Modeling and Control of Eye Movements with Musculotendon Dynamics

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Abstract

Recent anatomical studies of extraocular muscles (EOM) demonstrate the stability of muscle paths. This is due to the fact that each rectus EOM passes through a pulley consisting of an encircling ring or sleeve of collagen. In this paper, the EOMs are modeled using the Hill type musculotendon complex and the effect of extraocular pulleys are studied. The model proposed by Martin and Schovanec[2] for horizontal eye movements is used as the basis. Then the extraocular pulleys are introduced in order to study the implications to the dynamic model of the eye.

Summary

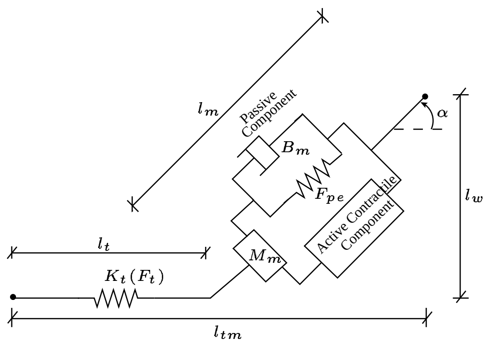
Modeling the eye plant in order to generate *saccadic* or *smooth pursuit eye movements*, has been the topic of research for a long time. Eye rotates with three degrees of freedom, i.e. horizontal, vertical and torsional. Previous studies that used modeling as a means of understanding the control of three dimensional eye movement have adopted two main approaches. The first approach focuses on the details of the properties of the extraocular muscles (EOMs)[2, 4], and the other focusing on control mechanisms for three dimensional eye movement using over simplified linear models with the details of the above EOM properties ignored [5]. About a decade ago, Miller [3] noticed the stability of rectus EOM belly paths during eye movements which provided a strong evidence that the EOM paths are constrained by pulleys. Each rectus EOM seemed to pass through a pulley consisting of an encircling ring or sleeve of collagen, located near the globe equator. Implications to ocular kinematics due to pulleys are discussed in this paper using the detailed ocular model first proposed by Martin and Schovanec[2] for horizontal saccadic eye movements. In the original model, the geometrical implications due to pulleys are not considered. In this paper, the dynamic model is modified to accomodate the pulleys.

The Hill-type model [1, 6] (Figure (A)) used for the musculotendon complex, has been shown to incorporate enough complexity while remaining computationally practical.

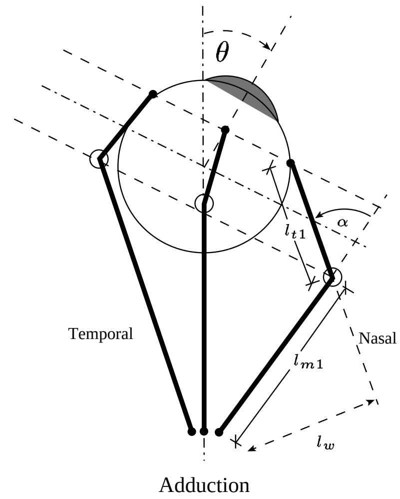
The pulley positions change as a function of the eye-rotation (see Figure (B)) in such a way that the volume of the muscles remain invariant[6], i.e., l_w remains constant. $l_w = l_m \sin\alpha$ gives

$$\dot{\alpha} = -\frac{\dot{l}_m}{l_m}\tan\alpha.$$

In Figure (B), musculotendon complex is shown in thick lines. It is assumed that the muscle extends from the 'annulus of Zinn' up to the pully and the tendon connects the muscle to the eye globe. The pully motion under the constraint that the muscle volume is invariant, uniquly defines its path. This is a direct consequence of the fact that the length of the muscle (l_m) is known for a



(A)



(B)

Figure 1: (A) Hill-type model of the musculotendon complex. (B)Superior view of the eye showing the shifts in horizontal rectus pulley positions. Pulleys are shown as *rings*.

given neural activation. The uniqe pully path also determines the length of the tendon l_t and the angle through which the tendon get inserted to the eyeglobe, hence enabling the calculation of the total torque on the eye.

Tendon dynamics expressed in the form $\dot{F}_t = K_t(F_t)\dot{l}_t$ where

$$K_t(F_t) = \begin{cases} k_{te}F_t + K_{tl} & 0 \leq F_t < F_{tc} \\ k_s & F_t \geq F_{tc} \end{cases}$$

Writing the force balance equation together with the neural contraction dynamics gives rise to a system of equations describing the dynamics of the eye in the form $\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2$.

References

[1] A. Hill. The heat of shortening and dynamic constants of muscle. *Proc. Roy. Soc. B.*, 126:136–195, 1938.

[2] C. Martin and L. Schovanec. The control and mechanics of human movement systems. *Progress in Systems and Control Theory*, 25:173–202, 1999.

[3] J. Miller. Functional anatomy of normal human rectus muscle. *Vision Res.*, 29:223–240, 1989.

[4] J. Miller and D. Robinson. A model of the mechanics of binocular alignment. *Computers and Biomedical Research*, 17:436–470, 1984.

[5] T. Raphan. Modeling control of eye orientation in three dimensions. 1. role of muscle pulleys in determining saccadic trajectory. *J. of Neurophysiology*, 79:2653–2667, 1998.

[6] F. Zajac. Muscle and tendon: Properties, models, scaling, and application to biomechanics and motor control. *CRC Critical Reviews in Biomedical Engineering*, 17:359–411, 1989.