

$$\begin{aligned} \text{a) } \sigma'(x) &= \frac{d\sigma(x)}{dx} = \frac{d}{dx} \frac{1}{1+e^{-x}} = \frac{d}{dx} (1+e^{-x})^{-1} = -(1+e^{-x})^{-2} \frac{d}{dx} (1+e^{-x}) \\ &= -(1+e^{-x})^{-2} \left( 0 + \frac{de^{-x}}{dx} \right) = -(1+e^{-x})^{-2} \cdot e^{-x} \frac{d(-x)}{dx} = -(1+e^{-x})^{-2} \cdot (-e^{-x}) \\ &= (1+e^{-x})^{-2} \cdot e^{-x} = \frac{e^{-x}}{(1+e^{-x})^2} \end{aligned}$$

$$\text{b) } \sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \underbrace{\frac{1}{1+e^{-x}}}_{=\sigma(x)} \frac{e^{-x}}{1+e^{-x}} = \sigma(x) \frac{1+e^{-x}}{1+e^{-x}} - \underbrace{\frac{1}{1+e^{-x}}}_{\sigma(x)} = \sigma(x) (1 - \sigma(x))$$