Practicing problems for the final exam for MTH 202, Spring 2018,

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1.1 Four Ways to Represent a Function:

1) The graph of a function f is given.

(a) State the value of $f(-1)$.

(b) For what values of x is $f(x) = 2$?

(c) State the domain and range of f.

(d) On what interval is f increasing?

(e) On what interval is f decreasing?

2.2 The limit of a function:

1) Explain in your own words what is meant by the equation $\lim_{x\to 2} f(x) = 5$. Is it possible for this statement to be true and yet $f(2) = 3$? Explain.

2) Explain what is meant to say that $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 7$. In this situation is it possible that $\lim_{x\to 1} f(x)$ exists? Explain.

2.3 Calculating limits using limit laws:

2) The graph of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

(a) $\lim_{x\to 2}[f(x) + g(x)]$ **(b)** $\lim_{x\to 1}[f(x) + g(x)]$ **(c)** $\lim_{x\to 0}[f(x)g(x)]$

(d) $\lim_{x\to -1} \frac{f(x)}{g(x)}$ **(e)** $\lim_{x\to 2}[x^3 f(x)]$ **(f)** $\lim_{x\to 1} \sqrt{3 + f(x)}$

10, 13, 14, 15, 16, 18,19) Evaluate the limit, if exists

$\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4}$ $\lim_{x\to 4} \frac{x^2+5x+4}{x^2+3x-4}$ $\lim_{t\to -3} \frac{t^2-9}{2t^2+7t+3}$

$\lim_{x\to -1} \frac{x^2-4x}{x^2-3x-4}$ $\lim_{h\to 0} \frac{(4+h)^2-16}{h}$ $\lim_{x\to -1} \frac{x^2+2x+1}{x^4-1}$

$\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7}$

25) Use the Squeeze Theorem to show that $\lim_{x\to 0} x^2 \cos(20\pi x) = 0$.

27) If $4x - 9 \le f(x) \le x^2 - 4x + 7$ for $x \ge 0$, find $\lim_{x\to 4} f(x)$.

2.4 Continuity:

26) Use continuity to evaluate $\lim_{x\to\pi} \sin(x + \sin x)$

2.5 Limits involving infinity:

4) For the function g below, state the following:

(a) $\lim_{x\to\infty} g(x)$ **(b)** $\lim_{x\to-\infty} g(x)$

(c) $\lim_{x\to3} g(x)$ **(d)** $\lim_{x\to0} g(x)$

(e) $\lim_{x\to-2^+} g(x)$

(f) The equation of the asymptotes.

6,9,10) Sketch the graph of a function that satisfies the following conditions:

 # 6) $\lim_{x\to0^+} f(x) = \infty$ $\lim_{x\to0^-} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = 1$ $\lim_{x\to-\infty} f(x) = 1$

 # 9) $f(0) = 3$ $\lim_{x\to0^-} f(x) = 4$ $\lim_{x\to0^+} f(x) = 2$ $\lim_{x\to-\infty} f(x) = -\infty$
$\lim_{x\to4^-} f(x) = -\infty$ $\lim_{x\to4^+} f(x) = \infty$ $\lim_{x\to\infty} f(x) = 3$

 # 10) $\lim_{x\to3} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = 2$ $f(0) = 0,$ f is even.

15, 17, 20, 22, 24, 31, 33) Find the limit.

 # 15) $\lim_{x\to-3^+} \frac{x+2}{x+3}$ **# 17)** $\lim_{x\to1} \frac{2-x}{(x-1)^2}$ **# 20** $\lim_{x\to\infty} \frac{3x+5}{x-4}$

 # 22 $\lim_{t\to-\infty} \frac{t^2+2}{t^3+t^2-1}$ **# 24** $\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}}$

 # 31 $\lim_{x\to-\infty}(x^4 + x^5)$ **# 33** $\lim_{x\to\infty} \frac{x+x^3+x^5}{1-x^2+x^4}$

2.7 Derivatives:

7) If $f(x) = 3x^2 - 5x$, find $f'(2)$ and use it to find an equation of the tangent line to the curve $y = 3x^2 - 5x$ at the point $(2, 2)$.

13) Find $f'(a)$ for $f(x) = 3 - 2x + 4x^2$.

2.8 The Derivative as a Function:

19, 20, 23) Find the derivative of each of the following functions using the definition of derivative. State the domain of the function and the domain of its derivative.

 # 19. $f(x) = \frac{1}{2}x - \frac{1}{3}$ **# 20.** $f(x) = 1.5x^2 - x + 3.7$ **# 23.** $g(x) = \sqrt{1+2x}$

2.9 What Does f' Say about f?:

11) The graph of f' of a function f is shown below.

(a) On what interval f is increasing or decreasing?

(b) At what values of x does f have a local maximum or minimum?

(c) On what interval f is concave upward or downward?

(c) State the x-coordinate(s) of the point(s) of inflection.

(e) If it is known that $f(0) = 0$, sketch a possible graph of f.

16) Sketch the graph of a function that satisfies all of the given conditions.

$f'(x) > 0$ for all $x \neq 1$, vertical asymptote $x = 1$, $f''(x) > 0$ if $x < 1$ or $x > 3$, $f''(x) < 0$ if $1 < x < 3$.

21) Suppose $f'(x) = xe^{-x^2}$.

(a) On what interval f is increasing? On what interval f is decreasing?

(b) Does f have a maximum of minimum value?

23) Let $f(x) = x^3 - x$. Use the facts that $f'(x) = 3x^2 - 1$ and $f''(x) = 6x$ to find:

(a) The intervals on which f is increasing or decreasing.

(b) The intervals on which f is concave upward or downward.

(c) The inflection points of f.

3.1 Derivatives and Polynomials and Exponential Functions:

3,4,6,11,14,19,21) Differentiate the function:

 # 3) $f(x) = 186.5$ **# 4)** $f(x) = \sqrt{30}$ **# 6)** $f(t) = \frac{1}{2}t^6 - 3t^4 + t$

 # 11) $G(x) = \sqrt{x} - 2e^x$ **# 14)** $R(x) = \frac{\sqrt{10}}{x^7}$ **# 19)** $y = \frac{x^4+4x+3}{\sqrt{x}}$

 # 21) $v = t^2 - \frac{1}{\sqrt[4]{t^3}}$

25) Find the equation of the tangent line and normal line to the curve $y = x^4 + e^x$ at the point $(0, 2)$.

44) On what interval is the function $f(x) = x^3 - 4x^2 + 5x$ concave upward?

3.2 The Product and Quotient Rules:

4, 5,6,10, 18) Differentiate:

#4. $g(x) = \sqrt{x}e^x$ **#5.** $y = \frac{e^x}{x^2}$ **#6.** $y = \frac{e^x}{1+x}$

#10. $R(t) = (t + e^t)(3 - \sqrt{t})$ **#18.** $f(x) = \frac{1-xe^x}{x+e^x}$

3.3 Rules of Changes in the Natural and Social Sciences:

1) A particle is moving according to a law of motion $s = f(t) = t^3 - 12t^2 + 36t$, where s is in meters and t in seconds.

(a) Find the velocity and acceleration as functions of t.

(b) When is the particle moving forward?

(c) When is the particle at rest?

(d) Find the acceleration after $5s$.

(e) Find the total distance traveled and displacement during the first $8s$.

(g) When is the particle speeding up? When is it slowing down?

3.4 Derivatives of Trigonometric Functions:

#1, 3,4,8,) Differentiate:

 # 1 $f(x) = \sin x + \frac{1}{2}\cot x$ **# 4** $g(x) = \sqrt{x}\sin x$ **# 8** $y = \frac{1+\sin x}{x+\cos x}$

 # 19-a Find an equation of the tangent line to the curve $y = x\cos x$ at $(\pi, -\pi)$.

3.5 The Chain Rule:

 # 1. $y = \sin(4x)$ **#3.** $y = (1 - x^2)^{10}$ **# 6** $y = \sin(e^x)$

7, 9,10,13,16,17,27,29) Find the derivative of the function:

 # 7. $F(x) = \sqrt[4]{1 + 2x + x^3}$ **# 9.** $g(t) = \frac{1}{(t^4+1)^3}$ **# 10.** $f(t) = \sqrt[3]{1 + \tan t}$

 # 13. $h(t) = t^3 - 3^t$ **# 16.** $y = e^{-5x}\cos(3x)$

 # 17. $g(x) = (1+4x)^5(3+x-x^2)^8$ **# 27.** $y = 2^{\sin \pi x}$ **# 29.** $y = \cot^2(\sin\theta)$

35) Find an equation of the tangent line to the curve $y = (1 + 2x)^{10}$ at the point $(0, 1)$.

39-(a)) If $f(x) = x\sqrt{2 - x^2}$, find $f'(x)$.

3.6 Implicit Differentiation:

3, 8, 11) Find dy/dx by implicit differentiation.

 # 3. $x^3 + x^2y + 4y^2 = 6$ **# 8.** $1 + x = sin(xy^2)$ **# 11.** $e^{\frac{x}{y}} = x - y$

15) Use implicit differentiation to find an equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

29, 30, 31) Find the derivative of the function. Simplify where possible.

 # 29. $y = \tan^{-1}(\sqrt{x})$ **# 30.** $y = \sqrt{\tan^{-1}(x)}$ **# 31.** $y = \sin^{-1}(2x + 1)$

3.7 Derivatives of Logarithmic Functions:

2,3,4,7, 10,18) Differentiate the function:

#2 $f(x) = \ln(x^2 + 1)$ **#3** $f(\theta) = \ln(\cos\theta)$

#4 $f(x) = \cos(\ln x)$ **#7** $f(x) = \log_2(1 - 3x)$

#10 $f(t) = \frac{1+\ln t}{1-\ln t}$ **#18** $y = [\ln(1 + e^x)]$

23) Find an equation of the tangent line to the curve $y = \ln(x^2 - 3)$ at the point $(2, 0)$.

27, 31) Use the logarithmic differentiation to find the derivative of the function.

 # 27 $y = (2x + 1)^5(x^4 - 3)^6$ **#31** $y = x^x$

3.8 Linear approximations and derivatives :

5,6) Find the linearization $L(x)$ of the function at a.

#5. $f(x) = x^4 + 3x^2$, $a = -1$ **#6.** $f(x) = \ln x, a = 1$

9) Find the linear approximation of the function $f(x) = \sqrt{1 - x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$.

17) Use a linear approximation to estimate **#17.**$(8.06)^{\frac{2}{3}}$

4.1 Related Rates:

1) If V is the volume of a cube with edge length x and the cube expands as time passes, find dV/dt in terms of dx/dt.

3) Each side of a square is increasing at a rate of $6cm/s$. At what rate is the area of the square increasing when the area of the square is $16cm^2$.

5) If $y = x^3 + 2x$ and $dx/dt = 5$, find dy/dt when $x = 2$.

9) If a snowball melts so that its surface area decreases at a rate of $1cm^2/min$, find the rate at which the diameter decreases when the diameter is $10cm$.

4.2 Maximum and Minimum Values:

8) Sketch the graph of a function f that is continuous on $[1,5]$ such that f has absolute minimum at 1, absolute maximum at 5, local maximum at 2 and local minimum at 4.

12-(b)) Sketch the graph of a function on $[-1,2]$ that has a local maximum but no absolute maximum.

13) (a) Sketch the graph of a function on $[-1,2]$ that has an absolute maximum but no absolute minimum.
(b) Sketch the graph of a function on $[-1,2]$ that is discontinuous but has both an absolute maximum and an absolute minimum.

23,24,29,32,35) Find the critical numbers of the following functions.

23. $f(x) = 5x^2 + 4x$ **24.** $f(x) = x^3 + x^2 - x$ **29.** $g(y) = \frac{y-1}{y^2-y+1}$

32. $G(x) = \sqrt[3]{x^2 - x}$ **35.** $f(x) = x \ln x$

37,38,43,46,48) Find the absolute maximum and absolute minimum values of f on the given interval. **37.** $f(x) = 3x^2 - 12x + 5,\ [0,3]$ **38** $f(x) = x^3 - 3x + 1,\ [0,3]$ **43** $f(t) = t\sqrt{4-t^2},\ [-1,2]$ **46.** $f(x) = x - 2\cos x,\ [-\pi,\pi]$ **48.** $f(x) = x - \ln x,\ [\frac{1}{2}, 2]$

4.3 Derivatives and the Shape of Curves:

#7,9,11) **(a)** Find the intervals on which is the function increasing or decreasing.

(b) Find the local maximum and minimum values of the function.

(c) Find the intervals of concavity and the inflection points.
7. $f(x) = x^3 - 12x + 1$ **9.** $f(x) = x - 2\sin x,\ 0 < x < 3\pi$ **11.** $f(x) = xe^x$.

24, 26) **(a)** Find the intervals of increase or decrease.

(b) Find the local maximum and minimum values.

(c) Find the intervals of concavity and the inflection points.

(d) Use the information from parts (a)-(c) to sketch the graph.

24. $B(x) = 3x^{2/3} - x$ **26.** $f(x) = \ln(x^4 + 27)$.

30, 34, & more) **(a)** Find the vertical and horizontal asymptotes.
(b) Find the intervals of increase or decrease.

(c) Find the local maximum and minimum values.

(d) Find the intervals of concavity and the inflection points.

(e) Use the information from parts (a)-(d) to sketch the graph.

30. $f(x) = \frac{x^2}{(x-2)^2}$ **34.** $f(x) = \frac{e^x}{1+e^x}$

$f(x) = \frac{x}{(x-1)^2}$ $f(x) = \frac{x^2+1}{x+2}$ $f(x) = \frac{x^2}{x^2-1}$ $f(x) = \frac{2x}{x^2-1}$

4.5 Intermediate Forms and 1'Hospital's Rule:

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

(a)$\lim_{t\to 0}\frac{e^t-1}{t^3}$ **(b)**$\lim_{x\to\infty}\frac{e^x}{x^3}$ **(c)**$\lim_{x\to 0^+} x^{x^2}$

4.6 Optimization Problems:

2) Find two numbers whose difference is 100 and whose product is minimum.

3) Find two positive numbers whose product is 100 and whose sum is minimum.

6) Find the dimension of a rectangle with area 1000 cm^2 whose perimeter os as small as possible.

9) If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

10) A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimension of the box that minimize the amount of material used.

4.9 Antiderivatives:

2,5,7,9,10,11) Find the most general antiderivative of the function (check your answer by differentiation).

2. $1 - x^3 + 12x^5$ **5.** $f(x) = \sqrt[3]{x} + \frac{5}{x^6}$ **7.** $f(u) = \frac{u^4+3\sqrt{u}}{u^2}$

9.$g(\theta) = \cos\theta - 5\sin\theta$ **10.** $f(x) = 3e^x + 7\sec^2 x$ **11.** $f(x) = 2x + 5(1-x^2)^{-1/2}$

16, 21, 24) Find f.

16. $f''(x) = 2 + x^3 + x^6$ **21.** $f'(t) = 2\cos t + \sec^2 t$, $-\pi/2 < t < \pi/2$, $f(\pi/3) = 4$

24. $f''(x) = 4 - 6x - 40x^3$, $f(0) = 2$, and $f'(0) = 1$

3) A particle moves with acceration function $a(t) = 5 + 4t - 2t^2$. Its initial velocity is $v(0) = 3m/s$ and its initial displacement is $s(0) = 10$. Find its position after t seconds.

5.2 The Definite Integrals:

31) The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

(a) $\int_0^2 f(x)dx$ **(b)** $\int_0^5 f(x)dx$ **(c)** $\int_5^7 f(x)dx$ **(d)** $\int_0^9 f(x)dx$

34, 37) Evaluate the integrals $\int_{-1}^{2} \sqrt{4-x^2}dx$ and $\int_{-1}^{2} |x|dx$ by interpreting it in terms of area.

39) Given that $\int_4^9 \sqrt{x}dx = \frac{38}{3}$, what is $\int_9^4 \sqrt{t}dt$?

40) Evaluate $\int_1^1 x^2 \cos x dx$.

5.3 Evaluating Definite Integrals:

2,4,8,14,23,24) Evaluate the integral.

(1) $\int_{-1}^{3} x^5 dx$ **(4)** $\int_{-2}^{0} (u^5 - u^3 + u^2)du$ **(8)** $\int_{\pi}^{2\pi} \cos\theta d\theta$

(14) $\int_1^9 \frac{3x-2}{\sqrt{x}}dx$ **(23)** $\int_{-1}^{1} e^{u+1}du$ **(24)** $\int_0^1 (1+x^2)^3 dx$

#29) What is wrong with the equation $\int_{-1}^{3} \frac{1}{x^2}dx = \frac{x^{-1}}{-1}\Big]_{-1}^{3} = -\frac{4}{3}$?

35) Evaluate the integral $\int_{-1}^{2} x^3 dx$ and interpret it as a difference of areas. Illustrate with a sketch.

37) Verify by differentiation that the formula $\int \frac{x}{\sqrt{x^2+1}}dx = \sqrt{x^2+1} + C$ is correct.

#41) Find the general indefinite integral $\int (1-t)(2+t^2)dt$.

5.4 The Fundamental Theorem of Calculus:

7,8,10,12,14,15) Use Part 1 of The Fundamental Theorem of Calculus to find the derivative of the function.

(7) $g(x) = \int_0^x \sqrt{1+2t}dt$ **(8)** $g(x) = \int_1^x \ln t dt$ **(10)** $F(x) = \int_x^{10} \tan\theta d\theta$

(12) $h(x) = \int_0^{x^2} \sqrt{1+r^3}dr$ **(14)** $y = \int_{e^x}^{0} \sin^3 t dt$ **(15)** $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}du$