# Identifying Generators of Neural Oscillations in the Presence of Noise: Gersch Causality versus Granger Causality

Mingzhou Ding

Program in Complex Systems and Brain Sciences

Center for Complex Systems, Florida Atlantic University

Boca Raton, Florida 33431-0991

A single random variable is characterized by its mean and variance. The degree of linear relationship between a pair of random variables is assessed by the correlation coefficient. For three random variables one can compute an additional quantity called partial correlation which measures the linear relation between a pair of variables after the influence of the third variable has been removed or"partialled out". This measure tells us whether the correlation between a pair of variables can be fully account for by the presence of the third one. Specifically, if the partial correlation between two previously correlated variables is zero, we say that the original correlation is spurious. Similar techniques exist for situations with more than three variables.

Neurobiological signals are often collected in the form of time series. For time series, in the spectral domain, the quantities analogous to variance, correlation coefficient and partial correlation coefficient are spectral power, ordinary coherence and partial coherence. The same discussion above regarding the utilities of these measures can be applied to time series at each frequency. In practice, it is often the case that symmetric interdependence measures like ordinary coherence is not completely satisfactory, and further dissections of the relations among a set of simultaneously recorded neural signals are desired to tease out the functional connectivity of complex neural networks. Recognizing the potential of partial coherence in realizing this goal, Gersch (1970), while investigating the proper identification of epileptic foci using multi-electrode (n=3) data, proposed that "one channel is said to drive the other channels if the first channel explains or accounts for the linear relation between the other two." Here the quantity computed is the partial coherence and we henceforth refer to this partial coherence based driver identification approach as the Gersch Causality. Over the years Gersch Causality has been employed explicitly or implicitly by many researchers as a way of identifying sources of driving or causal influence.

Another way of assessing the causal relations between a pair of random time series stems from the idea of Wiener (1956). Acknowledging the importance of temporal ordering in the inference of causal relations from a pure statistical point of view, Wiener proposed that, for two simultaneously measured time series, one series can be called causal to the other if we can better predict the second time series by incorporating the knowledge of the first one. This concept was later adopted and formalized by Granger (1969) in the context of linear regression models of stochastic processes. Specifically, if the variance of the prediction error for the second time series at the present time is reduced by including past measurements from the first time series in the linear regression model, then the first time series can be said to cause the second time series. Granger Causality has received a great deal of attention and has been applied widely in the econometrics literature. Recent work has also examined its applicability to the study of neural signals.

For neurobiological data a complicating factor is that the time series picked up by a sensor is inevitably a mixture of the signal of interest (e.g. theta oscillation in the hippocampus) and other unrelated processes collectively referred to as the measurement noise. The effectiveness of the two different causality measures, the Gersch Causality and the Granger Causality, in this situation is the question we address in this work. We study experimental recordings from the limbic system of the rat during theta oscillations and show that the application of partial coherence and Gersch Causality leads to contradicting conclusions whereas the Granger Causality gives consistent results. We hypothesize that the observed phenomena can be explained by the differential levels of signal to noise ratio. We confirm the hypotheses by performing a simulation involving multiple time series. The main point of our work is that, despite its wide use, partial coherence based Gersch Causality is extremely sensitive to measurement noise and often leads to erroneous results. The Granger Causality, on the other hand, appears to be robust against moderate levels of noise.


1. Gersch W. and Goddard G. V. (1970) Epileptic focus location: Spectral analysis method, Science, 169:701-702

2. Granger C. W. J. (1969) Investigating casual relations by econometric models and cross-spectral methods, Econometrica, 37: 424-438

3. Wiener N. (1956) The theory of prediction, in Modern Mathematics for Engineers, Series 1, E. F. Beckenback ed., Chapter 8