

# Subthreshold cross-correlations between cortical neurons: A reference model with static synapses

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## Abstract

The structure of cross-correlations between subthreshold potentials of neo-cortical neurons was recently examined. Characteristic features included broad widths and significant peak advances. It was suggested that dynamic synapses shape these cross-correlations. Here a reference model is developed comprising leaky integrators with static synapses. The forms of the subthreshold correlations are derived analytically for two different forms of synaptic input: steady drive and populations bursts. For the latter case the model captures the widths seen in experiment. However, the model could not account for the peak advance. It is concluded that models with static synapses lack the necessary biological details for describing cortical dynamics.

*Keywords:* Neocortex, Microcircuit, Correlations, Subthreshold

## 1. Introduction

Exploring correlated activity between neurons is essential for the understanding of how information is processed in cortex. A recent experimental study examined correlations of subthreshold voltage due to the network activity in an active cortical slice [5]. The subthreshold voltage responds to the activity of many thousands of presynaptic cells and contains more information than the spike trains [3, 6, 7], which have been the subject of previous correlation studies (see [8] for an information-theoretic approach). In the slice experiment [5] the cortical network was in one of two states, either a steady-firing state or a state in which the network had intermittent bursts of activity. In each experiment subthreshold activity was measured from two neurons. The experimental paradigm involved the injection of a hyperpolarizing current bringing

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the neurons near to the reversal potential for inhibition, thereby keeping the neurons in the subthreshold regime and isolating the excitatory input. For the case where the network was in the steady firing state the cross-correlation was sharply peaked and centered near zero. However, for the case of population bursts the characteristic features of the cross-correlation were broad widths ( $100 \rightarrow 500\text{ms}$  for the case of population bursts) and a significant advance to the peak ( $50 \sim 100\text{ms}$  between pyramidal and certain interneuron types).

Here it is examined if a simple model comprising a pair of leaky integrators with static synapses is sufficient to capture these basic experimental features. The paper closes with an examination of the descriptive scope of models with static synapses and of possible extensions that might allow for a better agreement with experimental observations.

## 2. The model

Two leaky-integrator neurons (having no spike mechanism) with membrane potentials  $V_1(t)$  and  $V_2(t)$ , receive input from a common and separate pool of presynaptic neurons. Only excitatory post-synaptic potentials (EPSPs) are considered, consistent with the experimental procedure described above. (Synaptic conductance changes will be ignored. However it will be shown later that our conclusions are not affected by this approximation.) The dynamics of the subthreshold potential, subject to a synaptic input  $I_{syn}$ , obey:

$$\tau_m \dot{V} = -V + RI_{syn}(t) \quad \text{where} \quad I_{syn}(t) = \frac{q}{\tau_f} \sum_{\{t_k\}} \Theta(t - t_k) \exp\left(-\frac{t - t_k}{\tau_f}\right).$$

The quantities introduced are;  $\tau_m$  the membrane time constant,  $R$  the neuronal input resistance,  $\tau_f$  the falling time constant,  $q$  is the total amount of charge in a synaptic current pulse and  $\{t_k\}$  the set of arrival times for all synaptic inputs. These parameters will be later subscripted by the labels 1, 2 for each of the neurons. The subthreshold potential above is measured from the reversal potential of the inhibitory drive (near  $-70\text{mV}$ ) and can be written as a summation of EPSPs  $\mathcal{E}(t)$

$$V(t) = \int_0^\infty ds \mathcal{E}(s) \sum_{\{t_k\}} \delta(s - (t - t_k)) \quad \text{with} \quad \mathcal{E}(t) = qR \frac{(e^{-\frac{t}{\tau_m}} - e^{-\frac{t}{\tau_f}})}{(\tau_m - \tau_f)} \Theta(t) \quad (1)$$

## 3. The cross-correlation function

Throughout this paper, the cross-correlation function is defined as follows

$$\mathcal{C}(\Delta) = \langle V_1(t)V_2(t + \Delta) \rangle - \langle V_1 \rangle \langle V_2 \rangle \quad (2)$$

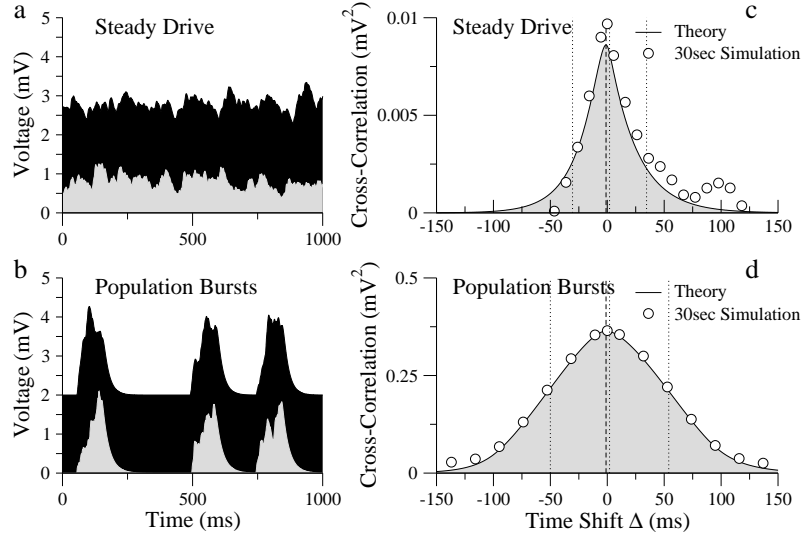


Figure 1: Results from numerical simulations. (LEFT) Voltage traces for the neuronal pair (neuron 1 is black and offset by 2mV, neuron 2 is grey) for the cases of steady Poissonian (a) and burst (b) synaptic drive. (RIGHT) The corresponding cross-correlations. The parameters used are  $\{\tau_{m1}=20, \tau_{f1}=5, \tau_{m2}=25, \tau_{f2}=2\}$ ms and  $q_1 R_1=q_2 R_2=3$ , for both cases. For the steady input the common and total rates were 50Hz and 200Hz respectively. For the population burst  $r_{cB} = 100\text{Hz}$ ,  $r_{sB} = 400\text{Hz}$ ,  $T_B = 100\text{ms}$  and  $T_{IBI} = 500\text{ms}$ . The peak (Eq. 5) of the cross correlation is  $-1.0\text{ms}$  (dashed line), and the mean is  $2\text{ms}$  (central dotted line). Neither of these are of the same magnitude as the experimentally observed peak shifts. Widths (defined as twice the standard deviation, shown by the flanking dotted lines) Poissonian drive (c) 64ms, population bursts (d) 104ms. The latter falls into the range seen in experiment.

where it is stressed that the amplitude of  $\mathcal{C}(\Delta)$  is *not* normalized. Inserting the form of the voltage given in equation (1) yields:

$$\mathcal{C}(\Delta) = \int_0^\infty \int_0^\infty ds_1 ds_2 \mathcal{E}_1(s_1) \mathcal{E}_2(s_2) \left[ r_c \delta(s_1 - s_2 + \Delta) + r_0 \rho(s_1 - s_2 + \Delta) - r_0^2 \right] \quad (3)$$

where  $r_c$  is the average common rate and  $r_0$  is the average total rate. The quantity  $\rho(t)$  is the cross-conditional probability of finding a pulse at a time  $t$  in the input of one neuron given a pulse at  $t=0$  in the input of the other.

**Steady synaptic drive.** For Poissonian input the cross-conditional density is constant  $\rho_S(t) = r_0$  giving the cross-correlation function for  $\Delta > 0$  as

$$\mathcal{C}_S(\Delta) = r_c \int_0^\infty ds \mathcal{E}_1(s) \mathcal{E}_2(s + \Delta) = r_c q_1 q_2 R_1 R_2 \left( M_{12} e^{-\Delta/m_2} - F_{12} e^{-\Delta/f_2} \right) \quad (4)$$

$$M_{12} = \frac{1}{(m_2 - f_2)} \frac{m_2^2}{(m_1 + m_2)(m_2 + f_1)}, \quad F_{12} = \frac{1}{(m_2 - f_2)} \frac{f_2^2}{(f_1 + f_2)(m_1 + f_2)}.$$

The notation has been lightened by writing  $m_1$  for  $\tau_{m_1}$  etc. The cross-correlation function for  $\Delta < 0$  takes a similar form, but with the indices and sign of  $\Delta$  reversed, i.e.  $X_{12}(\Delta)$  is replaced by  $X_{21}(-\Delta)$ .

*The peak and mean advance.* The time difference  $\Delta_S^*$  at which the cross-correlation function peaks has the same sign as  $m_2 f_2 - m_1 f_1$ . When  $\Delta_S^* > 0$

$$\Delta_S^* = -\frac{m_2 f_2}{(m_2 - f_2)} \log \left( \frac{m_2}{f_2} \frac{(f_1 + f_2)(m_1 + f_2)}{(m_1 + m_2)(m_2 + f_1)} \right) \quad V_1 \text{ leads } V_2 \quad (5)$$

The advance can also be measured in terms of the moments of  $\mathcal{C}_S(\Delta)$ . The mean advance is  $\overline{\Delta}_S = (f_2 + m_2) - (f_1 + m_1)$  and of the order of the membrane and synaptic time constants ( $\sim 10$ ms). This is an order of magnitude less than that seen in experiment: the model cannot reproduce the experimental results. An example is given in Fig. 1 (panel c) where the peak is negative and the mean positive, but both located close to zero.

*The width.* This is obtained from the moments of the cross-correlation:

$$\overline{(\Delta - \overline{\Delta}_S)^2} = m_1^2 + f_1^2 + m_2^2 + f_2^2 \quad (6)$$

Hence the width, measured as the square root of this quantity, is less than twice the longest membrane or synaptic time-scale ( $\sim 10$ s of ms: see Fig. 1 panel c) and again this is less than that seen in experiment.

**Population bursts.** The population bursts are modeled here as a Poissonian distributed box-car function, with the burst frequency  $r_B = r_{Bc} + r_{Bs}$  where  $r_{Bc}$  and  $r_{Bs}$  are the common and separate rates within the burst. The burst length is  $T_B$  and the average interval between the centers of bursts is  $T_{IBI}$ . The low spike rate between bursts is approximated as being equal to zero. The required cross-conditional density  $\rho(t)$  for this type of presynaptic input is

$$\rho_B(t) = \begin{cases} r_B \left(1 - \left|\frac{t}{T_B}\right|\right) + r_0 & 0 < |t| < T_B \\ r_0 & T_B < |t|. \end{cases} \quad (7)$$

The average rate  $r_0$  is a function of the inter-burst interval  $r_0 = r_B T_B / T_{IBI}$  as is the common rate  $r_c = r_{Bc} T_B / T_{IBI}$ . The correlation function can be written in terms of a convolution with  $\mathcal{U}_S = \mathcal{C}_S / r_c$ :

$$\mathcal{C}_B(\Delta) = \mathcal{C}_S(\Delta) + r_B r_0 \int_{-T_B}^{T_B} d\Delta' \mathcal{U}_S(\Delta + \Delta') \left(1 - \frac{|\Delta'|}{T_B}\right). \quad (8)$$

Performing the integration gives the cross correlation for  $\Delta > 0$  as:

$$\begin{aligned}
\frac{\mathcal{C}_B(\Delta)}{r_0 q_1 q_2 R_1 R_2} = & \frac{r_c}{r_0} \left( M_{12} e^{-\Delta/m_2} - F_{12} e^{-\Delta/f_2} \right) \\
& + \frac{2r_B}{T_B} \left( M_{12} m_2^2 e^{-\Delta/m_2} (\cosh(T_B/m_2) - 1) \right. \\
& - \frac{2r_B}{T_B} \left( F_{12} f_2^2 e^{-\Delta/f_2} (\cosh(T_B/f_2) - 1) \right) \\
& + \frac{r_B}{T_B} \Theta(T_B - \Delta) (T_B - \Delta + (m_2 + f_2) - (m_1 + f_1)) \\
& + \frac{r_B}{T_B} \Theta(T_B - \Delta) \left( M_{21} m_1^2 e^{-(T_B - \Delta)/m_1} - F_{21} f_1^2 e^{-(T_B - \Delta)/f_1} \right) \\
& - \frac{r_B}{T_B} \Theta(T_B - \Delta) \left( M_{12} m_2^2 e^{(T_B - \Delta)/m_2} - F_{12} f_2^2 e^{(T_B - \Delta)/f_2} \right) \quad (9)
\end{aligned}$$

The cross-correlation for  $\Delta < 0$  is obtained by reversing all indices  $\{1, 2\} \rightarrow \{2, 1\}$  and the sign of  $\Delta$ .

*The peak and mean advance.* In the convolution equation (8), the width of the burst is much greater than the steady-firing cross correlation function. Hence  $\Delta_B^* \simeq \Delta_S^*$  where the small corrections would come from any skew in  $\mathcal{C}_S$  and it is seen that the burst model also cannot explain the large peak advances seen in experiment. The mean advance  $\overline{\Delta}_B$  is also equivalent to the steady-input result  $\overline{\Delta}_S$  given above. See Fig. 1 panel (d) for an example.

*The width.* The width is obtained approximately by using equation (8):

$$\overline{(\Delta - \overline{\Delta})_B^2} \simeq (m_1^2 + f_1^2 + m_2^2 + f_2^2) + \frac{T_B^2}{6} \quad (10)$$

noting that the common input (first term in Eq. 8) is small compared to the within-burst correlations. The model therefore captures the large widths seen in experiment. This is seen in the example given in Fig. 1 (panel d) in which even relatively short bursts (100ms) give a standard deviation that is almost twice the size of the Poissonian case.

#### 4. Discussion

In this study an analytical approach was developed to examine the neuronal and synaptic mechanisms that underlie features of the cross-correlation seen between cortical neurons [5]. The model comprised a pair of leaky integrator neurons with static synapses subject to steady input or population bursts. We demonstrated that the duration of the population bursts largely determines the width of the cross-correlation (Eq. 10). It was also seen in experiment that the peak of the cross-correlation showed a significant shift from zero ( $50 \sim 100$ ms

for the cases of correlations between pyramidal and certain interneuron types). However, Eq. (5) and the derived formula for the mean cannot produce peak shifts of this order with realistic synaptic kinetics and membrane parameters. It is concluded the model with static synapses and passive membranes is not sufficiently detailed to explain the dynamics of cortical microcircuits.

The model ignored the large synaptic conductance changes seen in neurons embedded in active networks [1]. These effects are well approximated by a shorter membrane time-constant, which would only reinforce our conclusions. Another mechanism ignored in this model is the dynamic nature of synapses [2, 4]. That this mechanism is likely to produce the peak shift is supported by the fact that the largest peak delays were seen between neurons in which one received facilitating and the other depressing synapses. The passive neuron models used here also neglected trans-membrane currents. In particular, the  $I_h$  current is strongly active at hyperpolarized membrane potentials and has the long time-scale necessary for a mechanism that might underlie the peak advance. These mechanisms need to be investigated to analyze which might underlie the observations and thereby better model the dynamics of the correlated activity in active cortical microcircuits, both in the isolated brain slice and *in vivo* [3, 6].

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