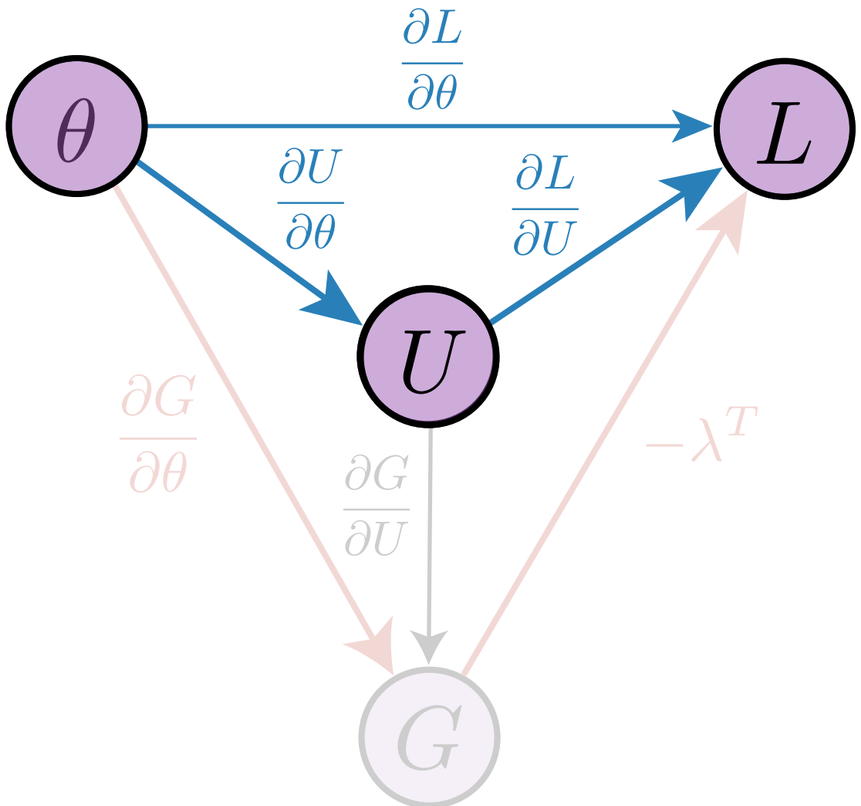
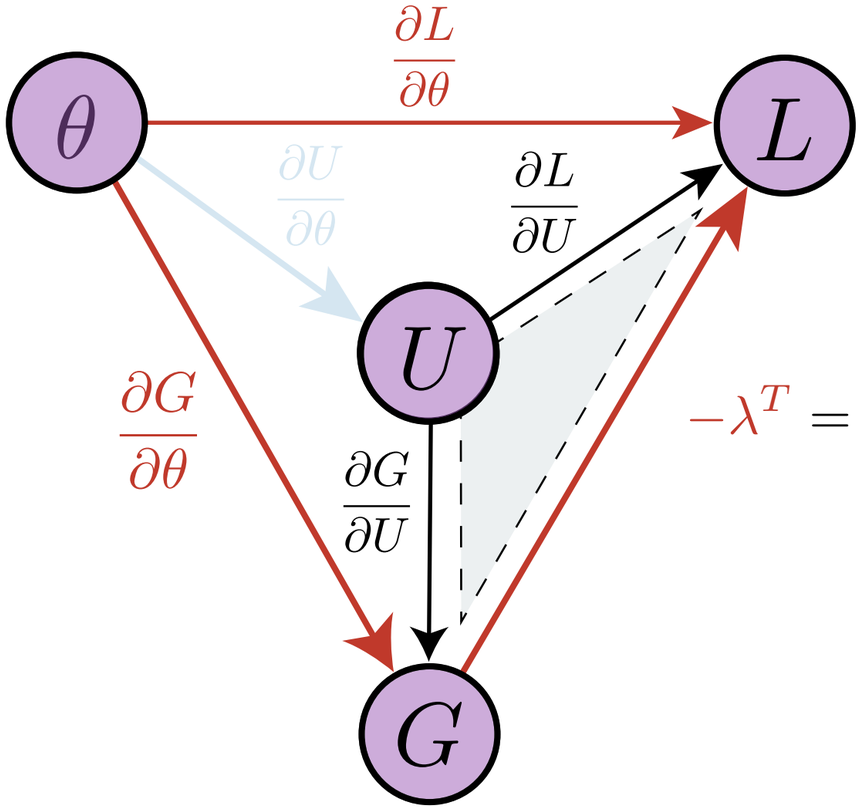
Direct gradient calculation



$$\frac{dL}{d\theta} = \frac{\partial L}{\partial \theta} + \frac{\partial L}{\partial U}\frac{\partial U}{\partial \theta}$$

Gradient based on adjoint



$$-\lambda^T = \frac{\partial L}{\partial G} = \frac{\partial L}{\partial U}\frac{\partial U}{\partial G}$$
$$= \frac{\partial L}{\partial U}\left(\frac{\partial G}{\partial U}\right)^{-1}$$

$$\frac{dL}{d\theta} = \frac{\partial L}{\partial \theta} + \frac{\partial L}{\partial G}\frac{\partial G}{\partial \theta} = \frac{\partial L}{\partial \theta} - \lambda^T\frac{\partial G}{\partial \theta}$$