Practice Exam 1 — MATH 426

Name: ______________________________

(1) Let $A = \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ -1 & -2 & 1 \end{pmatrix}$, $x = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$. Perform the following operations. If the operation cannot be performed explain why not.
  (a) Ax
  (b) $x^T x$
  (c) xx^T
  (d) $x^T Ax$

(2) Let $A = \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ -1 & -2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1- & 0 & 4 \\ -2 & 1 & 2 \\ 0 & -2 & 1 \end{pmatrix}$. Perform the following operations. If the operation cannot be performed explain why not.
  (a) AB
  (b) BA
  (c) A^T
  (d) BA^T

(3) Given that $A = \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ -1 & -2 & 1 \end{pmatrix}$,$B = \begin{pmatrix} -1- & 0 & 4 \\ -2 & 1 & 2 \\ 0 & -2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -1- & 1 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 1 \end{pmatrix}$
$E = \begin{pmatrix} -2- & 1 & 4 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{pmatrix}$. Answer the following questions:
  (a) Multiply $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} C & 0 \\ 0 & D \end{pmatrix}$ in an efficient manner
  (b) Multiply $\begin{pmatrix} A^T & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} C^T & 0 \\ 0 & D \end{pmatrix}$ in an efficient manner
  (c) Multiply $\begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix} \begin{pmatrix} B & 0 \\ 0 & D \end{pmatrix}$ in an efficient manner
  (d) Multiply $\begin{pmatrix} A & 0 \\ 0 & B^T \end{pmatrix} \begin{pmatrix} D & 0 \\ 0 & C \end{pmatrix}$ in an efficient manner

(4) What is the flop count if we take a number and multiply by another number and then divide by 3?

(5) If we derive an algorithm that is $\mathcal{O}(n^3)$ that solves the system $Ax = b$. How would the length of time (approximately) to compute x be effected if instead of a 100 by 100 matrix A we had a 300 by 300 matrix A?

(6)

(7) Given $x = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 4 \end{pmatrix}$. Calculate the following:
  (a) $\|x\|_1$
  (b) $\|x\|_2$
  (c) $\|x\|_\infty$

(8) Given $A = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}$. Find the following.
  (a) $\|A\|_1$

(b) $\|A\|_2$
(c) $\|A\|_\infty$

(9) Given $A = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}$ and $\kappa_p(A)$ is the condition number using the p-norm. Find the following.
(a) $\kappa_1(A)$
(b) $\kappa_2(A)$
(c) $\kappa_\infty(A)$

(10) Solve the system of equations $Ax = b$ using Gaussian elimination:
(a) $A = \begin{pmatrix} 2 & 1 \\ 4 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
(b) $A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 0 & 1 \\ 1 & -2 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$
(c) $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ 2 & -2 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

(11) Assume that we know $A = LU$ is a LU-decomposition and we want to solve the system $Ax = b$. Answer the following functions:
(a) What are the conditions on L and U that make the decomposition unique?
(b) The system $Ly = b$ is solved using what kind of substitution? And the system $Ux = b$ is solved using was kind of substitution?
(c) What must be true about A to guarantee we can get this kind of decomposition (Gaussian elimination without pivot)?

(12) Solve the system of equations $Ax = b$ using an LU-decomposition. To receive full credit you must clearly state the algorithm used to find L and U.
(a) $A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
(b) $A = \begin{pmatrix} 6 & 18 & 3 \\ 2 & 12 & 1 \\ 4 & 15 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$
(c) $A = \begin{pmatrix} -1 & 1 & 2 \\ -2 & 4 & 7 \\ -3 & 11 & 23 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

(13) Given that $A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 5 & 6 \\ 2 & 6 & 45 \end{pmatrix}$. Use this matrix to answer the following:
(a) Find R such that $A = R^T R$ (and R is upper triangular) using the inner product formulation.
(b) Find R such that $A = R^T R$ (and R is upper triangular) using the outer product formulation.
(c) Find R such that $A = R^T R$ (and R is upper triangular) using the bordered formulation.