

# **FIRING RATE MODULATION BY BACKWARD PROPAGATING POTENTIAL WAVES**

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## **SUMMARY**

### **Objectives**

The paper analyses the effect of backward propagating potential waves on the firing rate input – output transformation of neurons. The analyses is carried out using a relatively simple computational model of neurons.

### **Context**

It is known that when neurons generate a spike at the axon hillock, backward propagating potential waves travel along the soma membrane of the neuron towards the dendrites (Pan & Colbert, 2001). It is also conjectured that such backward propagating potential waves contribute to the regulation of the efficacy of the synaptic transmission (Senn et al., 2001, Stuart & Hausser, 2001). Consequently, it is interesting to study how backward propagating potential waves influence the functionality of the neurons.

### **Method**

A simple multi-compartment numerical neuron model with one dimensional structure was used to carry out the investigation.

The model supposes that the backward propagating potential wave has a significant non- $\text{Na}^+/\text{K}^+$  (e.g.,  $\text{Ca}^{2+}$ ) component, which travels by the mediation of non-voltage-dependent channels. The evolution of this potential wave is described in terms of magnitude and phase components by the equations:

$$\varphi_C(x-1, t+1) = \varphi_C(x, t) \tag{1}$$

$$r_C(x-1, t+1) = r_C(x, t) \cdot e^{-\beta_{ck} \cdot \frac{1}{v_c}} \tag{2}$$

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The initial conditions are given by the equations

$$\varphi_C(x_H, t+1) = \begin{cases} 0 & \text{if } out\_spike = true \\ \varphi_C(x_H, t) + \frac{1}{k_C} & \text{if } out\_spike = false \text{ and } \varphi_C(x_H, t) < \frac{3\pi}{2} \\ \frac{3\pi}{2} & \text{if } out\_spike = false \text{ and } \varphi_C(x_H, t) = \frac{3\pi}{2} \end{cases} \quad (3)$$

$$r_C(x_H, t+1) = \begin{cases} 1 & \text{if } out\_spike = true \\ r_C(x_H, t) \cdot e^{-\beta_C \cdot \frac{1}{v_C}} & \text{if } out\_spike = false \text{ and } \varphi_C(x_H, t) < \frac{3\pi}{2} \\ 0 & \text{if } out\_spike = false \text{ and } \varphi_C(x_H, t) = \frac{3\pi}{2} \end{cases} \quad (4)$$

where  $x_H$  is the axon hillock location, and  $k_C$  and  $v_C$  are the parameters (phase change and spatial speed) of the wave propagation process.

The values of the backward propagating potential wave along the neuron's membrane are given by the equation

$$C(x, t) = r_C(x, t) \cdot \cos(\varphi_C(x, t)) \quad (5)$$

The incoming synaptic excitation generates a forward propagating  $Na^+/K^+$  potential wave that is described by the equations:

$$\varphi_{NK}(x+1, t+1) = \varphi_{NK}(x, t) \quad (6)$$

$$r_{NK}(x+1, t+1) = (r_{NK}(x, t) + \lambda C(x, t)) \cdot e^{-\beta_{NK} \cdot \frac{1}{v_{NK}}} \quad (7)$$

The initial conditions are given by the equations:

$$\varphi_{NK}(0, t+1) = \begin{cases} 0 & \text{if } in\_spike = true \\ \varphi_{NK}(0, t) + \frac{1}{k_{NK}} & \text{if } in\_spike = false \text{ and } \varphi_{NK}(0, t) < \frac{3\pi}{2} \\ \frac{3\pi}{2} & \text{if } in\_spike = false \text{ and } \varphi_{NK}(0, t) = \frac{3\pi}{2} \end{cases} \quad (8)$$

$$r_{NK}(0, t+1) = \begin{cases} 1 & \text{if } in\_spike = true \\ r_{NK}(0, t) \cdot e^{-\beta_{NK} \cdot \frac{1}{v_{NK}}} & \text{if } in\_spike = false \text{ and } \varphi_{NK}(0, t) < \frac{3\pi}{2} \\ 0 & \text{if } in\_spike = false \text{ and } \varphi_{NK}(0, t) = \frac{3\pi}{2} \end{cases} \quad (9)$$

The values of the  $Na^+/K^+$  potential wave along the neuron's membrane are given by the equation

$$NK(x, t) = r_{NK}(x, t) \cdot \cos(\varphi_{NK}(x, t)) \quad (10)$$

The analysis of the effects of the backward propagating potential waves was carried out through numerical simulations, evaluating the effects on the input – output transformation of regular firing rates for several parameter settings.

## **Results**

The analysis shows that the presence of backward propagating potential waves has a strong effect on the firing rate transformation function of the neurons. To evaluate the effects the three ranges of firing rate transformations (exponential, linear and logarithmic; Markram et al., 1998) were considered. The results indicate that for a domain of parameter combinations we get expanded exponential and logarithmic ranges, and shortened linear range. For another domain of parameter combinations we get extended linear range and shortened exponential and logarithmic ranges. The analysis of these parameter domains show, that for appropriate combinations of spatial and phase-change speeds (resulting matching phase) of the two travelling waves, facilitation occurs, increasing the output firing rate, and expanding the exponential and logarithmic ranges at the expense of the linear range. On the other side for different combinations (resulting non-matching phase) depression occurs, decreasing the output firing rate, and expanding the linear transformation range at the expense of the other two.

## **Significance**

The presented work investigates the effects of backward propagating potential waves on the firing rate transformation function of the neurons. The analysis shows how neuro-modulatory substances may act to change this transformation function of a neuron by changing the ion permeability properties of the neuron's cell membrane. The analysis also shows that such neuro-modulatory effects may cause the change from facilitation of firing rate transmission to depression and inversely, allowing extended adaptability and functionality to neural circuits.

## **References**

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