Real-Time Estimation of Predictive Firing Rate

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Abstract

Population vector analysis of multi-unit firing code in the motor cortex provides a real-time estimate of the desired voluntary movement direction and trajectory used in neuroprosthesis for assisting paralyzed patients. Accurate estimation of instantaneous firing rate in each epoch used in the population vector calculation is crucial. Yet real-time instantaneous firing rate is often unknown until the next spike has fired. We have developed statistical based estimator for predicting the waiting time for the next spike to fire. Instantaneous firing rate is computed based on the conditional probability density function (pdf), providing a better estimate of the intended movement trajectory.

Keywords: spike train analysis; probability density function; firing rate

1. Introduction

Population vector analysis of multi-unit firing code in motor cortical neurons is a measure developed by Georgopoulos et *al*. [1 - 4] to estimate movement direction and trajectory based on a vector sum of the weighted-average firing rate with respect to the "preferred movement direction" of each individual neuron in the ensemble. This measure is a powerful means to predict the movement trajectory to the intended target. In other words, the spike-firing rate of the motor cortical neurons can be decoded to represent the intended voluntary arm movement using the population vector analysis.

Simultaneous recording of spike trains recorded from 64-channel electrodes implanted in the motor cortex provided a means for real-time prediction of arm movement trajectory [5]. This real-time prediction of arm movements based on population vector analysis of multiple spike trains will be implemented as a neuroprosthesis so that a robotic arm can be moved by "mental efforts" of the patient implanted with 64-electrodes in the motor cortex.

One of the challenges in obtaining the firing rate of the spike train in real-time is due to the typically low firing rate in motor cortical neurons. Because of the low firing rate, or the "sparse code," accurate estimate of the true firing rate of each individual neuron is important so that the population vector sum can be computed accurately. This challenge is usually a non-issue when the population vector is computed off-line, based on *a posteriori* information of the spike firings, since the instantaneous firing rate can always be computed within a given spike train.

But when real-time estimate of the population vector is needed, the instantaneous firing rate for each neuron is required. Traditionally, the population vector is computed incrementally for each epoch of fix time-interval, such as every 30 msec. The mean firing rate for each neuron is then computed for each epoch, and the population vector based on the preferred direction of each neuron is computed using this mean firing rate for each epoch.

Using this method, estimation errors exist at the boundary of the epoch because spikes do not often fall on the epoch boundary. The true instantaneous firing rate can be computed if the two boundary-delimiting spikes exist (see Fig. 1). In off-line analysis, the boundary-delimiting spikes are known; thus, there would not be any ambiguous estimation of the mean firing rate for the epoch.

[INSERT FIGURE 1 HERE]

But in real-time analysis, the future delimiting spike is unknown for the current epoch calculation of mean firing rate (see Fig. 1). Thus, it necessitates the estimation of the waiting time for the next spike to occur. The accuracy of this estimation will influence on the estimate of the mean firing rate for the current epoch used in the population vector calculation. Since the overall firing rate is low, which is typically found in motor cortex, the error in estimating the waiting-time for the unknown future spike can be large.

There are many existing methods for resolving this ambiguity. One of them is to use a Gaussian filter to convert the spike firings into firing rate, which is effectively a smoothing function for the discrete spike rate. This method is highly dependent on the parameter used for the Gaussian filter, thus it is user-dependent.

Another method commonly used is to extrapolate the next firing interval based on the previous firing rate (or firing interval). This is often a good estimate since it is dependent on the neuron's own firing rate rather than an arbitrary parameter.

The current paper proposes an alternate predictor of the waiting-time for the unknown future spike so that the instantaneous firing rate of the neuron during the current epoch can be better estimated.

2. Theoretical Methods - Adaptive Statistical Estimation

One of the better predictors of future events is based on the history past events. In other words, since we are recording on-going spikes from the neuron in real-time, we can collect the firing statistics of the neuron. The interspike-interval (ISI) statistic characterizes the probability of firing of a next spike based on the firing of a current (reference) spike. Normalizing the ISI by the number of spike gives the probability density function (pdf) for the neuron, which will be used as the predictive function for the waiting-time for the next spike firing.

Let the spike train be, *a(t),* recorded from the neuron with a total of N spikes be represented by

$$a(t) = \sum_{n=1}^{n=N} \delta(t - t_n) \tag{1}$$

where t_n is the occurrence times of n-th spikes in spike train *a(t)*, and $\delta(t)$ is a delta function denoting the occurrence of a spike at time t.

The ISI, τ_n, is defined as the time-interval between two adjacent spikes in the spike train:

$$\tau_n = t_n - t_{n-1} \tag{2}$$

Then, the probability density function (pdf) of the first-order ISI is defined as:

$$p(\tau) = \frac{1}{N} \sum_{n=1}^{N} \delta(\tau_n - \tau) \tag{3}$$

and the probability distribution function, PDF, which is given by

$$P(\tau) = \int_0^{\tau} p(t)dt \tag{4}$$

Since the spike train is recorded in real-time, the total number of spikes, N, increases with time. Thus, the pdf function also changes with time, with a better approximation when the sampling size, N, increases with time.

3. Statistical Estimation

Given that the firing pdf for the neuron is known, the probability of the next spike firing can be estimated for the future spikes. We also have additional information of the firing probability, since we are predicting the next spike firing at the boundary-delimiting conditions. Because the delimiting condition is that a spike did not fire at the end of the epoch-window period, the probability of next spike firing becomes the conditional probability given that a waiting-time, $\Delta\tau$, has elapsed since the last spike and a spike fired at the next time increment, □□+ □t. In other words, the probability given that a spike had not fired for □□ is essentially 1□P(□).

4. Comparison between current method and other traditional methods

There are many other alternate methods for computing the mean firing rate. One of the most common methods is simply compute the mean firing rate (number of spikes fired within the epoch-window divided by the window-length) truncating the extra time between spikes at both ends of the epoch-window. This always results in over-estimating the firing rate since the extra times at the boundary condition are ignored.

Another traditional method is to use the averaged firing rate during the epoch as the predictor to extrapolate the future firing rate for the next spike. In other words, it uses the mean firing rate as predicted firing rate for the next spike. This method is better than the truncated firing rate method above since it uses the past averaged firing rate as the predictor for the future spike rate.

In comparison, the present method provides a better estimate of the future spike rate since it takes advantage of the instantaneous firing rate for the prediction rather than the mean firing rate. The current method provides a means to increase the statistical significance as N (total number of spikes in the sample size) increases with real-time on-line recording session. Furthermore, the predicted estimated rate is operator-independent and self-adaptive. The future firing rate is estimated entirely based on how long a spike had not occurred at the end-boundary.

Given these advantages, real-time estimate of the firing rate at each epoch can be performed with higher accuracy. With the improved accuracy of estimated instantaneous firing rate for each individual neuron, the vector sum of the population ensemble can be more accurately represented. With a better representation of the population vector, the intended movement trajectory can be predicted more accurately for sparse-code encoding in the cortex.

References

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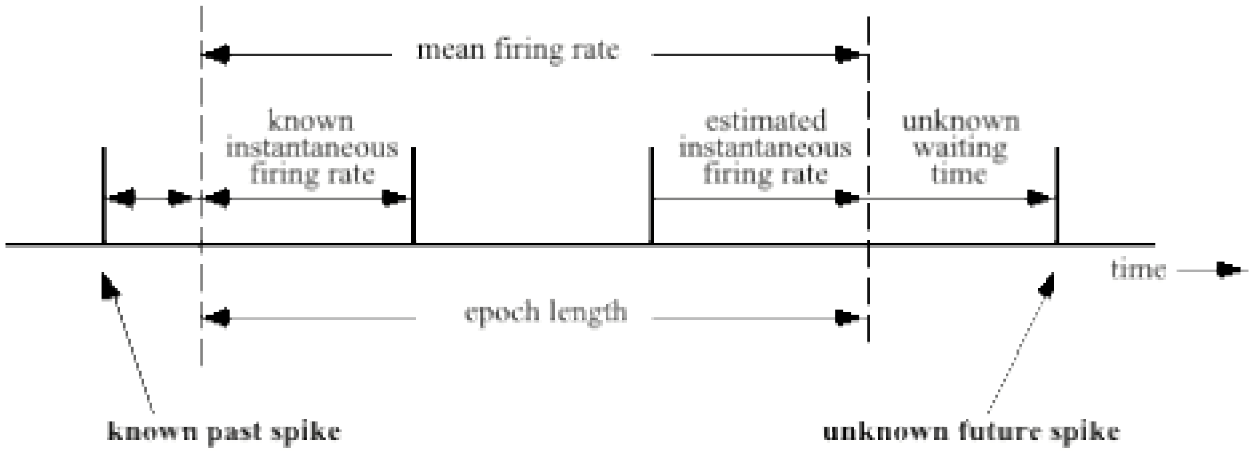
Biosketch

Dr. David C. Tam is an associate professor at the University of North Texas. He holds a Ph.D. in physiology, and three B.S. degrees in computer science, physics and astrophysics, all from the University of Minnesota. His current research interest is developing statistical multiple spike trains analysis techniques and applying these analytical methods to both simulated and experimental neurophysiological data in using emotional behavioral paradigms to understand the

signal processing functions of the central nervous system (CNS) as well as extracting the encoding schemes used by the CNS.

Figure Legends

Figure 1. Diagram showing the knowledge of boundary-delimiting spikes needed to compute the mean firing rate. The waiting-time for the future spike is unknown in real-time when the mean firing rate is computed for the current epoch.


mean firing rate
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