Oscillatory modes in a neuronal network model with transmission latency

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Abstract

We analyzed the oscillatory activities in a neuronal network model as the basis of synchrony of the activities in the brain. The model consists of two groups of neurons that are interconnected. One group is composed of an excitatory and an inhibitory neuron which are expressed by Hodgkin-Huxley equations. The network shows different phase-locked oscillations and other activities depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission latency. In the present paper we show the oscillatory modes in symmetrical and asymmetrical networks with the mutual coupling from the inhibitory neuron in each group to the excitatory neuron in the other group.

Keywords: neuronal network model, oscillation, synchrony, Hodgkin-Huxley equations, latency

1 Introduction

The synchronization and phase-locked activities in different regions in the brain is one of the important features of information processing by temporal coding in the central nervous system[2]. To clarify the fundamental mechanism of the temporal coding [8], it is very significant to elucidate the phase-locked activities of the neuronal networks with different coupling configurations [4]. We analyzed the oscillatory activities in a neuronal network model which consists of two groups of neurons to investigate the basic characteristics of synchronization. The neurons are coupled with synaptic delay. Each group consists of excitatory and inhibitory neurons that are interconnected. The neurons that are synchronized in activities can be represented by a single neuron in the theoretical model. In the present simulation, therefore, each group in the model network are composed of an excitatory neuron and an inhibitory neuron for simplicity of analysis. The neurons are formulated by Hodgkin-Huxley equations[3].

We have demonstrated that in the network with mutual coupling of excitatory neurons in both groups, oscillatory activities of not only the in-phase mode which seems to be trivial but also anti-phase mode and the periodic solutions in which the phase difference of the oscillation in two groups of the neurons continuously changes with the coupling strength and latency [5].

In the present paper we show the results of the analysis of the oscillatory modes in symmetrical and asymmetrical networks with the mutual coupling from the inhibitory neuron in each group to the excitatory neuron in the other group. We demonstrate the regions of some parameters in the network in which different oscillatory solutions exist stably and show a variety of bistability in activities.

2 A Neuronal Network Model

The dynamics of the membrane potentials of the neurons is formulated as

$$C_M \frac{d\mathbf{V_k}}{dt} = \mathbf{I_{ext,k}} + \mathbf{I_{ion,k}} + \sum_{j=1}^{2} \mathbf{W_{kj}} \mathbf{I_{syn,j}} \quad (k = 1, 2), \tag{1}$$

where

$$\mathbf{V_k} = \begin{pmatrix} V_{ek} \\ V_{ik} \end{pmatrix}, \quad \mathbf{I_{ext,k}} = \begin{pmatrix} I_{ext,ek} \\ I_{ext,ik} \end{pmatrix},$$

$$\mathbf{I_{ion,k}} = \begin{pmatrix} I_{ion,ek} \\ I_{ik} \end{pmatrix}, \quad \mathbf{I_{syn,k}} = \begin{pmatrix} I_{syn,ek} \\ I_{syn,ik} \end{pmatrix}.$$

In the above, $I_{ext,rk}$, $I_{ion,rk}$ and $I_{syn,rk}$ $\quad (r = e, i)$ denote the external input current, the ion currents of the neuron and the synaptic current from the corresponding neuron in the network, respectively. The subscripts e and i denote excitatory and inhibitory neurons respectively. The ion currents $I_{ion,rk}$ are formulated conventionally. Some formulations have been proposed for synaptic coupling [1], [6], [7]. The synaptic current $I_{syn,rk}$ is expressed here as

$$\begin{aligned} I_{syn}(V) = \sum_J A_{syn}(J) w \{ &\exp(\frac{-t + t_s(J) + \tau_d}{\tau_+}) \\ &- \exp(\frac{-t + t_s(J) + \tau_d}{\tau_-}) \}, \end{aligned} \tag{2}$$

where w is the coupling coefficient between the corresponding neurons and an element of the coupling matrix

$$\mathbf{W_{kj}} = \begin{pmatrix} a_{kj} & -c_{kj} \\ b_{kj} & -d_{kj} \end{pmatrix}.$$

The J-th instant when the membrane potential increases and exceeds -30mV is denoted by $t_s(J)$ and the synaptic delay is denoted by τ_d. If the current of a previous firing is larger than 0.01, it is added to the present current and otherwise neglected. The factor $A_{syn}(J)$ is expressed as

$$A_{syn}(J) = \begin{cases} 1 & (if \;\; t \geq t_s(J) + \tau_d) \\ 0 & (else) \end{cases}. \tag{3}$$

In the present simulations the values of the external input current $I_{ext,rk}$ and parameters C_M, τ_+ and τ_- are set as

$$I_{ext,rk} = 10[\mu\mathrm{A}] \;\; (r = e, i, \; k = 1, 2),$$
$$C_M = 1[\mu\mathrm{F}/cm^2], \; \tau_+ = 3[\mathrm{ms}], \; \tau_- = 1[ms].$$

3 Results

We analyzed the symmetrical networks of two identical groups. We made the coupling coefficients between the neurons in the groups denoted as $a_{11} = a_{22} = a_0$, $b_{11} = b_{22} = b_0$, $c_{11} = c_{22} = c_0$, and $d_{11} = d_{22} = d_0$. The bidirectional coupling coefficients between groups are equal and denoted as $a_{12} = a_{21} = a_1$, $b_{12} = b_{21} = b_1$, $c_{12} = c_{21} = c_1$, and $d_{12} = d_{21} = d_1$. The coupling coefficients inside the group are set as

$$a_0 = 10, \; b_0 = 9, \; c_0 = 10, \; d_0 = 0$$

when they are fixed. By the coupling given by a_0 it is assumed that a number of excitatory neurons are interconnected to act synchronously.

We investigated different configurations of mutual coupling between the groups. The network shows a variety of phase-locked oscillations and other oscillatory activities depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission latency. As in a number of oscillatory systems composed of two components, the present model generates in-phase mode, in which the phase difference of oscillation in two components is 0 degrees, and anti-phase mode, in which the phase difference is 180 degrees. We showed in a previous paper

[5] the solution in which the two groups oscillate in continuously changing phase difference with the parameters of coupling and latency. This mode is referred to as intermediate-phase mode. In some networks chaotic oscillations and quasiperiodic oscillations are also observed. In the latter the phase difference of the oscillation gradually changes with time. This mode is referred to as varying phase mode. Fig. 1 shows the phase portrait and time series of an example of the varying phase mode.

In the present paper we show the results of the analysis of a network with one typical and symmetrical type of coupling. Both excitatory units are coupled bidirectionally, $c_1 \neq 0$, $a_1 = b_1 = d_1 = 0$. It is referred to as EI coupling. We analyzed the regions of oscillatory solutions in symmetrical networks with different values of coupling strength c_0 and c_1, and synaptic delay τ_d as well as asymmetrical networks in the sense that $c_{11} \neq c_{22}$. The regions of oscillatory solutions that are found in the model network are illustrated in Fig. 2. Each illustration consists of two rows. The upper row shows the ranges of the oscillatyory solutions that emerge with the initial conditions where the in-phase mode would be likely to occur. The lower shows those with the initial conditions where the anti-phase mode would be likely to occur. The parameter value shown in the right end shows the one at which the analysis is started.

In the symmetrical networks In-phase mode and anti-phase mode coexist in large range of parameters c_0, c_1 and τ_d. There exists intermediate-phase mode in which the phase difference of the oscillation in the two groups vaies from 0 deg. to about 80 degree with the change of the parameters c_0, c_1 and τ_d. It coexists with in-phase mode or anti-phase mode. With the synaptic delay larger than a critical value only in-phase mode exits. Chaotic activities are also observed with small values of c_0. It coexists with anti-phase mode. In the asymmetrical network in which c_{11} and c_{22} are different, the mode coexistence is similar to the case of the symmetrical network. In Figure 2 (d) the center of the rows corresponds to the symmetrical network. The region of in-phase mode becomes smaller and that of intermediate-phase mode becomes larger. Varying phase mode emerges with the small values of b_{22} or large values of b_{22}, in other words, when the asymmetricity is large.

4 Conclusions

We analyzed the oscillatory activities in a neuronal network model as the basis of synchrony of the activities in the brain. The model consists of two groups of neurons that are interconnected. One group is composed of an excitatory and an inhibitory neuron which are expressed by Hodgkin-Huxley equations. The network shows different phase-locked oscillations and other oscillatory activities depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of synaptic latency.

In this paper we showed the regions of oscillatory modes in the coupling strength and synaptic delay in symmetrical and asymmetrical networks with the mutual coupling from the inhibitory neuron in each group to the excitatory neuron in the other group. The model possesses a variety of bistability in its activities. The rich variety of the results suggests that it is necessary to make further analysis of the networks with different coupling configurations and different asymmetricities.

Acknowledgments
The authors are most grateful to K. Kawamoto for his help with computer simulation.

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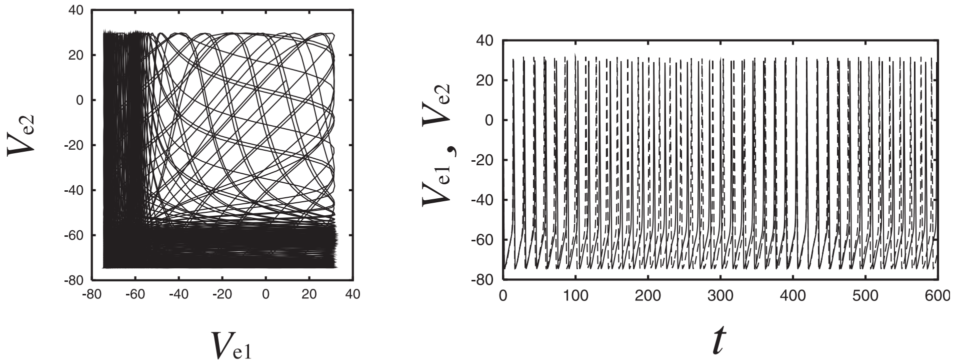
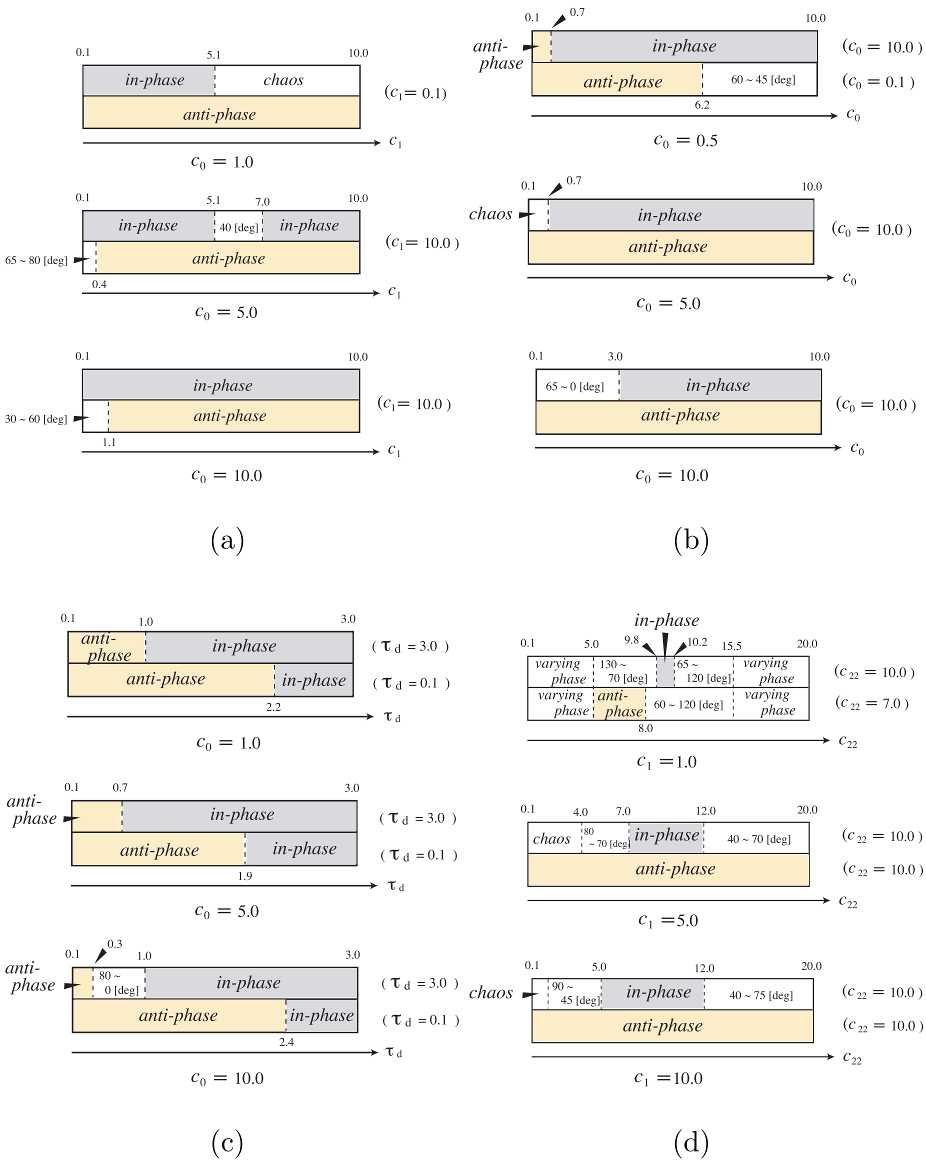
Figure 1 An example of phase portrait and time series of varying phase mode.

Figure 2 The regions of different oscillatory modes in EI coupling, (a), (b) and (c) symmetrical networks, (a) and (b) $\tau_d = 1$, (c) $c_1 = 5$, (d) an asymmetrical network, $\tau_d = 1$, $c_{11} = 10$.



V_{e2}
V_{e1}
V_{e1}, V_{e2}
t

Fig. 1.



in-phase
chaos
anti-phase
(c_1= 0.1)
c_0 = 1.0
40 [deg]
65 ~ 80 [deg]
(c_1= 10.0)
c_0 = 5.0
30 ~ 60 [deg]
c_0 = 10.0
(a)
anti-phase
60 ~ 45 [deg]
(c_0 = 10.0)
(c_0 = 0.1)
c_0 = 0.5
65 ~ 0 [deg]
(b)
(τ_d = 3.0)
(τ_d = 0.1)
80 ~ 0 [deg]
(c)
varying phase
130 ~ 70 [deg]
65 ~ 120 [deg]
60 ~ 120 [deg]
(c_{22} = 10.0)
(c_{22} = 7.0)
c_1 =1.0
80 ~ 70 [deg]
40 ~ 70 [deg]
c_1 =5.0
90 ~ 45 [deg]
40 ~ 75 [deg]
c_1 =10.0
(d)

Fig. 2.