The Effect of Correlation on Population Decoding

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The present study uses a neural field model to investigate the effect of correlation in neural responses on population decoding. It confirms that, as the width of correlation increases, the Fisher information saturates and does no longer increase in proportion to the neural density. However, we prove that as width increases further, wider than $\sqrt{2}$ times the effective width of the turning function, the Fisher information increases again without limit. Moreover, we clarify the asymptotic efficiency of the maximum likelihood inference type of decoding methods for correlated neural signals, which has important meaning in practice.

Oral Presentation Preferred

Summary

1 Introduction

Population coding is a paradigm to represent stimuli by using the joint activities of a number of neurons, which is widely used in the sensor and motor areas of the brain. Experiments have revealed that the neural activities in a population code are often correlated. It is therefore of great importance to theoretically understand how population coding is affected by correlation.

A widely argued issue in the literature is how correlation changes decoding accuracy. Based on analyzing the Fisher information (whose inverse, called the Cramér-Rao bound, is the optimal accuracy one can achieve.), [1, 3] have shown that the correlation can either increase or decrease decoding accuracy, depending on the length scale. The correlation also modifies the asymptotic behavior of Fisher information. When no correlation exists, or correlation is within a local range of the population, the Fisher information is proportional to the neural density. As the correlation length increases (but limited to a certain value, as explained later), the Fisher information saturates and does no longer increase with the neural density. This property has lead to the suspicion on the utility of using a large population to improve the decoding accuracy.

In this work, based on a new prototype model for the encoding process, we first re-investigate the effect of correlation on the Fisher information. The study not only re-discoveries the main results in the literature in a simpler way, but also, reveals a new important feature on the asymptotic behavior of Fisher information. It states when the correlation length increases to be wider than $\sqrt{2}$ times the effective width of the tuning function, the Fisher information increases again without limitation. This finding, together with others, appeals for experimental confirmation on the range of neural correlation in the brain.

We then investigate the effect of correlation on particular decoding methods. Three strategies are studied. All of them are formulated as the maximum likelihood inference (MLI) type (including the Center of Mass method), whereas, they differ in the amount of encoding information being used. We confirm that, the method, which uses the knowledge of the tuning function but neglects the correlation, is a good compromise between computational complexity and decoding accuracy [2]. Moreover, we clarify the asymptotic efficiencies of the three method for correlated signals, which has important meaning in practice.

2 The Encoding Model

We consider a one-dimensional neural field, in which neurons are located with uniform density ρ. The activity of neuron at position c (i.e., the preferred stimulus) is denoted by $r(c)$. The neural activities, $\mathbf{r} = \{r(c)\}$, are Gaussian pair-wise correlated, that is,

$$Q(\mathbf{r}|x) = \frac{1}{Z}\exp\{-\frac{\rho^2}{2\sigma^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[r(c)-f(c-x)]h^{-1}(c,c')[r(c')-f(c'-x)]dcdc'\}, \tag{1}$$

where x is the stimulus. The tuning function $f(c-x)$ is given by

$$f(c-x) = \frac{1}{\sqrt{2\pi}a}\exp^{-(c-x)^2/2a^2}, \tag{2}$$

where a is the tuning width.

We choose the covariance function $h(c, c')$ to be of the Gaussian form,

$$h(c,c') = \rho(1-\beta)\delta(c-c') + \rho^2\beta e^{-(c-c')^2/2b^2}, \tag{3}$$

where b is the correlation length.

This correlation model serves to be a good prototype for theoretic study, showing advantage of giving a clear picture of the results. It contains a number of important cases.

- *No Correlation*: When $b = 0$, neurons are uncorrelated.
- *Local Correlation*: When b is of order $1/\rho$, that is, $b = m/\rho$ for a fixed m, neurons are correlated only within m neighboring neurons.
- *Short-range Correlation*: When $1/\rho \ll b < \sqrt{2}a$, neurons are correlated over a short range.
- *Wide-range Correlation*: When $b \geq \sqrt{2}a$, neurons are correlated over a wide range.

- *Uniform Correlation*: When $b \longrightarrow \infty$, neurons are uniformly correlated with strength β.

3 The Main Results

Fisher Information. We calculate the Fisher information for the encoding model $Q(\mathbf{r}|x)$. Fig.1 shows how the Fisher information behaves as the correlation width changes (the neural density is fixed). It first decreases drastically when the correlation length is small, and increases again when the length is large.

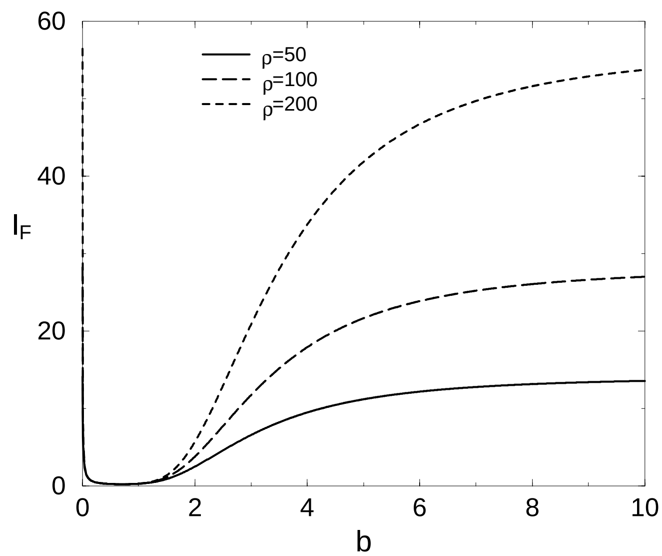

Figure 1: The change of the Fisher information with the correlation length.

Fig.2 shows the asymptotic behavior of Fisher information in different correlation cases. In the case of short-range correlation, the Fisher information is of finite value even when ρ goes to infinity. This result agrees with [1, 3]. For the wide-range correlation case, the Fisher information increases again in proportional to ρ. This a new interesting finding.

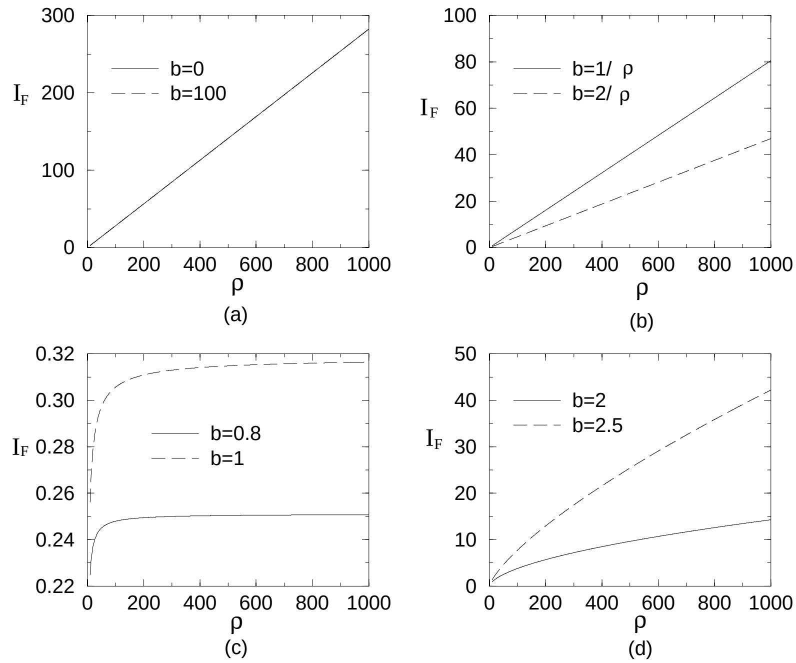

Figure 2: The asymptotic behavior of Fisher information in different correlation cases. (a) No correlation and Uniform correlation ($b = 100$, approximated as infinity). The two curves coincide. (b) Limited range correlations. (c) Short-range correlations. (d) Wide-range correlations.

The Performance of Decoding Methods. Three decoding methods are studied. All of them are formulated as the MLI type, i.e., a solution of $\nabla \ln P(\mathbf{r}|\hat{x}) = 0$.

- The first method is the conventional MLI, referred to as FMLI, which utilizes all of the encoding information, i.e., the decoding model is the true encoding one,

$$P(\mathbf{r}|x) = Q(\mathbf{r}|x). \tag{4}$$

- The second method, referred to as UMLI, utilizes the information of the tuning function, but neglects the neural correlation, that is,

$$P(\mathbf{r}|x) = \frac{1}{Z_U} \exp\{-\frac{\rho}{2\sigma^2} \int_{-\infty}^{\infty} [r(c) - f(c-x)]^2 dc\}. \tag{5}$$

	FMLI	UMLI	COM
No Correlation	AS	AS	AS
Local Correlation	AS	AS	AS
Short-range Correlation	Non-AS	Non-AS	Non-AS
Wide-range Correlation	Non-AS	Non-AS	Non-AS
Uniform Correlation	AS	AS	AS

Table 1:

- The third method, referred to as COM, does not utilize any information of the encoding process, but instead it assumes an incorrect but simple tuning function,

$$P(\mathbf{r}|x) = \frac{1}{Z_C} \exp\{-\frac{\rho}{2\sigma^2} \int_{-\infty}^{\infty} [r(c) - \tilde{f}(c-x)]^2 dc\}, \qquad (6)$$

  where $\tilde{f}(c-x) = -(x-c)^2 +$const is the presumed tuning function. Note that this method is the Center of Mass strategy.

Table.1 lists the asymptotic behaviors of the three methods, where AS denotes asymptotic efficient and Non-AS vice versa. When a method is non-asymptotic efficient, its decoding error is of the Cauchy type and difficult to be quantified. For FMLI, it is not asymptotically efficient in the case of short- and wide- range correlations. The Cramér-Rao bound is not achievable in this case. One should be careful for carrying analysis based on the Fisher information.

Fig.3 compares the decoding errors of FMLI, UMLI and COM. It shows that UMLI has a larger error than that of FMLI, but a lower error than that of COM.

References

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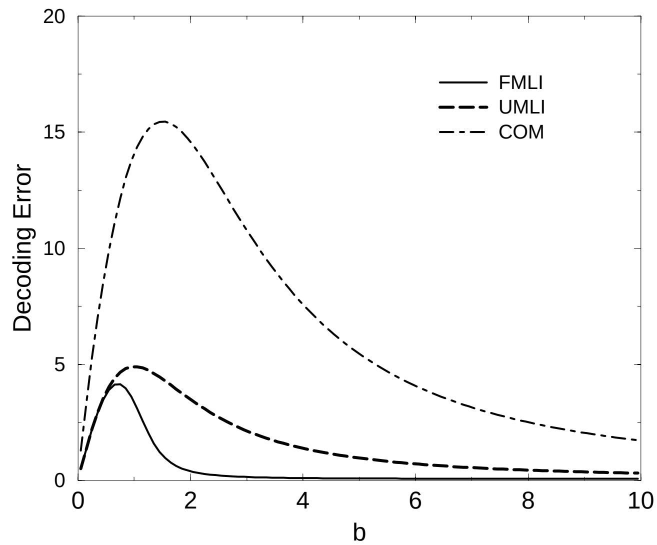

(b)

Figure 3: Comparing the decoding errors of FMLI, UMLI and COM for different correlation lengths.