

Computing Conditional Probabilities in a Minimal CA3 Pyramidal Neuron

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Abstract

The CA3 region of the hippocampus seems to be a sequence predicting recoder; thus each CA3 neuron must compute something that approximates conditional probabilities which forecast its own firing. Here we compare a sensibly derived neural network model, based on local conditional probabilities to an existing model of CA3. Due to the divisive inhibition used in the existing model, the excitation equations of the two models are similar and interesting analogies are highlighted. In addition to their similarities, both models demonstrate deficiencies, but in differing aspects of their behavior.

Keywords: Hippocampus; McCulloch-Pitts; Conditional Probability; Forecasting; CA3

1. Introduction

Considering the hippocampus as a sequence prediction device incorporates most existing explanations of its purpose. As a sequence prediction device, the hippocampus is able to forecast what will happen next given an input sufficiently similar to one encountered before. Within the hippocampus, the CA3 region is

paramount in this forecasting ability, with other regions helping to recode the input and decode the output.

In the conditional probability based model for the hippocampus, the CA3 region generates the probability that a particular event will occur in the immediate future, given that a particular event or sequence of events has just occurred. This probability is encoded in the firing pattern of the CA3, and information regarding previous events will be provided through the inputs to the CA3, both externally from the entorhinal cortex and dentate gyrus, as well as recurrently from itself. One method of decoding the CA3 forecast is to assume that the same CA3 pyramidal neurons that fire when an event occurs also fires to signal that an event is expected to occur.

In one model of the CA3 region [5] the pyramidal neurons receive input from both an external source (corresponding to the entorhinal cortex and dentate gyrus) and recurrently from each other. In this model the external source can force the neuron to fire, but the recurrent input (that is the input from other neurons in the CA3 region) must be sufficiently strong in order to induce the neuron to fire. Specifically, the internal activation due to recurrent input is modeled as

$$y_j(t) = \frac{\sum_{i=1}^n w_{ij} c_{ij} z_i(t - \Delta t)}{\sum_{i=1}^n w_{ij} c_{ij} z_i(t - \Delta t) + K_{FB} \sum_{i=1}^n z_i(t - \Delta t) + K_{SH} + K_{FF} \sum_{i=1}^n x_i(t)}, \quad (1)$$

where w_{ij} represents the strength of the synapse from neuron i to neuron j , c_{ij} is a binary variable representing whether a synapse from neuron i to neuron j exists, z_i is a binary variable representing whether neuron i fired in the previous computational interval $(t - \Delta t)$, x_i is a binary input indicating whether neuron i received external

forcing input, and K_{FB} , K_{SH} , and K_{FF} are the feedback, shunting, and feedforward inhibitions, respectively. In this model, neuron j fires if either $y_j(t) > \theta$ (where θ is typically one-half), or if $x_j(t) = 1$ (that is, if neuron j receives external forcing input.)

By contrast, starting from a model based entirely on conditional probabilities, an equation similar to equation 1 can be derived,

$$y_j(t) = \frac{\sum_{i=1}^n c_{ij} v_{ij} z_i(t - \Delta t)}{\sum_{i=1}^n c_{ij} v_{ij} z_i(t - \Delta t) + K_0(t - \Delta t) + K_{1(j)}(t - \Delta t) + K_{2(j)}(t - \Delta t)}, \quad (2)$$

for which the neuron fires if $y_j > 1/2$ or if it is forced by external input. Interpretations of the variables in equation 2 will be given later.

2. Necessary Probabilities and Statistics

Mathematically, the necessary forecasting calculation for the idealized CA3 neuron is $P(Z_j(t) = 1 \mid \vec{Z}(t - \Delta t) = \vec{z})$, where $Z_j(t)$ is the contribution (output) of neuron j to determining whether or not an event is expected to occur, and $\vec{Z}(t - \Delta t)$ are inputs to the CA3. For this to be a forecasting calculation, the event $\vec{Z}(t - \Delta t) = \vec{z}$ (corresponding to what “has just occurred” in the introduction) must precede the event $Z_j(t) = 1$ (corresponding to what “will occur in the immediate future”). This time ordering of events is implicit for the remainder of this discussion, so t will be dropped from the equation.

For a single neuron, the number of afferent neurons in \vec{Z} is on the order of 10,000, so even if the individual components of \vec{Z} are constrained to be binary, there are $2^{10,000}$ possible values that \vec{Z} can attain. Therefore, most values of \vec{Z} will never be

experienced, so $P(Z_j = 1 \mid \vec{Z} = \vec{z})$ will not be calculable simply by examining prior history. However, according to Bayes' Theorem,

$$P(Z_j = 1 \mid Z = z) = \frac{P(Z = z \mid Z_j = 1)P(Z_j = 1)}{P(Z = z \mid Z_j = 1)P(Z_j = 1) + P(Z = z \mid Z_j = 0)P(Z_j = 0)}, \quad (3)$$

which can be combined with an assumption of approximate conditional independence,

$$P^*(Z = z \mid Z_j = 1) \approx \prod_{i=1}^n P(Z_i = z_i \mid Z_j = 1), \quad (4)$$

where $P^*(\vec{Z} = \vec{z} \mid Z_j = 1)$ indicates the approximation that the components of \vec{Z} are independent of one another given the output of neuron j .

The individual probabilities $P(Z_i = z_i \mid Z_j = 1)$ can be estimated by the observed average value of Z_i , prior to neuron j firing (that is, prior by Δt) since the values it can take on are only zero and one. Specifically,

$$\begin{aligned} P(Z_i = 1 \mid Z_j = 1) &= \bar{Z}_{ij(1)}, & P(Z_i = 0 \mid Z_j = 1) &= 1 - \bar{Z}_{ij(1)}, \\ P(Z_i = 1 \mid Z_j = 0) &= \bar{Z}_{ij(0)}, \text{ and } & P(Z_i = 0 \mid Z_j = 0) &= 1 - \bar{Z}_{ij(0)}, \text{ where} \end{aligned} \quad (5)$$

$$\bar{Z}_{ij(1)} \stackrel{\text{def}}{=} E[Z_i \mid Z_j = 1] \text{ and } \bar{Z}_{ij(0)} \stackrel{\text{def}}{=} E[Z_i \mid Z_j = 0]. \quad (6)$$

It is important to note that there is no direct relationship between statistics $\bar{Z}_{ij(0)}$ and $\bar{Z}_{ij(1)}$. Similarly,

$$P(Z_j = 1) = \bar{Z}_j, \text{ where } \bar{Z}_j \stackrel{\text{def}}{=} E[Z_j]. \quad (7)$$

Since the z_i 's are binary, the preceding equations can be combined as

$$P^*(Z_j = 1 | Z = z) \approx \frac{\bar{Z}_j \prod_{i=1}^n \bar{Z}_{ij(1)}^{z_i} (1 - \bar{Z}_{ij(1)})^{1-z_i}}{\bar{Z}_j \prod_{i=1}^n \bar{Z}_{ij(1)}^{z_i} (1 - \bar{Z}_{ij(1)})^{1-z_i} + (1 - \bar{Z}_j) \prod_{i=1}^n \bar{Z}_{ij(0)}^{z_i} (1 - \bar{Z}_{ij(0)})^{1-z_i}}. \quad (8)$$

3. McCulloch-Pitts Neuron

At this point equation 8 has some superficial similarities to McCulloch-Pitts neurons, but uses products instead of summation. However, logarithmic manipulation allows equation 8 to be rewritten in a form that conforms more closely to models of the CA3 region based on McCulloch-Pitts neurons. First of all, dividing the numerator and denominator by the numerator yields

$$P^*(Z_j = 1 | Z = z) \approx \left[1 + \frac{(1 - \bar{Z}_j)}{\bar{Z}_j} \prod_{i=1}^n \left(\frac{1 - \bar{Z}_{ij(0)}}{1 - \bar{Z}_{ij(1)}} \right) \left(\frac{\bar{Z}_{ij(0)} (1 - \bar{Z}_{ij(1)})}{\bar{Z}_{ij(1)} (1 - \bar{Z}_{ij(0)})} \right)^{z_i} \right]^{-1}. \quad (9)$$

Examining the odds instead of the probability further simplifies equation 9 to

$$\frac{P^*(Z_j = 1 | Z = z)}{P^*(Z_j = 0 | Z = z)} \approx \frac{\bar{Z}_j}{(1 - \bar{Z}_j)} \prod_{i=1}^n \left(\frac{1 - \bar{Z}_{ij(1)}}{1 - \bar{Z}_{ij(0)}} \right) \left(\frac{\bar{Z}_{ij(1)} (1 - \bar{Z}_{ij(0)})}{\bar{Z}_{ij(0)} (1 - \bar{Z}_{ij(1)})} \right)^{z_i}. \quad (10)$$

To satisfy its role of calculating conditional probability, the neuron fires if the odds exceeds some threshold (ϕ), where $\phi = 1$ is a logical choice. This requirement can be written as

$$\frac{\bar{Z}_j}{(1 - \bar{Z}_j)} \prod_{i=1}^n \left(\frac{1 - \bar{Z}_{ij(1)}}{1 - \bar{Z}_{ij(0)}} \right) \left(\frac{\bar{Z}_{ij(1)} (1 - \bar{Z}_{ij(0)})}{\bar{Z}_{ij(0)} (1 - \bar{Z}_{ij(1)})} \right)^{z_i} > \phi \quad (11)$$

Taking the natural log of both sides of equation 11 yields

$$\ln \left(\frac{\bar{Z}_j}{(1 - \bar{Z}_j)} \prod_{i=1}^n \left(\frac{1 - \bar{Z}_{ij(1)}}{1 - \bar{Z}_{ij(0)}} \right) \left(\frac{\bar{Z}_{ij(1)} (1 - \bar{Z}_{ij(0)})}{\bar{Z}_{ij(0)} (1 - \bar{Z}_{ij(1)})} \right)^{z_i} \right) > \ln \phi, \quad (12)$$

which can be simplified by the substitutions

$$\begin{aligned} c_{ij} v_{ij} &\stackrel{\text{def}}{=} \ln \left[\bar{Z}_{ij(1)} (1 - \bar{Z}_{ij(0)}) / \bar{Z}_{ij(0)} (1 - \bar{Z}_{ij(1)}) \right], & K_0 &\stackrel{\text{def}}{=} \ln \phi, \\ K_{1(j)} &\stackrel{\text{def}}{=} \ln (1 - \bar{Z}_j / \bar{Z}_j), \text{ and} & K_{2(j)} &\stackrel{\text{def}}{=} \sum_{i=1}^n c_{ij} \ln (1 - \bar{Z}_{ij(0)} / 1 - \bar{Z}_{ij(1)}), \text{ to} \\ & & \sum_{i=1}^n c_{ij} v_{ij} z_i &> K_0 + K_{1(j)} + K_{2(j)}, \end{aligned} \quad (13)$$

where the c_{ij} term explicitly indicates that the calculations are only valid over the afferent inputs to neuron j . In turn, equation 13 can be rewritten as

$$\begin{aligned} 2 \sum_{i=1}^n c_{ij} v_{ij} z_i &> \sum_{i=1}^n v_{ij} z_i + K_0 + K_{1(j)} + K_{2(j)}, \text{ or} \\ y_j(t) &\stackrel{\text{def}}{=} \frac{\sum_{i=1}^n c_{ij} v_{ij} z_i (t - \Delta t)}{\sum_{i=1}^n c_{ij} v_{ij} z_i (t - \Delta t) + K_0 (t - \Delta t) + K_{1(j)} (t - \Delta t) + K_{2(j)} (t - \Delta t)} > \frac{1}{2}, \end{aligned} \quad (14)$$

where the explicit dependence on time has been reintroduced for clarity. Equation 14 now fits many models that use McCulloch-Pitts neurons [2, 4], with a threshold fixed at one-half.

4. Biological Interpretation

Therefore, the only pieces of information required for a neuron to create reasonable forecasts of the future are the state of its afferent neurons ($z_i(t - \Delta t)$, which is transmitted through its synapses), the expectation of a spike (\bar{Z}_j) in a computational interval, the expectation of a recent spike in its afferent neurons when the neuron

fires ($\bar{Z}_{ij(1)}$), and the expectation of a recent spike in its afferent neurons when the neuron is quiescent ($\bar{Z}_{ij(0)}$). Mechanisms exist for the neuron to generate approximations for each piece of necessary information.

The expectation of a spike is stored in the neuron's postsynaptic excitability. That is, higher expectation of a spike leads to lower excitability. This naturally arises out of the role that $K_{1(j)}$ plays in equation 14. Specifically, increasing the value of \bar{Z}_j increases the value of $K_{1(j)}$, which decreases the value of y_j .

The expectation of a recent spike in an afferent neuron given that this neuron will be firing ($\bar{Z}_{ij(1)}$) is the statistical equivalent to the post-synaptic modification rule describe in Levy (1996) [2] and elsewhere. The post-synaptic modification rule only modifies the synaptic weights when the post-synaptic neuron (Z_j) fires. When the post-synaptic neuron fires, the weights are strengthened if the pre-synaptic (afferent) neuron fired in the preceding interval, and weakened if the pre-synaptic neuron was quiescent. Furthermore, Levy, Colbert, and Desmond (1990) [3], demonstrate that the modification rule

$$w_{ij}^+(t + \Delta t) = w_{ij}^+(t) + \mu z_j(t) (z_i(t - \Delta t) - w_{ij}^+(t)) \quad (15)$$

under assumptions of stationarity and ergodicity leads to $w_{ij}^+ \rightarrow \bar{Z}_{ij(1)}$ as $t \rightarrow \infty$.

Similarly, the expectation of a recent spike in an afferent neuron when this neuron is quiescent ($\bar{Z}_{ij(0)}$) is the equivalent of habituation. In the familiar sense, the synapse becomes habituated to afferent input if it is not followed by post-synaptic firing, and this habituation decays in the absence of afferent input [3]. The synaptic modification equation of this habituation-like process is

$$w_{ij}^-(t + \Delta t) = w_{ij}^-(t) + \mu(1 - z_j(t))(z_i(t - \Delta t) - w_{ij}^+(t)) \quad (16)$$

which leads to $w_{ij}^- \rightarrow \bar{z}_{ij(0)}$ under the same assumptions as those made for equation 15.

5. Discussion

Comparing equation 1 to equation 2 requires first a comparison of the summation term that appears in the numerator (as well as in the denominator). These summations are clearly similar with respect to their use of the afferent input information, but there remains a significant difference in the weights attributed to these inputs. (The c_{ij} term is implicitly included in the v_{ij} terms.) For example, the w_{ij} terms are limited to the range $[0,1]$, but the v_{ij} terms in equation 2 are theoretically unbounded. Rules governing the shedding of ineffectual inputs discussed by Adelsberger-Mangan and Levy (1994) [1] will limit v_{ij} to be positive, but there is no upper bound to its size. (However, the biological implementation would necessarily have an upper bound.) Additionally, the v_{ij} terms incorporate habituation effects, but the w_{ij} terms do not.

Furthermore, the divisive inhibition terms in equations 1 and 2 show more differences than they do similarities. For example, equation 2 has no dependence in these terms on recurrent (feedback) or external (feedforward) inputs. Conversely, the inhibition terms in equation 1 are constants, although there are variants of equation 1 that effectively allow K_{FB} to dynamically adapt.

A real problem in the conditional probability model is that firing dynamics governed by equations 2, 15, and 16 are unstable. Specifically, neurons that fire more

than other neurons will have smaller values of $K_{I(j)}$, which will make them more likely to fire in the future. As a system this instability is increased since if the activity of the system as a whole increases, the $K_{I(j)}$ and $K_{I(j)}$ terms will decrease while the v_{ij} terms increase, which in turn causes the activity to increase even more. Inhibitory neurons are one mechanism that could be added to this model to control these activity fluctuations.

Only a flavor of the underlying biology is given here and interested readers are directed to Levy, Colbert, and Desmond (1990) [3] for a more rigorous treatment.

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