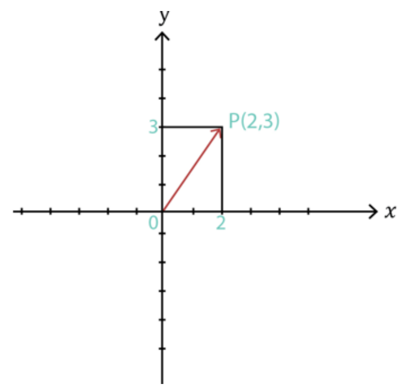
UNIT 3 VECTORS IN THREE DIMENSIONS

3.1 Three Dimensional Vectors

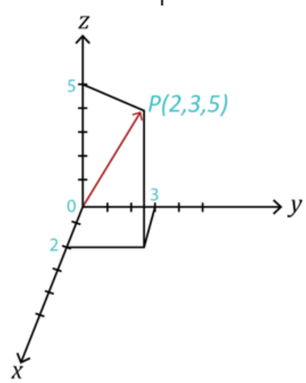
3-DIMENSIONAL SPACE

To this point, we have been working with vectors in 2-dimenisional space. Now, we will expand our discussion to 3-dimensional space.

The **2-dimensional coordinate system** is built around a set of two axes that intersect at right angles and one particular point called the origin. Points in the plane are described by ordered pairs (x, y) and vectors in standard position by $\langle x, y\rangle$.

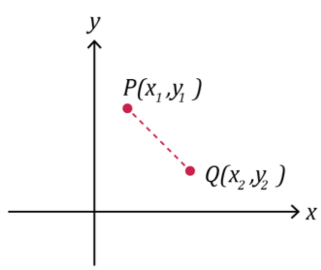


The **3-dimensional coordinate system** is built around a set of three axes that intersect at right angles and one particular point again called the origin. Points in the plane are described by ordered triples (x, y, z) and vectors in standard position by $\langle x, y, z\rangle$.



THE DISTANCE BETWEEN TWO POINTS IN 2 & 3-DIMENSIONAL SPACE

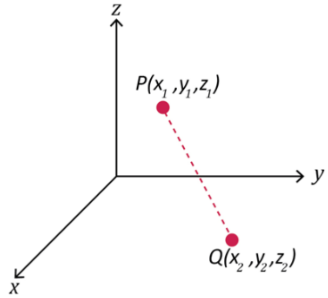
In **two-dimensional space**, the distance d between two points say $P(x_1, y_1)$and $Q(x_2, y_2)$ is given by the distance formula



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In **three-dimensional space**, the distance d between two points say $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Example (1)

The distance between the two points $P(2, 2, 5)$ and $Q(5, 6, 2)$ is

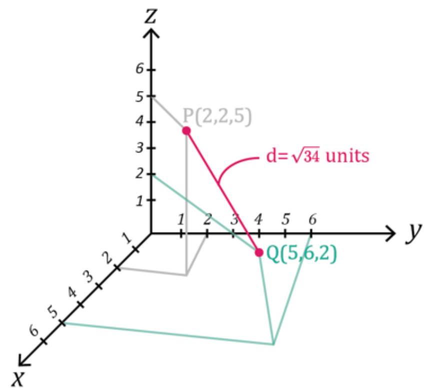
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$d = \sqrt{(5-2)^2 + (6-2)^2 + (2-5)^2}$$

$$d = \sqrt{(3)^2 + (4)^2 + (-3)^2}$$

$$d = \sqrt{9 + 16 + 9}$$

$$d = \sqrt{34} \approx 5.8 \text{ units}$$



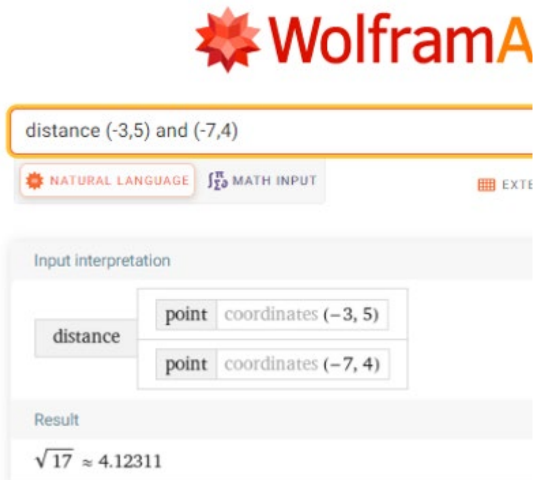
The distance between the two points $P(2, 2, 5)$ and $Q(5, 6, 2)$ is $\sqrt{34} \approx 5.8$ units.

USING TECHNOLOGY

We can use technology to find the distance between points.

Go to www.wolframalpha.com.

To find the distance between the two points $(-3, 5)$ and $(-7, 4)$ enter distance $(-3, 5)$ and $(-7, 4)$ in the entry field. Wolframalpha tells you what it thinks you entered, then tells you its answer. In this case, $\sqrt{17} \approx 4.12311$.

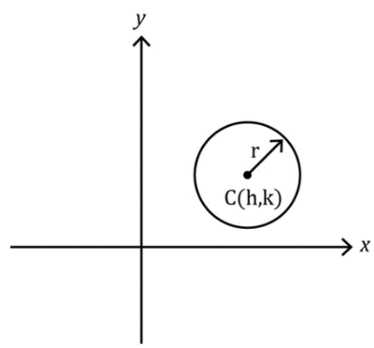


THE EQUATION OF A CIRCLE AND A SPHERE

We can use the distance formulas to get equations of circles and spheres.

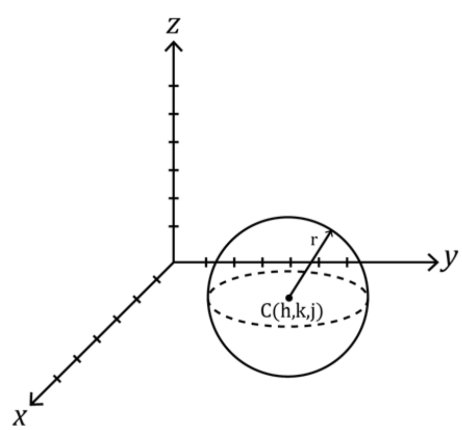
The center-radius form of a circle with center at the point $C(h,k)$ and radius r is

$$(x-h)^2+(y-k)^2=r^2$$



The center-radius form of a sphere with center at the point $C(h,k,j)$ and radius r is

$$(x-h)^2+(y-k)^2+\ (z-j)^2=r^2$$



Example (2)

To write the equation of a circle that has as its center the point $C(4,7)$ and radius 8, we use the center-radius form $(x-h)^2+(y-k)^2=r^2$ with $h=4, k=7$ and $r=8$.

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-4)^2+(y-7)^2=8^2$$

$$(x-4)^2+(y-7)^2=64$$

Example (3)

To write the equation of a sphere that has as its center the point $C(4,7,1)$ and radius 8, we use the center-radius form $(x-h)^2+(y-k)^2+\ (z-j)^2=r^2$ with $h=4, k=7, j=1$ and $r=8$.

$$(x-h)^2+(y-k)^2+\ (z-j)^2=r^2$$

$$(x-4)^2+(y-7)^2+\ (z-1)^2=8^2$$

$$(x-4)^2+(y-7)^2+\ (z-1)^2=64$$

TRY THESE

1. Find the distance between the two points $(2,4)$ and $(-3,6)$. Round to one decimal place.

ANS: $\sqrt{29}\ \approx 5.4$ units

2. Find distance between the two points $(-3,5,-6)$ and $(7,-4,2)$. Round to one decimal place.

ANS: $7\sqrt{5}\ \approx 15.6$ units

3. Write the equation of a circle that has as its center the point $C(2,9)$ and radius 1.

ANS: $(x-2)^2+(y-9)^2=1$

4. Write the equation of a sphere that has as its center the point $C(-2,5,-7)$ and radius 4.

ANS: $(x+2)^2+(y-5)^2+\ (z+7)^2=16$

NOTE TO INSTRUCTOR

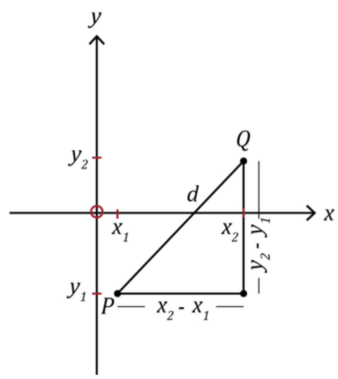
Consider deriving the formula for the distance between two points. Let the two points be $P(x_1, y_1)$ and $Q(x_2, y_2)$. Draw the two points and use the Pythagorean Theorem.

By the Pythagorean Theorem,

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Take square roots to get

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



The equation of a circle comes from the distance formula by using one of the points, say $P(x_1, y_1)$, as the center $C(h, k)$ and the other point, say $Q(x_2, y_2)$, as a general point (x, y) on the circle. A circle is defined as a closed plane curve consisting of all points (x, y) at a given distance r from a point within the curve. Use the distance formula replacing d with r and $(x_2 - x_1)^2$ with $(x - h)^2$ and $(y_2 - y_1)^2$ with $(y - k)^2$.

Consider working through these problems as examples.

1. Find the distance between the two points $(3, -5)$ and $(2, -3)$. Round to one decimal place.

ANS: $\sqrt{5} \approx 2.2$ units

2. Find the distance between the two points $(-1, -2, -3)$ and $(4, -6, 1)$. Round to one decimal place.

ANS: $\sqrt{57} \approx 7.5$ units

3. Write the equation of a circle that has as its center the point $C(3, 6)$ and radius 2.

ANS: $(x - 3)^2 + (y - 6)^2 = 4$

4. Write the equation of a sphere that has as its center the point $C(-4, -3, 7)$ and radius 3.

ANS: $\left(x - (-4)\right)^2 + \left(y - (-3)\right)^2 + (y - 7)^2 = 3^2$
$(x + 4)^2 + (y + 3)^2 + (y - 7)^2 = 9$