Phase transition between reactive and predictive eye movements is confirmed with nonlinear forecasting and surrogates

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Abstract

We previously demonstrated that there is an abrupt (rather than smooth) transition between reactive and predictive modes of eye-movement tracking of target lights (a phase transition). We also found evidence that the sequence of eye movements in the reactive mode was independent, while those in the predictive mode were correlated and possibly formed a random fractal sequence. Here we confirm this finding by quantifying the rate of decay of nonlinear forecasting when applied to these data, and develop an extension to small data sets using surrogate data. Verification of these statistical properties in small data sets will be of great use in situations, such as the clinical testing of neurological patients, in which testing time is limited due to subject discomfort or other considerations.

1. Introduction

In previous work we demonstrated that there is a phase transition between two different modes of oculomotor tracking [6]. Subjects were asked to follow, with their eyes, visual targets that jumped horizontally at various rates. Subjects made rapid eye movements known as saccades, which are highly accurate and move the eyes rapidly to their new position. When target pacing was low (below 0.5 Hz), saccade latency (time between target jump and start of saccade) was approximately 180 msec; subjects waited for each target jump and reacted with a saccade. When target pacing was fast (above 0.6 Hz), saccade latency decreased dramatically, to the point where subjects made saccades that anticipated the target jumps. We call these two distinct behaviors reactive and predictive saccades. The transition from reactive to predictive as pacing frequency increases (and vice versa) is not smooth but abrupt, and occurs at about 0.5 Hz. (This result is closely related to earlier work on phase transitions in finger-tapping [2].)

Our initial study also presented evidence for a difference in the statistical scaling properties of saccade latencies in the reactive and predictive modes. The power spectra of the series of latencies for reactive saccades was relatively flat, suggesting an uncorrelated or white noise process. The spectra of the predictive saccade latency series decayed as a function of frequency, in a power-law fashion. The decay exponent was in a range that suggested that the latencies form a random fractal sequence, indicative of long-term correlations between latencies. These scaling results were found also with rescaled-range analysis (Hurst exponent).

Here, we confirm and extend this result through the use of nonlinear forecasting or prediction [1]. Nonlinear forecasting is a way to predict subsequent events in a time series, based on a reconstruction of system trajectories in state space. It has been shown that forecasting quality decays as a function of forecasting horizon (time into the future over which the forecast is made) in a characteristic manner for random fractal processes. We show that forecasting confirms that predictive saccade latencies indeed form a random fractal sequence, and we demonstrate a way to apply this methodology to small data sets through the use of surrogate data. The ability to demonstrate these mathematical characteristics in small data sets will allow the use of this

method in a wide variety of cases where testing time is limited. Such circumstances include testing of clinical neurological patients for disorders in the ability to make predictive movements, and testing in different gravity levels during parabolic aircraft maneuvers where each level is maintained for only 20-25 seconds [5]. (At a rate of 1.0 Hz there are two saccades per second; 1000 trials takes more than eight minutes, while 50 trials takes only 25 seconds.)

2. Methods

To obtain data on saccades at different pacing rates, subjects were seated in a dark room and presented with two red LED targets spaced horizontally 15 deg to the left and right of center. The LEDs were illuminated alternately at rates from 0.2 Hz to 1.0 Hz. The subject was instructed simply to look at each light when it came on. Eye movements and LED position were recorded, and a series of saccade latencies was extracted by comparing the two time series.

In experiments designed to look for a phase transition, 50 trials (target steps) were presented at frequencies of 0.2, 0.3, 0.5, 0.7, 0.9, and 1.0 Hz, monotonically increasing and then decreasing. These produced small data sets, which demonstrated phase transitions but are too small for reliable spectral or Hurst analysis. They also present problems with forecasting, to be explored below. To obtain large data sets more suitable for some analyses, 1000 trials were presented to the same subjects at 0.3 and 0.9 Hz only, in a separate session.

For the large data sets, power spectra were found by Fourier analysis. Lines were fit to the spectra via linear regression to obtain scaling exponents. Hurst rescaled-range analysis was also carried out on the large data sets, in order to verify power-law scaling found in the spectra [3].

Nonlinear forecasting was carried out in a standard manner as described by many others [1] and as used in our previous work on other aspects of eye-movement control [4]. In this procedure, time-delay reconstruction is used to generate trajectories in a state space, and a local linear fit is made starting at each point in turn, in order to forecast subsequent points. The forecast points are compared to the actual future points in the series via correlation coefficient, as a function of the number of time steps (actually trials) into the future over which the forecast is made. Decay of forecasting quality with time step is quantified by linear regression.

Surrogate data techniques [7] were used to assess the numerical reliability of the forecasting results. This is a technique in which some aspect of a given data set is used to generate a number of random data sets (the surrogates), based on a model of the data. For example, the model could be that the original data set is a white noise process with a given mean and variance; in this case the surrogates would consist of white noise processes each with the same mean and variance as the original data set. Measurements (such as forecasting) carried out on the surrogates would indicate whether or not that particular model reproduces the results from the original data set, and therefore whether or not the model is an accurate reflection of the original (as far as that specific measurement is concerned). In this work, we generate surrogates of two kinds. First, we simply shuffle the values in the data set; this is based on a model of the data as a white noise process. Second, we find the frequency spectrum, shuffle the phases of the frequency components, and produce a time series via inverse Fourier transform (phase-shuffle); this models the data as a random process with a given linear correlation structure, or autocorrelation (since phase-shuffling does not change the magnitude or power spectrum, and therefore leaves the autocorrelation unchanged). We compare the results of forecasting applied to the original data sets, and to both of these surrogates, below.

3. Results

Figure 1 (based on Fig. 4 in [6]) shows power spectra for reactive and predictive saccades for one subject (N=1000 trials). There is a distinct difference between the two conditions. The spectrum for reactive saccades is approximately flat, indicative of white noise or uncorrelated trials. The spectrum for predictive saccades decays linearly on this log-log scale, indicative of power-law scaling.

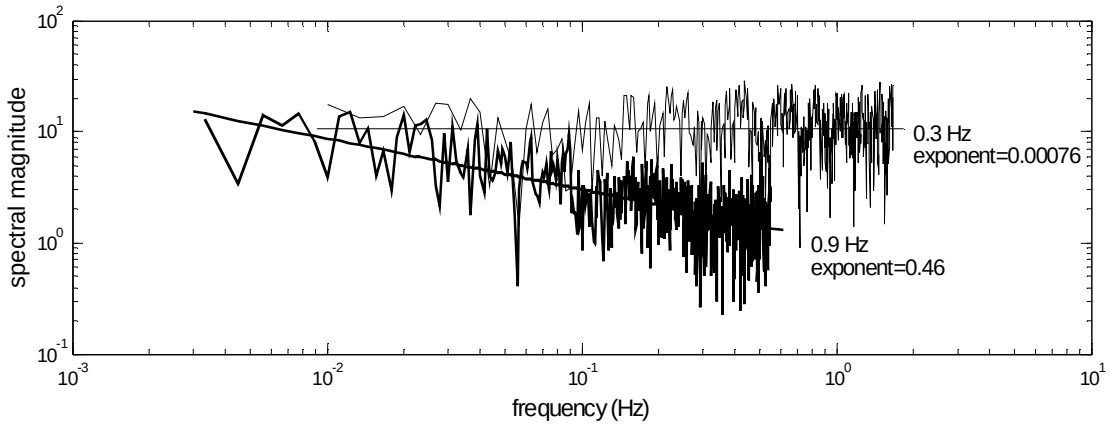

Figure 1. Power spectra for sequences of reactive (0.3 Hz) and predictive (0.9 Hz) saccades, from 1000 trials of each. Slope of lines fit to each spectrum are indicated as decay exponents.

Figure 2 shows forecasting quality (correlation coefficient *r* between sets of forecast and actual values) as a function of number of steps into the future over which the forecasting is carried out, for the case of predictive saccades (pacing at 0.9 Hz, N=1000). In Fig. 2A the forecasting quality clearly decays with increasing forecasting step. This result is re-plotted in the form of log(1-*r*) vs. forecasting step in Fig. 2B, and as a function of log(forecasting step) in Fig. 2C. The decay of forecasting quality is linear with respect to log(forecasting step), which indicates that the data form a random fractal process [8].

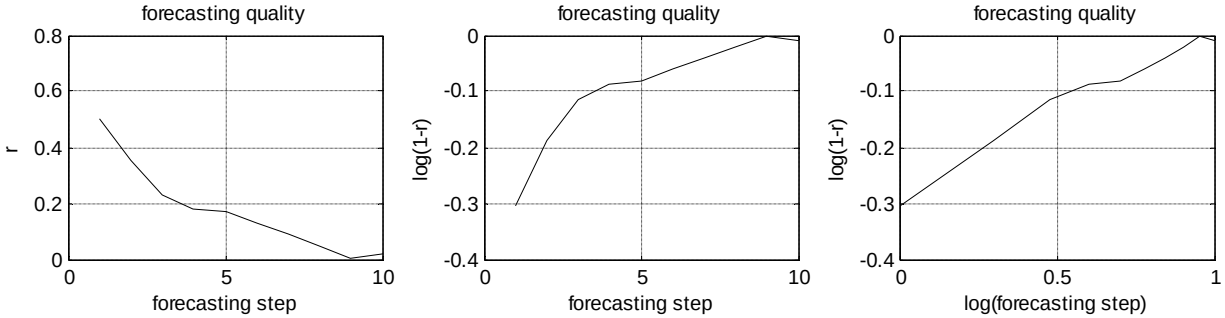

Figure 2. Forecasting of predictive (0.9 Hz) saccades, from 1000 trials. First graph shows forecasting quality (correlation coefficient *r*) as a function of forecasting step into the future. Other graphs show log(1-*r*) as a function of forecasting step, and log(1-*r*) as a function of log(forecasting step).

Analysis of surrogate data based on these large data sets confirms the finding of a fractal process at 0.9 Hz pacing, and uncorrelated "noise" at 0.3 Hz pacing. In Fig. 3 are the results of both the shuffle and the phase-shuffle surrogates, applied to reactive (0.3 Hz) and predictive (0.9 Hz) saccades, from one subject.

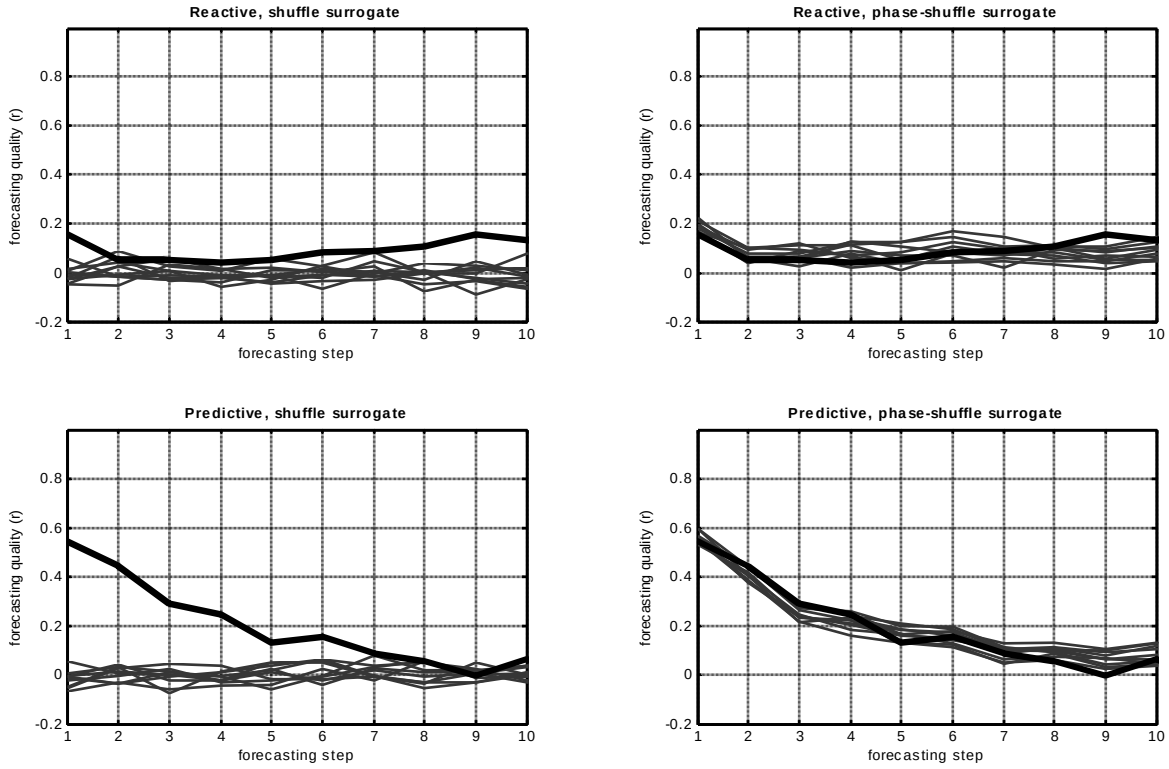

Figure 3. Results of surrogate data analysis for one subject, for reactive saccades (top two graphs) and predictive saccades (bottom two graphs). In each case, a shuffle surrogate (left) and a phase-shuffle surrogate (right) are shown. Forecasting of the original data set in each case is given by the thick line. Results from ten surrogates of each type are lighter thin lines.

This analysis is based on two considerations. First, the shuffle surrogate should not make forecasting any worse (relative to the original data set) if the underlying data set is uncorrelated noise. Second, the phase-shuffle surrogate should not make the forecasting any worse if the underlying data set is a stochastic process with a given correlation structure (of which a fractal random process is one possibility). The top two graphs in Fig. 3 show that indeed the forecasting of reactive saccade latencies is not significantly different from the forecasting of either type of surrogate based on this data set. This suggests that the series of reactive saccade latencies can be modeled as an uncorrelated noise process. The bottom left graph in Fig. 3 shows that the forecasting of predictive saccade latencies is much better than is the forecasting of surrogates formed from this data set by random shuffling of the data; this indicates that these data are not adequately modeled as an uncorrelated noise sequence. The bottom right graph in the figure shows that the forecasting of predictive saccades latencies is almost identical to the forecasting of surrogates that are formed from this data set by phase-shuffling; this indicates that these data can be modeled as a stochastic process with a given autocorrelation function (time-correlation structure). This is consistent with the identification of these data as a fractal random process, as found above. Thus forecasting results for large data sets can distinguish between reactive and predictive modes of oculomotor tracking.

Now we turn our attention to the attempt to obtain analogous results with small data sets (N=50). Our approach is again to use forecasting of surrogates, not to compare to the original data set, but to help identify any power-law scaling. For each pacing frequency (0.2, 0.3, 0.5, 0.7,

0.9, 1.0 Hz), 100 surrogates were produced of each type (shuffle, phase-shuffle). A line was fit to the surrogate forecasts in each case, via linear regression. Fig. 4 presents the results, for shuffle (top) and phase-shuffle (bottom) surrogates. The forecasts for the phase-shuffle surrogates in the predictive mode (bottom graphs, 0.7 Hz and above) exhibit a clear decay; this is different from all of the other cases (all shuffle surrogates, and phase-shuffle surrogates in the reactive mode).

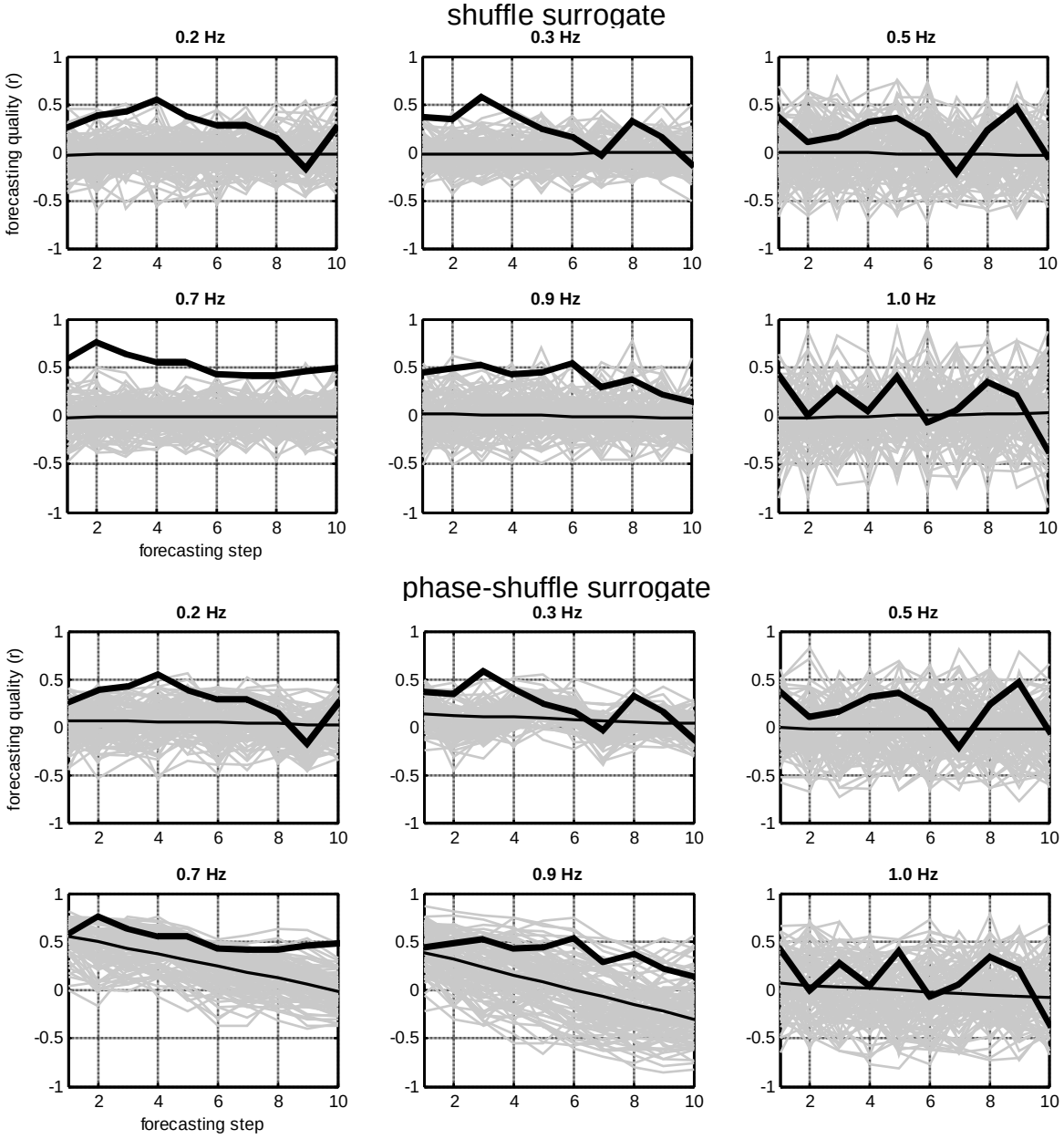

Figure 4. Forecasting of shuffle (top) and phase-shuffle (bottom) surrogates. Each surrogate is based on a sequence of 50 saccade latencies at each pacing frequency. Surrogate results are shown as ten thin gray lines in each graph; thin dark line is best linear fit to the surrogates. Original data forecasting results are thicker dark lines in each case.

Although forecasting with such small data sets cannot confirm power-law scaling, forecasting of their surrogates does. This is of value in demonstrating a change in behavior commensurate with the phase transition. Thus we can verify that there are two different statistical behaviors in

short data sets. Our earlier work suggests strongly that these are uncorrelated noise and long-term correlated data, but we cannot conclusively demonstrate this with the method at hand.

4. Discussion

The difference in statistical properties between reactive and predictive saccades is a recent finding. We are just beginning to explore its implications. One promising interpretation is the following. When target steps come far apart, the subject responds to each one independently, with no attempt (or no ability) to use information on the timing of past saccades to trigger subsequent saccades. This leads to low predictability for reactive saccades, and scaling that suggests that the saccade latencies form a white noise process. On the other hand, when target steps come close together, there is insufficient time to react to each individual step when programming a saccade, and the timing performance of previous saccades must be considered in triggering subsequent saccades. This leads to correlations in the latency series of predictive saccades. The specific scaling properties (random fractal sequence) suggest that these correlations are "long-term" – that the correlations between trials decay gradually with time. This in turn suggests that neural mechanisms rely heavily on past performance in the programming of predictive motor acts.

The methodology described here allows for application to patient data, phase-transition data, and other short data sets (on the order of 50 rather than 1000 trials). This allows the use of short data sets in further investigations of reactive/predictive transitions.

Acknowledgements

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