar{N} . The picture remains basically the same for larger networks, but note, that storage of a reasonable number of patterns requires small pattern sizes. Then the interaction values may fall considerably below the maximum value of $N \ln 2$. As we will show elsewhere, *overlapping* patterns, e.g., random patterns, in that regime again result in interactions that reach a moderately high fraction of the maximally possible values.

In summary the derived formulas show that spatial stochastic interaction in auto-associative Willshaw networks and hetero-associative synfire chain networks are equal and reach moderate values. Temporal interaction is nonvanishing only in networks comprising hetero-associative propagation of activity. In [4,5] we argued that neural systems may try to maximize temporal interaction. In that case activity propagation along neural pathways should be the rule rather than persistent attractor states.

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