"Predictive Learning in Rate-Coded neuronal Networks"

Bernd Porr and Florentin Wörgötter
bp1@cn.stir.ac.uk
worgott@cn.stir.ac.uk
Department of Psychology, University of Stirling, Stirling FK9 4LA, Scotland
Phone: +44 01786 466369, Fax: +44 01786 467641

Poster presentation preferred

Abstract

A novel approach is developed for learning of continuous signals in neural networks. Input signals are band-pass filtered before being summed at an output unit. A new learning rule is devised utilizing the temporal derivative of the output. The initial development of the weights is calculated and simulation results are shown to demonstrate the performance. The system can handle patterns with long temporal duration and it can process multiple inputs. Both features link our approach to ”classical conditioning” where inputs from different sensorial modalities lead to learning even if presented with a long temporal interval between them.

Poster presentation preferred

Poster presentation preferred

Predictive Learning in Rate-Coded neuronal Networks: A theoretical approach towards classical conditioning

Bernd Porr and Florentin Wörgötter
Department of Psychology, University of Stirling, Stirling FK9 4LA, Scotland
bp1@cn.stir.ac.uk
worgott@cn.stir.ac.uk

Classical conditioning takes place on rather long time scales (seconds) and requires two input stimuli, the unconditioned and the conditioned stimulus, which normally come from different sensorial modalities. It can be interpreted as a form of predictive temporal learning where the unconditioned stimulus acts as a predictor of the conditioned stimulus.

Recent theoretical approaches towards temporal learning rest on spiking neurons (Gerstner et al., 1996; Kempter et al., 1999). These models are often highly realistic but usually elude from a rigorous mathematical analysis. Also one finds that long time scales and multiple inputs are hard to treat with these models. On the other hand there exists a long tradition of models with linear or so called rate coded neurons which allow an easier treatment (Miller, 1996). Thus in our approach we have chosen a rate code description as the level of abstraction and present a theoretical framework for predictive (temporal) learning which is able to handle time-continuous input signals (rate-functions) of arbitrary shape. To this end our “neurons” will act as damped oscillator circuits and a new learning rule is developed which utilizes the temporal change (the derivative) of the output to modify the weights. This system has the feature, that very few components are required to cover large time intervals and it allows to use multiple inputs, which are the two critical prerequisites for ”classical conditioning”.

We consider a system of N units h receiving inputs x and producing outputs u. The input units connect with weights ρ to one output unit v (Fig. 1A). All input units are in principle equivalent but we will use h_0 to denote the one unit which transmits the unconditioned stimulus. The output v is then given as:

$$v(t) = \rho_0 u_0(t) + \sum_{i=1}^{N} \rho_i u_i(t) \qquad \text{with} \tag{1}$$

$$u_i(t) = x_i(t) * h_i(t) \tag{2}$$

where $*$ denotes the convolution operation. The transfer function h in Eq. 2 shall be that of a standard *resonator* which transforms δ-pulses input into a damped oscillation. We implemented

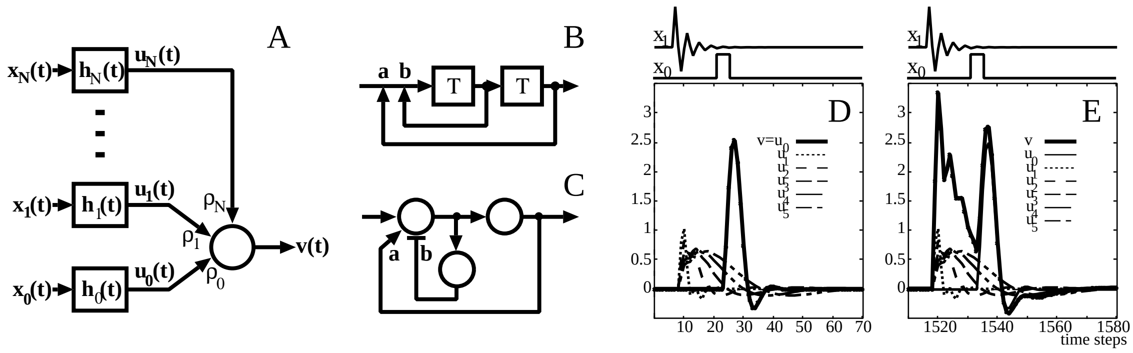

Figure 1: A) The basic circuit in the time domain. B) An IIR filter which represents h_i. C) A neuronal representation of an IIR filter. D,E) Simulation results for a typical classical conditioning paradigm. Stimuli were repeated every 100 time-steps. D) at the first stimulus presentation ($t = 0$), E) at the 15th stimulus presentation ($t = 1500$).

the resonator as an infinite impulse response (IIR) filter which is shown in Fig. 1B. This filter consists of two delay lines with unit delay T. The outputs of these delay lines are fed back to the input of this circuit. The two weights a and b determine the characteristics of the filter. In Fig. 1C we show a possible implementation of such a filter on a neuronal basis. Due to the fact that the weight a is greater and the weight b smaller than zero we need one inhibitory and one excitatory pathway. Note that appropriate neuronal transmission delays are needed in order to simulate the resonator characteristic correctly. This can be achieved by cascading the neurons (not plotted).

> We state our goal as: After learning, the output unit shall produce a well discernible signal v (e.g., of high amplitude and steeply rising) in response to the earliest occuring conditioned stimulus $x_j, j \geq 1$.

Learning (viz. weight change) takes place according to a Hebb-like rule (Eq. 4)

$$\rho_i \rightarrow \rho_i + \Delta\rho_i \qquad i = 1, 2, 3... \tag{3}$$

$$\Delta\rho_i(t) = \mu u_i(t) v'(t) \tag{4}$$

Before showing simulation results for continuous input signals we will briefly consider the mathematical properties of this system and find that one can compute the solution for the initial development of the weights (i.e., for t=0) when assuming δ-function inputs. One obtains:

$$\Delta\rho_i(T) = \mu \frac{1}{2\pi} \int_{-\infty}^{+\infty} -i\omega V(-i\omega) U_i(i\omega) d\omega, \tag{5}$$

for the development of the weights where V and U_i are the LAPLACE transforms of v and u_i (see Eq. 1). Quite unusual for such network models we observe, that this integral can be evaluated analytically by the method of residues when applying PLANCHERELS theorem, which is, however, beyond the focus of this short article.

At this stage, however, it is important to note that signal sampling theory allows to transfer this result, which was obtained with δ-function inputs, to continuous signals of arbitrary shape. This is shown in a simulation where we also demonstrate that the system can deal with rather long temporal intervals and still produce brisk "expectation potentials". To this end we implemented 5 resonators with different frequencies $h_1, \ldots, h_5$ which all get their input from the conditioned stimulus x_1 (Fig. 1D,E). The unconditioned stimulus x_0 still drives only one resonator (h_0). The conditioned stimulus is represented by a sine wave burst (e.g., ringing a bell) and the unconditioned stimulus by a square wave pulse (e.g., switching on a light). Initially the output v coincides with the unconditioned stimulus (D). After learning it is shifted forward and rises with the occurrence of the conditioning stimulus.

In summary, we have designed a system for temporal learning which captures the essential characteristics of classical conditioning and is able to treat continuous time-functions. This can be helpful when trying to understand the neuronal processes underlying classical conditioning in a more detailed way. In addition, such systems might become of technical relevance extending the concept of predictive filters to systems with multi-modal inputs. Furthermore our (linear) approach is embedded in signal theory which offers the advantage of an extended analytical treatment.

References

Gerstner, Kempter, van Hemmen, and Wagner (1996). A neuronal learning rule for sub-millisecond temporal coding. *Nature*, 383:76–78.

Kempter, R., Gerstner, W., and van Hemmen, J. L. (1999). Hebbian learning and spiking neurons. *Physical Review E*, 59:4498–4514.

Miller, K. D. (1996). Receptive fields and maps in the visual cortex: Models of ocular dominance and orientation columns. In Donnay, E., van Hemmen, J., and Schulten, K., editors, *Models of Neural Networks III*, pages 55–78. Springer-Verlag.