Stochasticity in Localized Synfire Chain

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Abstract

We report on stochastic evolutions of firing states through feedforward neural networks with Mexican-Hat type connectivity (the MH network). The variance in connectivity, which depends on the pre-synaptic neuron, generates a common noisy input to post-synaptic neurons. We develop a theory to describe the stochastic evolution of the localized synfire chain driven by a common noisy input. The development of a firing state through neural layers does not converge to a certain fixed point but keeps on fluctuating. Stationary firing states except for a non-firing state are lost, but an almost stationary distribution of firing state is observed.

Key words: Localized Synfire Chain, Stochastic Evolution Equations, Order Parameters, Fourier Modes.

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1 Introduction

A homogeneous feedforward network has been proposed as a simple model of transmitting a synchronous activity and has been intensively studied theoretically [1–5]. Stochastic evolution of propagating activity through a feedforward network has been studied recently [6]. Activity evolves through the network is described as the stochastic process. The stochasticity comes from properties of network structure, i.e., the sparseness of synaptic connections, or variance of connectivity weights. When connectivity is sparse, the number of spikes that a post-synaptic neuron can accept varies [6]. Even when connectivity is not sparse, variance in connection weight produces distributed input to post-synaptic neurons [7].

Activity in a homogeneous feedforward network is uniform. The brain, however, is not homogeneous. A feedforward network with Mexican-Hat (MH) type connectivity has been studied more recently [8,9]. For convenience, we will refer to this network as *the MH network* in this paper. The MH network demonstrates stable propagation of an localized activity, but the effect of common noise originating connectivity variance in a MH network has not yet been studied.

The present paper examines the MH network to study stochastic evolution of a localized activity driven by a common noise. Our strategy is describing the evolution of the firing states through order parameter equations by using the cosine function to represent the MH type connectivity [10].

2 Model

The network model used in this paper is depicted in Fig. 1, and described as

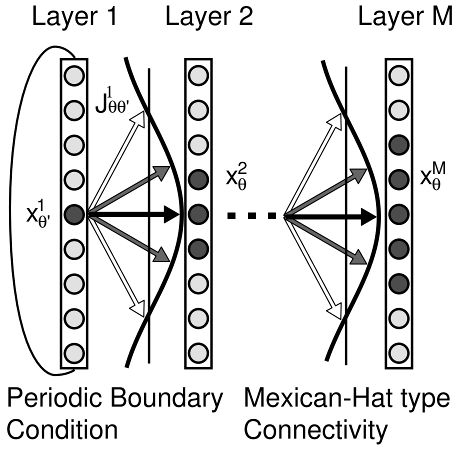

Fig. 1. Network architecture. Each layer consists of N units of neurons arranged in one-dimensional layer with periodic boundary condition. Output of x_θ takes value of 1 (firing), or 0 (silent). M sheets of neural layers have feedforward connection.

$$x_\theta^{l+1} = \Theta\left(h_\theta^{l+1}\right) = \Theta\left(\Sigma_{\theta'} J^l_{\theta\theta'} x^l_{\theta'} - h\right), \tag{1}$$

where $x_\theta^{l+1} = \{0, 1\}$ is the output of the neuron on the $(l + 1)$th layer at position θ. h_θ^{l+1} is the internal state of the neuron. One neuron at θ', where $\theta' = \{-\frac{\pi}{2}, -\frac{\pi}{2} + \frac{\pi}{N}, -\frac{\pi}{2} + \frac{2\pi}{N}, ..., \frac{\pi}{2} - \frac{\pi}{N}\}$, is making a synapse on a next-layer neuron at θ with connectivity $J^l_{\theta\theta'}$. Here, h is a threshold. Θ is a step function. Each layer consists of N neurons with a periodic boundary, and feedforward connection of MH type connectivity $J^l_{\theta\theta'}$. $J^l_{\theta\theta'}$ is described by a cosine function [10] as follows:

$$J^l_{\theta\theta'} = -\frac{J_0}{N} + \frac{J_2}{N}\cos(2(\theta - \theta')) + w^l_{\theta\theta'} + w^l_{\theta'}, \tag{2}$$

where J_0 is a parameter of homogeneous connectivity, and J_2 is the amplitude of MH type connectivity. $w^l_{\theta\theta'}$ and $w^l_{\theta'}$ are fluctuations in connectivity described as the Gaussian distribution. $w^l_{\theta\theta'} \sim \mathcal{N}(0, \Delta^2/N)$, and $w^l_{\theta'} \sim \mathcal{N}(0, \delta^2/N)$. $w^l_{\theta\theta'}$ means connectivity variance between a pair of pre- and post-synaptic neurons, which is independent of each connection. Here we introduce $w^l_{\theta'}$, which is heterogeneity of a pre-synaptic neuron represented as connectivity shift depending only on a pre-synaptic neuron. If $w^l_{\theta'} > 0$ and the neuron at position θ' fires, the probability of emitting a spike in all the post-synaptic neurons is increased. Therefore, $w^l_{\theta'}$ is a source of common noise and the correlations of each neural activities. We define common noise as the sum of all the firing neuron's heterogeneity $w^l_{\theta'}$. Common noise is literally common to all the post-synaptic neurons, and fluctuates as the composition of firing neuron changes.

3 Theory: Evolution Equations for the Order Parameters

This paper introduces three order parameters, r^l_0, r^l_{2c}, and r^l_{2s} as the 0th and 2nd coefficients of the Fourier transformation of the firing state at the lth layer. The order parameters are defined as follows:

$$r^l_0 = \frac{1}{N}\Sigma_\theta x^l_\theta, \quad r^l_{2c} = \frac{1}{N}\Sigma_\theta \cos(2\theta) x^l_\theta, \quad r^l_{2s} = \frac{1}{N}\Sigma_\theta \sin(2\theta) x^l_\theta, \tag{3}$$

where r^l_0 is the mean firing rate of the lth layer and r^l_{2c} and r^l_{2s} are inhomogeneous parameters that represent localized activity around the $\theta = 0$ and, $\theta = \pi/4$. We use the rotation-invariant order parameter $r^l_2 = \sqrt{(r^l_{2c})^2 + (r^l_{2s})^2}$ to plot firing states here after.

In the thermodynamical limit, by substituting the equations (3) into the equation (1), and using the central limit theorem, we obtain another representation of the internal state as

$$\begin{aligned} h = &-J_0 r^l_0 + J_2 \left(r^l_{2c}\cos(2\theta) + r^l_{2s}\sin(2\theta) \right) \\ &+ \Delta\sqrt{r^l_0}\tilde{z}^l_\theta + \delta\sqrt{r^l_0}\tilde{\eta}^l - h, \end{aligned} \tag{4}$$

where $\tilde{z}^l_\theta \sim \mathcal{N}(0,1)$ and $\tilde{\eta}^l \sim \mathcal{N}(0,1)$. We utilized the fact that, in the limit of infinite neuron number N, the sums of noisy inputs are reduced the Gaussian noise. Note that $\tilde{z}^l_\theta$ is a Gaussian noise independent of θ', but $\tilde{\eta}^l$ only depend on layer l and is common to all neurons in the $l+1$th layer. Although $\tilde{\eta}^l$ is the same as what is called common input, we refer to $\tilde{\eta}^l$ as *common noise* because it is described as Gaussian.

Using the above notations, we can derive the order parameter equations for the evolution of firing states. Let $\boldsymbol{r} = (r_0, r_{2c}, r_{2s})^t$. Assuming that $\tilde{\eta}^l$ is given, we can derive a map from $\boldsymbol{r}^l$ to $\boldsymbol{r}^{l+1}$ as,

$$\boldsymbol{r}^{l+1} = F(\boldsymbol{r}^l, \tilde{\eta}^l) = \frac{1}{\pi}\begin{bmatrix} \int_{-\pi/2}^{\pi/2} \mathrm{d}\theta \frac{1}{2}\mathrm{erfc}(\frac{\tilde{z}^l_\theta(\tilde{\eta}^l)}{\sqrt{2}}) \\ \int_{-\pi/2}^{\pi/2} \mathrm{d}\theta \cos(2\theta)\frac{1}{2}\mathrm{erfc}(\frac{\tilde{z}^l_\theta(\tilde{\eta}^l)}{\sqrt{2}}) \\ \int_{-\pi/2}^{\pi/2} \mathrm{d}\theta \sin(2\theta)\frac{1}{2}\mathrm{erfc}(\frac{\tilde{z}^l_\theta(\tilde{\eta}^l)}{\sqrt{2}}) \end{bmatrix}, \tag{5}$$

where

$$\tilde{z}^l_\theta(\tilde{\eta}^l) = -\frac{-J_0 r^l_0 + J_2(\cos(2\theta) r^l_{2c} + \sin(2\theta) r^l_{2s}) + \delta\sqrt{r^l_0}\tilde{\eta}^l - h}{\Delta\sqrt{r^l_0}} \tag{6}$$

We obtained one set of evolution equations for every $\tilde{\eta}^l$. Note that $\tilde{\eta}^l$ is not averaged out over the spatial integration, and self-averaging of $\boldsymbol{r}^l$ breaks down. Therefore, $\boldsymbol{r}^l$ becomes a stochastic variable. Given probability distributions of the lth layer activity $\boldsymbol{r}^l$, and normalized common noise $\tilde{\eta}^l$, the probability distribution on the $l+1$th layer activity is written as,

$$\begin{aligned} p(\boldsymbol{r}^{l+1}) &= \int_{\mathrm{R}} \mathrm{d}\boldsymbol{r}^l \int \mathrm{d}\tilde{\eta}^l p(\boldsymbol{r}^{l+1}|\boldsymbol{r}^l, \tilde{\eta}^l) p(\boldsymbol{r}^l, \tilde{\eta}^l), && (7) \\ &= \int_{\mathrm{R}} \mathrm{d}\boldsymbol{r}^l \int \mathrm{d}\tilde{\eta}^l \delta(\boldsymbol{r}^{l+1} - F(\boldsymbol{r}^l, \tilde{\eta}^l)) p(\boldsymbol{r}^l, \tilde{\eta}^l), && (8) \\ &= \int_{\mathrm{R}} \mathrm{d}\boldsymbol{r}^l K(\boldsymbol{r}^{l+1}, \boldsymbol{r}^l) p(\boldsymbol{r}^l), && (9) \end{aligned}$$

where $K(\boldsymbol{r}^{l+1}, \boldsymbol{r}^l)$ is a kernel function,

$$K(\boldsymbol{r}^{l+1}, \boldsymbol{r}^l) = \int_{-\infty}^{\infty} \mathrm{d}\tilde{\eta}^l p(\tilde{\eta}^l) \delta(\boldsymbol{r}^{l+1} - F(\boldsymbol{r}^l, \tilde{\eta}^l)). \tag{10}$$

where region R is the area $\boldsymbol{r}^l$ exists. Simultaneous distribution $p(\boldsymbol{r}^l, \tilde{\eta}^l)$ can be divided into $p(\boldsymbol{r}^l)p(\tilde{\eta}^l)$. Therefore the conditional probability $p(\boldsymbol{r}^{l+1}|\boldsymbol{r}^l, \tilde{\eta}^l)$, which is a deterministic map from a given $\boldsymbol{r}^l$ and $\tilde{\eta}^l$ to $\boldsymbol{r}^{l+1}$, can be separately integrated.

Here, we have seen that the pre-synaptic dependent heterogeneity $w^l_{\theta'}$ is the origin of stochastic activity propagation. Alternative origin of stochasticity is sparseness of connection [6]. Although the origin of stochasticity is different, the expression of the consequential evolution equations are the same. In both cases, in the limit of $N \to \infty$, the noise component of an input to one post-synaptic neuron is divided into two part; independent Gaussian noise and common noise. It correspond to the $\tilde{z}^l_\theta$ and $\tilde{\eta}^l$, in this paper.

4 Effect of Common Noise

The effect of common noise on uniform activity in a homogeneous feedforward network was examined by Amari et al. [6]. In the MH network, uniform activity is also stable depending on parameters J_0, J_2 and Δ. The activity is almost the same as that observed in a homogeneous feedforward network. Since the localized activity is the characteristic phenomenon in the MH network,we have mainly studied the effect of common noise with the parameter region where the localized activity is stable.

We calculated 10000 computer simulations and the analytical distribution with an initial condition of $p(r^1_0, r^1_2) = \delta(r^1_0 - 0.5)\delta(r^1_2 - 1/\pi)$. The probability distribution of $p(r^l_0, r^l_2)$ at the 20th layer for parameter $(J_0, J_2) = (-0.75, 3)$

is shown in Fig. 2A. The distribution of firing rate over many trials in the homogeneous feedforward network has bimodal distribution in both high firing rate and low firing rate regions [6]. Here, the MH network has a bimodal distribution in the uniform firing and the localized firing regions. Fig. 2D is a bird's eye view of the distribution. This indicates that firing states fluctuate between uniform and localized activities.

When uniform connectivity parameter J_0 is small compared to Fig. 2A, localized activity seems to be dominant (Fig. 2E). However, when MH type connectivity parameter J_2 is small, the analytical solution only exhibits uniform activity. However, the distribution calculated by computer simulations with $N = 3000$, soaked out from the $r_2 = 0$ line. This inconsistency between the theory and simulations results from the finite size effect of N and is clearly observed in the low r_2 region. This inconsistency is, however, limited to low r_2 region and does not affect the entire shape of distribution severely. The intersections at $r_0^{20} = 0, 0.33, 0.66, 0.99$, and $r_2^{20} = 0.05, 0.14, 0.22$ in Fig. 2B,C illustrate the consistency between the theory and the simulations.

5 Discussion

The introduction of common noise made the evolution of the firing state stochastic, and the probability distribution $p(r_0^l, r_2^l)$ have a broad distribution. The firing state distribution in a feedforward homogeneous network has a bimodal structure with peaks at both high and the low firing rates. The activity of one layer is sometimes synchronized, but desynchronized at the other times [6].

Uniform, localized activity, and non-firing state were observed in the MH network for the parameter $(J_0, J_2) = (-0.75, 3)$. This means that activity

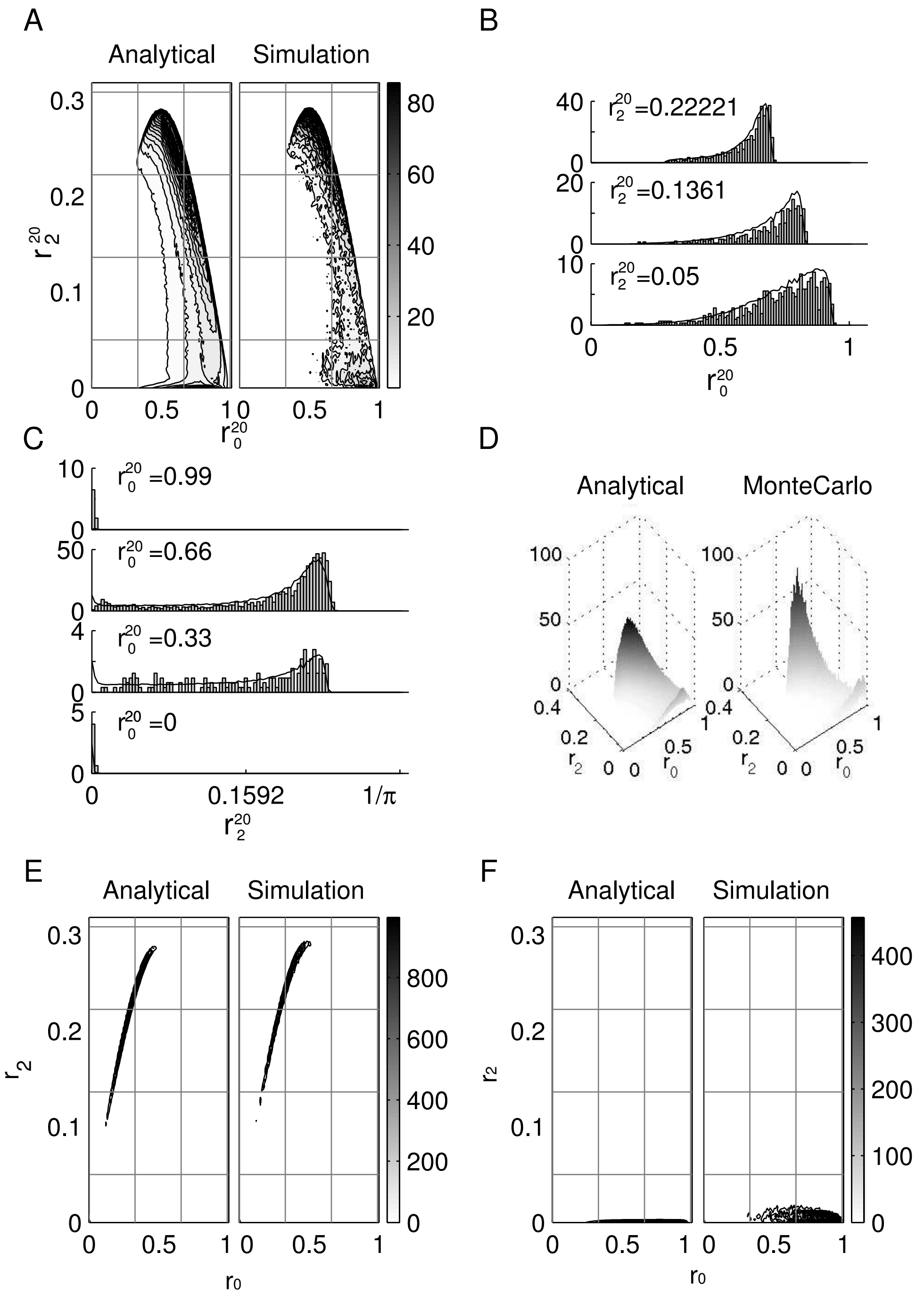

Fig. 2. Probability distribution of r_0^{20}, r_2^{20}. (A): contour plot of probability distribution obtained from analytical and simulation, with parameter $(J_0, J_2) = (-0.75, 3)$. (B) and (C): intersections of Probability distribution of r_0^{20}, r_2^{20} at $r_0^{20} = 0, 0.33, 0.66, 0.99$, and $r_2^{20} = 0.05, 0.14, 0.22$. (D): bird's eye view of (A). (E) and (F): contour plot for the parameters $(J_0, J_2) = (0.2, 3)$, and $(-0.75, 1)$.

is occasionally uniform, and sometimes localized. There are also parameters where localized activities are exclusively observed.

Noise sources may exist in the real neural system other than connection weight variance and sparseness. For example, ongoing activity [11] and thermal noise can be alternative source of firing state fluctuation. Themal noise is usually independent from the activity of a network. The ongoing activity might have dynamical relations with the network activity, but such the recurrent network is hard to analyse. On the other hand, the common noise originated in the connection weight variance has intriguing property; it depends on its network firing rate. The noise intensity increases as the firing rate increases. The effect of common noise in the LIF neuron system is not considered so far. Therefore, it would be intriguing to study the relationships between the variance of inter-spike intervals and the common noise.

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