Quantitative Information Transfer through Layers of Spiking Neurons Connected by Mexican-Hat-type Connectivity.

Kosuke Hamaguchi* and Kazuyuki Aihara

Department of Complexity Science and Engineering,

Graduate School of Frontier Science, the University of Tokyo, Tokyo, 113-8654, Japan.

Abstract

In this paper, quantitative information transmission through the layer connected by Mexican-Hat(MH) type connectivity is examined. On the low noise level, a network structure of all-to-all feed-forward connection cannot transmit quantitative information on one single shot of propagating spike volley. MH-type connection and localized input enables network to transmit stable quantitative information on a correlated spike volley. Under a low background noise condition, input amplitude can be encoded as the number of spikes and transmitted through the network.

1 Introduction

How neurons code information is a central issue in neuroscience. A firing rate of a neuron or neuron population is regarded as the main information carrier in the sensory system. In contrast, recent studies show that precisely timed spikes may play a functional role in cortical neural coding[1, 2]. The former is called rate coding, and the latter spatiotemporal spike coding. Which coding paradigm is used in a brain is still a controversial issue.

It is important to study the nature of neural signal transmission through networks of spiking neuron, because synchrony and correlation of spikes cannot be studied by firing-rate based models. The simplest network architecture is layers of spiking neurons, which have no lateral connection within the layer and are connected to the next layer in the feed-forward manner. When a spiking activity propagates through such a network, the firing tends to synchronize. This is called syn-fire chain[3] and has been intensively studied [4, 5, 6]. With low background noise, quantitative information is destroyed after propagation through a few layers (syn-fire mode). In other words, the propagating spikes either synchronize or die out. However, in the presence of a noisy background current, quantitative information can be propagated through the layers (rate-mode) [7, 8].

The syn-fire mode retains synchrony but loses quantitative information. In contrast, rate-mode transmits quantitative information but loses temporary structure. Here we pose a question: Is quantitative information transmission keeping synchrony or correlation is possible?

We are going to solve this question by introducing spatial structure in connection weight distribution. So far, a simple feed-forward network model with spatially uniform connection have mainly been used. This assumption of uniformity is useful for analysis, but it is not biologically realistic. Biological neural layers consist of excitatory neurons and inhibitory neurons. Their interaction can be approximated as Mexican-Hat (MH) type connectivity: excitation between nearby neurons and weak inhibition between distant neurons. We analyze the firing propagation through the feed-forward neural layers connected by MH-type connectivity with no lateral connection.

When one layer of a MH-type neural network is considered, several interesting phenomena are observed. In the McCulloch-Pitts neuron model, Amari showed that a so-called localized-activity can exist in such a layer. The condition of moving-

*Corresponding author.
e-mail: hammer@sat.t.u-tokyo.ac.jp (K. Hamaguchi)

localized activity was also provided [9]. Solitary waves propagation in a two dimensional layer of spiking neurons was also studied [10] However, feed-forward network connected by MH-type weighting has not fully been studied, although van Rossum et al. has shown that the information of a position of moving stimulus can be transmitted onto the subsequent layers [8].

In the present paper, we examine the property of a feed-forward network of integrate-and-fire neurons with MH type connectivity. First, we show that stable quantitative information transmission is depend on the input spatiotemporal structure. Second, by assuming a spatially gaussian-type strength input, we show that this network can encode amplitude information by one shot of propagating correlated spikes.

2 Model

The neuron model used in this paper is the integrate-and-fire neuron, which is described as

$$\frac{\mathrm{d}}{\mathrm{d}t}v_k(x,t) = -\frac{v_k(x,t)}{\tau_m} + I_k(x,t) + Noise(\mu,\sigma) \tag{1}$$

$$I_k(x,t) = \sum_{x'} w(|x-x'|)\alpha(t-t(x')). \tag{2}$$

We refer to $v_k(x,t)$ as a membrane potential at position x on the kth layer at time t. All the neurons are assumed to be homogeneous and thus have the same membrane time constant τ_m. Background Ornstein-Uhlenbeck Process noise with time constant 2msec. is also introduced with mean $\mu = 1$ and standard deviation (SD) σ. Neuron x fires when $v_k(x,t) = \theta$ AND $\frac{\mathrm{d}v_k(x,t)}{\mathrm{d}t} > 0$ are satisfied. θ is a firing threshold membrane potential. The factor $w(|x-x'|)$ is a measure of the synaptic efficacy from neuron x' on the $k-1$th layer to neuron x on the kth layer. $w(x)$ is described as

$$w(x) = W_0(1 - \frac{x^2}{2\sigma_{\mathrm{MH}}^2})\exp(-\frac{x^2}{2\sigma_{\mathrm{MH}}^2}) \tag{3}$$

$\alpha(t-t(x))$ is the time course of the postsynaptic current (PSP) from the neuron x. Here α is described with an exponential decay constant τ_s as follows:

$$\alpha(s) = \frac{q}{\tau_s}exp(-\frac{s}{\tau_s})\mathcal{H}(s). \tag{4}$$

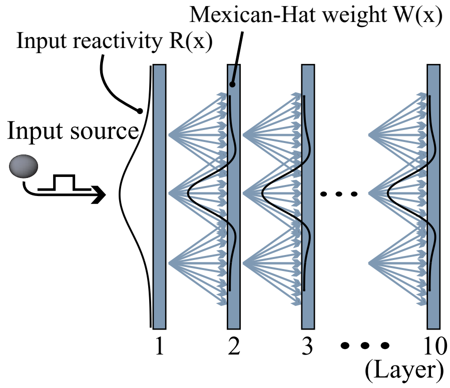

Figure 1: Network architecture of the model. The layer of integrate-and-fire neurons are connected in an MH-type fashion. Each layer contains 300 neurons. Mexican-Hat connectivity has σ_{MH} of 30 units; one post-neuron has 60 excitatory synapses. Input gaussian reactivity has $\sigma_G = 0.02$ so that minimum reactivity is set to 0.01. Input is 10 ms long and simulation time is 50 ms long.

EPSP and IPSP differ only in sign. $\mathcal{H}(s)$ is the Heaviside step function with $\mathcal{H}(s) = 1$ for $s > 0$ and $\mathcal{H}(s) = 0$ for $s \leq 0$.

The network architecture is shown in Fig. 1. Each layer contains N neurons on a one-dimensional line with a periodic boundary condition; the 1st neuron and the Nth neuron are next to each other. The kth layer receives input from the $k-1$th layer neurons through MH-type connectivity. Input layer (first layer) neurons accept external input $I_1(x,t)$.

3 Results and Discussion

Dependence of propagation stability on input structure

First, we show the dependence of firing stability on spatiotemporal structures of inputs. As examples, two input types are used. Input stimulus is square wave of τ_{stim} (ms), spatially localized around $\frac{N}{2}$th neuron with length L, and is described as

$$I_1(x,t) = \begin{cases} I_0(\mathcal{H}(t(x)) - \mathcal{H}(t(x)-\tau_{stim})) & x \in D \\ 0 & x \notin D \end{cases}, \tag{5}$$

where $t(x) = \frac{x}{v}$ and region $D = [\frac{N-L}{2}, \frac{N+L}{2}]$. This stimulus moves from 0 to N in input layer with velocity v, and firing waves on input layer follow it with same velocity v. A study of a one-layer neural sheet with MH connections [10] shows that stability of a traveling wave depends on its velocity and threshold parameters. When the velocity of the wave is too fast or slow, an input to the next layer becomes weak. Here we choose intermediate level of speed ($v = 25[unit/ms]$), and for comparison, spatially uniform firing ($v = \infty[unit/ms]$) was studied.

In both velocities, L units are activated but differ in spike timing. Figure 2 (a,b) shows how spike activity propagates through the network with varying L. When activity moves at an appropriate velocity, the number of spikes is robustly propagated, whereas spatially uniform firing is not (Figure 2 (b)) .

Figure 2 (c,d) shows the raster-plot of several L cases. It suggests that robust firing propagation is accomplished by a temporary changing input. When a neuron was fired uniformly, the propagated firing patterns became unstable because excitation and inhibition counterbalance each other.

The counterbalance of excitatory and inhibitory inputs can be easily formulated. If all the L neurons fire within a small time variance around $t = 0$, the input current $I(x, t)$ to the next layer is formulated as follows:

$$I(x,t) = \int_{-L}^{L} \alpha(t + \delta t(x'))W(x - x')\mathrm{d}x'. \tag{6}$$

When $\delta t(x') \to 0$,

$$I(x,t) = \alpha(t)(N^{(1)}(x - L, \sigma_{\mathrm{MH}}) - N^{(1)}(x + L, \sigma_{\mathrm{MH}})), \tag{7}$$

where $N^{(1)}(\mu, \sigma) = -\mu \exp(-\frac{\mu^2}{2\sigma^2})$.

Equation 7 describes what is known as the edge-sharpening effect. The longer L, the less an input at position $x = 0$. However, under initial condition variation, not only the edge but also other localized activities appear (Fig. 2 (d)). Thus, the uniformly activating input is not suitable for quantitative information coding, and the traveling wave causes more robust propagation.

Stable propagation of input amplitude. We examine more natural input structure than the

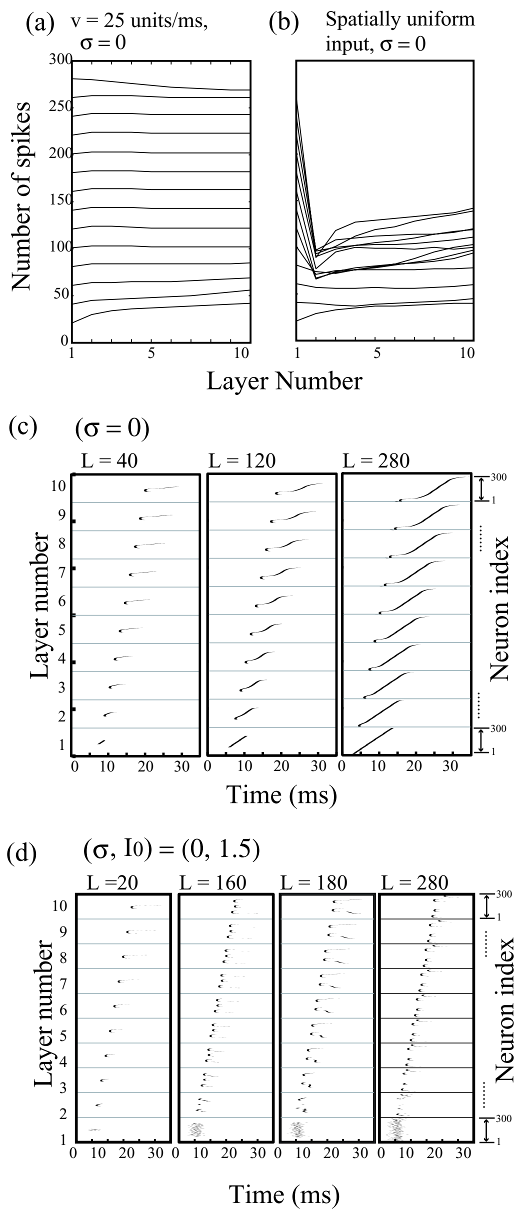

Figure 2: Propagation of the number of spikes: (a) with an appropriate velocity ($v = 25[unit/ms]$), the number of spikes are robustly propagated. (b) spatially uniform firing ($v = \infty[unit/ms]$) results unstable propagation. (c)raster-plots of (a). Whereas the firing patterns are gradually decaying, The number of spikes are propagated in robust manner. (d) raster-plots of (b). Firing patterns are very sensitive to the initial membrane potential fluctuation. All the above results have no background noise.

one used . We assume that neural responses are correlated, with strength specified by a gaussian function for the preferred stimuli. This gaussian type reactivity naturally produces traveling activity on an input layer, and changes the area of active region depending on the input amplitude.

Assuming that the input is a square wave of τ_{stim} (ms), we formulate the input as follows:

$$I_1(x,t) = I_0 \exp(-\frac{x^2}{2\sigma_{\mathrm{G}}^2})(\mathcal{H}(t) - \mathcal{H}(t-\tau_{stim})) \quad (8)$$

Let us assume that $v(x,0) = 0$. Then, the firing time of each neuron is

$$t(x) = \tau_m log(\frac{I_0 exp(-\frac{x^2}{2\sigma_G^2})}{I_0 exp(-\frac{x^2}{2\sigma_G^2}) - \theta}) \quad (9)$$

$$0 \le |x| \le \sqrt{2\sigma^2 ln(\frac{I_0}{\theta})}, \quad (10)$$

where $2\sqrt{2\sigma^2 ln(\frac{I_0}{\theta})}$ is the firing region of the input layer. This means that input amplitude I_0 can be encoded as the firing area of the network. Under this condition, we examined how locally activating stimuli propagate through the network.

Figure 3 (a) illustrates the transition capability for quantitative information on various W_0s and SD σs. The correlation coefficient of I_0 and the number of spikes of the 10th layer are measured. The strength of synaptic connectivity does not affect the qualitative behavior of the network, but noise amplitude have serious effect on it. For small noise variance ($\sigma = 0$), the response and input is proportional. For moderate noise variance ($\sigma = 2$), where spontaneous firing rate is in order of 1Hz, the response still retains high linearity but gradually decaying. As for high noise variance ($\sigma = 4$), where spontaneous firing rate is more than 10Hz, linearity is completely lost.

The detail of propagation at the point $(\sigma\, W_0) = (0, 12)$ is shown in Fig. 3 (b). It shows the propagation of spike number through the network. Each line corresponds to the input $I_0 = [1, 1.2, 1.4, ..., 2.8, 3]$ from below. The amplitude information is encoded as the number of spikes on the first layer, and subsequent layer's number of spikes reflects the original quantitative information quite linearly (Corr. coeff. = 0.9583). These novel phenomena are not observed in the all-to-all connected network (Figure 3 (c)). All the amplitude I_0 more than 1.2 is encoded as sharply packeted 300 spikes.

Further detail of transmission is shown as raster-plots. Three different amplitude I_0s are shown in Figure 3(d-f). The raster-plot of the input layer (first layer) is almost identical to the result of equation.9. The bigger the I_0 becomes, the wider the firing area. Firings propagated like a ripple on a layer and were copied onto the subsequent layers, but decay of ripple shape was observed as is in Fig. 4 (c).

Figure 4 shows the propagations under background noise conditions. Under moderate noise level (noise $\sigma = 2$), the network still remains its input-output linearity (Figure 3 (a), see point $(\sigma, W_0) = (2, 12)$. Corr. coeff. = 0.8044). However some firing volleys accidentally grew to more than their original amount (Fig. 4 (a)). This uncertainty reduces the reliability of information, thus multiple spike volleys would be necessary for reliable calculation. The mechanism of this instability is not clear. Noise-induced activity was observed in the high noise region (noise $\sigma = 4$)(Fig. 4 (f-h), Corr. coeff. = -0.6334). Thus, in the high noise region, the quantitative information coding by one shot of spike volleys is not realistic.

4 Conclusion

The all-to-all connection network cannot encode quantitative information on a single shot of synchronized spikes. Using a MH-type feed-forward network, we have examined the possibility of stable quantitative information transmission on a single shot of spikes. First, we have shown that movement of a stimulus point stabilizes quantitative information coding. Second, on the low noise level, we have found that introduction of spatially gaussian-type reactivity results in linear input amplitude and output spike number. By way of conclusion, quantitative information on a single shot of correlated spikes is obtained in wide range of parameters. The mechanism of linearity is, however, not yet thoroughly been explained. Whereas high background noise does not show this property, still rate-mode transmission would be possible. So far we have used only square wave input. The future challenge with this system will be to examine robust transmission with more general types of input.

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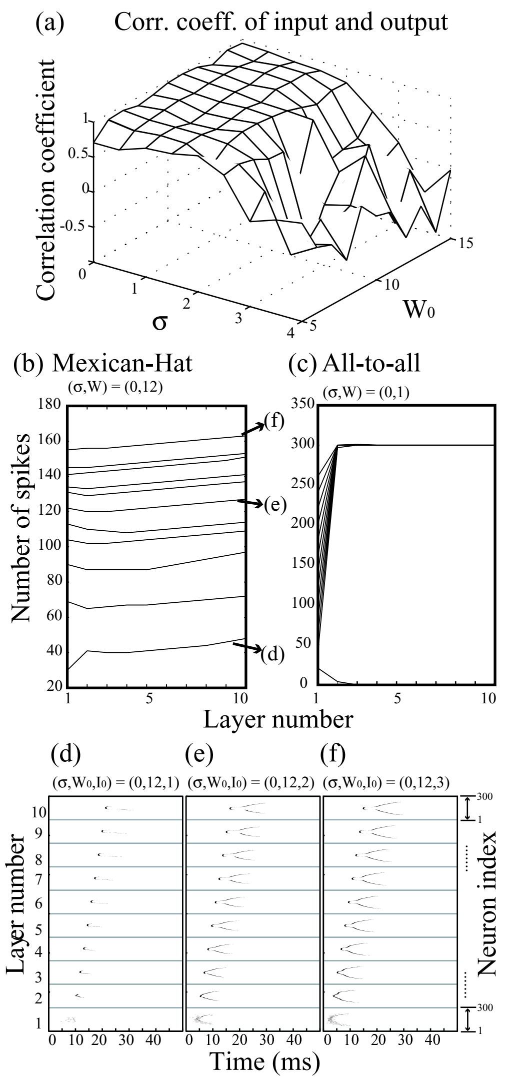

Figure 3: Propagation of spikes under no background noise condition. (a)The correlation coefficient of I_0 and number of spikes of 10th layer are measured.(b)The development of number of spikes with Mexican-Hat type connectivity. (c) same as *b*

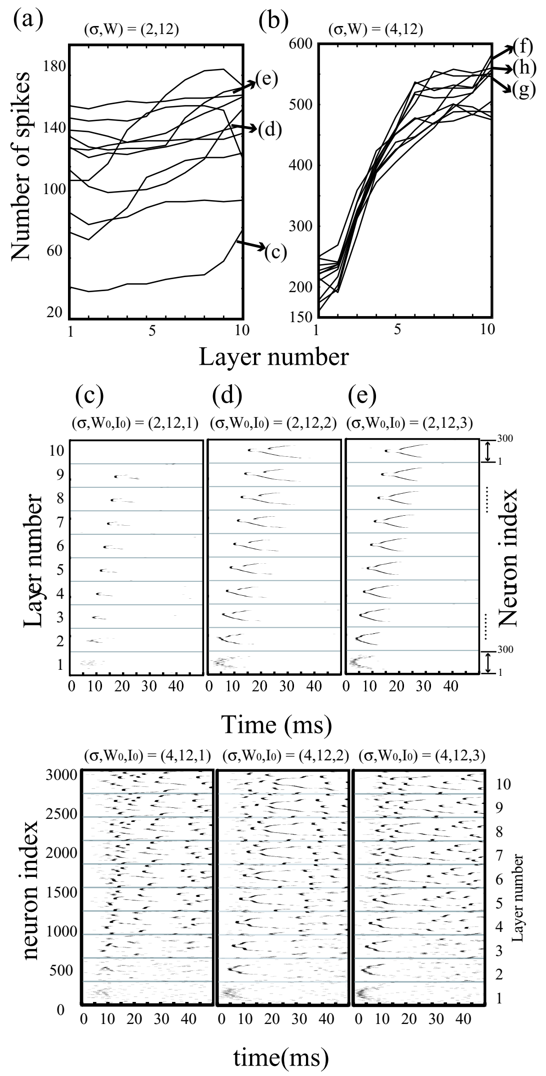

Figure 4: Propagation of spikes under the existence of background noise. (a,b) shows the development of propagating number of spikes. They are less stable than zero-noise limit. (c,d,e) is the raster-plot of (a) with different input amplitude I_0, and (f,g,h) is for (b).