

On the Variability of Cortical Neural Responses: A Statistical Interpretation

Sheng Li and Si Wu

Department of Informatics, University of Sussex, UK

Abstract

Neural responses in the cerebral cortex exhibit tremendous variability. Understanding the origin and the functional meaning of this variability is of critical importance for our understanding of neural coding. The present study investigates in the neural response variability from the view of statistical inference. We would show that high variability can also arise due to the inferential sensitivity in neural coding. This view is supported by the simulation on the representation of nature images.

Key words: neural coding; neural response variability; statistical inference

1 Introduction

External stimuli are encoded in neural responses in the brain. Neural responses have a unique feature that all spikes are of the same stereotyped shape, with the variable quantity being the time interval between successive spikes (ISI). The pattern of ISIs is then being the medium to convey the stimulus information. It is expected that if the relationship between stimuli and the patterns of ISI is revealed, the neural cipher is broken. Motivated by this idea, experiments have been recorded a large quantity of data on neuronal responses in different areas of the brain. The finding on the responses of a single neuron in the cortex, however, leads to a quite puzzling result: the pattern of ISI seems to be generated by a random process; its correlation with the stimulus is negligible. Actually, the variability of neural response is rather high, when it is measured by the variance-mean ratio of ISI. The value of the ratio is typically within the range of (1, 1.5), and it is comparable or even larger than the corresponding value of the random Poisson process (see e.g., (4; 3)).

Email addresses: `sheng@sussex.ac.uk` (Sheng Li), `siwu@sussex.ac.uk` (Si Wu).

Giving a reasonable justification of the origin and the functional meaning of this high variability is significantly important for our understanding of neural coding. A large volume of work has been done on this issue. In terms of the origin, there are two possibilities being suggested. One concerns the irregularity at the single neuron level, where the stochasticity comes from the unreliable release of neuro-transmitter at synapses, the randomness in the channel gating, and the fluctuations in membrane potential (see e.g., (2)). The other concerns the irregularity arises at the network level. For instance, as shown by (3; 5), irregular firing can be generated by the balanced contribution between excitatory and inhibitory inputs. In terms of the function meaning that justifies why the brain works in this highly variable dynamic regime, there are also many discussions. For instances, it is suggested that the irregularity enables neural system to response fast to external stimulus, it provides a mechanism similar to stochastic resonance to improve the signal-to-noise ratio and it serves as a framework for encoding the uncertainty of stimulus. Although these studies answered our concern in many aspects, it is still not fully convincing as why neural activity has to be such irregular in the first place. Anyhow, in principle the aforesaid functional advantages can also be implemented in deterministic systems. The missing part might directly reason out the irregularity of neural response with its goal of information encoding.

In the current study we are investigating another potential source of irregularity in neural response. This view is motivated by a general property in statistical inference, that is, under certain circumstance the inferred result is be very sensitive to noise. An intuitive illustration of this consequence is shown in Fig.1. Fig.1a presents a natural image. Two pictures those are independently down-sampled from this image are shown in Fig.1c and Fig.1d. While using a large sampling rate, their discrepancy is small, and it is almost visually indistinguishable. We may expect that their neural representations have similar values. The result turns out to be a surprise, as shown in Fig.1b, the neural representations are significantly different in the level of individual neural activities (the details in Sec.3). This implies that neural representation can be very sensitive to input noise (Here the notion of noise refers to the minor discrepancy between two pictures). The inferential sensitivity is what we will attribute as a source of variability in neural activities. We will analyze and quantify the sensitivity in neural coding in the following sections.

2 Statistical Inferential Sensitivity

The ultimate goal of neural coding is to represent and retrieve external stimuli in a noisy environment, which, mathematically, can be expressed as statistical inference. According to our interests are focused on the general properties of neural coding, a simple model that captures the fundamental features of neural

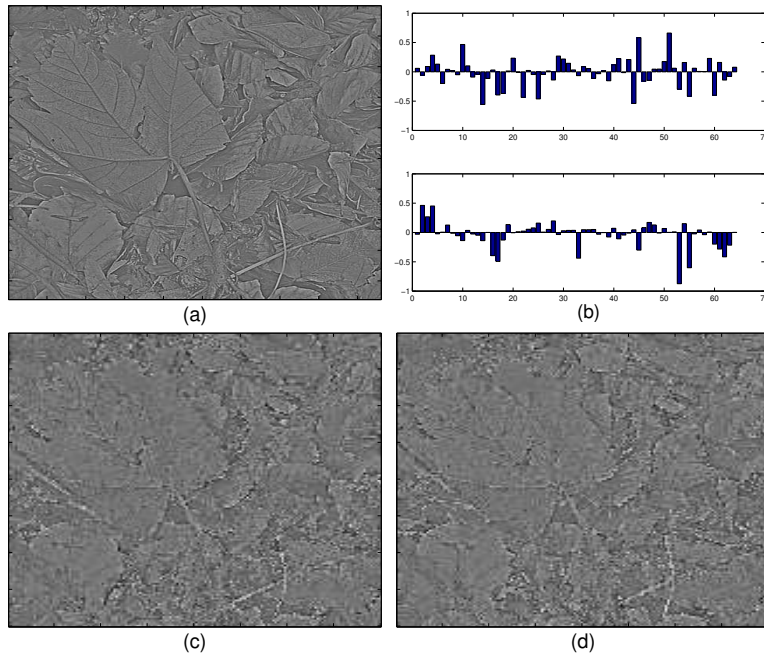


Fig. 1. An illustration of inferential sensitivity. (a) A natural image; (b) The neural representations for the two sampled pictures in (c) and (d); (c) and (d) Two copies of independent sampling the image in (a) with the sampling rate $1/9$.

inference is considered. The results to be provided below are still applicable to general cases. We consider that external stimulus is encoded by a linear superposition of basis functions (e.g. (1)),

$$I(\mathbf{x}) = \mathbf{a} \cdot \boldsymbol{\phi}(\mathbf{x}) = \sum_i a_i \phi_i(\mathbf{x}), \quad (1)$$

where $I(\mathbf{x})$ denotes the stimulus, and $\boldsymbol{\phi}(\mathbf{x}) = \{\phi_i(\mathbf{x})\}$ the basis functions which are related to the neuronal receptive fields. The coefficients $\mathbf{a} = \{a_i\}$ represents the neural responses, which are the representation of the stimulus in the basis set $\boldsymbol{\phi}$. In practice, the data observed by the estimator is noisy, which can be written by the true value plus a random variable,

$$y_i = I(x_i) + \varepsilon_i, \quad \text{for } i = 1, \dots, N, \quad (2)$$

where N is the number of data points being observed. The task of neural estimator is to infer the neural representation \mathbf{a} from the observed input value $y(\mathbf{x})$.

To illustrate the idea, we further simplify the model (just for the purpose of being mathematically trackable) by considering there are only two neurons and the basis functions are Gaussian (Fig.2a), and the noise form is also independent Gaussian. Because of the noise, the finite number of data points,

the intrinsic structures of the stimulus, and the decoding method used, the inferred result will generally deviate from the true representation. We can formally prove that in the above model setting, the minimum variability of neural response is given by,

$$P(a_1, a_2) = \frac{1}{Z} \exp\left\{-\frac{1}{2} \mathbf{a}^T \boldsymbol{\Sigma} \mathbf{a}\right\}, \quad (3)$$

where the vector $\mathbf{a} = (a_1 - a_1^0, a_2 - a_2^0)$, with a_1^0, a_2^0 being the true representation, and the matrix $\boldsymbol{\Sigma}$ is the Fisher information

$$\boldsymbol{\Sigma} = \frac{1}{\sigma^2} \begin{pmatrix} \sum_i \phi_1(x_i)^2 & \sum_i \phi_1(x_i)\phi_2(x_i) \\ \sum_i \phi_1(x_i)\phi_2(x_i) & \sum_i \phi_2(x_i)^2 \end{pmatrix} \quad (4)$$

The minimum variability means that for any un-biased estimator with the covariance matrix $\boldsymbol{\Omega}$ of the estimation, $\boldsymbol{\Omega} \geq \boldsymbol{\Sigma}^{-1}$, according to the Cramér-Rao bound. The above distribution quantifies the range and the corresponding probability where the inferred result will fall when a noisy stimulus is presented. Intuitively, the broader the distribution, the larger the fluctuation in the estimation can be expected. High variability also implies the inferred results for two independent samplings of the same stimulus can be dramatically different.

The marginal distributions of (4) quantify the response variability of individual components. They are calculated to be

$$P(a_i) = \frac{1}{Z_i} \exp\left\{-\frac{1}{2\tau_i^2} (a_i - a_i^0)^2\right\}, \quad \text{for } i = 1, 2, \quad (5)$$

with τ_1^2 (similarly for τ_2^2) given by

$$\tau_1^2 = \frac{\sigma^2 \sum_i \phi_2(x_i)^2}{\sum_i \phi_1(x_i)^2 \sum_i \phi_2(x_i)^2 - (\sum_i \phi_1(x_i)\phi_2(x_i))^2}. \quad (6)$$

From eq.(4) and (6), we see that the overlap between the basis functions, measured by $OP = \sum_x \phi_1(x)\phi_2(x) / \sqrt{\sum_x \phi_1(x)^2 \sum_x \phi_2(x)^2}$, plays a crucial role in determining the magnitude of the inferential sensitivity. For a fixed noise level, the larger the overlap, the higher the variability is. The value of variability can even be arbitrarily large depending on the amount of overlap (Fig.2b). This overlapping effect is also intuitively understandable. In the overlapping region, the contributions from different basis functions on the generation of data are obscured. As a result data points in this region are less informative in ‘indistinguishing’ the activities of neurons. More overlap implies more ambiguity.

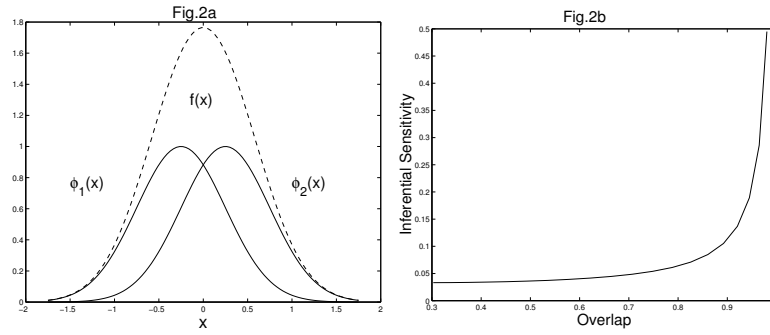


Fig. 2. An illustration of a simple model study. (a) The stimulus and the basis functions; (b) Variability vs. Overlap. The noise level is fixed.

3 The Neural Response Variability

The above observation on the inferential sensitivity turns out to have an important implication on neural encoding. Typically, stimuli, such as natural images, have complicated global structures that cover a large portion of the input space. In order to infer such those features from noisy data, information from different sites need to be integrated. This computationally requires receptive fields of neurons to be largely overlapping as is evident in experiment. From the opinion of the function approximation, this overlapping is essential for accurately interpolating function values between basis centers. Thus, neural responses are inevitably sensitive to input noise. In biological systems, noise is ubiquitous, e.g., it might come from the fluctuation of external stimulus, be induced during the transfer of neural signals or generated by the network dynamics itself. These noises, even small, can still lead to large variability in neural responses due to the inferential sensitivity.

Simulation Experiment. To confirm the above analysis, we simulate the encoding of natural image. The noisy input is generated by a repeatedly and independently down-sampling fixed natural scene. This down-sampling process mimics the time-varying input stream in real-time computation in neural systems. The inference strategy we use is the sparse coding algorithm (1). In the simulation, five different sampling rates are considered. Here the sampling rate is defined as the percentage of data points used in the inference. At each sampling rate, 100 copies were sampled.

Fig.1 illustrate the effect of inferential sensitivity, which confirms that minor discrepancy in input can lead to dramatic changes at the output. It is worth to point out that we did not calculate the variance-mean ratio to compare with experimental data. This is because in the sparse coding model (1), neural activity can be negative value, which makes the mean extremely small. It is unfair to compare in such case. But consider the mean of the absolute value of neural activities is in the order of (0, 0.2) (not shown here), the variance values in the Table.1 indicate the irregular behaviors.

Table 1

Variance vs. Sampling Rate

Sampling rate	1/4	1/9	1/16	1/25	1/36
Variance	0.0306	0.0496	0.0576	0.0615	0.0627
Mean	0.0004	0.0010	0.0013	0.0006	0.0014

4 Conclusions

In the current study we investigate a source of irregularity in neural response from the perspective of statistical inference. It shows that irregularity can arise due to the inferential sensitivity of neural encoding. More precisely, it is the overlap between neuronal receptive fields accounted for this sensitivity. This overlapping, however, is unavoidable due to the fact that the neural estimator needs to extract global smooth structure of external input. This interpretation is different from the others in the literature, which is more concentrating on dynamical properties of single neurons or networks. All these sources, however, are not necessarily contradictory, actually they supplement with each other and give us a even more complete picture of how neural response variability is generated.

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