Development of joint ocular dominance and orientation selectivity maps in a correlation-based neural network model

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Abstract

A new correlation-based model for ocular dominance and orientation selectivity columns is proposed. This model can be used to obtain realistic maps for either the ocular dominance maps or orientation selectivity maps. It can also be used to generate joint maps that take into account the extra requirements related to the singularities of the orientation selectivity map and the relative angles of intersection between the contour lines of the two maps. For future research we propose the use of stochastic processes in generating cortical maps and modification of Hebbian cortical interaction terms to make the model more biologically plausible.

Keywords: ocular dominance, orientation selectivity, pattern formation.

1 Introduction

Ocular dominance (OD) and orientation selection (OS) are among the most studied phenomena that occur in the visual cortex. They are interesting since they show the extent to which organization of the neural circuitry and its functions are subject to environmental interactions. Feature-based algorithms and correlation-based learning techniques have been widely by theoretical neuroscientist to model OD and OS (Goodhill 1993, reviewed by Erwin 1995 and Swindale 1996).

The fact that the ocular dominance columns and orientation selectivity columns are not independent of each other is also very interesting. The singularities from the orientation selectivity columns tend to be situated in the center of the ocular dominance bands (Maldonado 2000). Also the contour lines from the orientation selectivity map tend to intersect the contour lines from the ocular dominance domains at 90-degree angles (Swindale 1996).

2 The single neuron model

The model for the ocular dominance starts with an individual neuron: we need to have a neuron that is a simple dynamical system that bifurcates in two states. The first state would represent the left ocular dominance; the other would represent the right one. We can accomplish this using the following equations for the neuron:

$$\tau_1 \frac{dV}{dt} = -V + (w_L I_L + w_R I_R)(1 - V) \tag{1}$$

$$\tau \frac{dw_L}{dt} = f(V \cdot I_L) - g(V) w_L I_L \tag{2}$$

$$\tau \frac{dw_R}{dt} = f(V \cdot I_R) - g(V) w_R I_R \tag{3}$$

In equations w_L represents the left connection and w_R the right one. The functions f and g are monotonically increasing functions on the interval $(0, \infty)$. Typical functions used were $f(x) = e^x$ and $g(x) = x$. If $\tau_1 << \tau$ then V will reach the steady state. Note that the normalization scheme that prevent the weights to grow indefinitely falls under "divisive" enforcement, as defined in Goodhill 1994.

$$V = \frac{w_L I_L + w_R I_R}{1 + w_L I_L + w_R I_R} \tag{4}$$

Under the environment's influence, after many exposures to signals of all orientations, the connections start to modify. Because the time scale for these modifications is small as compared to the time scale of each signal we can take into account only the averaged effects of them. Therefore the equations are going to be:

$$V = \frac{w_L I_L + w_R I_R}{1 + w_L I_L + w_R I_R} \tag{5}$$

$$\tau \frac{dw_{L_{ij}}}{dt} = f(< V I_L >) - g(< V >) < I_L > w_{L_{ij}} \tag{6}$$

$$\tau \frac{dw_R}{dt} = f(< V I_R >) - g(< V >) < I_R > w_R \tag{7}$$

Assuming $< I_L >= 1$ and $< I_R = 1 >$ we make the following approximations:

$$< \frac{w_L I_L + w_R I_R}{1 + w_L I_L + w_R I_R} I_R > \approx < \frac{w_L < I_L I_R > + w_R < I_R I_R >}{1 + w_L + w_R} > \tag{8}$$

$$< \frac{w_L I_L + w_R I_R}{1 + w_L I_L + w_R I_R} I_L > \approx < \frac{w_L < I_L I_L > + w_R < I_R I_L >}{1 + w_L + w_R} > \tag{9}$$

We are going to use the following convention: $< I_L I_L >= C_{LL}$, $< I_L I_R >= C_{LR}$, $< I_R I_L >= C_{RL}$ and $< I_R I_R >= C_{RR}$. Typical values used in our model were: $C_{LL} = 1$, $C_{LR} = 0.5$, $C_{RL} = 0.5$ and $C_{RR} = 1$. With these the equations become:

$$\tau \frac{dw_L}{dt} = f(\frac{C_{LL} w_L + C_{RL} w_R}{1 + w_L + w_R}) w_L - g(\frac{w_L + w_R}{1 + w_L + w_R}) w_L \tag{10}$$

$$\tau \frac{dw_R}{dt} = f(\frac{C_{RL} w_L + C_{RR} w_R}{1 + w_L + w_R}) w_R - g(\frac{w_L + w_R}{1 + w_L + w_R}) w_R \tag{11}$$

The initial values for w_L and w_R are random and small in magnitude. It is simple to show that the variable with the larger magnitude will win over the smaller one (figure 1.).

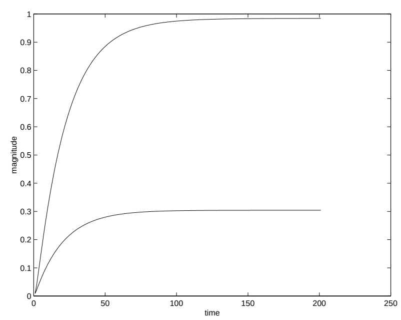

Figure 1: Evolution of the connections weights vs time. The connections with initial larger magnitude will eventually win.

3 Ocular dominance model

The visual cortex is modeled as a two dimensional array of neurons where each neuron receives inputs originating from both eyes. These are correlated both in what concerns the orientations selectivity and ocular dominance properties, represented in this model by the connection weights of the neurons. Now the equations characterizing the evolution of a single neuron (eq 10 - 11), can be used for all neurons in the array.

$$V_{ij} = \frac{w_{L_{ij}} + w_{R_{ij}}}{1 + w_{L_{ij}} + w_{R_{ij}}} \tag{12}$$

$$\tau \frac{dw_{L_{ij}}}{dt} = f(\frac{C_{LL} w_{L_{ij}} + C_{RL} w_{R_{ij}}}{1 + w_{L_{ij}} + w_{R_{ij}}}) w_{L_{ij}} - g(\frac{w_{L_{ij}} + w_{R_{ij}}}{1 + w_{L_{ij}} + w_{R_{ij}}}) w_{L_{ij}} \tag{13}$$

$$\tau \frac{dw_{R_{ij}}}{dt} = f(\frac{C_{RL} w_{L_{ij}} + C_{RR} w_{R_{ij}}}{1 + w_{L_{ij}} + w_{R_{ij}}}) w_{R_{ij}} - g(\frac{w_{L_{ij}} + w_{R_{ij}}}{1 + w_{L_{ij}} + w_{R_{ij}}}) w_{R_{ij}} \tag{14}$$

Since there is no structural information in this system, this model cannot produce pattern formation yet, therefore we need to introduce a interaction term. In order to have pattern formation we would like to have a term that will influence the dynamical evolution of the system from a random cortical map towards an ocular dominance one. This can be implemented by letting each neuron receive inputs from the other neurons. The magnitude of the inputs received is going to be decreasing with distance obeying a gaussian function (eq 15). The influence of those inputs is intended to make the neuron weights more similar to those of the neighbors.

$$J(x,y) = A_1 e^{-(\frac{x^2}{2\sigma_1^2} + \frac{\beta_1 y^2}{2\sigma_2^2})} - A_2 e^{-(\frac{x^2}{2\sigma_3^2} + \frac{\beta_2 y^2}{2\sigma_4^2})} \tag{15}$$

where $\sigma 1$, $\sigma 2$, $\sigma 3$ and $\sigma 4$ are the constants controlling the two-dimensional mexican hat, and β_1 and β_2 are controlling the degree of anisotropy. A typical one-dimensional mexican hat is represented in figure 2.

Adding the convolution term equation (eq 1) will become now:

$$\tau \frac{dV_{ij}}{dt} = -V_{ij} + (w_{L_{ij}} I_L + w_{R_{ij}} I_R)(1 - V_{ij}) + \epsilon \sum_k (J_{i-k,j-l} V_{kl}) \tag{16}$$

where ϵ is a small parameter. If τ is small we have

$$0 = -V_{ij} + (w_{L_{ij}} I_L + w_{R_{ij}} I_R)(1 - V_{ij}) - \epsilon \sum_{k,l} (J_{i-k,j-l} V_{kl}) \tag{17}$$

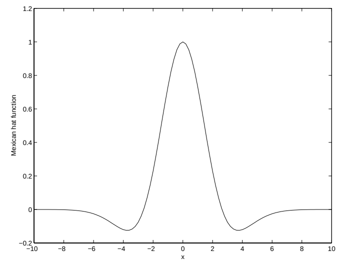

Figure 2: 1D mexican hat

We are going to use perturbation theory to find the solution up to the first order. Let $V_{ij} = V_{0_{ij}} + \epsilon V_{1_{ij}}$. In the 0^{th} order we have:

$$V_{0_{ij}} = (w_{L_{ij}} I_L + w_{R_{ij}} I_R)/(1 + (w_{L_{ij}} I_L + w_{R_{ij}} I_R)); \tag{18}$$

Solving for the first order we have:

$$0 = -(V_{0_{ij}} + \epsilon V_{1_{ij}}) + (w_{L_{ij}} I_L + w_{R_{ij}} I_R)(1 - (V_{0_{ij}} + \epsilon V_{1_{ij}})) + \epsilon \sum_{k,l} (J_{i-k,j-l}(V_{0_{kl}} + \epsilon V1_{kl})) \tag{19}$$

from which we can find the solution up to the first order:

$$V_{ij} = V_{0_{ij}} + \epsilon \frac{\sum_{k,l}(J_{i-k,j-l} V_{0_{kl}})}{1 + (w_{L_{ij}} I_L + w_{R_{ij}} I_R)} \tag{20}$$

Having the solution for V_{ij} we can calculate all the needed averages $< V_{0_{ij}} >$, $< V_{0_{ij}} I_L >$ and $< V_{0_{ij}} I_R >$, for which we are going to use approximations (1 - 2) again. We also need $< V_{1_{ij}} >$, $< V_{1_{ij}} I_L >$ and $< V_{1_{ij}} I_R >$:

$$< V_{1_{ij}} >= \frac{\sum_{k,l}(J_{i-k,j-l} < V_{0_{kl}} >)}{1 + (w_{L_{ij}} + w_{R_{ij}})} \tag{21}$$

$$< V_{1_{ij}} I_L >= \frac{\sum_{k,l}(J_{i-k,j-l} < V_{0_{kl}} I_L >)}{1 + (w_{L_{ij}} + w_{R_{ij}})} \tag{22}$$

$$< V_{1_{ij}} I_R >= \frac{\sum_{k,l}(J_{i-k,j-l} < V_{0_{kl}} I_R >)}{1 + (w_{L_{ij}} + w_{R_{ij}})} \tag{23}$$

The model for ocular dominance is now completely defined. Using these equations we simulated ocular dominance pattern formation. Typical results are presented in figure 3.

The anisotropic ocular dominance maps can be simulated by making the cortical interaction function J also anisotropic. Simulations with anisotropy much greater in one dimension are presented in figure 4.

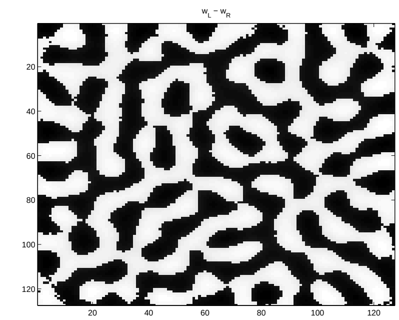

Figure 3: Ocular Dominance on a 128 by 128 domain

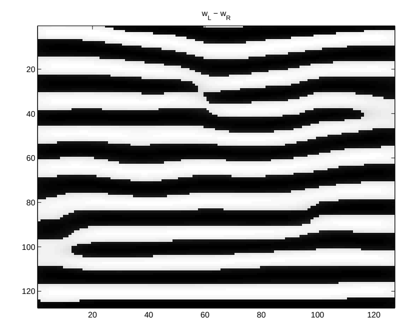

Figure 4: Anisotropic ocular dominance on a 128 by 128 domain

4 Orientation selectivity columns

The ocular dominance model can be easily adapted for orientation selectivity pattern formation. Instead of doing the sum over left and right in equations 2 - 3 we are going to sum over all orientations. Again we have maximum correlation between signals of equal orientation and the correlations decrease when the angle between orientations get larger. As opposed to the ocular dominance maps, it is possible now to have maps containing pinwheels. These are areas of the map where regions of all different orientations meet at one point.

$$\tau \frac{dV_{ij}}{dt} = -V_{ij} + \sum_{\phi} (w_{\phi_{ij}} < I_\phi >)(1 - V_{ij}) \tag{24}$$

$$\tau_1 \frac{dw_{\phi_{ij}}}{dt} = f(< V_{ij} I_\phi >) - g(< V_{ij} >) < I_p hi > w_{\phi_{ij}} \tag{25}$$

In our simulations we typically used the following formula for the correlations between orientations:

$$C(\theta-\phi) = [\frac{(1+cos(\theta-\phi))}{2}]^n, n \in \mathbf{Z} \tag{26}$$

The shape of the function C is presented in figure 5.

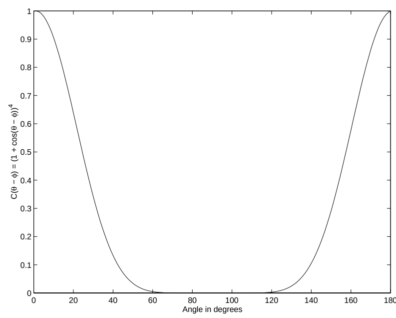

Figure 5: Orientation correlation function, for $n = 4$

The resulting maps contain the desired pinwheels, as one can see in figure 6. The density of the pinwheels is directly related to the spatial characteristics of the mexican hat.

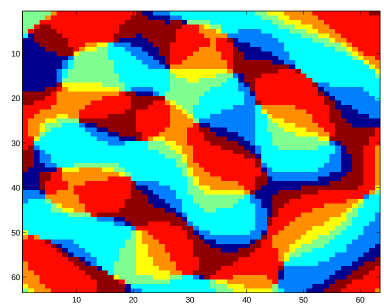

Figure 6: Orientation domains

5 Joint model

We can now build a joint model in a very simple fashion:

$$< I_{od1\ \phi1} I_{od2\ \phi2} >= C_{od1\ od2} C_{\phi1\ \phi2} \tag{27}$$

where $C_{od1\ od2}$ is defined in the ocular section, equation 8 - 9 and $C_{\phi1\ \phi2}$ is defined in the orientation selectivity section, equation 26.

Joint maps were formed and the ocular dominance contour lines tended to intersect the orientation selectivity lines at 90^o angles, although they also passed through some of the singularities, as presented in figure 7.

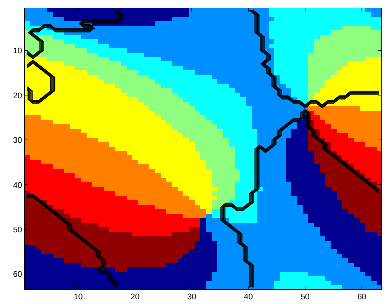

Figure 7: Joint model without the interaction terms

We fixed this problem using a similar approach to one used by Swindale 1992. We added an interaction term that slowed down the recruitment process in a region where a singularity would exist.

$$A(i,j) = \frac{1}{1+\epsilon \sum_{x_1,\ y_1} J(i-k, j-l)\cos(\phi_{ij} - \phi_{kl})} \tag{28}$$

$$\tau_1 \frac{dw_{OD\ \phi_{ij}}}{dt} = \left(f(< V_{ij} I_{OD\ \phi} >) - g(< V_{ij} >) < I_{OD\ \phi} > w_{OD\ \phi_{ij}}\right) A_{ij} \tag{29}$$

This slows down direct competition in that region and, consequently, the winners are likely to recruit in a slower fashion all the region. A typical result is presented in figure 8. This method would work under the assumption that the orientation selectivity map develops before orientation columns (Crowley 2000, Piepenbrock 1996).

6 Proposed future research

We would like to use gaussian functions for the decaying cortical interaction functions, as they are more biologically plausible than the mexican hat interaction functions. This would make the density of the pinwheels of the orientation selectivity cortical maps more relevant.We would also like to investigate to what extend the use of stochastic processes, by using equations 6 - 7 instead of the approximations 10 - 11, would improve the quality of cortical maps.

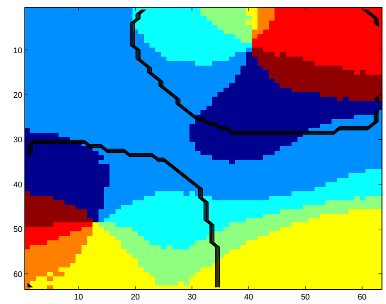

Figure 8: Joint model with the interaction term

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