Dynamics of human sleep EEG

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Abstract

Several investigators of EEG time series reported a rejection of the null hypothesis of a linear stochastic system for epochs longer than 10s. We investigate whether this rejection of the null hypothesis caused by non-linearity or by non-stationarity. Our approach is a combination of autoregressive modelling and surrogate data testing. It is shown that the fraction of subsegments, for which the null hypothesis has to be rejected increases with the length of the subsegments and is related to the fluctuations of the parameters.

Key words: autoregressive modelling, human sleep EEG, non-linearity, non-stationarity

1 Introduction

During the last decades many attempts were made to apply methods from nonlinear dynamics to analyse EEG time series. It turned out, however, that the assumption that the EEG can be described by a deterministic chaotic system did not hold. Today most researchers describe the EEG as a stochastic process. It is, however, still an open question to which extent one has to consider nonlinear effects.
If the EEG is given as a time series $x(t), x(t+\Delta t), \ldots, x(t+N\Delta t)$ a very general ansatz for a discrete time model is given by

$$x(t+\Delta t) = f(x(t), x(t-\Delta t), \ldots, x(t-m\Delta t), \vec{p}, \vec{\xi}) \ ,$$

where $\vec{p}$ is a parameter vector, and $\vec{\xi}$ represents a normal-distributed white noise term. The system is considered as non-stationary, if the parameters depend on time and non-linear, if the function f is non-linear. Note that this distinction is not unique. In certain cases it might be possible to transform

a linear model with time dependent parameters into a stationary non-linear model by increasing m (see also [1]).
For time series of human sleep EEG, the surrogate data analysis produced evidences for the rejection of the null hypothesis of a linear stochastic process (see e.g. [2,3]). These analyses were performed on segments with a length larger than 10s (16s in [3] and $20.5, 41, 82$ and 164s in [2]). Because the null hypothesis includes also stationarity, its rejection can be both due to non-linearity and non-stationarity and the test cannot distinguish between these two possibilities.
We approach the relation between non-linearity and non-stationarity by modelling the dynamics on short time scales by autoregressive models. It was shown in [4] that autoregressive modelling of short segments up to a duration of $1s$ can catch essential features of human sleep EEG.
By performing a surrogate data analysis, first a test is performed on the hypothesis, that $1s$ segments of EEG time series were generated by linear stochastic processes, i.e. in our case by an autoregressive model. If the test would reject this hypothesis, non-linearity and non-stationarity has to be taken into account already on this time scale. In a second step the segment length is increased and the surrogate data analysis is repeated. If the rejection of the null hypothesis of a linear stochastic process for the long segments is due to non-stationarity, its probability will increase with the length of the subsegments. We expect that the significance of the rejection of the null hypothesis is related to the fluctuations of the parameters of the autoregressive models in the subsegments.

2 Data and Methods

The sleep EEG of 4 healthy male subjects were recorded during 8 h in the sleep laboratory. The EEG signals from C_3-A_2-derivation were conditioned by the following analog filters: a high-pass filter (-3 dB at 0.16 Hz), a low-pass filter (-3 dB at 102 Hz, <-40 dB at 256 Hz), and a notch filter (50 Hz). The data were sampled with a frequency of 512 Hz, digitally filtered (low-pass FIR filter, -3 dB at 49 Hz), and stored on a PC with a resolution of 128 Hz. Sleep stages were scored visually according to the criteria of Rechtschaffen and Kales [5]. 361 segments with a duration of 1 min (7680 points) were selected for REM sleep, sleep stage 2 and slow-wave sleep (SWS) (sleep stage 3 and 4) after visual inspection for lack of obvious non-stationarity due to sleep stage transitions and artefacts. The segments from sleep stage 2 we split in two classes: Class 2A consists of segments with sharp waves larger than $-100\mu V$ which are mostly vertex waves or K-complexes, while the second class, 2B, comprises segments with small or no spikes.

The segments were divided into subsegments with length ranging from $T = 1-30s$ and autoregressive(AR) models were estimated using Burgs algorithm. To avoid over-fitting on the short segments the model order was fixed to six . For every subsegment 100 surrogate data segments were produced by iterating the model with randomly shuffled residuals. The model was tested by estimating the correlation sum both for the original and the surrogate data. The correlation sum is given by

$$C(r) = \frac{2}{N \cdot (N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(r - ||\vec{X}_i - \vec{X}_j||) \; .$$

$\vec{X}_i = (x(i), x(i-\tau), \ldots, x(i-(m-1)\tau))$ are the state vectors and $||\cdots||$ denotes the maximum norm. The delay time τ was chosen as $\tau = 3\Delta t$ with Δt the sampling time. The embedding dimension was fixed to $m = 3$. The length scale r was set to $r = 0.5\sigma_x$, whereas σ_x is the standard deviation of the subsegment. Then the quantity

$$S = \frac{\ln C_{orig}(r) - \langle \ln C_{surr}(r) \rangle}{\sigma_{surr}}$$

is used as test statistic, where σ_{surr} denotes the standard deviation of $\ln C(r)$ of the surrogates. Because the distribution of this statistic is not Gaussian, the significance levels were numerically estimated by using a set of realisations of an AR-model, with coefficients typical for EEG time series.

3 Results

Firstly we tested subsegments of length $T = 1s$. The fraction of original segments for which the null hypothesis was rejected, is only slightly larger than for a set of time series produced by the typical AR-model. The accurate numbers for the four classes are given in the first line of table 1.
In particular neither sharp waves nor sleep spindles or slow waves are leading to a significant rejection of the null hypothesis given by the autoregressive model. However, their occurrence can be detected in the coefficients of the autoregressive models of the corresponding subsegments(cf. also [4]).
If the length of the subsegments is increased, the fraction of rejected segments increases dramatically and reaches values between 70% and 100% for the whole $60s$ segments.

The first reason for this increase is that the expectation value of the standard deviation of the estimate of $\ln C$ for the surrogates σ_{surr} decreases approxi-

segment length T [s]	REM	Stage 2A	Stage 2B	SWS
1	2.9%	4.3%	3.3%	2.2%
2	5.4%	13.1%	8.0%	3.7%
4	8.6%	31.6 %	20.5%	7.2%
10	24.3%	73.6%	58.3%	26.1%
30	54.4%	98.2%	94.8 %	71.7%
60	73.7%	100%	100%	86.9%

Table 1
Fraction of subsegments for which the null hypothesis is rejected ($p < 0.01$).

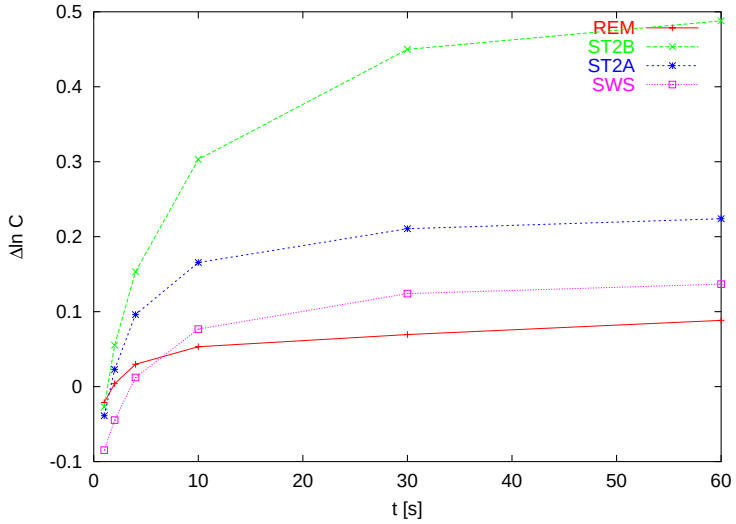

Fig. 1. Expectation values of $\ln C - \langle \ln C \rangle_{surr}$ for different segment lengths.

mately proportional to $T^{-1/2}$. This phenomenon can be observed also for surrogate data tests in nonlinear deterministic systems. A second possible reason for this increase is that the expectation value of the difference $\ln C - \langle \ln C \rangle_{surr}$ itself becomes larger, what can be interpreted as an effect of non-stationarity, because for a sufficiently large sampled stationary system it is independent of the length of the segment. Fig. 1 shows the averaged behaviour of these differences for the four groups depending on the length of the subsegments.
It is clearly seen that the strongest increase occurs for the two classes from sleep stage 2, which can be related to the occurrence of sharp waves and sleep spindles. However, also in slow wave sleep and REM sleep significant non-stationarity is detected.

The analysis of the fluctuation of the parameters of the AR-models supports this findings. Fig. 2 shows the standard deviations for the fluctuation of the parameters in the segments - again the fluctuations are much larger in sleep stage 2 than in SWS and REM sleep.

The transitions between different sleep stages are also reflected in the dy-

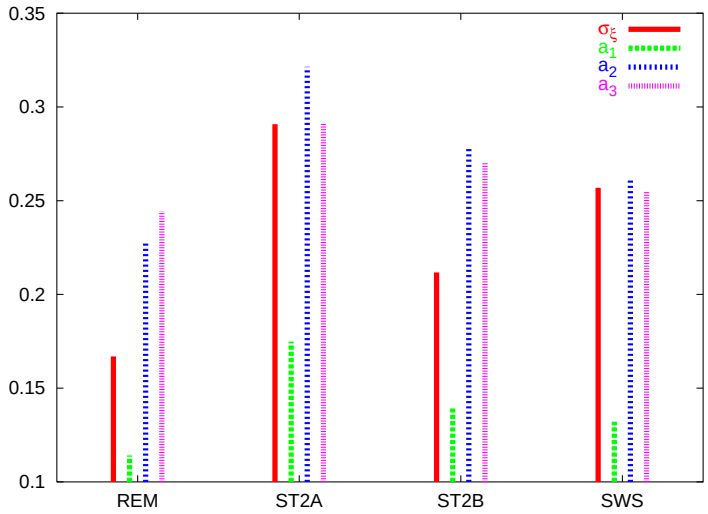

Fig. 2. Standard deviation of the distribution in the 60s segments of four parameters (the standard deviation of the residuals σ_ξ and the three first AR-coefficients) estimated in the 1s subsegments.

namics of the AR-coefficients. As already shown by several authors, e.g. [4,6], this effect can be used for automatic sleep stage detection algorithms. Note that the non-stationarities discussed above applied to time scales in the second range, non-stationarities related to sleep stage transitions correspond to time scales of minutes.
Moreover, the dynamics of the AR-coefficients is also related to the time course of spectral power, which is widely used for the characterisation of sleep dynamics. However, our analysis shows, that the most pronounced changes in the parameters of the autoregressive models cannot be directly related to transitions between sleep stages. Although some transitions occured synchronously we also encountered sleep stage transitions without major changes in the parameters and steep transitions in the parameter space without corresponding sleep stage transitions. See Fig. 3 as an example.

4 Conclusion

Our results support the hypothesis that the nonlinear signatures found in segments of human sleep EEG with duration longer than 10s is caused rather by parameter fluctuations than by static nonlinearities. These parameter fluctuations render the EEG segments non-stationary. However, this does not exclude the possibility that the fluctuations themselves might be described by at least approximately stationary processes on time windows of a duration of a few minutes between the stage transitions.
This would lead to the picture of a hierarchy of dynamical processes and time scales, in which the non-stationary parameters on shorter times scales are the

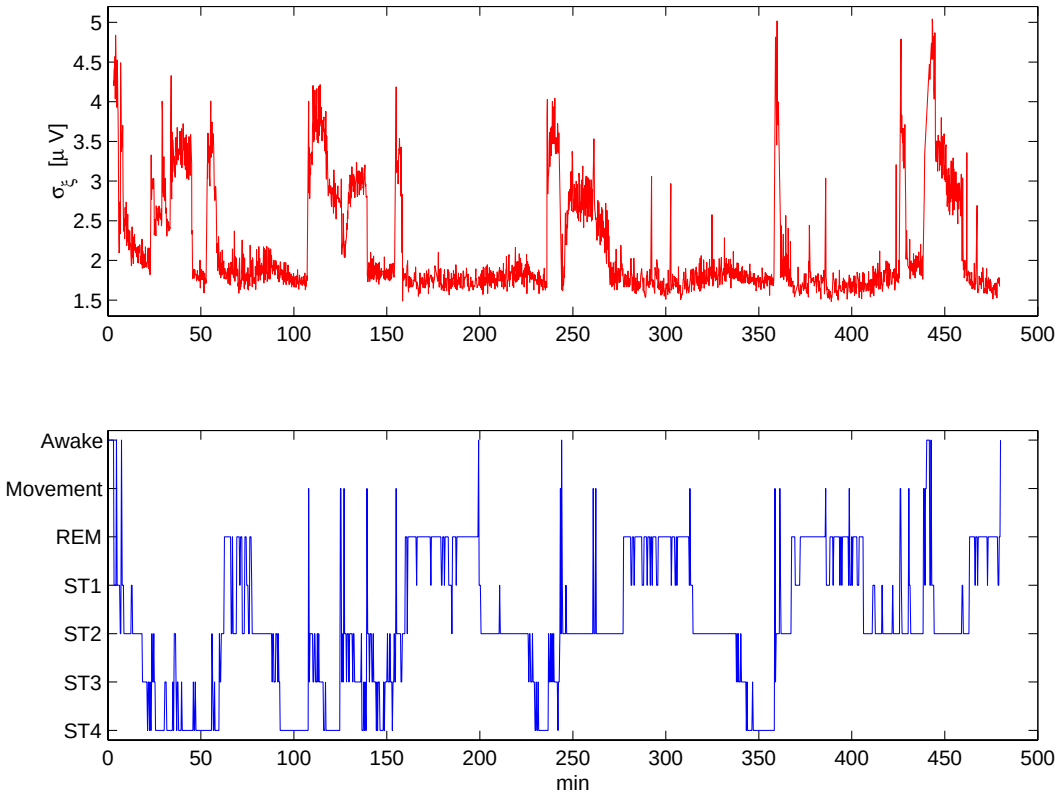

Fig. 3. Standard deviation σ_ξ of the residuals of the AR-models for non-overlapping $T = 10s$ segments and sleep structure according to Rechtschaffen/Kales criteria scored for $T = 20s$ segments(lower panel). In the upper panel all segments scored as awake, movement or with artefacts were removed.

state variables of processes on longer time scales which are governed again by non-stationary parameters. The present analysis was restricted to only two time scales: the time scale of the typical patterns of sleep EEG, i.e. spindles, K-complexes and slow waves and next longer time scale which we related to the fluctuations of the parameters describing the dynamics on the shorter time scale.
Moreover, we showed that this approach allows also to study the dynamics on the time scale of the transitions between sleep stages and may reveal transitions which are not included in the traditional concept of sleep stages.

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