Switching Between Memories
in Neural Automata with Synaptic Noise

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Abstract

We studied analytically and numerically stochastic neural automata. Activity fluctuations and synaptic noise evolve as driven by a different temperature, and synaptic intensities move randomly trough a set of previously–learned patterns. The network thus exhibits different retrieval phases, including switching between attractors and intermitency phenomena. This behavior seems relevant to biological systems.

Understanding how the processing of information in neural media is influenced by the biophysical processes that take place at the synaptic level is an open question in computational neurobiology and neural–networks modelling. In particular, the effect of synaptic dynamics and noise on complex neural functions such as associative memory is not yet well understood. In relation to this, it has been reported that short–term synaptic depression has a main role in the ability that some systems exhibit to switch between stored memories.[1] The same behavior ensues assuming dynamics of the neuron thresholds to fire.[2] The origin of the switching mechanism is in both cases at a sort of "fatigue" of the postsynaptic neuron under repeated presynaptic stimulation. This destabilizes the current attractor which may result in a transition to a new attractor.

In order to fit these findings into a general picture concerning the role of noise in associative memory tasks, we propose to study a family of stochastic *neural automata*. The models involve two independent competing dynamics, one for neurons and the other for synapses. There is a network of N (binary) neuron variables, $s_x = \pm 1$, and any two of them join by synapses of intensity $j_{xy} \in \Re$; $x, y = 1, \ldots, N$. We write $S \equiv \{s_x\}$ and $J \equiv \{j_{xy}\}$, and assume that time is discrete and that the probability of configuration (S, J) evolves according to

$$P_{t+1}(S, J) = \sum_{S', J'} \omega(S', J' \rightarrow S, J) \, P_t(S, J)$$

with transition probability $\omega(S, J \rightarrow S', J') = \omega_J^0(S \rightarrow S') \, \omega_{S'}^1(J \rightarrow J')$. Here, $\omega_{S'}^1(J \rightarrow J') = \Psi[\beta_1 \Delta H(S')]$, $\Psi(X)$ is a generic function, which for simplicity we take to satisfy detailed balance,[3] and $\omega_J^0(S \rightarrow S') = \prod_x \Psi[\beta_0 \Delta H(S)]$.

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$\Delta H(S)$ stands for the *energy cost* of the corresponding transition and, for simplicity, we assume the Hopfield *energy* function, $H_J(S) = -\frac{1}{2}\sum_{x,y} J_{xy}s_x s_y$. These choices correspond to the parallel updating (*Little dynamics*) that characterizes cellular automata. As a particular case of this, dynamics may proceed by the master equation [3] with probability per unit time $\sum_x \prod_{y\neq x} \delta\left(s_y, s'_y\right)$ $\delta\left(s_x, -s_x\right) \Psi\left[\beta_0 \Delta H(S)\right]$ for the transition $S \to S'$. This corresponds to sequential updating of the network (*Glauber dynamics*).

This model essential differs from some apparently close proposals already discussed in the literature, e.g., [4, 5, 6]. First, it assumes the same time scale for changes in S and J. On the other hand, the choice here for $\omega\left(S, J \to S', J'\right)$ amounts to drive neurons and synapses by two different temperatures, $\beta_0^{-1} \equiv T_0$ and $\beta_1^{-1} \equiv T_1$, respectively. Furthermore, we introduce the realistic assumption that memory is a global dynamic phenomenon. More specifically, dynamics is determined at each time step by one of the relevant patterns, say ν, in the sense that $H_J(S)$ is in practice given by $-\frac{1}{2}\sum_{x,y} J_{xy}^\nu s_x s_y$ at the given time. As an example, in order to have the Hebbian learning rule as a reference, we chose $J_{xy}^\nu = \kappa \xi_x^\nu \xi_y^\nu$ where $\xi_x^\nu = \pm 1$ are the variables defining the νth relevant pattern (one of the P *memorized* patterns) and κ is a constant. Under the present conditions, the energy cost associated to the transitions $s_x \to -s_x$ and $\nu \to \nu'$ is, respectively, $\Delta H(S) \simeq 2\kappa N s_x \xi_x^\nu m^\nu$ and $\Delta H(S) = -\frac{1}{2}\kappa N^2[(m^{\nu'})^2 - (m^\nu)^2]$, where m^μ is the overlap between the current state S and pattern ν.
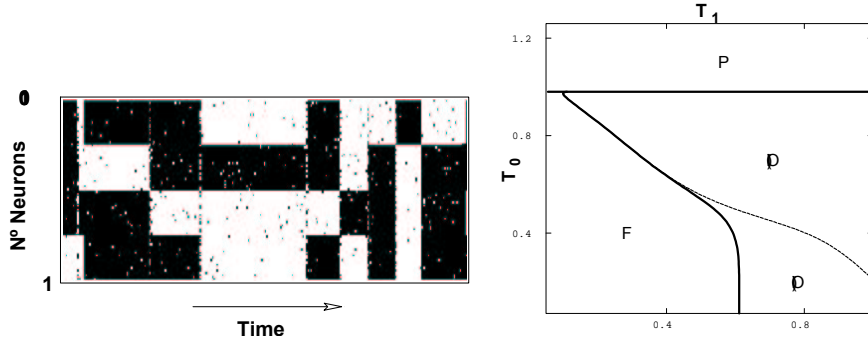
Left: Raster plot showing activity versus time for $N = 100$ neurons, $P = 4$ patterns, $T_0 = 0.9T_0^c$ and $T_1 = T_1^*$. Right: Phase diagram for $N = 16384$ and $P = 3$ showing phases P, F and O as defined in the main text. O(I) [O(II)] corresponds to temporally correlated [uncorrelated] jumps between the patterns, respectively. Here, $\kappa = P/N$, $T_1^* = 0.07$ and $T_0^c = 1$ for the model normalizations, and Little dynamics.

A preliminary study of this model has revealed a rather intriguing situation. Qualitatively different regimes occur depending on the parameter values T_0 and T_1 and the type of updating considered. For Glauber dynamics and $T_1 \to 0$, one may recover the familiar Hopfield model, while $T_1 \to \infty$ leads to the case of very fast fluctuating synapses.[5] For Little dynamics and very large but finite *temperature* T_1, three different regimes occur that are similar to the ones described in [1] and [2]. Namely, as illustrated in the figures, there is a ferromagnetic phase (**F**) in which one of the relevant patterns is the stable solution, a paramagnetic phase (**P**) without associative memory, and an oscillatory phase (**O**) in which the system constantly switches from one relevant pattern to the other.

The latter might illustrate the flexibility observed in biological systems to respond efficiently to the external changing stimuli.[7] Most interesting is also the fact that statistics concerning the jumping between attractors in the oscillating phase are consistent with the occurrence of intermittency.[8] We are presently studying further the consequences of the neural automata defined above.

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