The Role of Colored Noise in Pulse Detection, a Leaky Integrate-and-Fire Model Study

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Abstract

The membrane potential of a typical cortical neuron is strongly fluctuating as a consequence of many intrinsically noisy inputs. Measurements and modeling studies suggest that this noise is colored, i.e. correlated in time. Neural information transmission and processing is based upon transient inputs. In this modeling study we investigate the detection of transient inputs, modeled as pulses, in a leaky-integrate-and-fire neuron subject to colored noise. We find that (colored) noise facillitates the detection of weak pulses. Given a certain variance of the membrane potential colored noise makes pulse detection more robust, for any pulse intensity, compared to white noise.

Key words: pulse detection, colored noise, stochastic resonance

1 Introduction

Pulse detection - as an abstract model of detection of transient inputs or coincidences - in single neurons has experienced increasing attention in the recent past, see e.g. [3,4]. In this contribution we investigate pulse detection in a model neuron subject to colored noise. For the role of noise characteristics in describing neural behaviour see e.g. [2]. We employ a leaky-integrate-and-fire neuron subject to additional colored noise. The colored noise is modeled as an Ornstein-Uhlenbeck process, the color determines the time structure of the noise and can be characterized by a single time constant, the correlation time constant, see eq. (2). Detection performance is quantified as the weighted difference between correctly detected input pulses and false positives, see eq. (1). Our results show that colored noise facillitates pulse detection of weak pulses and makes pulse detection more robust for any pulse intensity. The further is a version of the phenomenon of stochastic resonance, for a general review see [1], for work in the context of neural computation see e.g. [5,7] and references therein. Sub-threshold pulses, i.e. pulses which are too weak to make the neuron fire alone, get detected by chance in the presence of membrane potential fluctuations, with increasing noise variance the performance does first increase until a maximum has been reached. For higher noise levels the performance deteriorates. We find that the optimal noise level increases with increasing noise correlation. Both effects - higher robustness and shift to higher optimal noise levels - are due to a significant decrease of the false alarm rate as a consequence of increasing the correlation in the noise. In section 2 we present the pulse detection scenario and how we quantify pulse detection performance. The model of the leaky integrate-and-fire neuron and the noise model are presented in section 3. Results will be shown in section 4, followed by conclusions.

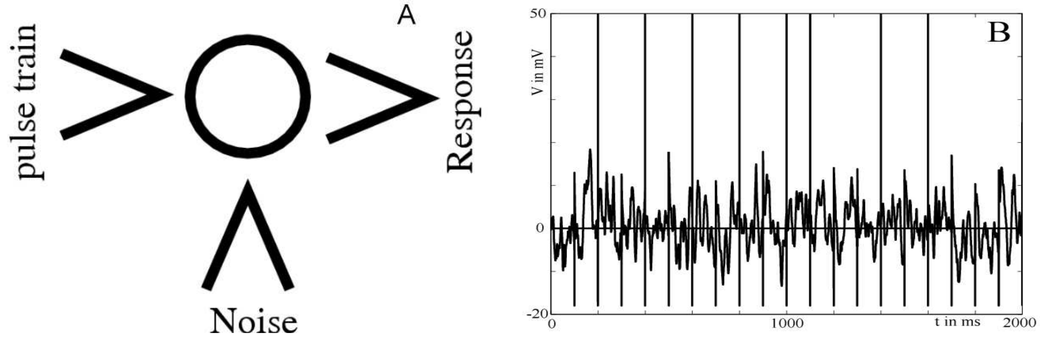

Fig. 1. **(A)** Genral set up: A single neuron receives a pulse train and colored noise inputs, for details see the model sections. **(B)** Membrane potential of a leaky-integrate-and-fire neuron, eq. (3), receiving colored noise input $\tau_X = 5$ms, eq. (2), and pulses of width $dt = 0.1$ms and height 18mV every 100ms. The timing of the pulses is indicated as vertical bars, ranging from 0mV to -18mV every 100ms. The threshold is at 20mV and reset at 0mV.

2 Pulse Detection Scenario

Figure 1 (A) displays the general set up for the simulations. A leaky integrate-and-fire neuron, eq. (3), receives additive colored noise, eq. (2), and a pulse train as inputs. The pulse train consists of thin (0.1ms) and high rectangular pulses, which are added to the membrane potential. In Fig. 1 (B) an example of the resulting membrane potential V is shown. The straight bars reaching from 0mV to -18mV indicate the timing of pulse application. Whenever the membrane potential reaches the threshold, 20mV, it is reset to 0mV. Such an event is called a spike and is indicated by a straight line reaching from 20mV up to 50mV. The neuron has succesfully detected a pulse (correct hit) if the membrane potential reaches the threshold immidiately after the occurence of a pulse. Any spikes which occur whithout a directly preceeding pulse are counted as false positives.

To quantify the quality of the response to the pulse trains, we employ the wheighted difference between the number of correct hits and false positives as a measure,

$$Q = \#(correct\ detected\ pulses) - \lambda\#(false\ positives), \tag{1}$$

where λ is a constant factor which weights the influence of false positives on Q compared to the correct hits. In contrast to other well-known measures like the signal-to-noise ratio, this measure reflects in a direct way the task relevance of the response and, thus, is well suited to capture the response behaviour of spiking systems. Other measures related to pulse detection can be found in [3] and references therein.

3 Leaky Integrate-and-fire Neuron and Noise Model

We model the cumulative effect of all spike trains - except for those which make up the pulses - as an Ornstein-Uhlenbeck process X, for an introduction see [6]. It corresponds to lowpass filtered white noise with a time-constant τ_X and diffusion coefficient D

$$\frac{dX(t)}{dt} = -\frac{1}{\tau_X} X(t) + \sqrt{D}\frac{dW(t)}{dt}, \tag{2}$$

where dW are the infinetesimal increments of the Wiener process, its time derivative corresponds to Gaussian white noise. In Fig. 2 (A) example traces of V are shown for $\tau_X = 0.1$ms and 10ms. Pulse trains are modeled as a series of rectangular peaks of width dt, and a given height. These are directly added to the membrane potential V. The membrane potential V of the leaky

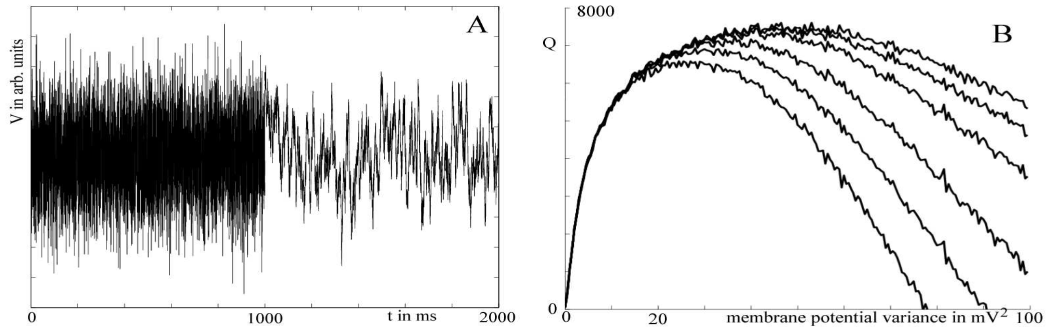

Fig. 2. **(A)** Membrane potential induced by colored noise with different time constants: Leaky-integrate-and-fire neuron, left $\tau_X = 0.1$ms, right $\tau_X = 10$ms, eq. (2). **(B)** Pulse detection performance, Q with $\lambda = 1$, eq. (1), as a function of the membrane potential variance in the leaky-integrate-and-fire neuron model for different values of τ_X. $\tau_X = 0.5, 1, 2, 5, 10, 20$ms. Note that increasing τ_X corresponds to higher optimal noise levels and better pulse detection performance for large membrane potential variance. Pulses are 50ms appart. 10000ms are simulated.

integrate-and-fire neuron model changes in time according to the differential equation

$$\frac{dV(t)}{dt} = -\frac{1}{\tau_V}V(t) + X(t) + pulsetrain, \tag{3}$$

where τ_V is the membrane time constant. Equation (3) describes the sub-threshold dynamics of the membrane potential. Once the membrane potential reaches the threshold, a spike is generated and the membrane potential is reset to $V_{reset} = 0$mV.

4 Results

Figure 2 (B) demonstrates the main effect of colored noise on pulse detection performance for a sub-threshold pulse. Pulse detection performance Q is

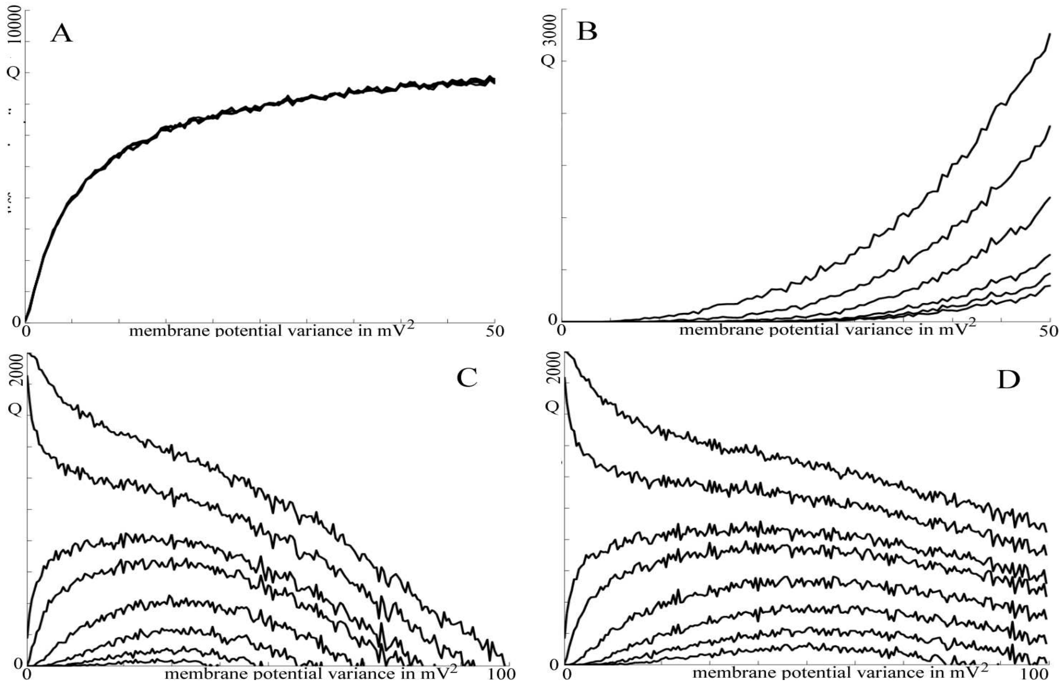

Fig. 3. **(A)** Correct detections as a function of the membrane potential variance in the leaky-integrate-and-fire neuron model for different $\tau_X = 0.5, 1, 2, ...$ms. **(B)** False positives as a function of the membrane potential variance in the leaky-integrate-and-fire neuron model for different $\tau_X = 0.5, 1, 2, ...$ms. **(C)**Pulse detection performance, Q with $\lambda = 1$, eq. 1, as a function of the membrane potential variance in the leaky-integrate-and-fire neuron model for different pulse heights, $\tau_X = 0.5$ ms, pulse heights $= 9, 11, 13, 15, 17, 19, 21, 23$. **(D)** As in (C) but $\tau_X = 10$ ms. In all subfigures pulses are 50ms appart. 10000ms are simulated.

plotet versus the membrane potential variance. Typical stochastic resonance curves emerge, with increasing variance the performance does improve up to a maximum, then it decreases again. Curves for different values of τ_X are plotted. Note that with increasing τ_X pulse detection gets more robust for high membrane potential variance.

In Fig. 3 (A) and (B) correct hits and false positives are displayed as a function of the membrane potential variance. Together they make up the curves from

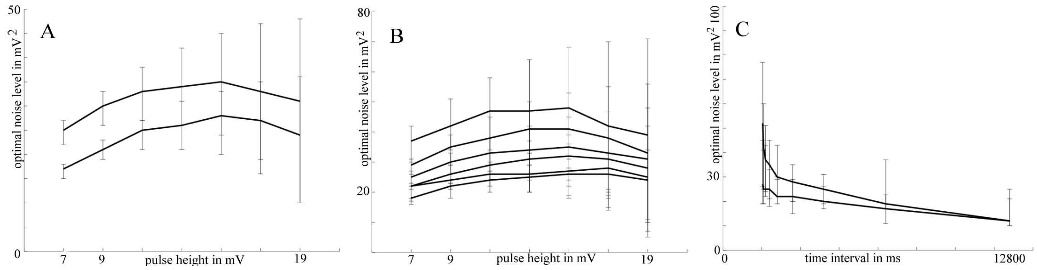

Fig. 4. **(A)** Optimal noise level for signal detection, in terms of Q with $\lambda = 1$, versus pulse height. Pulse heights are $7, 9, 11, 13, 15, 19,$ for smaller pulse heights no maximum does exist for $\lambda = 1$. For pulse heights $>= 20$ the optimal noise level is zero. The upper curve is for $\tau_X = 10\ ms$, the lower one for $\tau_X = 0.5$ms. **(C)** Optimal noise level for signal detection, in terms of Q versus pulse height. Bars correspond to all noise levels which correspond to 90% performance or more. Pulse heights as in (A). The different curves correspond to differnt λs, from top to bottom $\lambda = 1, 2, 5, 10, 20$, $\tau_X = 10$ms. In subfigures (C) and (D) the pulses are 100ms appart. **(C)** Optimal noise level for signal detection, in terms of Q with $\lambda = 1$, versus time interval. Time intervals are $50, 100, 200, 400, 800, 1600, 3200, 6400, 12800$ms. The upper curve is for $\tau_X = 10$ms, the lower one for $\tau_X = 0.5$ms. Pulse height is 18.

Fig. 2 (B), see eq. (1). There is no effect of changing τ_X on the correct hits, Fig. 3 (A), but on the false positives, Fig. 3 (B). With increasing τ_X the false positive rate goes down.

Figures 3 (C) and (D) show the pulse detection performance Q versus membrane potential variance for several pulse heights. For weak, sub-threshold, pulses typical stochastic resonance curves emerge. If the input pulses are strong enough to make the neuron fire without additional noise, performance is best in the absence of membrane potential fluctuations. For subfigure (C) and (D) we chose $\tau_X = 0.1$ms in (C) and $\tau_X = 10$ms in (D). Signal detection performance is clearly improved in (D), the negative effect of large membrane potential fluctuations on Q is less pronounced.

Figure 4 (A) shows the optimal noise level, in terms of membrane potential variance, as a function of the pulse height (sub-threshold pulses only, for super-threshold pulses it is best to have no noise). Bars indicate a noise level that would correspond to 90% performance in terms of Q indicating the width of the stochastic resonance curves. The lower curve corresponds to $\tau_X = 0.5$ms the upper one to $\tau_X = 10$ms. Note that the curves are non-monoton functions of the pulse height. Increasing τ_X shifts the optimal noise levels to higher values. In Fig. 4 (B) we have the same setting as in (A) but for several values of λ, see eq. (1). Increasing λ decreases the optimal noise level.

Figure 4 (C) shows the optimal noise level as a function of the time difference between pulses. Bars indicate a noise level that would correspond to 90% performance in terms of Q. The lower curve corresponds to $\tau_X = 0.5$ms the upper one to $\tau_X = 10$ms. For large time intervals the optimal noise levels coincide.

5 Conclusion

In this contribution we have demonstrated, that the color of the membrane potential fluctuations has a significant impact on the pulse detection performance of a leaky integrate-and-fire model neuron. Increasing the correlation of the additive noise makes pulse detection more robust for any pulse intensity. For sub-threshold pulses the optimal membrane potential variance is shifted to higher values. Both effects are due to a decreasing false alarm rate with increasing noise correlation.

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