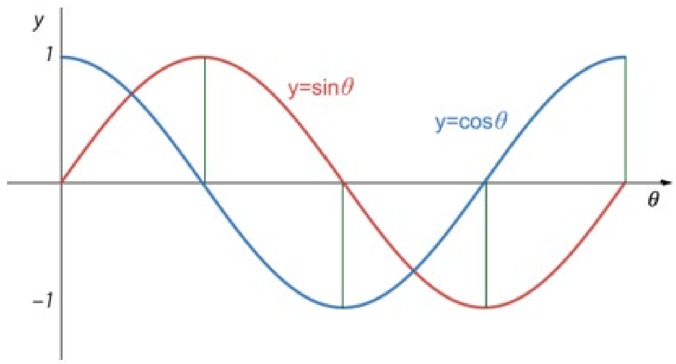
5.5 Amplitude and Period of the Sine and Cosine Functions

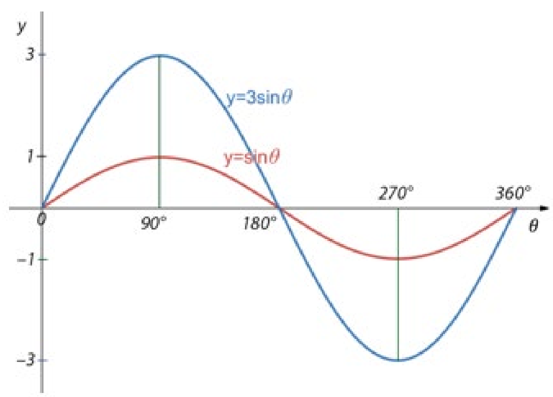
AMPLITUDE

We have seen how the graphs of both the sine function, $y = \sin\theta$ and the cosine function $y = \cos\theta$, oscillate between -1 and $+1$. That is, the heights oscillate between –1 and 1.

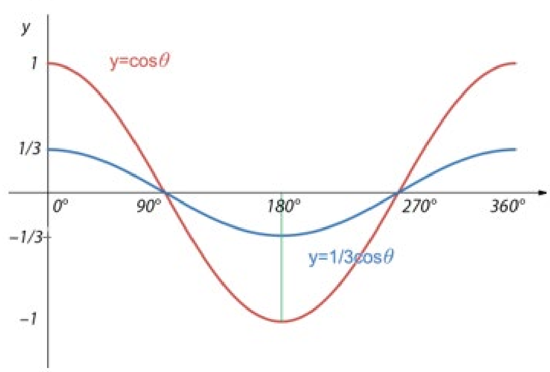


The height from the horizontal axis to the peak (or through) of a sine or cosine function is called the **amplitude** of the function. Each of the curves $y = \sin\theta$ and $y = \cos\theta$ has amplitude 1.

If we were to multiply the sine function $y = \sin\theta$ by 3, getting $y = 3\sin\theta$, each of the sine values would be multiplied by 3 making each value 3 times what it was. Each height would be tripled. The amplitude of $y = 3\sin\theta$ is 3.



If we were to multiply the cosine function $y = \cos\theta$ by $1/3$, getting $y = 1/3\cos\theta$, each of the cosine values would be multiplied by 1/3 making each value 1/3 of what it was. Each height of $y = \cos\,\theta$ would be 1/3 of what it was. The amplitude of $y = 1/3\cos\theta$ is 1/3.



THE AMPLITUDE OF $y = A\sin\theta$ AND $y = A\cos\theta$

Suppose A represents a positive number. Then the **amplitude** of both $y = A\sin\theta$ and $y = A\cos\theta$ is A and it represents height from the horizontal axis to the peak of the curve.

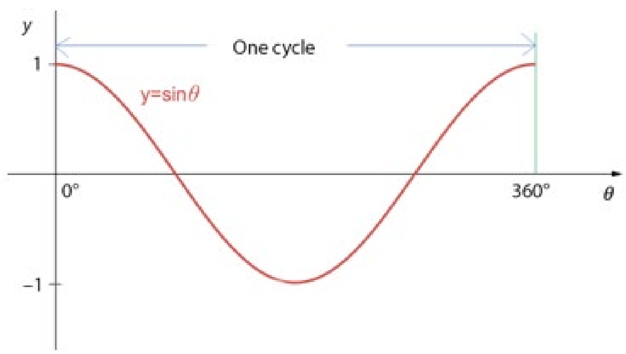
Examples

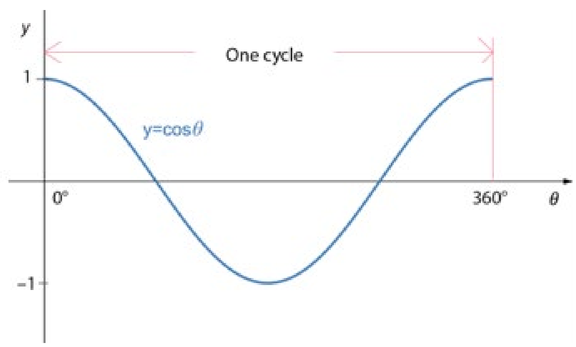
The amplitude of $y = 5/8\sin\theta$ is 5/8. This means that the peak of the curve is 5/8 of a unit above the horizontal axis.

The amplitude of $y = 3\sin\theta$ is 3. This means that the peak of the curve is 3 units above the horizontal axis.

PERIOD

Both the sine function and cosine function, $y = \sin\theta$ and $y = \cos\theta$, go through exactly one cycle from 0° to 360°. The **period** of the sine function and cosine functions, $y = \sin\theta$ and $y = \cos\theta$, is the "time" required for one complete cycle.



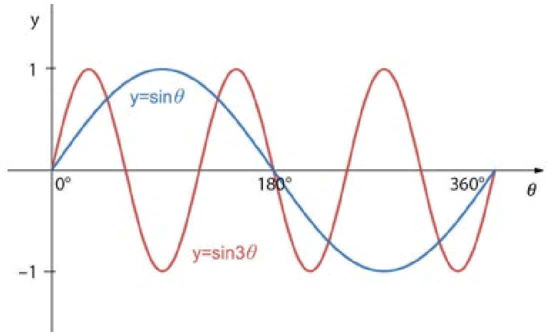


An interesting thing happens to the curves $y = \sin\theta$ and $y = \cos\theta$ when the angle θ is multiplied by some positive number, B. If the number B is greater that 1, the number of cycles on 0° to 360° increases for both $y = \sin\theta$ and $y = \cos\theta$. That is, the peaks of the curve are closer together, meaning their periods decrease. If the number B is strictly between 0 and 1, the peaks of the curve are farther apart, meaning their periods increase.

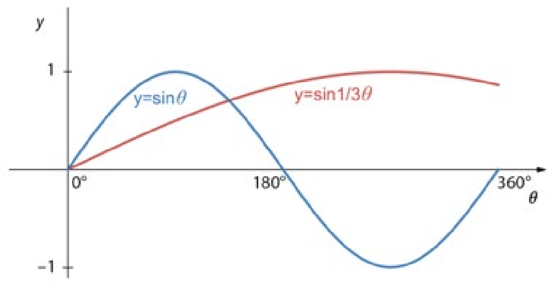
THE PERIOD OF $y = \sin(B\theta)$ AND $y = \cos(B\theta)$

Suppose B represents a positive number. Then the **period** of both $y = \sin(B\theta)$ and $y = \cos(B\theta)$ is $\frac{360°}{B}$. As B gets bigger, $\frac{360°}{B}$ gets smaller and the period increases.

If we were to multiply the angle in the sine function $y = \sin\theta$ by 3, getting $y = \sin 3\theta$, each of the angle's values would be multiplied by 3 making each value 3 times what it was. Each angle would be tripled and there would be 3 cycles in the interval 0° to 360°.
The period of $y = \sin 3\theta$ is $\frac{360°}{3} = 120°$.
The period of $y = \sin 3\theta$ is smaller than that of $y = \sin\theta$.



If we were to multiply the angle in the sine function $y = \sin\theta$ by $1/3$, getting $y = \sin\left(\frac{1}{3}\theta\right)$. Each of the angle's values would be multiplied by 1/3 making each value 1/3 what it was and there would be only 1/3 of a cycle in the interval 0° to 360°.
The period of $y = \sin\left(\frac{1}{3}\theta\right)$ is $\frac{360°}{1/3} = 360° \times 3 = 1080°$.
The period of $y = \sin\left(\frac{1}{3}\theta\right)$ is greater than that of $y = \sin\theta$.



USING TECHNOLOGY

We can use technology to help us construct the graph of a sine or cosine function.

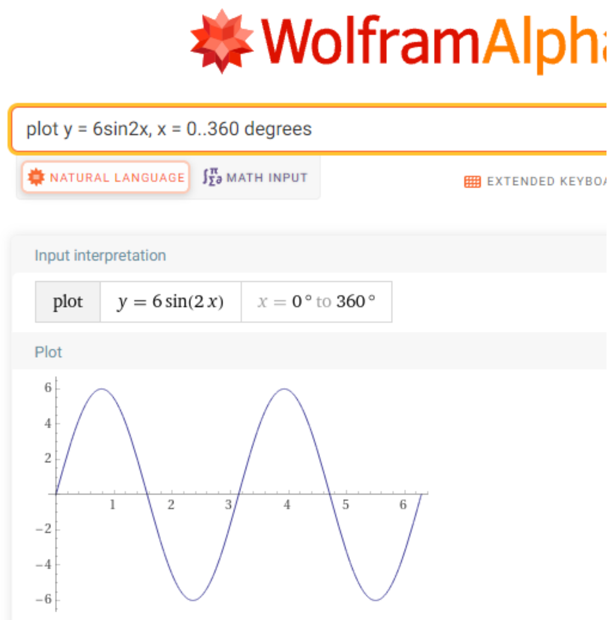
Go to www.wolframalpha.com.

Example (1)

Plot two complete cycles of $y = 6\sin 2\theta$ from 0° to 360°.

Type plot y = 6sin2x, x = 0..360 degrees in the entry field.

WolframAlpha tells you what it thinks you entered, then produces the graph.



You can see that WolframAlpha has plotted two complete cycles from 0° to 360° with amplitude 6.

Example (2)

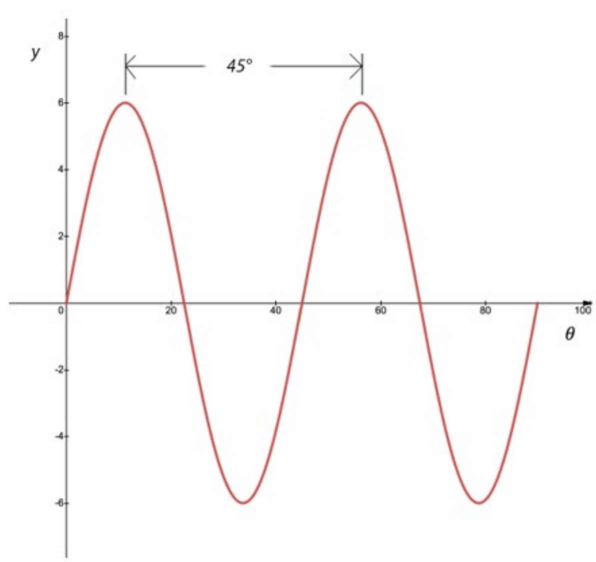
Find the period of $y = 6\sin 8\theta$.

We just need to evaluate $\frac{360°}{B}$ with $B = 8$.

$\frac{360°}{8} = 45°$

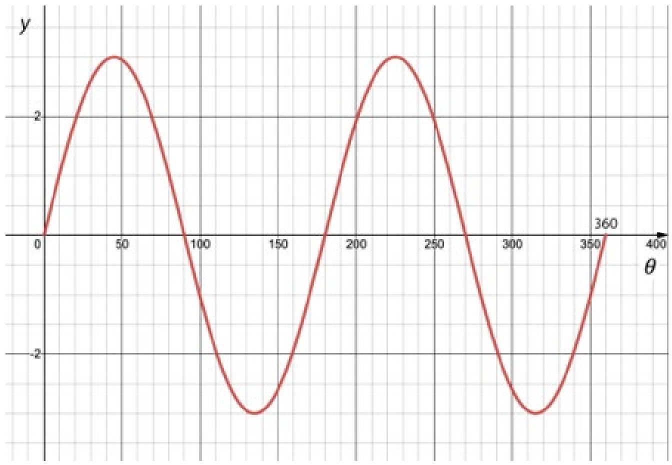
The period of $y = 6\sin 8\theta$ is $45°$

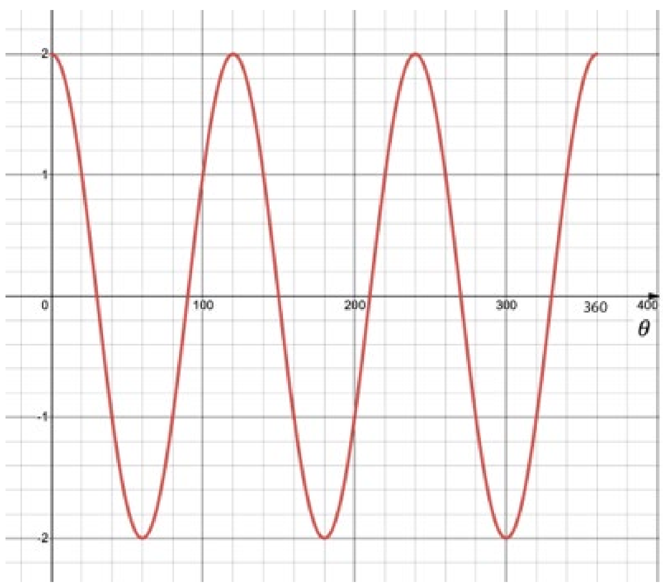
The graph of $y = 6\sin 8\theta$ helps us visualize this 45° period. You can see that the peaks differ by 45°.

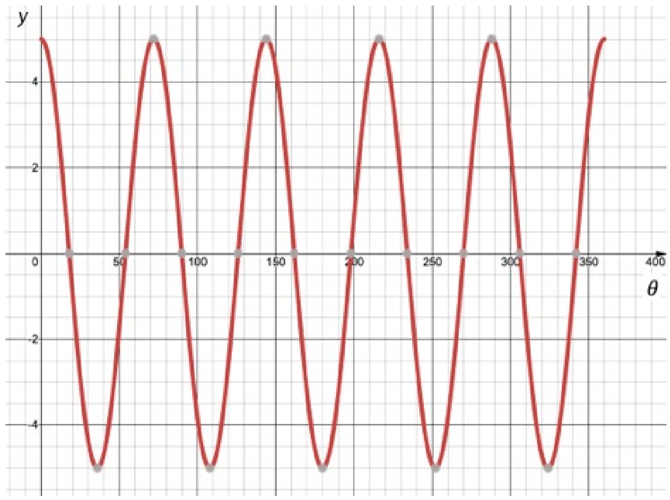


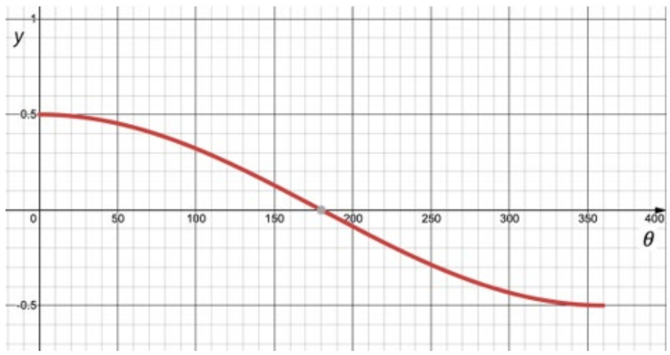
TRY THESE

1. Write the equation of each graph.









2. How many complete cycles are there in the graph of $y = 4\cos(3\theta)$ from 0° to 360°? What is the period and amplitude of this function?

ANS: 3 complete cycles. Period is $\frac{360°}{3} = 120°$. Amplitude is 4.

3. How many complete cycles are there in the graph of $y = 5\sin\left(\frac{4}{5}\theta\right)$ from 0° to 360°? What is the period and amplitude of this function of

ANS: $\frac{4}{5}$ of a complete cycle. Period is $\frac{360°}{4/5} = 360° \times \frac{5}{4} = 450°$. Amplitude is 5.

4. Write the equation of a sine curve that has amplitude 15 and period 50°. You need to specify both A and B in $y = A\sin(B\theta)$. Keep in mind that the period of this function is $\frac{360°}{B}$.

ANS: $y = 15\sin(7.2\theta)$, where $\frac{360°}{B} = 50° \rightarrow B = \frac{360°}{50°} = 7.2$

5. Write the equation of a cosine curve that has amplitude 100 and period 12°. You need to specify both A and B in $y = A\cos(B\theta)$. Keep in mind that the period of this function is $\frac{360°}{B}$.

ANS: $y = 100\cos(30\theta)$, where $\frac{360°}{B} = 12° \rightarrow B = \frac{360°}{12°} = 30$

6. Write the equation of a cosine function that has amplitude 3 and makes two complete cycles from 0° to 180°.

ANS: $y = 3\cos\,(4\theta)$ We need to specify both A and B in $y = A\cos(B\theta)$. Since the amplitude is 3, $A = 3$. Since the curve makes two complete cycles from 0° to 180°, it must make 4 complete cycles from 0° to 360°. So, $B = 4$.

7. Write the equation of a sine function that has amplitude 4 and makes three complete cycles from 0° to 90°.

ANS: $y = 4\sin\,(12\theta)$ We need to specify both A and B in $y = A\cos(B\theta)$. Since the amplitude is 4, $A = 4$. Since the curve makes three complete cycles from 0° to 90°, it must make 12 complete cycles from 0° to 360°. So, $B = 12$.

NOTE TO THE INSTRUCTOR

First, consider presenting the **amplitude** of the sine and cosine function.

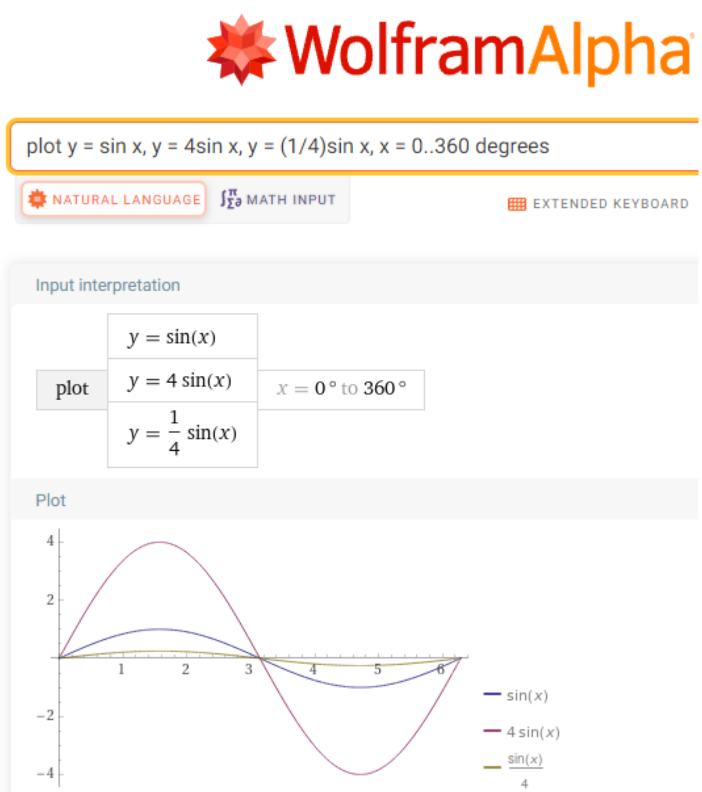
> Ask what would happen if we multiplied $y = \sin\theta$ by 4.

If we were to multiply the sine function $y = \sin\theta$ by 4, getting $y = 4\sin\theta$, each of the sine values would be multiplied by 4 making each value 4 times what it was. Each height would be quadrupled. The amplitude of $y = 4\sin\theta$ is 4.

> Now discuss what would happen if we multiplied $y = \sin\theta$ by 1/4.

If we were to multiply the sine function $y = \sin\theta$ by $\frac{1}{4}$, getting $y = \frac{1}{4}\sin\theta$, each of the sine values would be multiplied by $\frac{1}{4}$, making each value $\frac{1}{4}$ of what it was. Each height of $y = \sin\theta$ would be $\frac{1}{4}$ of what it was in $y = \sin\theta$. The amplitude of $y = \frac{1}{4}\sin\theta$ is $\frac{1}{4}$.

To compare the graphs, use WolframAlpha or Desmos to construct the graphs of $y = \sin\theta$, $y = 4\sin\theta$, and $y = \frac{1}{4}\sin\theta$ all on the same coordinate system.



Now present the **period** of the sine and cosine function.

Suppose B represents a positive number. Then the period of both $y = \sin(B\theta)$ and $y = \cos(\mathrm{B}\theta)$ is $\frac{360°}{B}$. As B gets bigger, $\frac{360°}{B}$ gets smaller and the period increases.

Ask what would happen if we were to multiply the angle θ by 4.

If we were to multiply the angle in the sine function $y = \sin\theta$ by 4, getting $y = \sin 4\theta$, each of the angle's values would be multiplied by 4 making each value 4 times what it was. Each angle would be quadrupled and there would be 4 cycles in the interval 0° to 360°. The period of $y = \sin 4\theta$ is $\frac{360°}{4} = 90°$. The period of $y = \sin 4\theta$ is smaller than that of $y = \sin\theta$.

Ask what would happen if we were to multiply the angle θ is multiplied by $1/4$.

If we were to multiply the angle in the sine function $y = \sin\theta$ by $1/4$, getting $y = \sin\left(\frac{1}{4}\theta\right)$. Each of the angle's values would be multiplied by 1/4 making each value 1/4 what it was and there would be only 1/4 of a cycle in the interval 0° to 360°. The period of $y = \sin\left(\frac{1}{4}\theta\right)$ is $\frac{360°}{1/4} = 360° \times 4 = 1440°$. The period of $y = \sin\left(\frac{1}{4}\theta\right)$ is greater than that of $y = \sin\theta$.

For comparison, you could use WolframAlpha or Desmos to construct the graph of each of these functions on the same coordinate system.