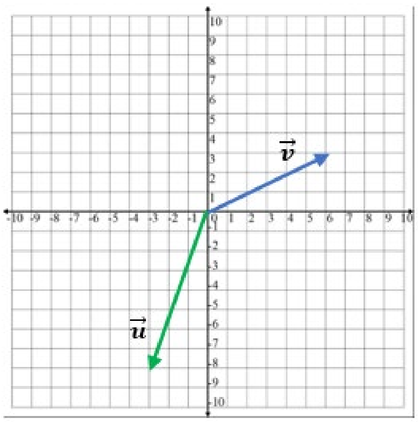
2.2 Addition, Subtraction, and Scalar Multiplication of Vectors

ADDITION & SUBTRACTION OF VECTORS

To add or subtract two vectors, add, or subtract their corresponding components.

Example (1)

To **ADD** the vectors $\vec{u}$ and $\vec{v}$, begin by writing each in component form.



$\vec{u} = \langle -3, -8 \rangle$ and $\vec{v} = \langle 6, 3 \rangle$

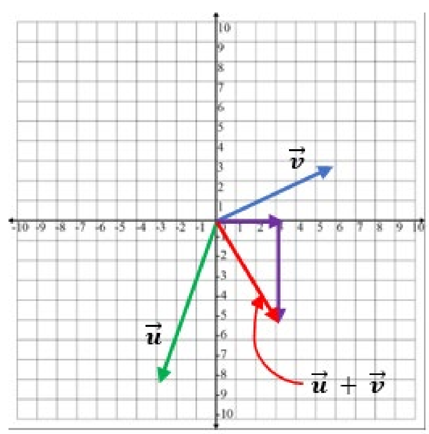
ADD their corresponding components.

$$\vec{u} + \vec{v} = \langle -3 + 6, -8 + 3 \rangle = \langle 3, -5 \rangle$$

So, $\vec{u} + \vec{v} = \langle 3, -5 \rangle$

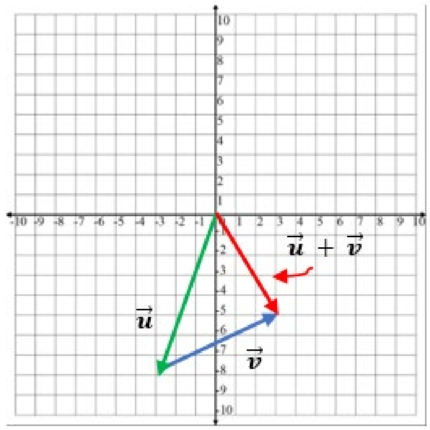
Now, graph this sum.

- Start at the origin.
- Since the horizontal component is 3, move 3 units to the *right*.
- Since the vertical component is -5, move 5 units *downward*.



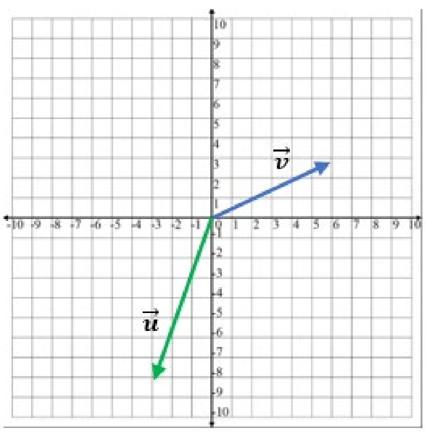
Example (2)

The addition of two vectors $\vec{u}$ and $\vec{v}$ can be demonstrated by placing the tail of one vector at the head of the other. Then connect the tail of $\vec{u}$ to the head of $\vec{v}$.



Example (3)

To **SUBTRACT** the vector $\vec{u}$ from the vector $\vec{v}$, begin by writing each in component form.



$\vec{u} = \langle -3, -8 \rangle$ and $\vec{v} = \langle 6, 3 \rangle$

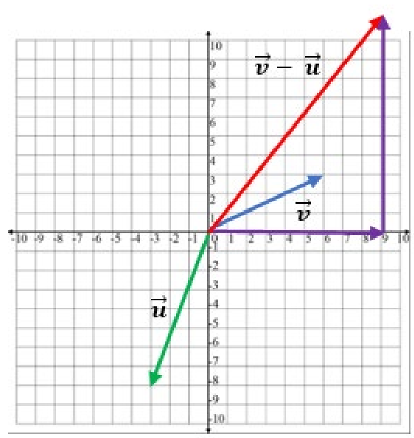
SUBTRACT the components of $\vec{u}$ from the corresponding components of $\vec{v}$.

$$\vec{v} - \vec{u} = \langle 6 - (-3), 3 - (-8) \rangle = \langle 6 + 3, 3 + 8 \rangle = \langle 9, 11 \rangle$$

So, $\vec{v} - \vec{u} = \langle 9, 11 \rangle$

Now, graph this sum.

- Start at the origin.
- Since the <u>horizontal component</u> is 9, move 9 units to the *right*.
- Since the <u>vertical component</u> is 11, move 11 units *upward*.



SCALARS

In contrast to a vector, and having both direction and magnitude, a SCALAR is a physical quantity defined by only its magnitude.

Examples are speed, time, distance, density, and temperature. They are represented by real numbers (both positive and negative), and they can be operated on using the regular laws of algebra.

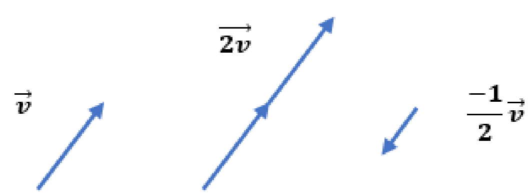
The term *scalar* derives from this usage: *a scalar is that which scales, resizes a vector*.

Scalar multiplication is the multiplication of a vector by a real number (a scalar).

Suppose we let the letter k represent a real number and $\vec{v}$ be the vector $\langle x, y\rangle$. Then, the scalar multiple of the vector $\vec{v}$ is

$$k\vec{v} = \langle kx, ky\rangle$$

To multiply a vector by a scalar (a constant), multiply each of its components by the constant.



1. Suppose $\vec{u} = \langle -3, -8\rangle$ and $k = 3$.

   Then $k\vec{u}$ =3$\vec{u}$ $= 3\langle -3, -8\rangle = \langle 3(-3), 3(-8)\rangle = \langle -9, -24\rangle$

2. Suppose and $\vec{v} = \langle 6, 3\rangle$ and $k = \frac{-1}{3}$.

   Then $k\vec{u} = \frac{-1}{3}\vec{u} = \frac{-1}{3}\langle 6, 3\rangle = \left\langle \frac{-1}{3}(6), \frac{-1}{3}(3)\right\rangle = \langle -2, -1\rangle$

3. Suppose and $\vec{u} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

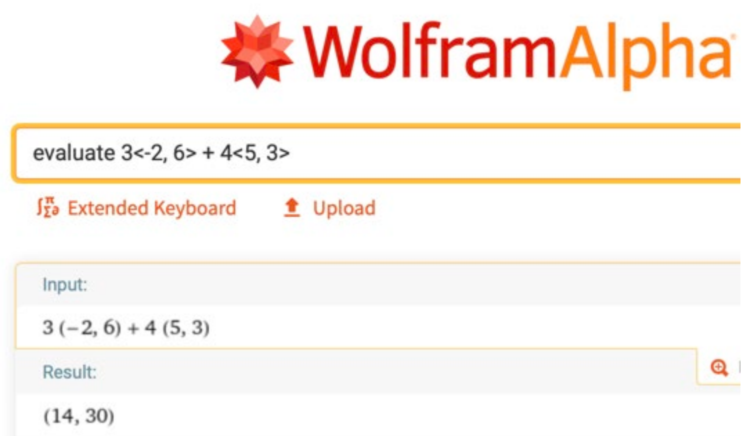
   Then $3\vec{u} + 4\vec{v} = 3\begin{bmatrix} -2 \\ 6 \end{bmatrix} + 4\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -6 \\ 18 \end{bmatrix} + \begin{bmatrix} 20 \\ 12 \end{bmatrix} = \begin{bmatrix} 14 \\ 30 \end{bmatrix}$

USING TECHNOLOGY

We can use technology to add and subtract vectors and to multiply a vector by a scalar.

Go to www.wolframalpha.com.

For the vectors $\vec{u} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, use WolframAlpha to find $3\vec{u} + 4\vec{v}$. Enter evaluate 3<-2, 6> + 4<5, 3> in the entry field. Wolframalpha tells you what it thinks you entered, then tells you its answer. In this case, $< 14, 30 >$.



TRY THESE

1. Find the sum of the two vectors $\vec{u} = \langle -5, 2 \rangle$ and $\vec{v} = \langle 10, -1 \rangle$.

2. Subtract the vector $\vec{u} = \langle -5, 2 \rangle$ from the vector $\vec{v} = \langle 10, -1 \rangle$.

3. Perform the operation $2\langle -5, 2 \rangle - 4\langle 1, 6 \rangle + 3\langle 4, -3 \rangle$.

NOTE TO INSTRUCTOR

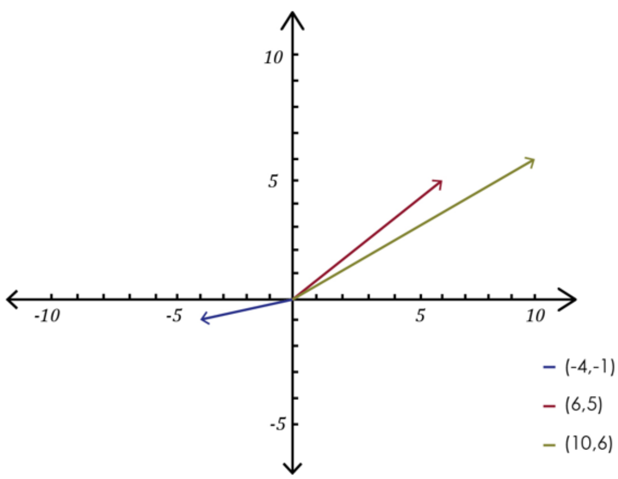
Consider showing these examples on the board.

1. Using the vectors $\vec{u} = \langle -4, -1 \rangle$ and $\vec{v} = \langle 6, 5 \rangle$, show addition using both the arrows originating at the origin and then by placing the tail of $\vec{u}$ onto the head of $\vec{v}$.

ANS: $\vec{u} + \vec{v} = \langle -4 + 6, -1 + 5 \rangle = \langle 2, 4 \rangle$

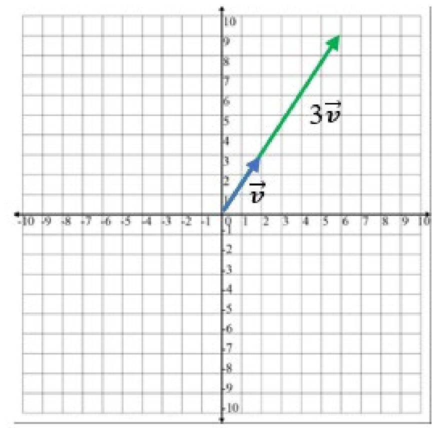
2. Subtract the vector $\vec{u} = \langle -4, -1 \rangle$ from the vector $\vec{v} = \langle 6, 5 \rangle$.

$$\vec{v} - \vec{u} = \langle 6 - (-4), 5 - (-1) \rangle = \langle 6 + 4, 5 + 1 \rangle = \langle 10, 6 \rangle$$



ANS: $\vec{u} - \vec{v} = \langle -10, -6 \rangle$

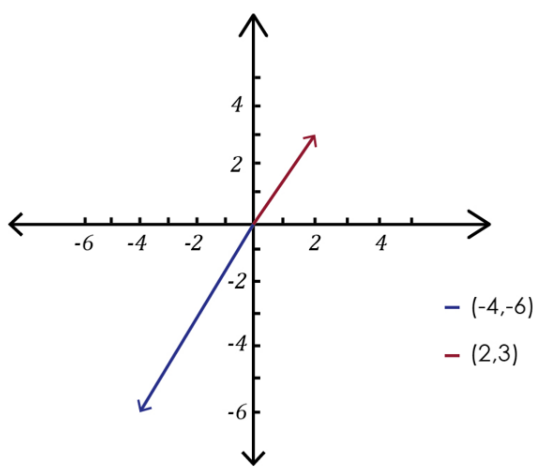
3. Multiply the vector $\vec{v} = \langle 2, 3\rangle$ by the scalar 3.



ANS: $3\vec{v} = 3\langle 2, 3\rangle = \langle 6, 9\rangle$

Draw $\vec{v}$ in one color and $3\vec{v}$ in another color. Point out how the length of vector $\vec{v}$ tripled. That is, $3\vec{v}$ should look 3 times as long as $\vec{v}$. It can be a bit hard to show because the vectors will appear to be on top of the other.

4. Multiply the vector $\vec{v} = \langle 2, 3\rangle$ by the scalar -2.



ANS:$-2\vec{v} = -2\langle 2, 3\rangle = \langle -4, -6\rangle$

Draw $\vec{v}$ in one color and $-2\vec{v}$ in another color. Point out how the length of vector $\vec{v}$ doubled and points in the opposite direction of $\vec{v}$. That is, -2$\vec{v}$ should look twice as long as $\vec{v}$ but pointing in the opposite direction.

5. Suppose and $\vec{u} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$and $\vec{v} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$. Find $5\vec{u} + 2\vec{v}$.

$$5\vec{u} + 2\vec{v} = 5\begin{bmatrix} -2 \\ 6 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 \\ 30 \end{bmatrix} + \begin{bmatrix} 10 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 36 \end{bmatrix}$$