

Spatiotemporal behavior in networks of Ca³ region in the hippocampus

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Abstract

A network of coupled neural oscillators can present different types of spatiotemporal behaviors depending on the coupling coefficients. However, small perturbations in the system could modify the expected behavior of the coupled system. We study the effect of time-correlated stochastic perturbations on the neural network activity when they are applied to the external current. Depending on the diffusion coefficient and the time correlation of the applied noise, an aperiodic synchronization among the cells of the network is achieved for a specific value of the noise intensity.

Key words: neural networks, pattern formation, synchronization, stochastic forcing

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1 Introduction

The emergence of spatiotemporal patterns like spirals, traveling waves or expanding rings, in assemblies of elements with nonperiodic behavior is no new in physics, biology or chemistry and now, this phenomenon has attracted the attention of the neuroscience field, moreover after observing these patterns in the cortex of animals and humans [6].

In this paper, we will analyze the patterns generated by an assembly of excitatory-inhibitory neurons diffusively coupled through the study of the emergence of synchronization associated to the different patterns. The phenomenon of synchronization is a common fact between living organisms and representative of their behavior. An example is the synchronization with prevalence of the gamma frequency range in the olfactory bulb and entorhinal cortex of various species, where these phenomena have been related to the integration of odor information [2]. Similar evidence is available for the sensorial and motor systems where neural synchronization has been discovered in monkeys and in humans [7]. From this point of view is interesting to understand the mechanism of synchronization in neural networks and to know the role of the different factors and parameters involved in this phenomenon [5].

On the other hand, the presence of noise in the neural system due to faults in synaptic coupling, perturbations in the ion channels of membrane potential, thermal fluctuations, etc., also plays an important role in the response of the neural systems [4], the most clear example is the stochastic resonance phenomenon observed in the sensorial system of the crayfish which allow them to detect the movement of the predators in turbulent streams [1]. We will focus

our studies in the analysis of the effects of time correlated gaussian noise on synchronization of a neural network model with excitatory and inhibitory neurons.

2 Model and analyzing tools

To characterize the individual cells or oscillators of the network, we use the recently introduced Morris-Lecar three-variable ordinary differential model [8]. The variables in this model correspond to membrane potentials for prototypical pyramidal cells (or excitatory cells) and inhibitory interneurons in order to mimic the CA3 region of the hippocampus. We are interested in this area because it is considered an important brain activity focus involved in spatial learning, where the pyramidal cells are neurons that carry information to "higher" areas, and in the epileptic seizures [8,11,6].

The system is described by the following set of coupled differential equations:

$$\begin{aligned}
\frac{dV_{ij}}{dt} &= g_{Ca}m_{\infty}(V_{ij} - V^{Ca}) - g_KW_{ij}(V_{ij} - V_{ij}^K) - \\
&\quad g_L(V_{ij} - V^L) + I - \alpha_{inh}Z_{ij} + coupling; \\
\frac{dW_{ij}}{dt} &= \frac{\phi(w_{\infty} - W_{ij})}{\tau_w}; \\
\frac{dZ_{ij}}{dt} &= b(cI + \alpha_{exc}V_{ij}),
\end{aligned} \tag{1}$$

with $m_{\infty}, w_{\infty}, \alpha_{exc}, \alpha_{inh}, \tau_w$ given in [8,9]. V_{ij} and Z_{ij} are the membrane potentials of the pyramidal and inhibitory cells, respectively, while W_{ij} is a relaxation factor which is the fraction of open potassium channels in the population of pyramidal cells, for the node $i, j = 1, \dots, N_x, N_y$, respectively ($N_x \times N_y$ number of cells). The third equation corresponds to the effect of inhibitory

neurons, while the first two equations, without the last two terms in the first equation, correspond to the classical Morris-Lecar model [10]. A more detailed description of all the parameters concerning the Morris-Lecar model, Eqs. (1), can be found in [8–10].

In the network each one of these individual oscillators is connected to their four nearest neighbours through its excitatory cells with *coupling*.

$$coupling = \frac{D}{4}(V_{i-1,j}^{t-delay} + V_{i+1,j}^{t-delay} + V_{i,j-1}^{t-delay} + V_{i,j+1}^{t-delay} - 4V_{i,j}^t) \quad (2)$$

Note that this coupling introduces a small time delay besides the diffusive coupling. The introduction of this delay is really important because the transmission of the signal along the axon needs some time in such a way neurons receive the signal after a short period of time after its emission from other neurons. This delay introduces a characteristic time scale into the model. The network was numerically integrated using an explicit Euler method with a time step of 0.01 time units (t.u.) per iteration, considering fixed boundary conditions and $delay = 1$ time step. Random initial conditions for all the variables were assigned to each cell in the network and with the chosen parameters each oscillator presents a chaotic bursting behavior. An important characteristic of the network is to know its synchronization level when the diffusion coefficient is changed and when small perturbations or modifications are introduced in the system. In this sense, to characterize the *degree of synchronization* among cells, we have calculated the phase synchronization among the cells of the network, this implies to determine the phase for each cell using spherical coordinates transform. In simple subsystems with three variables, we can define the phase making a transform of the coordinate system to the spherical coordinates. Under this transformation, phase ϕ appears in a natural way. Then,

in order to characterize the degree of synchronization among cells of the two dimensional array, we introduced the time-averaged quantity K

$$K = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \left[\frac{1}{(N-2)^2} \sum_{i,j=2}^{N-1} \frac{1}{4} (\|\phi_{i-1,j}(t) - \phi_{i,j}(t)\| + \|\phi_{i+1,j}(t) - \phi_{i,j}(t)\| + \|\phi_{i,j-1}(t) - \phi_{i,j}(t)\| + \|\phi_{i,j+1}(t) - \phi_{i,j}(t)\|) \right] \quad (3)$$

where $\phi_{i,j} = \arg(V_{ij}, W_{ij})$ and $\|\cdot\|$ represents the Euclidean distance. This function is positive definite and vanishes when all the cells in the network are globally synchronized.

As we mention in the Introduction, when to simulate neuronal systems, it is important to consider the presence of a noisy component in order to explain certain behaviors observed in vitro experiments. This component can be due to several reasons, in this paper, we analyze the effect of a globally correlated spatial stochastic component on the applied external current I such that $I(t) = I_0 + \xi(t)$ where $\xi(t)$ represents a colored Gaussian noise of zero mean whose dynamics is given by $\dot{\xi} = -\tau^{-1}\xi + \tau^{-1}\xi_w(t)$. The Ornstein-Uhlenbeck stochastic process $\xi(t)$ is driven by the white Gaussian noise $\xi_w(t)$ with $\langle \xi_w(t) \rangle = 0$ and $\langle \xi_w(t) \xi_w(t') \rangle = 2A\delta(t-t')$. The correlation function of $\xi(t)$ is an exponential function given by, $\langle \xi(t) \xi(t') \rangle = \frac{A}{\tau} \exp\left(\frac{-|t-t'|}{\tau}\right)$, where τ is the correlation time, A is the noise amplitude and $\sigma = \sqrt{A/\tau}$ is the noise dispersion. In the limit $\tau \rightarrow 0$ and finite $\tau\sigma^2$, the white-noise limit $\xi_w(t)$ is recovered [3].

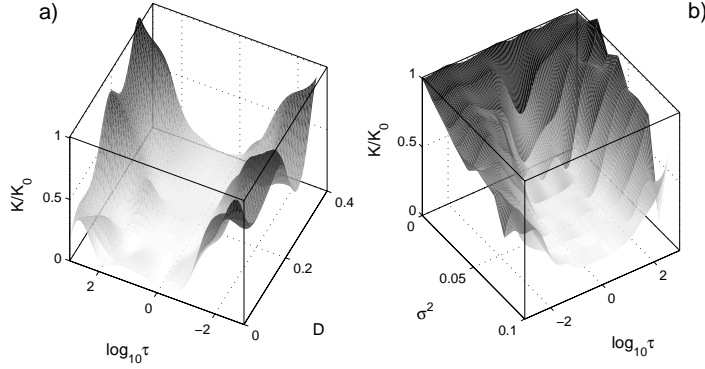


Fig. 1. a) Dependence of K/K_0 as a function of $\log_{10}\tau$ and the coupling diffusion coefficient D . $\sigma = 0.05$ and fixed boundary conditions were considered in Eq. (1). K_0 corresponds to the value of K for the case without noise, there is a different K_0 for each value of D . b) Dependence of K/K_0 as a function of $\log_{10}\tau$ and the noise dispersion. In this case, $D = 0.2$ and fixed boundary conditions were considered in Eq. (1).

3 Results

Our numerical simulations show that this 2D system of neurons with local interactions can display a rich repertoire of collective excitations such as asynchronous chaotic oscillations, expanding rings, and rotating spirals depending on the value of the connectivity, D . This could be expected as one of the features for patterns formations in a diffusive medium is the presence of two or more time or space scales in the activity of the media and we have this in our system which presents chaotic bursting behavior.

If we measure the level of synchronization of the network with different values of the coupling we can observe significant differences associated to the formation of patterns. However, if we consider the presence of small stochastic

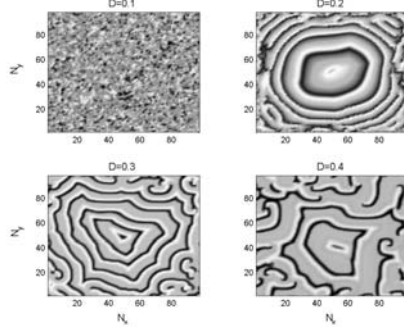


Fig. 2. Spatial patterns on the network for different values of the diffusion coefficient D . Network of 100×100 cells, $\sigma^2 = 0.05$ and fixed boundary conditions. The snapshots correspond to a single instant of time $t = 190$ t.u.. The value of time correlation, τ , is the value for which K/K_0 has a minimum in Fig. 1.

perturbations on the external current of each oscillator we find a synchronized aperiodic activity of the network for a critical value of the time correlation of the forcing, this synchronization destroys the patterns and homogenizes all cells of network in a same aperiodic behavior above some noise dispersion. This effect is shown in Fig. 1a where we plot the degree of synchronization K/K_0 as a function of the time correlation, and coupling strength, when the noise perturbs the current $I(t)$ with a small value of the noise dispersion. We looked for a similar minimum in the phase synchronization with the variation of the noise dispersion, however, this minimum only occurs with the time correlation of the noise but not with the dispersion value. Figure 1b shows the phase synchronization obtained as a function of the time correlation, and the noise dispersion for a given value of D . In both figures 1a and 1b it is possible to observe a peak, seen as a local minimum for some $\tau = \tau_R$.

We can show the behavior of the system along different longitudinal and transverse sections of the figure 1, so in figure 2 we plot four snapshots of the system for different coupling coefficients conserving fixed τ and σ^2 . we can see how

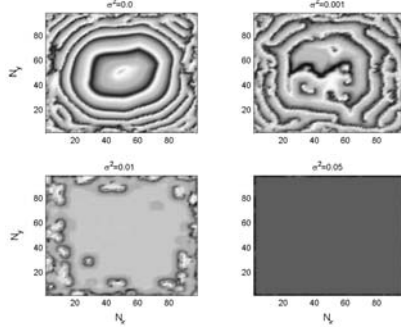


Fig. 3. Spatial patterns on the network for different values of the dispersion σ^2 . Network of 100×100 cells. The rest of parameters as in figure 2

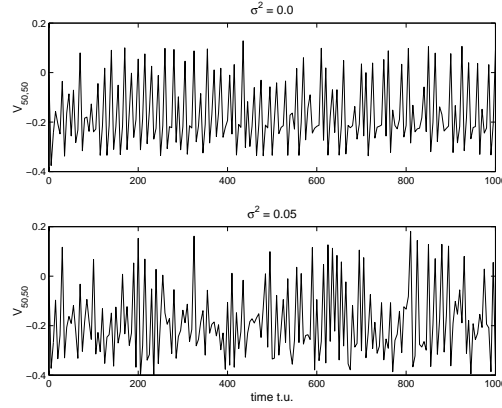


Fig. 4. Temporal evolution of a particular cell of the network with $D = 0.2$. a) $\sigma^2 = 0$ b) $\sigma^2 = 0.05$. The rest of parameters as in figure 3

this 2D system of neurons display a rich repertoire of patterns depending on the value of the connectivity, D . If now, we fix τ and D and vary the value of σ^2 we can observe the synchronization process of the network due to the presence of the noise and as a result the patterns disappear above some threshold value of the noise dispersion. Moreover, if we represent the temporal evolution of the cells in figure 4 we can see that the neurons conserve their aperiodic activity during the synchronization. That is, the synchronization is not due to a collapse of the neural activity but all cells match in the same active behavior.

In conclusion, the observed behavior shows a synchronization of the network

in aperiodic behavior with the help of tiny noisy perturbations on the external signal. This effect is really important, the presence of noise in a neural system can achieve an almost complete synchronization among the different cells. This could help us to understand certain processes like the emergence of epileptic seizure or the reception of weak signals in the sensorial systems, two situations where the noise could be harmful or beneficial, respectively. The study of the influences of noisy signals on control or anticontrol of the dynamical behavior is open and is really interesting because with small intensities it is possible to achieve great effects and in this sense the destructive traces produced over the cells by high voltages could be avoided.

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