Influence of topology on a neural networks performance

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Abstract

We have studied and compared the computational properties of an attractor neural network (ANN) with different topologies. Although highly connected neural networks present a better performance, the excessive number of synaptic connections make them biologically unrealistic and evolutionary unlikely. Here we show that the capacity of ANN with scale-free topology to stored and retrieve P binary patterns is higher than those of a highly-diluted random Hopfield network with the same number of synapses. This capacity increases when only nodes with high connectivity degree are considered.

Summary

There is at present a great interest in the study of evolving networks, in particular, networks with a scale–free topology [Albert and Barabasi(2002)]. In these networks, the probability $P(k)$ that a particular node has k neighboring nodes follows a power-law distribution (see figure 1). This implies that the network exhibits a relatively high number of nodes with small connectivity (*boundary*) and only a few number of nodes (*hubs*) with a connectivity that scales with the size of the network. A particularly interesting feature of scale–free networks is that they present the *small-world* property, that is, the average path length between two nodes is very small compared with the network size.

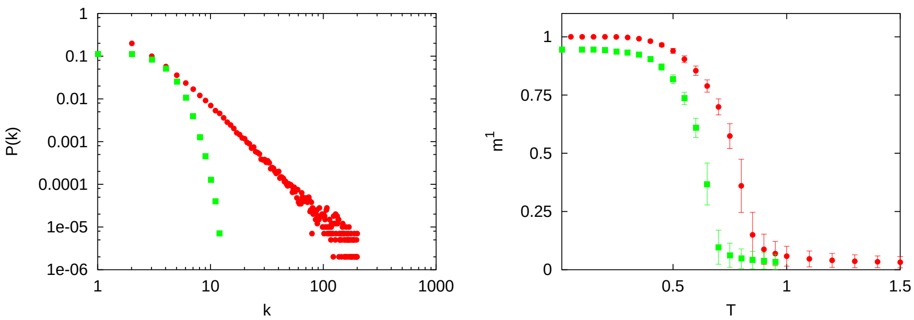

Figure 1: **Left**: A comparison between the connectivity probability distribution for a scale–free network (filled circles, right) and a highly–diluted Hopfield network (filled squares, left). The data correspond to an everage over 100 networks with $N = 4000$ neurons each. **Right**: Overlap curves at finite temperature for a scale–free network (filled circles, right) and a highly–diluted Hopfield network (filled squares, left). The data corresponds to $N = 1600$ neurons, $P = 1$ and $m = m_0 = 3$.

Evolving networks with complex topology appear to be very common in nature and, specially, in biology. Neuronal growing networks are good examples of biological systems with a complex topology that exhibit the small-world property. This fact has recently been demonstrated in a set of *in-vitro* experiments of growing cultured neurons [Shefi *et al.*(2002)]. Although a lot of work has been done in the last few years concerning the features of scale–free networks, it has only very recently been reported on the role of such an architecture in the performance of ANN [Stauffer *et al.*(2003)]. The authors show in this paper, by means of a set of numerical simulations, that a scale–free neural network is able to retrieve a pattern with a lower computer–memory cost as compared with the fully–connected Hopfield neural network. They also find a similar performance with a biologically unrealistic nearest–neighbor hypercubic Ising lattice.

We here study the effect of topology in associative memory tasks. Our starting point is a Barabassi-Albert (BA) evolving network [Albert and Barabasi(2002)] with N nodes and $m(N-m_0)$ links. Here, m_0 is the initial number of nodes generating the network, $m \leq m_0$ is the number of links that are added during the evolution at each time step, and N is the final number of nodes in the network. In order to build a neural system over such a network, we place a binary neuron, $s_i = 1$ or 0, at each node and "store" P binary patterns, $\xi^\mu \equiv \{\xi_i^\mu = 1 \text{ or } 0\}$, $\mu = 1, \ldots, P$, by associating a synaptic intensity ω_{ij} at each links according to the Hebbian learning rule, namely, $\omega_{ij} = \frac{1}{N}\sum_{\mu=1}^{P}(2\xi_i^\mu - 1)(2\xi_j^\mu - 1)$.

A direct comparison of this scale–free neural network (SFNN) and the standard Hopfield neural network cannot be made because the second case involves N^2 synaptic connections. A hypercubic Ising lattice has the same number of synapses than the SFNN; however, real neural systems exhibit more complex neuron connectivity than the Ising network. Consequently, we compared the performance of the SFNN with that of a *highly-diluted* Hopfield network (HDHN). The HDHN is obtained from the standard Hopfield network by randomly erasing synapses until only $m(N-m_0)$ synapses remain, i.e., the number of synapses scales with N instead of N^2. The connectivity distribution of the HDHN is showed in figure 1. The main differences between this distribution and the corresponding one for a SFNN, as illustrated in figure 1, is that the latter has no typical value for the connectivity while the HDHN distribution presents a maximum and, consequently, is characterized by a (typical) mean connectivity. The performance of the two networks is compared in figure 1 for $P = 1$. This clearly shows that the retrieval of information is better (at any temperature) for the SFNN than for the HDHN, even a zero temperature where any thermal fluctuation is avoided. In general, we find a good retrieval of information for $P = 1$ and not so good as P is increases in the SFNN, as also it occurs for the HDHN. In addition, we observe that the performance of the SFNN increases significantly, as compared with that of the HDHN, if one considers only the retrieval of information concerning neurons with a connectivity degree higher than some number, namely k_0, i.e., the hubs.[Torres *et al.*(2003)]. This finding is interesting because it reflects the negative role of the boundary and the positive role of the hubs on the SFNN during each retrieval experiment.

In conclusion, we have shown that the topology of a neural network has a main role in the processes of memorization and retrieval of patterns. In particular, neural networks with scale–free topology may exhibit a better performance than Hopfield–like networks with the same number of synapses distributed randomly over the network. Though the capacity of the SFNN to retrieve information is observed to decrease as the number of stored patterns increases, this capacity increases when fluctuations due to the nodes with small number of neighbor are neglected. Our study can be useful to understand the role of regions with different connectivity degrees in real neural systems during memorization and retrieval of information. In particular, it may help our understanding of how fluctuations or perturbations produced by some diseases in the typical number of synapses of some brain areas can affect the processing of information and memorization in these regions.

References

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