How information-geometric measure depends on underlying neural mechanism? [1]

Masami Tatsuno [a,2] , Masato Okada [a]

[a] *Laboratory for Mathematical Neuroscience, Brain Science Institute, RIKEN, 2-1 Wako-shi, Saitama 351-0198, Japan*

Abstract

Information-geometric measure was recently proposed to analyze higher order correlation among spike firing. However the detailed property of this quantity has not yet been elucidated. Here we demonstrated that information-geometric measure was dependent both on the underlying connections and on the local inputs. We also showed that information-geometric measure had a monotonic relation to cross-correlation function, depicting a similar feature of neural firing. This model-based approach, integrated with an analytical tool such as information-geometric measure, will give a powerful tool for computational and experimental neuroscience.

Key words: Information Geometry; Correlation Function; Spikes; Neural Firing

1 Introduction

Recent advance of experimental technique enables us to access a large number of simultaneously recorded neuron data. There have been a number of researches to single out higher order correlations, and relate them to information representation and behavior [3] [7]. To this end information-geometric measure was recently proposed, and it will provide a useful tool for spike train analysis [2] [5] [6].

However, the inverse problem "if a certain correlation is observed, what is an underlying mechanism for that?" has not yet been answered because of its

[1] This work was supported by Research Grant 626-52045 from RIKEN.
[2] Corresponding author. Tel.: +81-48-467-9664; fax: +81-48-467-9693.
E-mail address: masami@brain.riken.go.jp (M. Tatsuno)

ill-posed nature. In this paper, we investigated the property of information-geometric measure and correlation function, using a coupled neuron model. We demonstrated the dependency of information-geometric measure both on the underlying connection and on the local input, and also showed that information-geometric measure has a monotonic relation to correlation function.

This paper is organized as follows. In section 2, the coupled neuron model and its basic property is introduced. After a brief introduction of information-geometric measure in section 3, its theoretical and numerical analysis is presented in section 4. In section 5, we summarize our study and discuss the future directions.

2 Model and Correlation Function

We consider the case of an isolated pair of neurons. This simple setting allows us to investigate the system with mathematical clarity, and it is instructive for further network studies.

Following Ginzburg and Sompolinsky [4], the state of each neuron at time t takes one of two states, denoted by $S_i(t) = 0, 1$, where $i = 1, 2$. The neurons are assumed to have a stochastic nature, and the transition rates for the ith neuron takes the form

$$w(S_i \to (1 - S_i)) = \frac{1}{2\tau}\{1 - (2S_i - 1)[2g(u_i) - 1]\}, \tag{1}$$

where $g(u)$ is a monotonically increasing, differentiable function such as $g(u) = (1 + \tanh(u))/2$. The local field acting upon the ith neuron at time t is

$$u_i(t) = J_{ij}S_j(t) + h_i, \tag{2}$$

where J_{ij} denotes the connection from the jth presynaptic neuron to the ith postsynaptic one, and h_i represents the local input. In the equilibrium limit, the average activities are given by

$$\langle S_i \rangle = \langle S_j \rangle \Delta g_i + g(h_i), \tag{3}$$

where $\Delta g_i = g(J_{ij} + h_i) - g(h_i)$.

The correlation function between the activities of two neurons is defined as

$$C_{ij}(t, t + \tau) = \langle (S_i(t) - \langle S_i(t) \rangle)(S_j(t + \tau) - \langle S_j(t + \tau) \rangle) \rangle, \quad \tau \geq 0, \tag{4}$$

and its equilibrium value $C_{ij}(\tau)$ is obtained by taking the $t \to +\infty$ limit. Here, to be more concrete, we consider $C_{ij}(\tau)$ in the case of $J_{21} = J$ and $J_{12} = 0$, where no free energy exists because of the asymmetric connection. Then, the solutions for auto-correlations $C_{ii}(\tau)$ and cross-correlations $C_{ij}(\tau)$ are easily obtained [4].

As an example, the cross-correlation $C_{12}(\tau)$ is shown in Fig. 1 for the case $J = 5$, $h_1 = 0$, $h_2 = 0$, $\tau_0 = 10$. Numerical data were obtained from 5×10^5 bins. The asymmetric shape demonstrates the unidirectional nature of the connections.

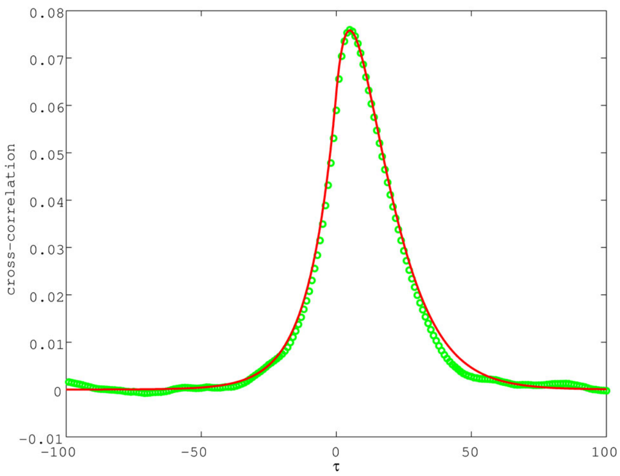

Fig. 1. Cross-correlation function for unidirectional connection from neuron 1 to neuron 2. Green circle represents numerical data and red solid curve theoretical prediction, respectively.

3 Information-geometric Measure

Information-geometric measure is defined as follows. For details, see refs [1] [2] [5] [6]. Let X_1 and X_2 are 2 binary variables and let $p = p(x_1, x_2)$, $x_i = 0, 1$, be its probability. Then, their joint probability is given by $p_{ij} = \mathrm{Prob}\{x_1 = i; x_2 = j\} \geq 0$, $i, j = 0, 1$, with a constraint $p_{00} + p_{01} + p_{10} + p_{11} = 1$. Let us now expand $\log p(x_1, x_2)$ in the polynomial of x_1 and x_2,

$$\log p(x_1, x_2) = \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 - \psi. \tag{5}$$

This is called a log-linear model and it is an exact expansion since x_i takes the binary values $0, 1$. The coefficients θ's and ψ are

$$\theta_1 = \log \frac{p_{10}}{p_{00}}, \quad \theta_2 = \log \frac{p_{01}}{p_{00}}, \quad \theta_3 = \log \frac{p_{11}p_{00}}{p_{01}p_{10}}, \quad \psi = -\log p_{00}. \tag{6}$$

Then, θ_3 is considered as information-geometric measure for 2 neuron interactions.

4 Results

We first examined how information-geometric measure θ's change when the unidirectional connection is modified. For the model introduced in section 2, we can derive theoretical curves,

$$\theta_1 = \log \frac{4 - 3\tanh(\Delta J)}{4 - \tanh(\Delta J)}, \quad \theta_2 = \log \frac{4 + \tanh(\Delta J)}{4 - \tanh(\Delta J)},$$

$$\theta_3 = \log \frac{(4 + 3\tanh(\Delta J))(4 - \tanh(\Delta J))}{(4 - 3\tanh(\Delta J))(4 + \tanh(\Delta J))}, \tag{7}$$

where $\Delta J = J_{21}$. The result is shown in Fig. 2 for the case $h_1 = 0$, $h_2 = 0$, $\tau_0 = 10$. The change in θ's reflects a modification in the underlying network architecture, that is the unidirectional connection J_{21}.

We next investigated the effect of the local inputs to information-geometric measure. If θ_3 reflects the underlying anatomical connection J_{21}, it is desirable that θ_3 remains constant for the increase of local input $h_i = \Delta h$ under the fixed J_{21}. The theoretical curves for θ_3 are obtained as,

$$\theta_3 = \log \frac{\langle S_1 S_2 \rangle (1 - \langle S_1 \rangle - \langle S_2 \rangle + \langle S_1 S_2 \rangle)}{(\langle S_1 \rangle - \langle S_1 S_2 \rangle)(\langle S_2 \rangle - \langle S_1 S_2 \rangle)}, \tag{8}$$

where $\langle S_1 \rangle = g(\Delta h)$, $\langle S_2 \rangle = g(\Delta h)(g(\Delta h + J_{21}) - g(\Delta h) + 1)$, $\langle S_1 S_2 \rangle = g(\Delta h)(g(\Delta h + J_{21}) + g(\Delta h)[g(\Delta h + J_{21}) - g(\Delta h) + 1])$, and $g(\Delta h) = (1 + \tanh(\Delta h))/2$. The result for $J_{21} = 5$ and 0 is shown in Fig. 3. Increase in θ_3 for $J_{21} = 5$ demonstrates that θ_3 is dependent not only on the underlying connections J_{21} but also on the local inputs.

Thirdly, we investigated time-delayed information-geometric measure $\theta_3(\tau)$ to verify if it reproduces cross-correlation. Using the relation $\langle S_1(t) S_2(t + \tau) \rangle =$

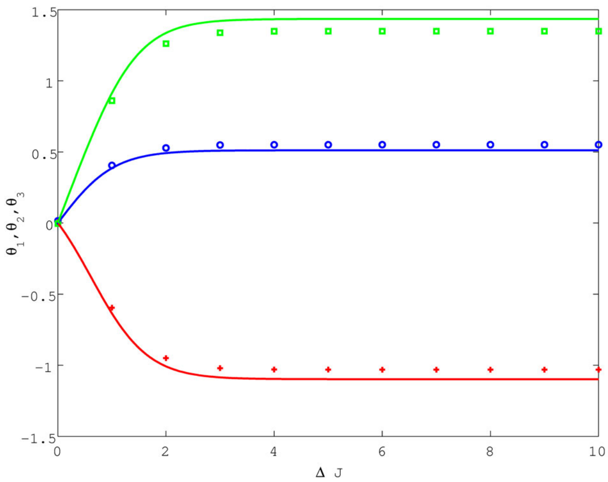

Fig. 2. Dependence of θ's on unidirectional connection J_{21}. Solid lines represent theoretical predictions, where green is θ_3, blue θ_2 and red θ_1 respectively. Green rectangle for θ_3, blue circle for θ_2 and red cross for θ_1 are obtained from numerical simulation.

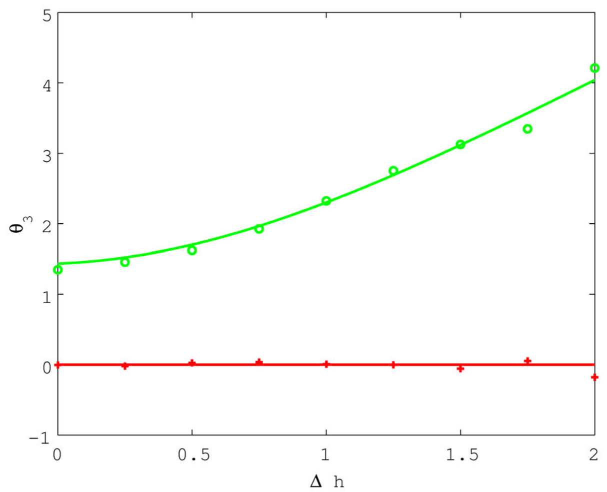

Fig. 3. Dependence of θ_3 on local input Δh. Solid lines are theoretical prediction, where green is for $J_{21} = 5$, and red for $J_{21} = 0$. Green circle for $J_{21} = 5$ and red cross for $J_{21} = 0$ are obtained from numerical simulation.

$C_{21}(\tau) + \langle S_1(t)\rangle\langle S_2(t+\tau)\rangle$, we can derive

$$\theta_3(\tau) = \log \frac{(C_{12}(\tau) + g(J_{21})/4 + 1/8)(C_{12}(\tau) - g(J_{21})/4 + 3/8)}{(-C_{12}(\tau) - g(J_{21})/4 + 3/8)(-C_{12}(\tau) + g(J_{21})/4 + 1/8)}, \quad (9)$$

where $C_{12}(\tau) = C_{12}(0)(1+2\tau/\tau_0)\exp(-\tau/\tau_0)$ for $\tau \geq 0$, and $C_{12}(\tau) = C_{21}(-\tau) = C_{12}(0)\exp(\tau/\tau_0)$ for $\tau < 0$. It is also easy to show that $d\theta_3/d\langle S_1 S_2\rangle > 0$. Therefore, θ_3 and C_{12} has a monotonic relation, depicting a similar feature of neural firing. The time-delayed θ_3 for the case $J_{21} = 5$ is shown in Fig. 4.

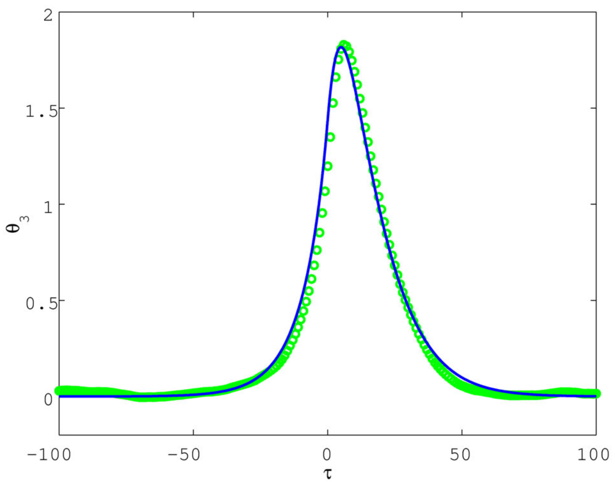

Fig. 4. Time-delayed θ_3 for unidirectional connection from neuron 1 to neuron 2. Green circle represents numerical data and blue solid curve theoretical prediction, respectively.

5 Summary

By investigating a coupled neuron model, we demonstrated that information-geometric measure is dependent both on the underlying connection and on the local input. We also investigated the relation between information-geometric measure and correlation function, showing that they are monotonically related and therefore depict a similar feature of neural firing. It is important to note that asymmetric nature of neural network can be detected in the framework of log-linear model, which is symmetric one, implying that information-geometric measure can be a good analytical tool for neural firing.

However to fully answer to an ill-posed question "if a certain correlation is observed, what is an underlying mechanism for that?", we need to integrate a model-oriented approach and an analytical tool for neural firing. Our study is the first step to this end, and it is very promising since our method is also applicable to more realistic networks consisting of excitatory and inhibitory populations.

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