Quantitative Information Transfer through Layers of Spiking Neurons Connected by Mexican-Hat-type Connectivity

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Abstract

A feedforward network with homogeneous connectivity cannot transmit quantitative information by one spike volley. In this paper, quantitative information transmission through neural layers connected by Mexican-Hat(MH) type connectivity is examined. It is shown that the intensity of an input signal can be encoded as a size of an active region in a neural layer.

Key words: Synfire Chain, Mexican-Hat, Quantitative Information

1 Introduction

Quantitative information is an essential factor to faithfully encode the outer world. For example, color vision, auditory perception, and nociception are normally accompanied by the signal intensity. However, the nature of neural

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information coding is an open question in neuroscience. In a sensory system, the firing rate of a neuron correlates with the signal intensity [8]. On the other hand, recent studies show that a synchronous activity is also a ubiquitous phenomenon in the brain (see review [5]). Recently, information coding in a feedforward network has been intensively studied in the context of the synfire chain [1,3,4,6]. Many feedforward networks considered so far are multilayered neural networks which have no lateral connections nor feedback loops. All the neural layers are homogeneously connected to their next layers in a feedforward manner. When a spiking activity propagates through such a network, the activity tends to be synchronized, otherwise the activity is rapidly dispersed. This result suggests that the information of a signal is finally reduced to 1-bit in the homogeneous feedforward neural networks. This raises the question of whether the quantitative information can be maintained through the feedforward networks.

The problem with the simple homogeneous feedforward networks is that the quantitative information is destroyed after the propagation of several layers. Several studies provide solutions to the problem. Introducing an appropriate level of additive Gaussian noise can bride an asynchronous firing mode and a synchronous firing mode, and the quantitative information on a signal intensity can be transmitted through the layers [7,9]. When weak noise is applied to the system, inter-synchronization interval can crudely encode an input signal [2].

On the other hand, it was shown that a feedforward network with Mexican-Hat-type (MH-type) connectivity can encode the analog information on the signal location [9]. However, encoding the quantitative information on the signal intensity was not yet studied in details. In this paper, we consider a possible mechanism of encoding the signal intensity by using the feedforward network with MH-type connectivity. For the sake of convenience, we will call the feedforward neural network with MH connectivity as the *MH network*.

2 Model

The neuron model used in this paper is the leaky integrate-and-fire neuron, which is described as

$$\frac{\mathrm{d}}{\mathrm{d}t} v(x_i^l, t) = -\frac{v(x_i^l, t)}{\tau} + I(x_i^l, t) + \xi, \tag{1}$$

where $v(x_i^l, t)$ is the membrane potential of the ith neuron at position x_i^l on the lth layer at time t. All the neurons are assumed to be homogeneous with the same membrane time constant τ. The background Ornstein-Uhlenbeck process noise ξ with time constant 2 ms is also introduced with the mean μ and the variance σ^2. θ is a threshold and the kth spike of the ith neuron occurs when $v(x_i^l, t_k(x_i^l)) = \theta$ is satisfied. An input $I(x_i^l, t)$ is described as follows:

$$I(x_i^l, t) = \sum_k \sum_j w\left(|x_i^l - x_j^{l-1}|\right) \alpha\left(t - t_k(x_j^{l-1})\right), \tag{2}$$

$$w(x) = W_0 \left(1 - \frac{x^2}{2\sigma_{\mathrm{MH}}^2}\right) \exp\left(-\frac{x^2}{2\sigma_{\mathrm{MH}}^2}\right). \tag{3}$$

The factor $w(|x_i^l - x_j^{l-1}|)$ is a measure of the synaptic efficacy from the jth neuron on the $(l-1)$th layer to the ith neuron on the lth layer. $w(x)$ is described as the second derivative of the Gaussian function. $\alpha(t - t_k(x_j^{l-1}))$ is the time course of the post-synaptic current (PSC) by the kth spike from the jth neuron on the $(l-1)$th layer. Here $\alpha(t)$ is described with an exponential decay constant α as $\alpha(t) = \alpha^2 t e^{-\alpha t} \mathcal{H}(t)$. $\mathcal{H}(t)$ is the Heaviside step function with $\mathcal{H}(t) = 1$ for $t > 0$ and $\mathcal{H}(t) = 0$ for $t \leq 0$.

The network architecture is shown in Fig. 1(A). Each layer contains $N = 300$ neurons on a one-dimensional layer with a periodic boundary condition.

The input layer neurons receive an external input $I_e(x,t)$. In the input layer, we assume that the responses of nearby neurons to an external signal are correlated with strength specified by the Gaussian function. It represents the property of the sensory cortex, where neighboring neurons usually show similar response properties to a specific stimulus. The external stimulus I_e to the first layer is described as $I_e(x,t) = I_0 \exp(-\frac{x^2}{2\sigma_{\mathrm{G}}^2})(\mathcal{H}(t) - \mathcal{H}(t - \tau_{\mathrm{stim}}))$, where I_0 is the signal intensity, and τ_{stim} is an input duration to be chosen so that one neuron emits at most one spike during the trial. Here we have omitted subscripts and superscripts for the simplicity.

3 Mapping signal intensity to active region in the input layer

Before examining the firing property of the MH network, the mechanism of converting the signal intensity into the active region is studied. Assuming that $v(x,0) = 0$, we get the first passage times in the input layer neurons as is shown in Fig. 1(B). The activity starts from the center of the input and spreads over the input layer. The active region is estimated as $2\sigma_{\mathrm{G}}\sqrt{2\log\left(\frac{\tau I_0}{\theta}\left(1 - \exp(-\frac{\tau_{\mathrm{stim}}}{\tau})\right)\right)}$ when $\sigma = 0$. Note that the active region depends on the signal intensity I_0. The stronger the intensity, the more the active region increases (see Fig. 1(C)).

4 Spatio-temporal spike pattern of the first spike clusters driven by a short duration input

To examine the coding property of the MH network quantitatively, we define a measure of information transmitted through the layers as *the number of spikes*. That is the sum of the number of spikes generated in a layer during one trial period and formulated as follows:

$$\text{the number of spikes} = \int \Sigma_j \delta(t - t_j)\mathrm{d}t, \tag{4}$$

where t_j is the firing time of all the spikes in one layer. When one neuron emits only one spike, this measure is approximately equal to the active region.

The characteristic rastergrams of propagating activities in the MH network are shown in Fig. 2(A) with changing the signal intensity I_0. There is no background noise variance ($\sigma = 0$) but mean input μ, while the initial distribution of the membrane potential is given as the Gaussian distribution with mean 7.5 [mV] and standard deviation $\sigma = 1.9$ [mV]. When a signal intensity is small (Fig. 2(A), $I_0 = 1.2$), it produces just a small active region in the first layer. This small firing cluster also generates a small firing cluster in the second layer, and it continues until it reaches the last layer while maintaining its activity region. As the signal intensity increases ($I_0 = 2.0$ and $I_0 = 3.0$), the propagating activity size is also increased. If the intensity is too small ($I_0 = 1.0$: not shown), the spike cluster is dispersed and unable to propagate its activity. Fig. 2(B) represents the evolution of the number of spikes in each layer with various input intensities. Each line is the mean of 100 trial data, and the top and the bottom of the error-bar represent for the three-fourth and one-fourth values of the data. Eleven lines correspond to the signal intensity $I_0 = [1.0, 1.2, 1.4, ..., 2.8, 3.0]$ from the bottom to the top. The $I_0 = 1.0$ case is hardly observed because the activity is too low. Since there is no background noise, error-bars represent the fluctuation of activities originated from the initial membrane distributions.

As a comparison to the MH network, the rastergrams of a homogeneous feedforward network are shown in Fig. 2(C). To control the initial layer's firing rate, L neurons in the input layer are forced to fire. When the input layer's firing rate is too small, neurons in subsequent layers are not activated ($L = 20$). When a signal intensity exceeds a certain threshold value, it can build up its activity to propagate through the subsequent layers ($L = 40$). The output of the homogeneous network is insensitive to the signal intensity (see Fig. 2(D)) because the response of the network to the $L \geq 40$ cases is almost the same.

5 Discussion

We have examined the coding property of the MH network by numerical simulations. Our result shows that the MH network can encode the quantitative information as the active region of the network. The information is stably transmitted through the layers. Here we have shown the zero noise case, but we have also confirmed that the stable quantitative information coding is achieved with the noise intensity $\sigma = 0 \sim 2$. We have exclusively considered the short duration signal case here. The response of the MH network to the longer duration signal is a future problem.

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Appendix

The parameters used in this work are as follows: membrane time constant $\tau =$

10[ms]; $\alpha = 2$ [ms] ; input time duration $\tau_{\rm stim} = 10$ [ms]; firing threshold $\theta = 15$ [mV]; the mean noise input $\mu = 1$; Mexican-Hat function's standard deviation $\sigma_{MH} = 15$ [units]; weight constant is $W_0 = 3$; refractory period is 5 [ms], and 1 [ms] synaptic delay is incorporated. The standard deviation of the input distribution is $\sigma_G = 50$ [units]. In the homogeneous network model, all the parameters other than the connectivity are the same as those in the MH network. The feedforward connection used in the homogeneous network is all-to-all connection, and the weight constant is 0.25.

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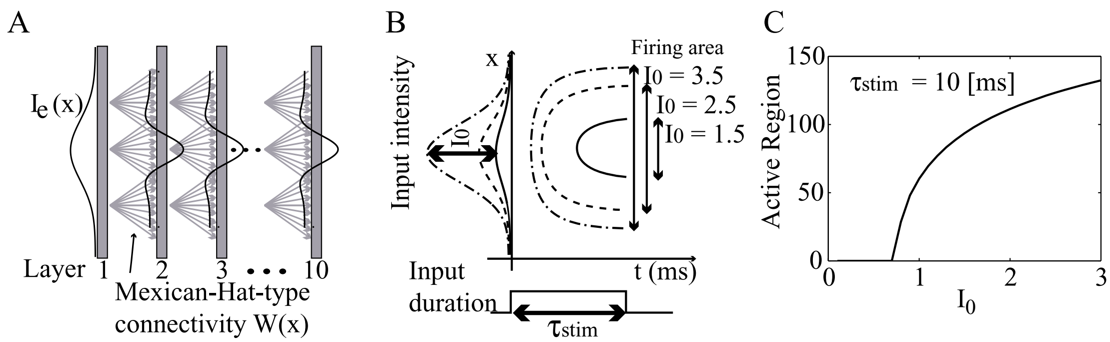

Fig. 1. (A): Network architecture. Layers of leaky integrate-and-fire neurons are feedforwardly connected with the MH-type connectivity. Each layer contains 300 neurons. Mexican-Hat connectivity has σ_{MH} of 15 [units]. One post-synaptic neuron accepts 30 excitatory inputs. σ_{G} is 50 [units]. (B): Spiking time curves of the input layer when the responses of the input layer neurons to the specific signal are described as a Gaussian distribution. (C): An active region in response to the short duration input with an intensity I_0.

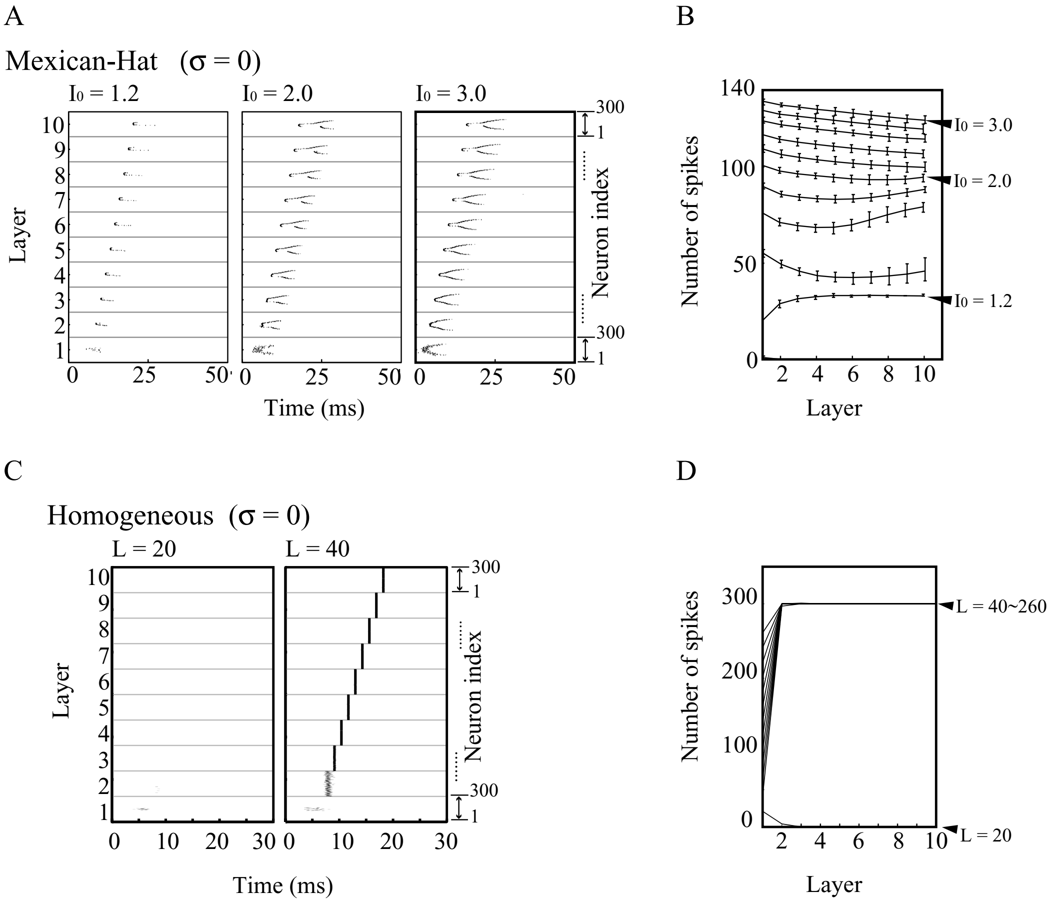

Fig. 2. Propagation of spikes under the no background noise condition. (A): Rastergram of the MH network. (B): The evolution of the *number of spikes* with changing the signal intensity I_0 in the MH network. (C):Rastergram of the homogeneous network. (D):The evolution of the *number of spikes* in the homogeneous network.