Time Encoding with Neuronal Ensembles

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Abstract

Time encoding is formal method of mapping amplitude information into a time sequence. The time sequence or code represents the information of an arbitrary bandlimited stimulus loss free. The average rate of the time code is proportional with the bandwidth of the stimulus. Stimuli of the visual, auditory and olfactory sensory systems, can not be mapped, however, into a single spike train because of the physical constraints on the spike rate of individual neurons. We show how to channelize and time encode natural stimuli into a neuronal ensemble without information loss.

Keywords: Time encoding, integrate and fire neurons, neuronal ensembles, frames, time decoding.

1 Extended Abstract

A key question in theoretical neuroscience is how to represent an arbitrary stimulus as a sequence of action potentials [1]. The temporal requirements imposed on this representation might dependent on the information presented to the sensory neurons. For example, the temporal precision of auditory processing involves measurements of interaural time delays with sub millisecond accuracy [5]. Rapid intensity transients appear to be a key stimulus feature for triggering precisely timed spikes [8]. The nervous system uses ensembles of neurons to encode information but direct experimental insights into the operation of biological neural networks is scarce [9]. We shall focus in this paper on an explicit, mathematically precise neural encoding mechanism called time encoding that exhibits strong computational capabilities and predictive power.

In [7] the question of stimulus representation was formulated as one of time encoding, i.e., as one of encoding amplitude information into a time sequence. There are two natural requirements that a time encoding mechanism should satisfy. The first is that the encoding should be implemented as a real-time asynchronous circuit. Secondly, the encoding mechanism should be invertible, that is the amplitude information can be recovered from the time sequence with arbitrary accuracy. A Time Encoding Machine is the realization of such an encoding mechanism.

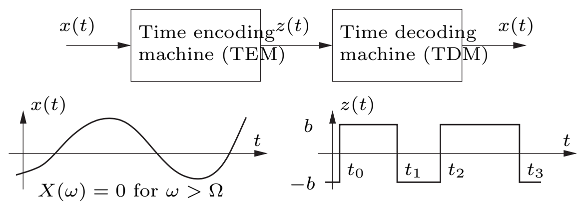

Figure 1: Time Encoding and Decoding.

Formally, a time encoding of a bandlimited function $x(t), t \in \mathbb{R}$, is a representation of $x(t)$ as a sequence of strictly increasing times $(t_k), k \in \mathbb{Z}$ (see Figure 1). Equivalently, the output of the encoder is a digital signal $z(t)$ that switches between two values $\pm b$ at times $t_k, k \in \mathbb{Z}$.

A Time Encoding Machine consisting of an "integrate and fire" neuron with feedback is invertible [7]. Under simple conditions, bandlimited stimuli encoded with the Time Encoding Machine can be recovered loss-free from the neural spike train at its output.

The Time Decoding Machine helps elucidate some of the key open questions of temporal coding. Stimuli encoded by a single integrate and fire neuron can be recovered loss-free from the neural spike train. Clearly, the recovery of the stimulus from a single running experiment is a defining biological requirement. The error introduced by dropping individual action potentials or by measurement jitter of the time of occurrence of action potentials can be explicitly evaluated. The resulting error is a measure that quantifies the importance of temporal coding. Stimulus time-delays in the sub millisecond range can be readily estimated.

The average rate of $z(t)$ is proportional with the bandwidth of the stimulus. Clearly the output of a neuron can not support large spike averages and a natural physical limit has to be imposed in our model. Using the theory of frames ([3], [6], [4], [10], [2]) we shall derive a channelization of the bandwidth of the stimulus that leads to a multidimensional time representation $(t_k^m)_{k\in\mathbb{Z}}, 1 \leq m \leq M$, where M is the number of channels. By choosing M, the peak spike rate of each integrate and fire neuron can be made arbitrarily small. Furthermore, the representation is invertible, i.e., the stimulus can be recovered from the multidimensional spike train loss free.

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