Multiplicative Gain Modulation for Linear and Non-linear Inputs

Volker Steuber[1] and R. Angus Silver

Department of Physiology
University College London
Gower Street
London WC1E 6BT
United Kingdom

Summary

Multiplicative gain modulations alter the sensitivity of a neuron to changes in its excitatory inputs. Such gain changes are observed frequently *in vivo* and are thought to play an important role for the processing of information in the brain [9]. Despite the apparent importance of gain modulations, the mechanisms by which they occur are not fully understood. Previous experimental and theoretical work has suggested three different scenarios that can lead to multiplicative gain changes:

(1) Shunting inhibition can reduce the gain under conditions where the variance of the input increases with the level of excitation [6].
(2) Increasing the frequency of balanced inhibitory and excitatory input can decrease the gain of a neuron in response to injected current [2].
(3) Inhibition or excitation alone can cause multiplicative gain changes if the relationship between driving input and subthreshold voltage is non-linear [7,8,3]. This can be caused by synaptic saturation [8] or by a non-linear dependence of input current on a stimulus parameter like contrast or eye position [7].

In all these cases, the presence of noise is essential because it introduces a power law dependence of firing rate f on mean voltage V between spikes [4,5]:

[1] Corresponding author, *E-mail address*: v.steuber@ucl.ac.uk

Preprint submitted to Elsevier Preprint *20 January 2004*

$$f = k\,V^{\alpha} \tag{1}$$

While it is generally agreed that a power law or an exponential relationship between mean voltage and firing rate is necessary for multiplicative gain modulation, it is not yet clear whether a second non-linear mapping between driving input and mean voltage is also required. Previous computer simulations [8] and mathematical analyses [7] have suggested that multiplicative gain modulation only occurs if the mean voltage depends on the driving input in a non-linear way. Murphy and Miller (2003) have shown that if the mean voltage V depends on the driving input p and the modulatory input m as

$$V = d(p) + m \tag{2}$$

the gain, i.e. the slope of the firing rate with respect to the driving input, is given by

$$\frac{\partial f}{\partial p} = k\alpha\,(d(p) + m)^{\alpha-1}\,\frac{\partial d(p)}{\partial p} \tag{3}$$

Combining equations (1), (2) and (3), the gain can be expressed as a function of firing rate f:

$$\frac{\partial f}{\partial p} = k\alpha\,(f/k)^{\alpha/(\alpha-1)}\,\frac{\partial d(p)}{\partial p} \tag{4}$$

Thus, the gain as a function of f can only depend on the modulatory input m if $d(p)$ is non-linear in p so that $\partial d(p)/\partial p$ is a function of m. This has been interpreted as evidence that a linear dependence

$$V = a\,p + m \tag{5}$$

can not result in multiplicative gain modulation. However, if a multiplicative gain change is defined as multiplicative scaling of the firing rate f as in references [9,2]

$$f = F(p, m) = G(p)\,H(m) \tag{6}$$

a change of gain as a function of f ceases to be a necessary criterion for multiplicative gain changes and these can occur for linear inputs p and m. This can easily be seen for an exponential dependence of firing rate on voltage

$$f = k\, e^V = k\, e^{ap}\, e^m \tag{7}$$

where the gain change is clearly multiplicative, yet the gain as a function of f is independent of m:

$$\frac{\partial f}{\partial p} = k\, a\, e^{ap}\, e^m = a\, f \tag{8}$$

Although a purely multiplicative gain change as defined in equation (6) requires an exponential relationship between V and f, we show that the same simplified model also exhibits approximately multiplicative scaling if the exponentiation is replaced by a power law. We then show that multiplicative gain modulation can occur in a leaky integrate-and-fire model

$$C\,\frac{dV}{dt} = -\frac{V - V_{rest}}{R} + I_p + I_m + I_{noise} \tag{9}$$

with driving input current I_p, modulatory input current I_m and Ornstein-Uhlenbeck current noise I_{noise} [10]

$$\frac{dI_{noise}}{dt} = -\frac{I_{noise} - I_0}{\tau} + \sqrt{D}\chi \tag{10}$$

where $D = 2\sigma^2/\tau$ is the noise diffusion coefficient, τ the noise time constant and χ Gaussian white noise with zero mean and unit standard deviation. Furthermore, we investigate the effect of conductance versus current inputs and non-linearities that are caused by rectifying input resistances [1], and relate our results to experimentally determined properties of cerebellar granule cells.

References

[1] L. Cathala, S. Brickley, S. Cull-Candy and M. Farrant, Maturation of EPSCs and intrinsic membrane properties enhances precision at a cerebellar synapse, J. Neurosci. 23 (2003) 6074 - 6085.

[2] F. S. Chance, L. F. Abbott and A. D. Reyes, Gain modulation from background synaptic input, Neuron 35 (2002) 773-782.

[3] B. Doiron, A. Longtin, N. Berman and L. Maler, Subtractive and divisive inhibition: effect of voltage dependent inhibitory conductances and noise, Neural Comp. 13 (2000) 227-248.

[4] D. Hansel and C. van Vreeswijk, How noise contributes to contrast invariance of orientation tuning in cat visual cortex, J. Neurosci. 22 (2002) 5118-5128.

[5] K. D. Miller and T. W. Troyer, Neural noise can explain expansive, power-law nonlinearities in neural response function, J. Neurophysiol. 87 (2001) 653-659.

[6] S. J. Mitchell and R. A. Silver, Shunting inhibition modulates neuronal gain during synaptic excitation, Neuron 38 (2003) 433-445.

[7] B. K. Murphy and K. D. Miller, Multiplicative gain changes are induced by excitation and inhibition alone, J. Neurosci. 23 (2003) 10040-10051.

[8] S. A. Prescott and Y. De Koninck, Gain control of firing rate by shunting inhibition: roles of synaptic noise and dendritic saturation, Proc. Natl. Acad. Sci USA 100 (2003) 2076-2081.

[9] E. Salinas and P. Thier, Gain modulation: a major computational principle of the central nervous system, Neuron 27 (2000) 15-21.

[10] G. E. Uhlenbeck and L. S. Ornstein, On the theory of Brownian motion, Phys. Rev. E 36 (1930) 823-841.