

Correlations in the background activity allow the use of single neuron learning rules in populations

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Abstract

Single neuron learning rules have received a lot of attention in recent years. In the central nervous system of higher animals single neurons rarely matter, and information is likely to be coded by populations of cells. Thus the question arises, if single neuron learning rules remain valid. In this paper we investigate the effect of correlations in the background activity on the information transmission properties of populations of neurons. We show that correlations in the background activity not only decrease information transmission but also immediately reduce the optimal noise level to that of the single neuron. As a consequence of this, single neuron learning rules remain valid in the context of population coding.

Key words: Stochastic Resonance, Population coding, Learning rule

1 Introduction

It is well established that background activity can influence the information transmission properties of cortical neurons [1] as, for example, in a stochastic resonance setting [2]. The term *stochastic resonance* is used for the phenomenon, that in a nonlinear dynamical system noise can improve the information transmission to a certain fraction (see [3] for a review), and has been extensively studied in the context of single neurons [4]. Recently, a learning rule was proposed that allows the neuron to adjust to the optimal noise level [5]. However, in the central nervous system of higher animals single neurons rarely matter, and information is likely to be coded by populations of cells, and the question arises, if single neuron learning rules remain valid.

In a recent studies we have shown, that the optimal noise level of a population of neurons with mutually independent noise sources depends on the

population size and that this dependency becomes weak, if the number of neurons increases [6]. Furthermore, for large populations the optimal noise level becomes almost independent of the number of neurons, which means, in principle, that single learning rules are applicable. But from a biological point of view, it is more reasonable to assume, that the noise between pairs of neurons is correlated in a certain way, because nearby neurons share a certain fraction of their input [7]. In this paper we investigate therefore, how correlations in the noise influence the information transmission properties of populations of neurons. We show, that even for small correlations in the noise, the optimal noise level immediately reduces to that of the single neuron. Which means that single neuron learning rules can still be used by the neurons in the populations to optimize information transmission.

This paper is organized as follows. First we introduce our model which consists of a summing array of parallel leaky integrate-and-fire neurons with common input and correlated noise and we briefly describe how we measured information transmission through the population. In section 3 we present the numerical results and we show, how correlations in the background activity alter information transmission properties of neural population. Section 4, finally, concludes with a brief discussion.

2 The model

2.1 The neural population

We consider a population of N leaky integrate-and-fire neurons as displayed in Fig. 1. The total input to each neuron i is the sum of the input signal I_{stim} and the individual correlated noise input $\frac{dW_i(t)}{dt}$. The entire output of the population is generated by summing over all individual outputs Y_i (called pooling in neurophysiological terminology) in every time step Δt .

The membrane potential V of the leaky integrate-and-fire neuron changes in time according to the following differential equation

$$C_m \frac{dV(t)}{dt} = -g_L(V(t) - E_L) + I_{stim}(t) + \sigma \frac{dW_i(t)}{dt}, \quad (1)$$

where $C_m = 0.5 \text{ nF}$ is the membrane capacitance, $g_L = 25 \text{ nS}$ the leak conductance of the membrane, $E_L = -74 \text{ mV}$ the reversal potential, I_{bias} a constant bias current, and $I_{stim}(t)$ an aperiodic Gaussian input stimulus, which is generated by a Fourier transform of a band-limited white noise power spectrum. Once the membrane potential reaches the threshold $V_{th} = -54 \text{ mV}$, a spike is immediately generated and the membrane potential is clamped to a reset value $V_{reset} = -60 \text{ mV}$ for an absolute refractory period $T_{ref} = 1.8 \text{ ms}$. Throughout

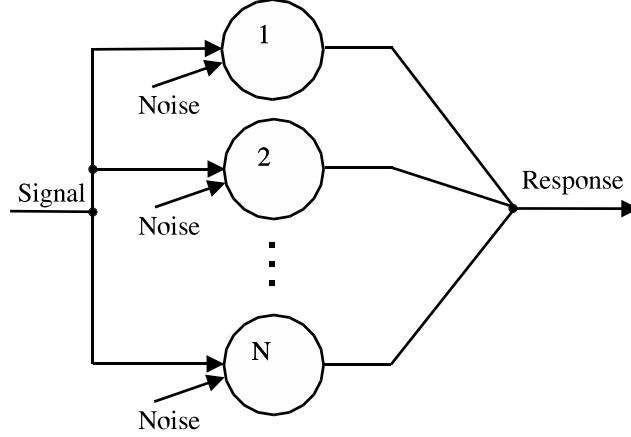


Fig. 1. A population of leaky integrate-and-fire neurons. Each neuron receives the same input signal and correlated Gaussian noise with zero mean and correlation matrix C . The entire output of the population is generated by summing over all individual outputs Y_i in every time step Δt .

this paper we used a bias current of $I_{bias} = 0.3 \text{ nA}$ and the standard deviation of the stimulus was equal to 0.05 nA . The threshold is located at $I_{bias} = 0.5 \text{ nA}$. The background activity is modeled by Wiener processes. The noise process $W_i(t)$ of each neuron is generated from N independent Wiener processes, which we multiply by a matrix A , so that the resulting noise processes $W_i(t)$ are correlated with a given correlation matrix C . The matrix A can be obtained from C with a method called cholesky decomposition. To the end of the paper we used a correlation matrix C , where the correlation factor c between any pair of neurons is the same.

$$C = \begin{pmatrix} 1 & c & c & \cdots & c \\ c & 1 & c & \cdots & c \\ c & c & 1 & \cdots & c \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c & c & c & \cdots & 1 \end{pmatrix} \quad (2)$$

2.2 Measuring the information transmission

We calculate the information transmitted through the population with a method recently described in literature [8]. Starting from the observed spike train, we calculate an estimate of the input signal by convolving the spike train with a filter h , so that the mean square error

$$\epsilon^2(h) = \langle |I_{stim}(t) - h * z(t)|^2 \rangle \quad (3)$$

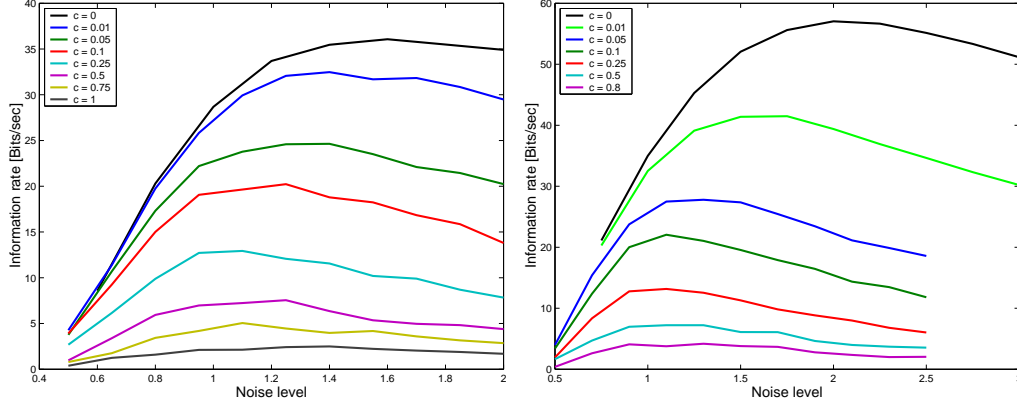


Fig. 2. a) displays the information rate of a population with $N = 100$ neurons. Different curves show different levels of correlations between pairs of neurons. From low correlations (top) to correlations about one (bottom). b) The same as in a) but for a population with $N = 1000$ neurons. The information rate was calculated from 409 samples of approximately 1 sec duration ($\Delta t = 0.1$ ms).

between the stimulus $I_{stim}(t)$ and its estimate $I_{est}(t) = h * z(t)$ is minimized. The non causal optimal linear filter h is calculated with a method called *Wiener-Kolmogorov filtering*. After that it is possible to obtain the information rate R_{info} with standard information theoretic methods as follows:

$$R_{info} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \log_2 \left[1 + \frac{S(\omega)}{N_{eff}(\omega)} \right]. \quad (4)$$

The power spectrum of the stimulus $S(\omega)$ and the power spectrum of the effective noise level in the estimate $N_{eff}(\omega)$ are obtained from segments of approximately one second, which are extracted from the stimulus and its estimate. We correct for systematic errors by introducing a frequency dependent gain $g(\omega)$, which we determine together with the noise components $n_{eff}(\omega)$ from the equation $I_{est}(\omega) = g(\omega)(I_{stim}(\omega) - n_{eff}(\omega))$ using linear regression. For further details see Rieke et al. [8].

3 Simulation results

Fig. 2a shows the information rate plotted against the noise level for a population of 100 neurons. The different curves display different degrees of correlation in the background activity. The top curve indicates the case of independent noise. Introducing correlations between pairs of neurons as described in chapter 2.1, we find a substantial decrease of the information rate even for small correlations in the noise. In addition, we find that not only the maximum of the information rate curve is reduced, but also the optimal noise level de-

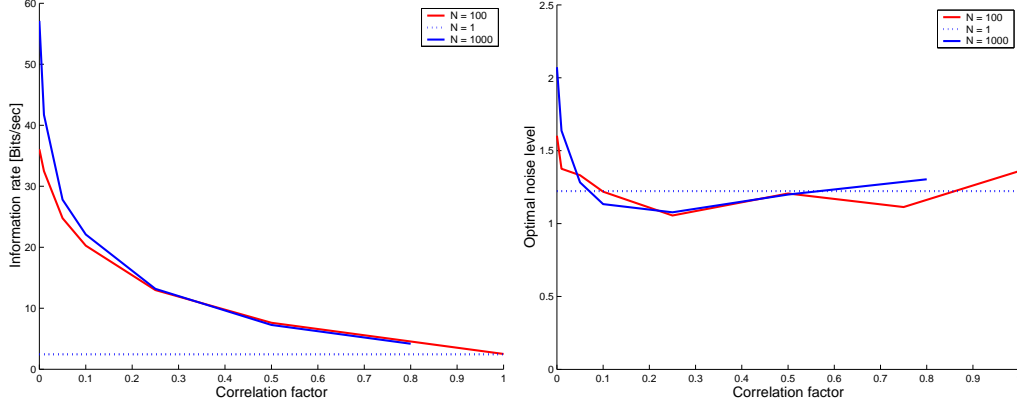


Fig. 3. a) The maximum of the information rate plotted against the correlation factor. b) The optimal noise level of the population plotted against the correlation factor. Legend: $N = 1000$: upper solid curve, $N = 100$: lower solid curve, single neuron: dotted line. Extracted from the previous figure.

creases with increasing correlation factors. Fig. 2b shows the same context for a population with 1000 neurons.

The role of the noise in this system is twofold. On the one hand it allows the transmission of sub-threshold signals but on the other hand it disturbs the information transmission. At the optimal noise level these two effects balance. An increase in population size allows the neuron to operate at a higher noise level, because the induced reliability due to the higher noise can be controlled by the population by pooling. In the case of correlated noise a smaller fraction of the noise can be averaged out with pooling, which leads to the decrease in information transmission and to a smaller optimal noise level. In the extreme case of a correlation factor $c = 1$, all noise processes are identical, which means that every neuron in the population gets the same input and will produce therefore the same output. In this case the information transmission curve of the population is the same as that one of the single neuron. Thus, the optimal noise level and the maximum of the information rate, reduce for increasing correlation factors to the corresponding single neuron values (dotted line), as shown in Fig. 2a and 2b, respectively. Remarkably, the optimal noise level decreases faster to the value of the single neuron than the maximum of the information rate. Even a small correlation factor of $c = 0.1$ leads to an optimal noise level of equal to that of the single neuron, meanwhile the maximum of the information rate is still much higher. Hence, such a regime allows neurons in the population to use single neuron learning rules to optimize the overall information transmission.

4 Discussion

We showed that even small correlations in the background activity alter the information transmission through populations dramatically. In the extreme case of identical noise inputs the information rate reduces to that of the single neuron. Interestingly, the optimal noise level decreases much faster to that of the single neuron than the maximum of information rate does. Although correlations in the background activity reduce the maximum of the information rate, a population of neurons can still benefit from this correlations in that sense that the neurons within the population can use single neuron learning rules to optimize the information transmission.

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