How does the information-geometric measure depend on underlying neural mechanisms? [1]

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Abstract

To analyze how information is represented among neuron groups, a recently presented information-geometric method (IGM) has attracted growing attention. However the detailed properties underlying the information-geometric measure have not yet been elucidated because of the ill-posed nature of the problem. Here the underlying neural mechanism of the information-geometric measure is investigated with an isolated pair of model neurons. For the symmetric network, the information-geometric measure is solely dependent on the underlying anatomical connections between the recorded neurons. For the asymmetric network, however, the information-geometric measure is dependent both on the intrinsic connections and on the external inputs to it. In other words, there are multiple neural mechanisms corresponding to the same information-geometric measure. In addition, the relation between IGM and conventional cross-correlation is also investigated.

Key words: Information Geometry; Correlation Function; Spikes; Neural Firing

1 Introduction

Development of an analytical method for neuronal population activity is critical to understand how information is represented in the brain [3] [7]. To this end, the novel analytical methods based on information geometry were proposed by Amari [1] [2], and were extended to be applicable to real neuron

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data by Nakahara et al. [5] [6]. The advantages of the information-geometric measure can be summarized as follows: (1) The modulation in mean firing rate and in coincident firing can be decomposed orthogonally. (2) Information carried by modulation in the mean firing rate and in coincident firing can be calculated. (3) Biologically plausible null hypothesis can be used for statistical test. (4) Systematic extension to any number of neurons is possible. Therefore, this method could be a powerful analytical tool for multiple spike data.

However, the possible underlying neural mechanisms for generating the information-geometric correlation components have not yet been elucidated because there could be multiple neural mechanisms producing the same information-geometric measure. For example, in the system of two neurons, the information-geometric measure corresponds to the network with symmetric connections, and the network with asymmetric connections is beyond the information-geometric measure. However, if the equilibrium state exists, the asymmetric network can be mapped onto symmetric networks. Using this property, the possible neural mechanisms underlying the information-geometric measure were investigated with an isolated pair of model neurons.

2 Information-geometric Measure

Let us briefly introduce the information-geometric measure. For details, see refs [1] [2]. Let X_1 and X_2 are 2 binary variables and let $p = p(x_1, x_2)$, $x_i = 0, 1$, be its probability. Their joint probability is given by $p_{ij} = \mathrm{Prob}\{x_1 = i; x_2 = j\} \geq 0$, $i, j = 0, 1$, with a constraint $p_{00} + p_{01} + p_{10} + p_{11} = 1$. Let us now expand $\log p(x_1, x_2)$ in the polynomial of x_1 and x_2 as follows,

$$\log p(x_1, x_2) = \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 - \psi. \tag{1}$$

This is called a log-linear expansion and it is an exact expansion since x_i takes the binary values $0, 1$. The coefficients θ's and ψ are given by,

$$\theta_1 = \log \frac{p_{10}}{p_{00}}, \quad \theta_2 = \log \frac{p_{01}}{p_{00}}, \quad \theta_3 = \log \frac{p_{11} p_{00}}{p_{01} p_{10}}, \quad \psi = -\log p_{00}. \tag{2}$$

θ_3 represents the information-geometric measure for 2 neuron interactions.

3 Model and Correlation Function

The isolated pair of model neurons was used in this study, because this simple setting allows us to investigate the system with mathematical clarity.

Following Ginzburg and Sompolinsky [4], the state of each neuron at time t takes one of two states, denoted by $S_i(t) = 0, 1$, where $i = 1, 2$. The neurons are assumed to have a stochastic nature, and the transition rates for the ith neuron takes the form

$$w(S_i \to (1 - S_i)) = \frac{1}{2\tau}\{1 - (2S_i - 1)[2g(u_i) - 1]\}, \tag{3}$$

where $g(u)$ is a monotonically increasing, differentiable function such as $g(u) = (1 + \tanh(u))/2$. The local field acting upon the ith neuron at time t is

$$u_i(t) = J_{ij} S_j(t) + h_i, \tag{4}$$

where J_{ij} denotes the connection from the jth presynaptic neuron to the ith postsynaptic one, and h_i represents the local input. In the equilibrium limit, the average activities are given by

$$\langle S_i \rangle = \langle S_j \rangle \Delta g_i + g(h_i), \tag{5}$$

where $\Delta g_i = g(J_{ij} + h_i) - g(h_i)$.

4 Results

The relationship between the information-geometric measure and the underlying neural mechanisms was elucidated in the equilibrium limit. The information-geometric measures,

$$\theta_1 = \log \frac{p_{10}}{p_{00}}, \quad \theta_2 = \log \frac{p_{01}}{p_{00}}, \quad \theta_3 = \log \frac{p_{11}p_{00}}{p_{01}p_{10}}, \tag{6}$$

were expressed by the underlying neural mechanisms,

$$p_{00} = 1 - \langle S_1 \rangle - \langle S_2 \rangle + \langle S_1 S_2 \rangle, \tag{7}$$
$$p_{01} = \langle S_2 \rangle - \langle S_1 S_2 \rangle, \tag{8}$$
$$p_{10} = \langle S_1 \rangle - \langle S_1 S_2 \rangle, \tag{9}$$
$$p_{11} = \langle S_1 S_2 \rangle, \tag{10}$$

where

$$\langle S_1 \rangle = \frac{g(h_1) + \Delta g_1 g(h_2)}{1 - \Delta g_1 \Delta g_2}, \tag{11}$$

$$\langle S_2 \rangle = \frac{g(h_2) + \Delta g_2 g(h_1)}{1 - \Delta g_1 \Delta g_2}, \tag{12}$$

$$\langle S_1 S_2 \rangle = \frac{1}{2}\{\langle S_1 \rangle g(J_{21} + h_2) + \langle S_2 \rangle g(J_{12} + h_1)\}. \tag{13}$$

For the symmetric network ($J_{21} = J_{12} = J$), the above equation reduced to $\theta_1 = 2h_1$, $\theta_2 = 2h_2$ and $\theta_3 = 2J$, and therefore the modulation in θ_3 reflects the change in the anatomical connections. On the other hand, for the asymmetric network, the underlying neural mechanisms cannot be determined uniquely, because $(\theta_1, \theta_2, \theta_3)$ are mapped from $(J_{21}, J_{12}, h_1, h_2)$. This ill-posed nature was depicted in Fig. 1.

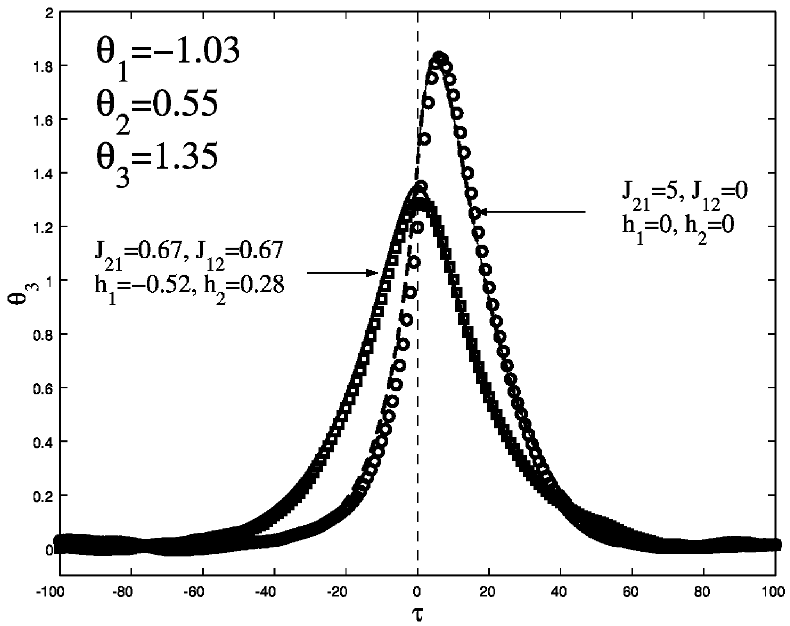

Fig. 1. Multiple neural mechanisms generate the same information-geometric measure parameters. The set of information-geometric measure parameters $(\theta_1, \theta_2, \theta_3)$=(-1.03, 0.55, 1.35) was generated from the two neural mechanisms parameters $(J_{21}, J_{12}, h_1, h_2)$=(5, 0, 0, 0) and $(J_{21}, J_{12}, h_1, h_2)$=(0.67, 0.67, -0.52, 0.28). The dotted and solid curves were obtained by theoretical calculation, and the circle and square by computer simulation.

The relationship between the information-geometric measure and the correlation function was also obtained. It was given by,

$$\begin{aligned}\theta_3(\tau) = &\log(C_{12}(\tau) + \langle S_1(t)\rangle\langle S_2(t+\tau)\rangle) \\ &+ \log(C_{12}(\tau) + 1 - \langle S_1(t)\rangle - \langle S_2(t+\tau)\rangle + \langle S_1(t)\rangle\langle S_2(t+\tau)\rangle) \\ &- \log(-C_{12}(\tau) + \langle S_1(t)\rangle - \langle S_1(t)\rangle\langle S_2(t+\tau)\rangle) \\ &- \log(-C_{12}(\tau) + \langle S_2(t+\tau)\rangle - \langle S_1(t)\rangle\langle S_2(t+\tau)\rangle),\end{aligned} \tag{14}$$

where,

$$C_{12}(\tau) = \langle (S_1(t) - \langle S_1(t) \rangle)(S_2(t+\tau) - \langle S_2(t+\tau) \rangle) \rangle, \quad \tau \geq 0, \qquad (15)$$

It is easy to show that these two measures have a nonlinear but monotonic relathionship. As was shown in Fig. 2, both measures are almost identical and therefore depict a similar feature of neural firing. However, the information geometric measure has an advantage because it is easily extended to more than three neuron interactions, it provides a biologically plausible statistical test, and it generates orthogonal decomposition correlation components.

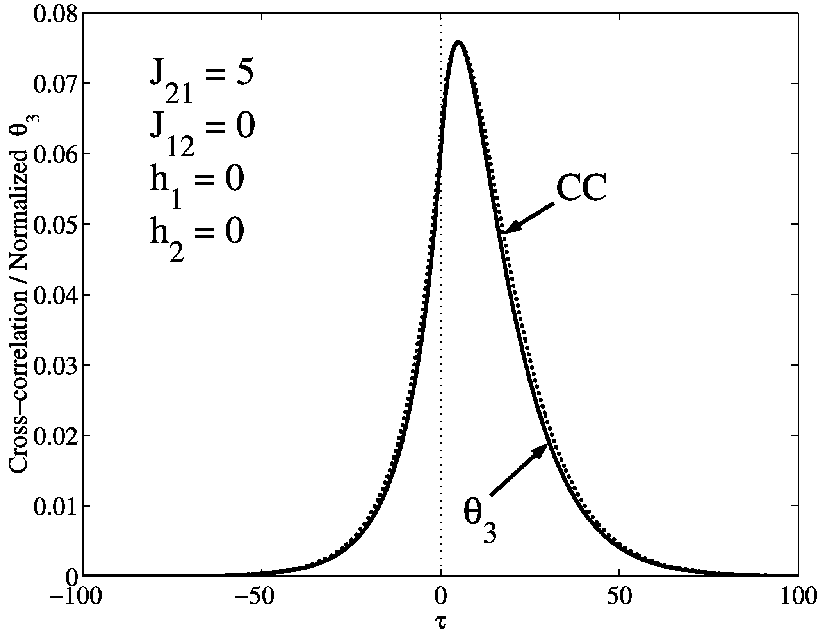

Fig. 2. Relationship between the information-geometric measure and the correlation function. Both the information-geometric measure and the correlation function were calculated for the neural mechanisms parameters $(J_{21}, J_{12}, h_1, h_2) = (5, 0, 0, 0)$. The information geometric parameter was normalized to have the same peak value with the correlation function.

5 Summary

The relationship between the information-geometric measure parameters and the possible neural mechanisms was elucidated using a pair of stochastic neuron model. The results are summarized as follows; For symmetric connections, θ_3 is only dependent on the anatomical connections ($J_{21} = J_{12}$), and θ_1 and θ_2 are only dependent on the local inputs h_1 and h_2, respectively. Therefore, the change in information-geometric parameter θ_3 reflects the change in the anatomical connections. For asymmetric connections, θ_3 is dependent both on the anatomical connections and on the local inputs. The multiple neural mechanisms $(J_{21}, J_{12}, h_1, h_2)$ can generate the same information-geometric measure parameters $(\theta_1, \theta_2, \theta_3)$, and therefore neural mechanisms cannot be determined uniquely from the information-geometric measure parameters. However, the shape of the time-delayed information-geometric measure parameters is informative to determine the direction of the connection. The relationship between the information-geometric measure parameters $\theta_3(\tau)$ and the correlation function $C_{\mathrm{ij}}(\tau)$ was also investigated, and the nonlinear but monotonic relationship was elucidated.

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