# MachLe - Olivier D'Ancona

## Evaluation Metrics

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN}$$
$$Precision = \frac{TP}{TP+FP}$$
$$Recall = \frac{TP}{TP+FN}$$
$$Specificity = \frac{TP}{TN+FP}$$
$$Fscore = \frac{2 \cdot Precision \cdot Recall}{Precision + Recall}$$
$$error\ rate = 1 - accuracy$$
$$macro\ average = \frac{1}{n}\sum_{i=1}^{n} avg_i$$

## Activation Functions

**Sigmoid** : $\sigma(x) = \frac{1}{1+e^{-x}}$

**Hyperbolic tangent** : $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

**Relu** : $\begin{cases} 0 & \text{si } x < 0 \\ x & \text{si } x \geq 0 \end{cases}$

**Gaussian** : $e^{-x^2}$

**Softmax** : $\frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}}$

## Neural Network

### Structure

**Biais** : $b$, An extra weight that can be learned using a learning algorithm. The purpose is to replace threshold.
**Input** : $I$, Input vector
**Weights** : $W$, Vector of weights

### Learning algorithm

1. Randomly initialize weights
2. Compute the neuron's output for a given input vector X
3. Update weights : $W_j(t+1) = W_j(t) + \eta\,(\hat{y_i} - y)\,x$ with $\eta$ the learning rate and $\hat{y_i}$ the desired output.
4. Repeat steps 2 and 3 for the number of epochs you need or until the error is smaller than a threshold.

## KNN

Hyperparameters :

- Number of neighbours $k$
- Distance metric
- normalization type
- strategy if no majority

Big $k$ :

## Bayes

$$P(C_k|x) = \frac{P(x|C_k) \cdot P(C_k)}{P(x)}$$

où

- $C_k$ : Classe ciblée
- $x$ : Évidence
- $P(C_k)$ : Probabilité a priori de la classe $C_k$
- $P(x|C_k)$ : probability of observing x given class j
- $P(C_k|x)$ : Probabilité a posteriori de la classe $C_k$ après observation de $x$
- $P(x)$ : Probabilité de l'évidence $x$

avec

$$P(x) = \sum_{\text{toutes classes } C_k} P(x|C_k) \cdot P(C_k)$$

### Classifier H/F:

- $P(C_f) = \frac{4}{70}, P(C_g) = \frac{66}{70}$
- $p(x|C_g) = 0.8, p(x|C_f) = 0.2$
- Calcul de $p(x)$ :

$$p(x) = 0.2 \times \frac{4}{70} + 0.8 \times \frac{66}{70}$$

- Calcul de $P(C_f|x)$ et $P(C_g|x)$ :

$$P(C_f|x) = \frac{0.2 \times \frac{4}{70}}{p(x)}, \quad P(C_g|x) = \frac{0.8 \times \frac{66}{70}}{p(x)}$$

(+)Can deal with imbalanced dataset, prior can be changed

## Linear Regression

Soit un tableau de données :
$x$ = Surface(g) , $y$ = Price(cm) , $x \cdot y$ , $x^2$

$$X = [1, Surface]$$

$$X^T X = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} = \begin{bmatrix} 7 & 38.5 \\ 38.5 & 218.95 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix} = \begin{bmatrix} 348 \\ 1975 \end{bmatrix}$$

$$\hat{\theta} = (X^T X)^{-1} X^T y = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} -2.67 \\ 9.51 \end{bmatrix}$$

$$\hat{y} = \theta_0 + \theta_1 x$$

### Matrix Inversion (2x2)

## Logistic Regression

$$h_\theta(x_n) = \sigma(x\theta^T)$$

- $h_\theta(x_n)$ : predicted value
- $\theta$ : model's parameters
- $X$ : input vector

**Goal** : Find the $\theta$ that maximizes the likelihood of the data.
**Loss** :

$$J(\beta) = -\frac{1}{n}\sum_{i=1}^{n} y_i \log(h_\theta(x_n)) + (1-y_i)\log(1-h_\theta(x_n))$$

## Normalization

**Min-max [0,1]** : $x' = \frac{(x - x_{min})}{(x_{max} - x_{min})}$

**Min-max [-1,1]** : $x' = 2 \cdot min\_max(x) - 1$
min-max doesn't handle outliers.

**Z-norm** : $x' = \frac{(x-\mu)}{\sigma}$

## Support Vector Machine

**Concept** : SVM finds the hyperplane that best separates different classes by maximizing the margin between the closest points of different classes (support vectors).
hw(x) = sign(b + wx)

### Formulation

$$\max_{\omega,b} \frac{1}{\|\omega\|} \quad \text{s.t.} \quad y_i(\omega \cdot x_i + b) \geq 1\,\forall i$$

where

- $\omega$ : Normal vector to the hyperplane
- $b$ : Bias term
- $x_i, y_i$ : Training data points and labels

### Kernel Trick

SVM can be extended to non-linearly separable data using kernel functions, which implicitly map input space to a higher-dimensional feature space

### Common Kernels

- Linear : $\langle x, x' \rangle$
- Polynomial : $(\gamma\langle x, x'\rangle + r)^d$
- Gaussian (RBF) : $e^{(-\gamma\|x-x'\|^2)}$

(+) Effective in high-dimensional spaces, Memory efficient, Versatile (different kernel functions)

(-) Sensitive to the choice of kernel and regularization parameters, Not suitable for very large datasets
hinge loss : $max(0, 1 - y_i(w \cdot x_i + b))$ (0 if correct classification) (1 if falls on the hyperplane) (>1 if misclassified)

### Objective function to min

$$\min_{\omega,b} \quad \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(\omega \cdot x_i + b))$$

where $C$ nutch the hinge loss term (how far are we predicting from ground truth) and regularization term (impeach big value, min w => maximize margin)

## Similarity Measures

$$Pearson = R^2 = 1 - \frac{\sum_1^n (y_i - \hat{y_i})^2}{\sum_1^n (y_i - \overline{y_i})^2}$$

$$Euclidian = \sqrt{\sum (I_1 - I_2)^2}$$

$$Manhattan = \sum |I_1 - I_2|$$

$$MSE = \frac{1}{N} \sum_1^n (y_i - \hat{y_i})^2$$

$$cosine\ similarity = \frac{A \cdot B}{\|A\| \cdot \|B\|}$$

$WSS = \sum_{i=1}^k \sum_{x \in C_i} d(x, \mu_i)^2$ Within-cluster-sum (distortion) is the sum of the squared distances between each point in a cluster $x_j$ and its cluster center.

## K-Means

1. Initialize $k$ centroids randomly.
2. Assign each point to the nearest centroid.
3. Recompute centroids as the mean of assigned points.
4. Repeat steps 2-3 until convergence.

minimize distortion : $J = \sum_{i=1}^k d(x_n, \mu_c)$

(+) Will converge
**(-)** Sensitive to initial conditions(size, density, distribution), Finds a local optimum

## Mean Shift Clustering

1. Choose bandwidth and initialize centroids.
2. Shift each centroid to the mean of points within the bandwidth.
3. Repeat until centroids converge.

(+) Can find clusters of arbitrary shape; robust to outliers.
**(-)** Computationally intensive; bandwidth parameter can be tricky to set.

## DB-Scan

1. Classify points as core, border, or noise based on density.
2. Form clusters around core points.
3. Assign border points to clusters or mark as noise.

(+) Identifies clusters of varying shapes; robust to noise.
**(-)** Sensitive to parameters; struggles with varying density clusters.

## Hierarchical Clustering

**Algorithm (Agglomerative)** :

1. Start with each point as a separate cluster.
2. Merge the closest pair of clusters.
3. Repeat step 2 until desired number of clusters is reached.

(+) No need to specify the number of clusters; intuitive dendrogram representation.
**(-)** Computationally expensive for large datasets; sensitive to outliers.

## Clustering

**Clustering** partitions data into clusters with high intra-class similarity and low inter-class similarity.
**Needs** : distance measure, criterion, algorithm.

### Partitions

**Distortion** : How close are we to a "centroid" defining the partition?
**Connectivity of points** : How close are points to each other?

### Elbow Method

Heuristic used in determining the number of clusters in a data set. It selects the value of $k$ that corresponds to the elbow of the curve (#cluster, WSS)

### Silhouette Coefficient

$$s = \frac{b - a}{\max(a, b)}$$

- $a$ is the mean distance between a sample and all other points in the same class (cohesion)
- $b$ is the mean distance between a sample and all other points in the next nearest cluster (isolation)

$s$ range is $[-1, 1]$ . A high value indicates that the object is well matched to its own cluster and poorly matched to neighboring clusters.

### Davies-Bouldin Index

$$DB = \frac{1}{k} \sum_{i=1}^k \max_{j \neq i} \frac{R_i + R_j}{d(C_i, C_j)}$$

- $R_i$ is the average distance between a point in cluster $C_i$ and all points in $C_i$ (cluster diameter))
- $d(C_i, C_j)$ is the distance between the centroids of $C_i$ and $C_j$

zero is the lowest possible score. Values closer to zero indicate a better partition.

## Decision Tree

A flowchart-like structure in which each internal node represents a test on a feature. Each leaf node represents a class label(decision taken after computing all features).

**Entropy** :

$$H(X) = -\sum_{i=1}^n p(x_i) \log_2 p(x_i)$$

where

- $p(x_i)$ : Probability of class $x_i$

**Information Gain** :

$$IG(X, Y) = H(X) - H(X|Y)$$

where

- $H(X)$ : Entropy of the parent node
- $H(X|Y)$ : Entropy of the child node

**Gini Impurity of a set** :
$G(X) = 1 - \sum_{i=1}^n p(x_i)^2$ where

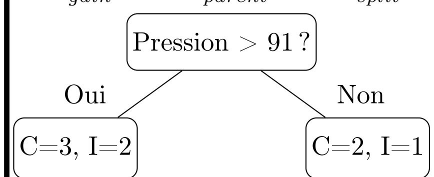
- $p(x_i)$ : is the proportion of points in a set that belongs to a class $i$ : $\frac{N_i}{N}$.

**G = 0.5** : maximum value of impurity, classes are balanced in the set.
**G = 0** : minimum value of impurity, all the values belong to a single class.

**Gini Impurity of a Split** :

$$Gini_{split} = \sum_{i=1}^n \frac{N_i}{N} G(X_i)$$

$$Gini_{gain} = Gini_{parent} - Gini_{split}$$




Première feuille :

$$Gini = 1 - \left(\frac{3}{3+2}\right)^2 - \left(\frac{2}{3+2}\right)^2 = \frac{12}{25}$$

Deuxième feuille :

$$Gini = 1 - \left(\frac{2}{2+1}\right)^2 - \left(\frac{1}{2+1}\right)^2 = \frac{4}{9}$$

**CART Algorithm** :

- Select the best attribute using IG or Gini
- Make that attribute a decision node and break the dataset into smaller subsets
- Recursively repeat the process on each

## Principal Component Analysis

### Kmeans

### Hierarchical clustering

## Autoencoders

### Kmeans

### Hierarchical clustering

## Convolutional Neural Networks

## Recurrent Neural Networks

## Dimensionality Reduction

## Reinforcement Learning

## Computational Complexity of ML Algorithms

| Algorithm | Assumption | Train Time/Space | Inference Time/Space |
|---|---|---|---|
| KNN (Brute Force) | Similar things exist in close proximity | $O(knd)$ / $O(nd)$ | $O(knd)$ / $O(nd)$ |
| KNN (KD Tree) | Similar things exist in close proximity | $O(nd\log(n))$ / $O(nd)$ | $O(k\log(n)d)$ / $O(nd)$ |
| Naive Bayes | Features are conditionally independent | $O(ndc)$ / $O(dc)$ | $O(dc)$ / $O(dc)$ |
| Logistic Regression | Classes are linearly separable | $O(nd)$ / $O(nd)$ | $O(d)$ / $O(d)$ |
| Linear Regression | Linear relationship between variables | $O(nd)$ / $O(nd)$ | $O(d)$ / $O(d)$ |
| SVM | Classes are linearly separable | $O(n^2d^2)$ / $O(nd)$ | $O(kd)$ / $O(kd)$ |
| Decision Tree | Feature selection by information gain | $O(n\log(n)d)$ / $O(\text{nodes})$ | $O(\log(n))$ / $O(\text{nodes})$ |
| Random Forest | Low bias and variance trees | $O(kn\log(n)d)$ / $O(\text{nodes} \times k)$ | $O(k\log(n))$ / $O(\text{nodes} \times k)$ |
| GBDT | High bias, low variance trees | $O(Mn\log(n)d)$ / $O(\text{nodes} \times M + \gamma_m)$ | $O(M\log(n))$ / $O(\text{nodes} \times M + \gamma_m)$ |