Oscillatory Modes in a Neuronal Network Model with Transmission Latency

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Abstract

We analyzed the oscillatory activities in a neuronal network model as the basis of synchrony of the activities in the brain. The model consists of two groups of neurons which are interconnected. One group is composed of an excitatory and an inhibitory neuron which are expressed by Hodgkin-Huxley equation. In the present study asymmetricity is introduced by nonidentical neuron groups. The network shows a variety of oscillatory modes depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission latency. The model generates different oscillations with the same parameter values of coupling and latency for different initial conditions.

Keywords: oscillation, phase-locked activity, synchronization, coupling, latency

1 Introduction

The synchronization of the neural activities in different regions in the brain is one of the important features of information processing in the central nervous system. To clarify the fundamental mechanism of the scynchronization, it is very significant to elucidate the phase-locked activities of the neuronal networks with different coupling configurations. We analyzed the oscillatory activities in a neuronal network model which consists of two groups of neurons to investigate the basic characteristics of synchronization. The neurons are coupled with transmission latency. Each group consists of excitatory and inhibitory neurons which are interconnected. In the simulation they are represented by an excitatory neuron and an inhibitory neuron for simplicity of analysis. The neurons are formulated by Hodgkin-Huxley equation.

The model generates oscillatory activities of not only the in-phase mode and anti-phase mode but also the periodic solutions in which the phase difference of the oscillation in two groups of the neurons continuously changes with the coupling strength and latency. Different oscillations emerge with the same parameter values of coupling and latency for different initial conditions.

In the present paper, we show the results of the analysis of the regions of periodic solutions in asymmetrical networks composed of nonidentical neuron groups.

2 A Neuronal Network Model

The dynamics of the membrane potentials of the neurons is formulated as

$$C_M \frac{d\mathbf{V_k}}{dt} = \mathbf{I_{ext,k}} + \mathbf{I_{ion,k}} + \sum_{j=1}^{2} \mathbf{W_{kj}} \mathbf{I_{syn,j}} \quad (k = 1, 2), \tag{1}$$

where

$$\mathbf{V_k} = \begin{pmatrix} V_{ek} \\ V_{ik} \end{pmatrix}, \quad \mathbf{I_{ext,k}} = \begin{pmatrix} I_{ext,ek} \\ I_{ext,ik} \end{pmatrix},$$

$$\mathbf{I_{ion,k}} = \begin{pmatrix} I_{ion,ek} \\ I_{ik} \end{pmatrix}, \quad \mathbf{I_{syn,k}} = \begin{pmatrix} I_{syn,ek} \\ I_{syn,ik} \end{pmatrix}.$$

In the above, $I_{ext,rk}$, $I_{ion,rk}$ and $I_{syn,rk} \quad (r = e, i)$ denote the external input current, the ion currents of the neuron and the synaptic current from the corresponding neuron in the network, respectively. The subscripts e and i denote excitatory and inhibitory neurons respectively. The ion currents $I_{ion,rk}$ are formulated conventionally. The synaptic current $I_{syn,rk}$ is expressed here as

$$\begin{aligned} I_{syn}(V) = \sum_J A_{syn}(J) w \{ &\exp(\frac{-t + t_s(J) + \tau_d}{\tau_+}) \\ &- \exp(\frac{-t + t_s(J) + \tau_d}{\tau_-}) \}, \end{aligned} \tag{2}$$

where w is the coupling coefficient between the corresponding neurons and the element of the coupling matrix

$$\mathbf{W_{kj}} = \begin{pmatrix} a_{kj} & -c_{kj} \\ b_{kj} & -d_{kj} \end{pmatrix}.$$

The J-th instant when the membrane potential increases and exceeds -30mV is denoted by $t_s(J)$ and the transmission latency is denoted by τ_d. If the current of a previous firing is larger than 0.01, it is added to the present current and otherwise neglected. The factor $A_{syn}(J)$ is expressed as

$$A_{syn}(J) = \begin{cases} 1 & (if\ \ t \geq t_s(J) + \tau_d) \\ 0 & (else) \end{cases} . \tag{3}$$

In the present simulations the values of the external input current $I_{ext,rk}$ and parameters C_M, τ_+ and τ_- are set as

$$I_{ext,rk} = 10[\mu\text{A}]\ (r = e, i,\ k = 1, 2),$$
$$C_M = 1[\mu\text{F}/cm^2],\ \tau_+ = 3[\text{ms}],\ \tau_- = 1[ms].$$

3 Results

We analyzed symmetrical networks of two identical groups and asymmetrical networks in which the groups are nonidentical. When the corresponding coupling coefficients in two groups are equal, we made the coupling coefficients between the neurons in the groups denoted as $a_{11} = a_{22} = a_0$, $b_{11} = b_{22} = b_0$, $c_{11} = c_{22} = c_0$, and $d_{11} = d_{22} = d_0$. The bidirectional coupling coefficients between groups are equal and denoted as $a_{12} = a_{21} = a_1$, $b_{12} = b_{21} = b_1$, $c_{12} = c_{21} = c_1$, and $d_{12} = d_{21} = d_1$. The coupling coefficients inside the group are set as

$$a_0 = 10,\ b_0 = 9,\ c_0 = 10,\ d_0 = 0$$

when they are fixed. By the coupling denoted by a_{11} and a_{22} it is assumed that a number of excitatory neurons are interconnected to act synchronously.

We investigated the following four configurations of mutual coupling between the groups;
(1) EE coupling: both excitatory units are coupled bidirectionally, $a_1 \neq 0$, $b_1 = c_1 = d_1 = 0$;
(2) IE coupling: the excitatory unit excites the inhibitory unit of the each other group, $b_1 \neq 0$, $a_1 = c_1 = d_1 = 0$;
(3) EI coupling: the inhibitory unit inhibits the excitatory unit of the each other group, $c_1 \neq 0$, $a_1 = b_1 = d_1 = 0$;
(4) II coupling: both inhibitory units are coupled bidirectionally, $d_1 \neq 0$, $a_1 = b_1 = c_1 = 0$.

The networks show a variety of phase-locked oscillations depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission latency. As in a number of oscillatory systems composed of two components, the present model generates in-phase mode, in which the phase difference of oscillation in two neuron groups is 0 degrees, and anti-phase mode, in which the phase difference is 180 degrees. In addition a remarkable oscillatory mode exists in the present model. In that solution the two groups oscillate in continuously changing phase difference with the parameters of coupling and latency. The mode is referred to as intermediate-phase mode. Chaotic oscillations and quasiperiodic oscillations are also observed. In the latter the phase difference of the oscillation gradually changes with time. It is referred to as varying phase mode.

We obtained the following results of the analysis of asymmetrical networks.
(1) EE-coupling
In the symmetrical network in-phase mode occurs in large range of the parameter a_0. In the asymmetrical network in which a_{11} and a_{22} are different, the region of a_{22} in which in-phase mode occurs become small and there exits the oscillatory mode in which the phase difference variance from about 23 degree to 120 degree is observed with the change of the coupling coefficient a_{22}. Chaotic oscillations also emerge with the larger values of a_{22} and a_1.
(2) IE-coupling
In the symmetrical network anti-phase mode exists in large range of parameters b_0 and b_1. In the asymmetrical network in which b_{11} and b_{22} are different, anti-phase mode exists as well in large range of parameter b_{22}. In-phase mode also occurs with relatively large range of b_{22}. In-phase mode, intermediate-phase mode and chaotic oscillations coexit with anti-phase mode in the respective regions of b_{22}. Which mode occurs depends on the initial conditions of the membrane potentials. Varying phase mode emerges with the small values of b_{22}.
(3) EI-coupling
In the symmetrical network in-phase mode and intermediate-phase mode coexist with anti-phase mode in large range of parameters c_0, c_1 and τ_d. mode exits. In the asymmetrical network in which c_{11} and c_{22} are different, the mode coexistence is same as the case of the symmetrical network. The region of in-phase mode becomes smaller and that of intermediate-phase mode becomes larger. Varying phase mode emerges with the small values of b_{22} or large values of b_{22}, in other words, when the asymmetricity is large. Chaotic activities are also observed with small values of c_{22}. It coexists with anti-phase mode. Which mode occurs depends on the initial conditions of the membrane potentials.
(4) II-coupling
In the symmetrical network anti-phase mode occurs in large range of parameters d_0, d_1 and τ_d. In the asymmetrical network in which d_{11} and d_{22} are different, anti-phase mode occurs in the large region of d_{22}. In addition in-phase mode occurs with relatively large range of d_1 and d_{22}. It coexists with anti-phase mode for the large range of the parameters.

4 Conclusions

We analyzed the oscillatory activities in a neuronal network model to elucidate the basis of synchrony of the activities in the brain. The model consists of two groups of neurons that are interconnected. The results of the analysis of some asymmetrical networks reveal that the model shows different phase-locked oscillations depending on the structure and intensity of interconnection between groups or coupling of neurons in the group, or the value of transmission latency. The oscillations include various periodic solutions in which the two groups oscillate not only in in-phase or anti-phase but also in continuously changing phase difference with the parameters of coupling and latency. Quasiperiodic solutions and chaotic oscillations are also observed. The rich variety of the results suggests that it is necessary to make further analysis of the networks with different coupling configurations and different asymmetricities.