2.5 Parallel and Perpendicular Vectors, The Unit Vector

PARALLEL AND ORTHOGONAL VECTORS

Two vectors $\vec{u} = \langle u_x, u_y \rangle$ and $\vec{v} = \langle v_x, v_y \rangle$ are **parallel** if the angle between them is 0° or 180°.

Also, two vectors $\vec{u} = \langle u_x, u_y \rangle$ and $\vec{v} = \langle v_x, v_y \rangle$ are parallel to each other if the vector $\vec{u}$ is some multiple of the vector $\vec{v}$. That is, they will be parallel if the vector $\vec{u} = c\vec{v}$, for some real number c. That is, $\vec{u}$ is some multiple of $\vec{v}$.

Two vectors $\vec{u} = \langle u_x, u_y \rangle$ and $\vec{v} = \langle v_x, v_y \rangle$ are **orthogonal** (perpendicular to each other) if the angle between them is 90° or 180°.

Use this shortcut: Two vectors are perpendicular to each other if their dot product is 0.

Example (1)

The two vectors $\vec{u} = \langle 2, -3 \rangle$ and $\vec{v} = \langle -8, 12 \rangle$ are parallel to each other since the angle between them is $180°$.

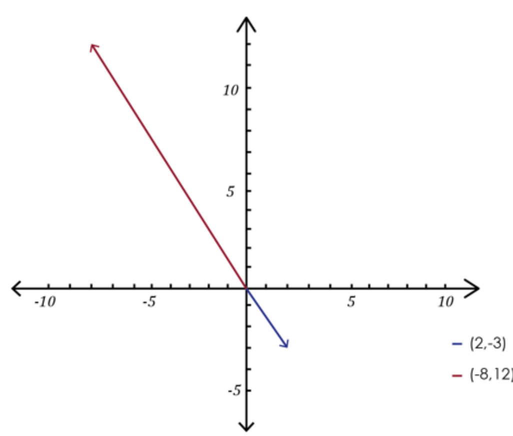
$$\theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$\theta = \cos^{-1} \frac{\langle 2, -3 \rangle \cdot \langle -8, 12 \rangle}{\sqrt{2^2 + (-3)^2} \cdot \sqrt{(-8)^2 + 12^2}}$$

$$\theta = \cos^{-1} \frac{2 \cdot (-8) + (-3) \cdot 12}{\sqrt{4 + 9} \cdot \sqrt{64 + 144}}$$

$$\theta = \cos^{-1} \frac{-52}{\sqrt{13} \cdot \sqrt{208}}$$

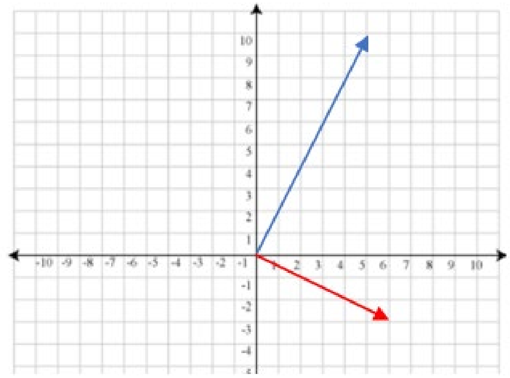
$$\theta = 180°$$



Example (2)

To show that the two vectors $\vec{u} = \langle 5,10 \rangle$ and $\vec{v} = \langle 6,-3 \rangle$ are orthogonal (perpendicular to each other), we just need to show that their dot product is 0.

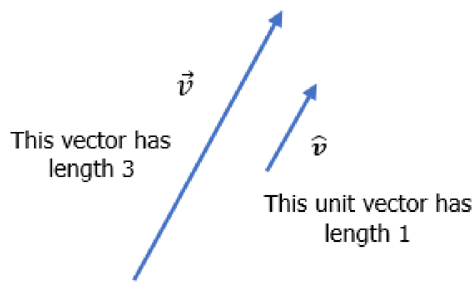
$$\langle 5,10 \rangle \cdot \langle 6,-3 \rangle = 5 \cdot 6 + 10 \cdot (-3) = 30 - 30 = 0$$



THE UNIT VECTOR

A unit vector is a vector of length 1.

A unit vector is a vector of length 1. A unit vector in the same direction as the vector $\vec{v}$ is often denoted with a "hat" on it as in $\hat{v}$. We call this vector "v hat."



The unit vector $\hat{v}$ corresponding to the vector $\vec{v}$ is defined to be

$$\hat{v} = \frac{\vec{v}}{||\vec{v}||}$$

Example (3)

The unit vector corresponding to the vector $\vec{v} = \langle -8, 12 \rangle$ is

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\hat{v} = \frac{\langle -8, 12 \rangle}{\sqrt{(-8)^2 + (12)^2}}$$

$$\hat{v} = \frac{\langle -8, 12 \rangle}{\sqrt{64 + 144}}$$

$$\hat{v} = \frac{\langle -8, 12 \rangle}{\sqrt{208}}$$

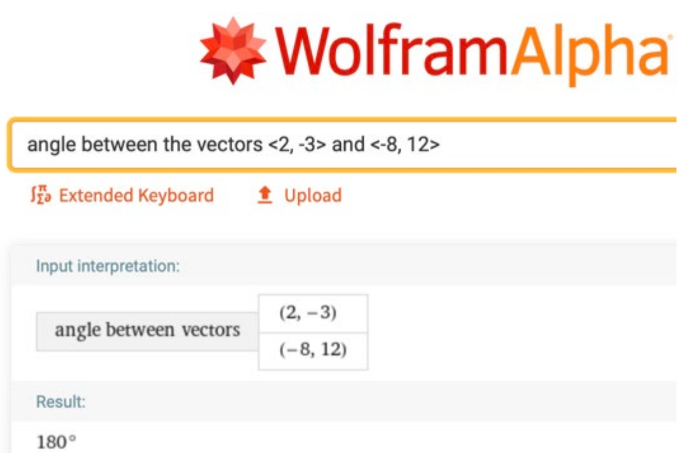
$$\hat{v} = \left\langle \frac{-8}{\sqrt{208}}, \frac{12}{\sqrt{208}} \right\rangle$$

USING TECHNOLOGY

We can use technology to find the angle θ between two vectors.

Go to www.wolframalpha.com.

To show that the vectors $\vec{u} = \langle 2, -3 \rangle$ and $\vec{v} = \langle -8, 12 \rangle$ are parallel**,** enter angle between the vectors <5, -3> and <2, 4> in the entry field. Wolframalpha tells you what it thinks you entered, then tells you its answer. In this case, $\theta = 180°$, indicating the two vectors are parallel.



TRY THESE

1. Determine if the vectors $\vec{u} = \langle 2, 16\rangle$ and $\vec{v} = \langle \frac{1}{2}, 4\rangle$ are parallel to each other, perpendicular to each other, or neither parallel nor perpendicular to each other.

ANS: Parallel

2. Determine if the vectors $\vec{u} = \langle 2, 1\rangle$ and $\vec{v} = \langle 3, -6\rangle$ are parallel to each other, perpendicular to each other, or neither parallel nor perpendicular to each other.

ANS: Perpendicular

3. Determine if the vectors $\vec{u} = \langle 7, 6\rangle$ and $\vec{v} = \langle 2, -1\rangle$ are parallel to each other, perpendicular to each other, or neither parallel nor perpendicular to each other.

ANS: Neither parallel nor perpendicular

4. Find the unit vector corresponding to the vector $\vec{v} = \langle 2, -1\rangle$.

ANS: $\hat{v} = \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

NOTE TO INSTRUCTOR

1. Show that $\vec{u} = \langle 1, 4\rangle$ and $\vec{v} = \langle 4, 16\rangle$ are parallel to each other.

Method 1

$$\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\,\|\cdot\|\vec{v}\,\|}$$

$$\theta = \cos^{-1}\frac{\langle 1,4\rangle\cdot\langle 4,16\rangle}{\sqrt{1^2+4^2}\cdot\sqrt{4^2+16^2}}$$

$$\theta = \cos^{-1}\frac{1\cdot 4+4\cdot 16}{\sqrt{1+16}\cdot\sqrt{16+256}}$$

$$\theta = \cos^{-1}\frac{68}{\sqrt{17}\cdot\sqrt{272}}$$

$$\theta = 0°$$

Method 2

Make sure your calculator is in degree mode, not radian mode.

Show that $\vec{u} = c\vec{v}$. Notice that $\vec{v} = 4\vec{u}$.

$$\langle 4, 16\rangle = 4\langle 1,4\rangle$$

2. Show that the vectors $\vec{u} = \langle 2,\ 1\rangle$ and $\vec{v} = \langle 3, -6\rangle$ are perpendicular to each other.

Method 1

$$\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\,\|\cdot\|\vec{v}\,\|}$$

$\theta = \cos^{-1}((\langle 2,1\rangle\cdot\langle 3,-6\rangle)/(2\text{^}2+1\text{^}2)\cdot\sqrt{}(3\text{^}2+$
$6\text{^}2\ ))$

$$\theta = \cos^{-1}\frac{2\cdot 3+1\cdot(-6)}{\sqrt{4+1}\cdot\sqrt{9+36}}$$

$$\theta = \cos^{-1}\frac{0}{\sqrt{5}\cdot\sqrt{45}} = 0$$

$$\theta = 90°$$

Method 2

The dot product of these two vectors is 0.

3. Find the unit vector corresponding to the vector $\vec{v} = \langle 4, 3 \rangle$.

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\hat{v} = \frac{\langle 4, 3 \rangle}{\sqrt{4^2 + 3^2}}$$

$$\hat{v} = \frac{\langle 4, 3 \rangle}{\sqrt{16 + 9}}$$

$$\hat{v} = \frac{\langle 4, 3 \rangle}{\sqrt{25}}$$

$$\hat{v} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$$