Modeling Neural Spatiotemporal Behavior

David Horn
School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv 69978, Israel
and
Brigitte Quenet
Lab. d'Electronique
Ecole Superieure de Physique et Chimie Industrielles
Paris 75005, France
and
Orit Kliper
School of Computer Science
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv 69978, Israel

February 6, 2002

Abstract

We propose a systematic way for constructing a neuronal model that produces spatiotemporal behavior of the type observed in locus olfaction. We solve the problem within the framework of a dynamic neural filter (DNF), a recurrent network that operates under constant inputs. Our system includes: 1. construction of hidden neuron states. 2. construction of the synaptic matrix and the inputs. 3. converting these solutions, achieved with binary variables, to a system of spiking neurons. The construction stages lead to a particular solution but many can, in principle, be accommodated by the data. It is interesting to have a systematic procedure of this kind since finding one

solution is a laborious process, although it leads just to a starting point in the search for a viable neuronal model.

1 Summary

An interesting issue in neuroscience is the question of spatiotemporal coding. Its existence has been demonstrated [3] in the locust olfactory system, where the spatiotemporal behavior of projection neurons encodes the odor presented to its receptor neurons. This transformation from odor-input to spatiotemporal activity occurs in the antennal lobe, that is the first module of the olfactory system. This system may therefore be regarded as a dynamic neural filter that turns spatial information distributed over its many glomeruli, that are fed by the receptor neurons, to specific spatiotemporal outputs. It is interesting to point out that, although this is a complicated biological system, it has an important simplifying feature that allows it to be represented by a simplified model of mathematical neurons, i.e. the fact that the activity of the projection neurons is limited to temporal bins defined by an oscillatory local field potential. Hence a model with binary neurons obeying Hopfield-Little dynamics [2] can provide a valid first order approximation of the spatiotemporal behavior of the system. We have presented this model in a previous work[4] and demonstrated how it can be used to generate the spatiotemporal behavior of projection neurons observed by Wehr & Laurent [3]. Here we develop it further and describe a systematic approach for its construction.

The dynamics of the model has the following structure

$$n_i(t+1) = H(h_i(t+1)) = H(\sum_j w_{ij} n_j(t) + R_i - \theta_i) \tag{1}$$

where w_{ij} is the synaptic coupling matrix, R_i is the external constant input (specifying odor activation) and θ_i is the threshold. H is the Heaviside step function taking the values 0 for negative arguments and 1 for positive ones. For simplicity we choose w_{ij} and R_i as

positive and negative integers, fixing $\theta_i = \frac{1}{2}$.

Consider the problem presented by Table 1, where we observe $K = 6$ series, representing the results of six odors, over $T = 4$ time steps in a problem of $N = 5$ neurons. The first two neurons correspond to the observed ones of Wehr and Laurent.

Table 1: Six spatio-temporal sequences defined for four time-steps in a system with five neurons.

time/code	1	2	3	4	5	6
1	1 1 0 0 0	1 0 0 0 0	1 1 1 0 0	1 0 0 0 0	1 0 1 1 0	1 0 0 0 0
2	1 1 0 0 1	1 1 0 0 1	1 1 1 1 0	1 0 1 0 0	1 0 0 0 0	1 1 1 0 0
3	1 1 0 1 1	1 1 0 1 1	0 1 1 1 0	0 1 1 0 0	1 1 1 0 0	0 1 1 1 1
4	0 0 0 1 0	0 0 0 1 0	0 0 1 1 0	0 1 1 1 0	1 1 1 1 1	0 0 0 1 0

The modeler is challenged to find a matrix $\mathbf{w}$ and six input vectors R that solve this problem. We have presented a solution to this problem in [5] based on the perceptron algorithm. The principle is to generalize the N-dimensional neural space to an $N + K$ dimensional one, such that each one of the K series is presented by one 1 and $K - 1$ null entries in this new axes. In this new space we define for each neuron i, a vector of perceptron weights $\vec{w}^i$

$$(\vec{w}^i)_j = w_{ij} \quad \text{for } j = 1, \cdots N \qquad (\vec{w}^i)_{N+k} = R_i^k - \theta_i \quad \text{for } k = 1, \cdots K. \tag{2}$$

We assume that the K series start from an initial null state, and we use the perceptron algorithm to construct these vectors $\vec{w}^i$, whose $(N + k)$th component includes the information about R_i^k.

Thus we obtain a synaptic weight-matrix, that serves to define a dynamic neural filter (DNF), as well as K inputs that lead to the sequences of Table 1. But this can work only if each column in Table 1 does not include repetitive states, and if no xor-like contradictions of perceptron dynamics occur. If such situations occur we have to add hidden neurons. In fact this is the case with the Wehr-Laurent series, described by the first two neurons in Table 1. Since these sequences cannot be generated by an $N = 2$ DNF, we added three hidden neurons in the example of Table 1, leading to a system that can be generated by a DNF.

When one adds hidden neurons one has also to decide what their states in each row and column will be. Hence we looked for a constructive algorithm that generates these states. We provide such an algorithm in this work. We show that for any problem of K binary sequences with T time steps one can find hidden neurons that allow for a DNF representation. Our algorithm constructs these states. In general, for high values of T and K we expect the total number of neurons (observed + hidden) to obey

$$N \geq \frac{1}{2}K(T-2) \tag{3}$$

which is derived from the Cover limit[6].

Once we generate these states, and construct the synaptic matrix and inputs, we are in a situation that allows us to attempt a slightly more realistic model, using spiking neurons. In an example that we will demonstrate, we work out a system of integrate-and-fire neurons whose synaptic matrix is taken from the solution to the problem of Table 1. The effect of the reverberating LFP is mimicked by an overall inhibition that allows for spike generation only during limited and periodic time windows. The synaptic interactions are endowed with a delay that matches the period of inhibition, thus generating a spiking pattern that conforms with the data of Table 1.

References

[1] Hertz, J., Krogh, A. & Palmer, R., G., 1991. *Introduction to the Theory of Neural Computation.* Addison Wesley Pub. Comp.

[2] P. Peretto, 1992. *An Introduction to the Modeling of Neural Networks.* Cambridge University Press.

[3] M. Wehr and G. Laurent, 1996. Odour encoding by temporal sequences of firing in oscillating neural assemblies. *Nature* 384, 162-166.

[4] B. Quenet, D. Horn, G. Dreyfus and R. Dubois, 2001. Temporal coding in an olfactory oscillatory model. *Neurocomputing* 38-40, 831-836.

[5] B. Quenet and D. Horn, 2002. The Dynamic Neural Filter: A Binary Model of Spatiotemporal Coding. Preprint.

[6] T. M. Cover, 1965. Geometrical and statistical properties of systems of linear inequalities with applications in pattern recognition. *IEEE trans. Elect. Comp.* **14** 326-334.

[7] B. Quenet, R. Dubois, S. Sirapian, G. Dreyfus and D. Horn, 2001. Two step modeling of spatiotemporal olfactory data: from binary to Hodgkin-Huxley neurons. Preprint.