Perceptual Geometry of Space and Form:

Visual Perception of Scenes And Places

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Short Abstract

The perceptual space is a medium in which perceptual geometry of natural surfaces, curves and objects is learned. Our paper provides evidence from neurobiology of memory and learning together with statistical learning theory for a theory of perceptual geometry of space and form based on the intelligent system's representation of memories of stimuli in aggregates and inferring statistical correlates and probabilistic structures within them. The concept of perceptual space interpolates between ideas of Helmholtz, Henri Poincare', and Einstein on the conceptual level, to modern psychophysical results such as the multi-scale theory of Koenderink, and modern neurobiology.

Extended Abstract

Introduction. In this paper, we introduce the concept of *perceptual space* as a medium to develop the perceptual geometry of the visual space and spatial organization of object forms. Rigorous study of the theoretical foundations of perceptual geometry is important for development of computational models for its exemplars. Indeed, design of psychophysical experiments to quantify perception of natural scenes and events depends very much on our understanding of mathematical and computational foundations of perceptual geometry, and in particular, the notion of *perceptual space*. In the same vein, electrophysiological recording from the cortex to study the underlying neuronal activation (as the experimental subject undergo natural environmental experiences) in the laboratory environment where precision instruments are optimally reliable require manipulation of parameters in rendering of natural scenes. We discuss the significance of perceptual space in cognitive science and in applications. The latter include modern imaging and broadcast technology, such as the fundamental theoretical and computational aspects of virtual presence, realistic scene rendering, realistic motion rendering, and related topics. We present simulation of results that apply the learning theoretic approach to test the theory in concrete cases of visual perception of forms of natural surfaces such as the human face. To our knowledge, all the results are new, and apply a fresh view of the past interpretation of geometry, from Helmholtz and Poincare' to Gibson and Koenderink. The success of our theory is expected within realistic expectation to be at least as good as present theories of visual perception, as the simulation results indicate consistent agreement with observations and approximation by the appropriate mathematical theories.

Neurobiological and Biobehavioral Relevance. The biological aspects of this research are two-fold: (a) biologically based representation of visual stimuli (images); (b) biologically based mechanisms for learning and memory. On the one-hand, our mathematical and computational development follow the biological and behavioral observations, and on the other hand, the *mathematical principles emerge* from the organisms capacity to learn, retain a representation in memory, recall the statistically preserved and frequently encountered perceptual relationships, and finally, form categorization based on dynamic correlations. The abstractions of such capacities are common between a wide range of living and artificially intelligent systems. The faculty of forming succinct rules from perceptual categorizations, however, depends on the higher-level cognitive assumptions about the system. Therefore, the emergent mathematical rules are computationally represented in terms of approximations to statistical relationships of parameters that are optimally statistically independent. The capacity for interpretation of these parameterizations, however, may or may not be available to the system. Nonetheless, our argument supports the view that the *perceptual geometry of space and form, that is, the collection of mathematical principles and their accompanying associations and inferences* (higher level computations) depend on the physical nature of the stimuli in the environment, as well as the type of transformation that the environmental stimuli undergo in the processing stages by the system's "brain."

Discussion. Perception of form and space are among fundamental problems in vision science. In recent cognitive and computational models of human perception, natural scenes are used systematically as preferred visual stimuli. Among key problems in perception of form, we have examined perception of geometry of natural surfaces and curves, e.g. as in the observer's environment. In previous papers, we have introduced the concept of the Gestalt of a surface, and examined the role of perception in estimation of some geometric quantities of natural surfaces through the proposed multi-scale multi-resolution Gestalt theory of surfaces. Besides a systematic mathematical foundation for a remarkably general framework, the advantages of the Gestalt theory of natural surfaces include a concrete computational approach to simulate or "recreate images" whose geometric invariants and quantities might be perceived and estimated by an observer. The computational modeling of the Gestalt of surfaces is proposed within statistical learning theory. The model depends on the probabilistic functions whose measurements (for each observer) are within standard psychophysics.

The theory of "natural curves" in contrast to the abstract theory of curves (e.g. in mathematics) is also influenced by the ecological and multi-scale theories of visual perception. Similar to surface theory, our approach is multi-scale, multi-resolution and through learning theoretic methods. Natural curves in the environment are perceived with the help of optical properties of surfaces. A variant of surface theory leads to fundamental concepts of "natural curvature" and "natural torsion", as statistically independent quantities in remarkable analogy with the differential geometric theory of space curves.

Conclusion. The perceptual space is a medium in which perceptual geometry of natural surfaces, curves and objects is learned, whether by the human observer or by any intelligent system capable of representing memories of stimuli in aggregates and inferring statistical correlates and probabilistic structures within them. The concept of perceptual space continues the earlier ideas of Helmholtz, Henri Poincare', and Einstein on the conceptual level, while it adheres to modern psychophysical results and the ensuing theories, such as the multi-scale theory of Koenderink. Parts of our arguments are already predicted by Henri Poincare' in his investigations of the psychological parameters influencing adaptation of geometric theories to interpret optics. Poincare' did not have access to the observations and measurements in modern neurobiology of learning and memory. As a result, his emphasis turned on the role of hypothesis in emergence of the geometric theories that rely on experience by the organism.