Response of a LIF neuron to inputs filtered with arbitrary time scale

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We study the effect of synaptic filtering in a leaky integrate-and-fire (LIF) neuron when the time constant τ_s of the synapse is large compared to the membrane time constant of the neuron ($\tau_m \sim 10\ ms$) of the neuron. While short τ_s could correspond to AMPA-like synaptic filtering ($\tau_{AMPA} \sim 1 - 5ms$), larger time scales are required to study both the effect of GABA and NMDA conductances ($\tau_{GABA} \sim 5 - 10ms$, $\tau_{NMDA} \sim 100ms$).

In this work we provide an analytical expression for the output firing rate of a LIF in which a white noise input is filtered by synapses with τ_s larger than τ_m. Although the formula is exact only for long τ_s, it is shown numerically that the same formula can be employed even for τ_s close to τ_m Additionally, we perform an interpolation in the firing rate between the short τ_s limit [Brunel and Sergi, 1998] and the long τ_s limit, obtained in this paper. This interpolating curve allows us to characterize the output firing rate of a LIF neuron for all values of the synaptic time constant.

A LIF neuron is defined by the equation

$$\dot{V} = -\frac{V}{\tau_m} + I(t) \tag{1}$$

where V is the membrane potential of the neuron, τ_m is its membrane time constant, and $I(t)$ is the input current. In the IF neuron model, a spike is evoked when V hits a threshold value Θ, and afterwards, it is reset to a value H. The presynaptic inputs are filtered by the synapses to give a current

$$\tau_s \dot{I}(t) = -I(t) + J_E \sum_{i=1}^{N_E} \sum_k \delta(t - t_i^k) - J_I \sum_{j=1}^{N_I} \sum_l \delta(t - t_j^l) \tag{2}$$

Here $t_{i(j)}^{k(l)}$ labels the time of the k-th (l-th) spike from the i-th excitatory (j-th inhibitory) pre-synaptic neuron. $N_{E(I)}$ and $J_{E(I)}/\tau_s$ are the number of inputs and the size of the post-synaptic current generated by a single spike from the excitatory (inhibitory) afferent populations. For simplicity we only consider a single synaptic type described by a synaptic time constant τ_s.

We take the diffusion approximation of equation (2) [Ricciardi, 1977, Tuckwell, 1988]

$$\tau_s \dot{I}(t) = -I(t) + \mu + \sigma\eta(t) \tag{3}$$

where μ and σ^2 are the mean and variance of the input current. $\eta(t)$ is a white noise with unit variance. From the system (1,3) we get a FPE that is solved by devising a novel perturbative technique in the parameter $1/\tau_s$. We find that the leading order for the output firing rate is

$$\nu_{out,0} = \frac{1}{\sqrt{2\pi}\tau_m} \int_{\hat{\Theta}/\gamma}^{\infty} dz \, \frac{e^{-z^2/2}}{log(\frac{\gamma z - \hat{H}}{\gamma z - \hat{\Theta}})} \tag{4}$$

where $\gamma = \sqrt{\tau_m/\tau_s}$, $\hat{\Theta} = \sqrt{2}(\Theta - \mu\tau_m)/\sigma\sqrt{\tau_m}$ and $\hat{H} = \sqrt{2}(H - \mu\tau_m)/\sigma\sqrt{\tau_m}$. This formula does not admit an expansion in powers of $1/\tau_s$ in the full space of input parameters. However, it is possible to have such an expansion in the suprathreshold regime of the input, that is, when $\mu\tau > \Theta$. Notice that in this regime, $\hat{\Theta} < 0$ and taking the limit of long τ_s in formula (4) leads to a finite output rate. It is then possible to expand the rate(4) in powers of $1/\tau_s$ obtaining, up to order $O(1/\tau_s)$,

$$\nu_{out,0} \sim \nu_r + \gamma^2 \tau_m \nu_r^2 [\tau_m \nu_s (\hat{\Theta}^{-1} - \hat{H}^{-1})^2 - \frac{1}{2}(\hat{\Theta}^{-1} - \hat{H}^{-2})] \tag{5}$$

where

$$\nu_r = \tau_m^{-1} log^{-1}(\frac{\hat{H}}{\hat{\Theta}}) \tag{6}$$

Notice that ν_r is the firing rate of a LIF neuron without driving noise in the suprathreshold regime. In the limit of long synaptic time constant, the input noise is filtered out, but still the mean depolarization is big enough to provoke firing of the neuron. When the input is in the subthreshold regime ($\mu\tau < \Theta$), the firing rate converges to zero as τ_s goes to infinity.

We now employ an interpolation between the short and long τ_s limits. In the short limit we use the firing rate formula given in Brunel and Sergi [1998] and Fourcaud and Brunel [2002] plus additional dependencies in τ_s, while for the long regime, we use formula (4). Numerical simulations are made by generating random walk samples from the stochastic equations defined in (1,3).

In Figure (1) the firing rate for a neuron in the subthreshold regime is plotted as a function of τ_s. The rate decreases monotonically as the synaptic time constant increases. In the suprathreshold regime (Figure (2)) the firing rate shows a minimum as a function of τ_s. The minimum position is also approximately predicted by the single formula (4).

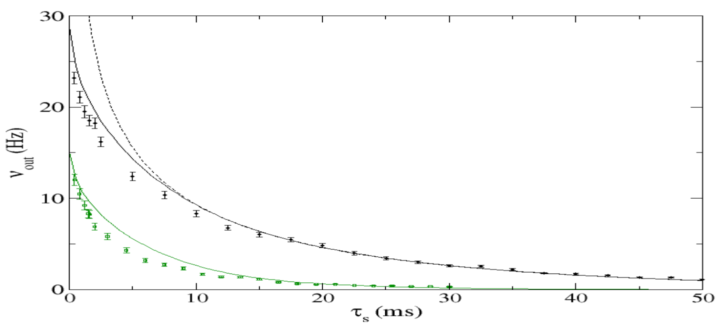

Figure 1: Output firing rate as a function of τ_s for a LIF neuron in the subthreshold regime. Parameters are: $\tau_m = 10ms$, $\Theta = 1$ and $H = 0$ (in arbitrary units), $\mu = 80s^{-1}$, and either $\sigma^2 = 12s^{-1}$, for the top curve, and $\sigma^2 = 4s^{-1}$, for the bottom curve. Full lines correspond to the rate predicted by the interpolation procedure with $\tau_{s,inter} = 15\ ms$. For comparison, we have plotted the firing rate predicted by the single formula (4). Discrete points are the simulation results with the same parameters.

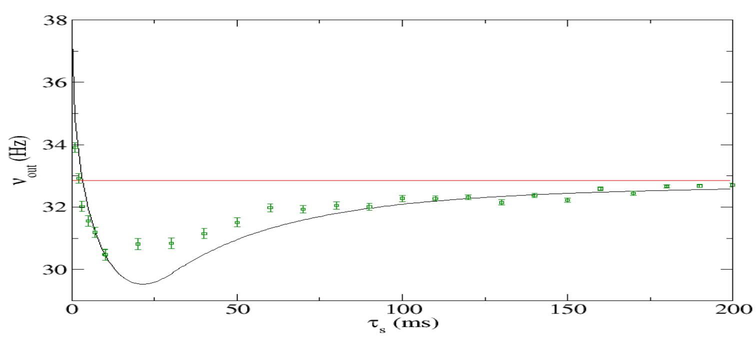

Figure 2: Output firing rate as a function of τ_s for a LIF neuron in the suprathreshold regime. Parameters are $\mu = 105s^{-1}$, $\sigma^2 = 1s^{-1}$, and the others are as in Figure (1). Full line corresponds to the rate predicted by the interpolation procedure with $\tau_{s,inter} = 30ms$. The straight line is the firing rate to which the full line approaches. Data come from simulations performed with the same parameters. The interpolating curve deviates from the simulation results only for intermediate values of τ_s.

References

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