3.3 Arithmetic on Vectors in 3-Dimensional Space

ADDITION & SUBTRACTION OF VECTORS

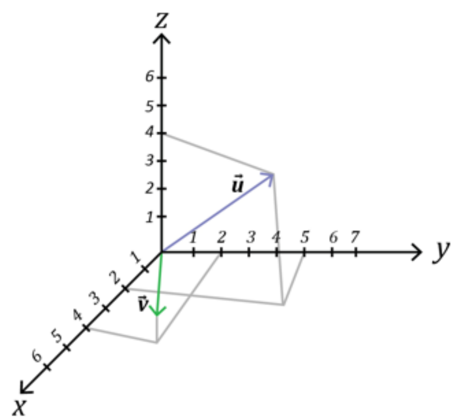
To add or subtract two vectors, add or subtract their corresponding components.

Example (1)

To **add** the vectors $\vec{u} = \langle 2, 5, 4 \rangle$ and $\vec{v} = \langle 4, 2, 1 \rangle$, add their corresponding components.

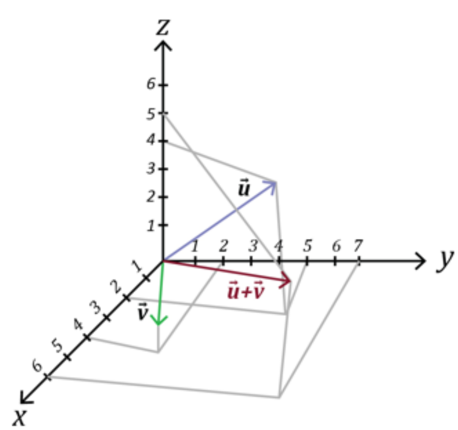
$$\vec{u} + \vec{v} = \langle 2+4, 5+2, 4+1 \rangle = \langle 6, 7, 5 \rangle$$

So, $\vec{u} + \vec{v} = \langle 6, 7, 5 \rangle$



Now, graph this sum. Start at the origin.

Since the x –component is 6, move 6 units in the x –direction.
Since the y –component is 7, move 7 units in the y –direction.
Since the z –component is 5, move 5 units upward.



Example (2)

To **subtract** the vectors $\vec{u} = \langle 2, 5, 4\rangle$ and $\vec{\boldsymbol{v}} = \langle 4, 2, 1\rangle$ subtract their corresponding components.

$$\vec{u} - \vec{v} = \langle 2-4, 5-2, 4-1\rangle = \langle -2, 3, 3\rangle$$

So, $\vec{u} - \vec{v} = \langle -2, 3, 3\rangle$

SCALAR MULTIPLICATION

Scalar multiplication is the multiplication of a vector by a real number (a scalar).

Suppose we let the letter k represent a real number and $\vec{v}$ be the vector $\langle x, y, z\rangle$. Then, the scalar multiple of the vector $\vec{v}$ is

$$k\vec{v} = \langle kx, ky, kz\rangle$$

Example (1)

Suppose $\vec{u} = \langle -3, -8, 5\rangle$ and $k = 3$.

Then $k\vec{u} = 3\vec{u} = 3\langle -3, -8, 5\rangle = \langle 3(-3), 3(-8), 3(5)\rangle = \langle -9, -24, 15\rangle$

Example (2)

Suppose $\vec{v} = \langle 6, 3, -12\rangle$ and $k = \frac{-1}{3}$.

Then $k\vec{u} = \frac{-1}{3}\vec{u} = \frac{-1}{3}\langle 6, 3, -12\rangle = \left\langle \frac{-1}{3}(6), \frac{-1}{3}(3), \frac{-1}{3}(-12)\right\rangle = \langle -2, -1, 4\rangle$

Example (3)

Suppose $\vec{u} = \begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$. Find $3\vec{u} + 4\vec{v} - 2w$.

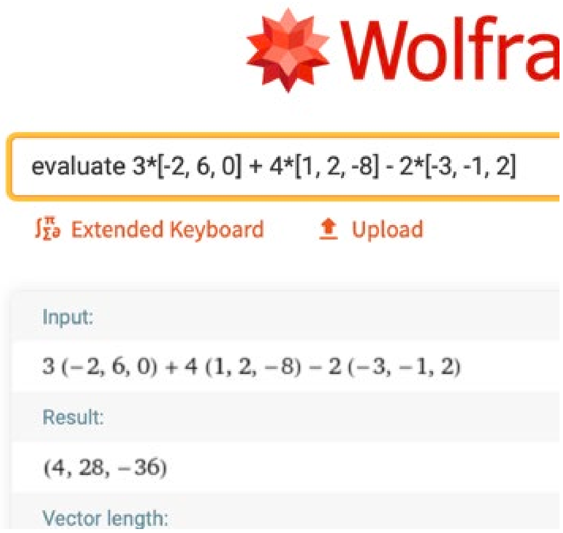
Then $3\vec{u} + 4\vec{v} - 2w = 3\begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix} - 2\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 18 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 8 \\ -32 \end{bmatrix} + \begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 28 \\ -36 \end{bmatrix}$

USING TECHNOLOGY

We can use technology to determine the value of adding or subtracting vectors.

Go to www.wolframalpha.com.

Suppose and $\vec{u} = \begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$. Use WolframAlpha to find $3\vec{u} + 4\vec{v} - 2w$. In the entry field enter evaluate 3*[-2,6,0] + 4*[1,2,-8] – 2*[-3,-1,2].



WolframAlpha answers $(4, 28, -36)$ which is WolframAlpha's notation for $\begin{bmatrix} 4 \\ 28 \\ -36 \end{bmatrix}$.

TRY THESE

1. Add the vectors $\vec{u} = \langle -3, 4, 6 \rangle$ and $\vec{v} = \langle 8, 7, -5 \rangle$.

ANS: $\vec{u} + \vec{v} = \langle 5, 11, 1 \rangle$

2. Subtract the vector $\vec{v} = \langle 8, 7, -5 \rangle$ from the vector $\vec{u} = \langle -3, 4, 6 \rangle$.

ANS: $\vec{u} - \vec{v} = \langle -11, -3, 11 \rangle$

3. Given the three vectors, $\vec{u} = \langle 2, 4, -5 \rangle$, $\vec{v} = \langle -3, 4, -8 \rangle$, and $\vec{w} = \langle 0, 1, 2 \rangle$, find $2\vec{u} + 3\vec{v} - 4\vec{w}$.

ANS: $2\vec{u} + 3\vec{v} - 4\vec{w} = \langle -5, 16, -42 \rangle$

4. Suppose and $\vec{u} = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$, find $4\vec{u} - 4\vec{v} - \vec{w}$.

ANS: $4\vec{u} - 4\vec{v} - \vec{w} = \langle 16, -13, -26 \rangle$

NOTE TO INSTRUCTOR

Consider working through these examples.

1. Add the vectors $\vec{u} = \langle 3, -4, 5\rangle$ and $\vec{v} = \langle -1, 4, 2\rangle$.

ANS: $\vec{u} + \vec{v} = \langle 2, 0, 7\rangle$

2. Subtract the vector $\vec{v} = \langle -5, 2, 1\rangle$ from the vector $\vec{u} = \langle -9, 4, -3\rangle$.

ANS: $\vec{u} - \vec{v} = \langle -4, 2, -4\rangle$

3. Given the three vectors, $\vec{u} = \langle 1, -2, -3\rangle$, $\vec{v} = \langle 4, 3, 2\rangle$, and $\vec{w} = \langle 1, -1, 1\rangle$,
   a. Find $2\vec{u} + 3\vec{v} - 4\vec{w}$.
   b. Find the length of the vector $2\vec{u} + 3\vec{v} - 4\vec{w}$.

ANS: a. $2\vec{u} + 3\vec{v} - 4\vec{w} = \langle 10, 9, -4\rangle$
b. $\sqrt{197}$

4. Suppose and $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -5 \\ 2 \\ 6 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 6 \\ 0 \\ -6 \end{bmatrix}$, find $3\vec{u} - 4\vec{v} - 2\vec{w}$.

ANS: $3\vec{u} - 4\vec{v} - 2\vec{w} = \langle 11, -5, -21\rangle$