UNIT 5 SOME BASIC TRIGONOMETRY

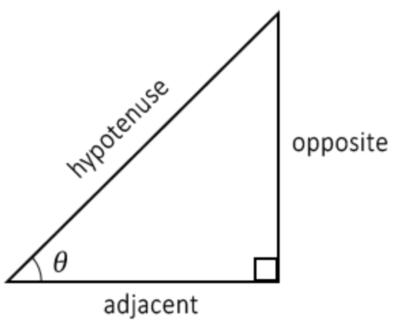
5.1 The Basic Trigonometric Functions

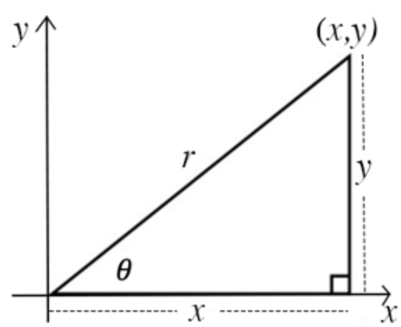
RIGHT TRIANGLE TRIGONOMETRY

There are six trigonometric functions associated with right triangles. Since our focus is on the mathematics of games, we will concentrate on only three of them, the sine function, the cosine function, and the tangent function.

> The sine function is useful for producing the vertical motion of an object and the cosine function for producing the horizontal motion.

The figures just below show right triangles with angle θ, and sides opposite angle θ, adjacent to angle θ, and the hypotenuse of the triangle.





The angle θ has two measures associated with it:

1. Its degree measure, which we can label $\theta°$, and
2. Its trigonometric measure.

> A trigonometric measure of an angle is a ratio (quotient) of two of the sides of the triangle.

We will discuss all three of these ratios, the sine, the cosine, and the tangent of an angle.

THE SINE OF AN ANGLE

In words: In a right triangle, the *sine* of angle θ is the ratio of the length of the side opposite θ to the length of the hypotenuse. We abbreviate the phrase "the *sine* of angle θ" with $\sin\theta$.

Then, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$. That is $\sin\theta = \frac{y}{r}$.

THE COSINE OF AN ANGLE

In words: In a right triangle, the *cosine* of angle θ is the ratio of the length of the side adjacent to θ to the length of the hypotenuse. We abbreviate the phrase "the *cosine* of angle θ" with $\cos\theta$.

Then, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$. That is $\cos\theta = \frac{x}{r}$.

THE TANGENT OF AN ANGLE

In words: In a right triangle, the *tangent* of angle θ is the ratio of the length of the side opposite θ to the length of the side adjacent to θ. We abbreviate the phrase "the *tangent* of angle θ" with $\tan\theta$.

Then, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. That is $\tan\theta = \frac{y}{x}$.

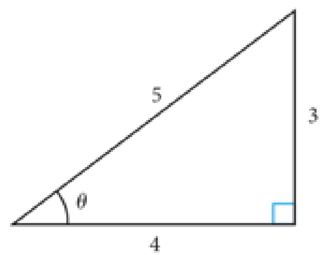
Example (1)

Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the 3-4-5 triangle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} = 0.6$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5} = 0.8$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4} = 0.75$$



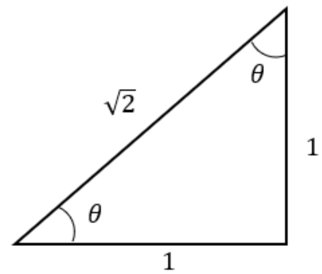
Example (2)

Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the triangle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \approx 0.7071$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \approx 0.7071$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$$



USING TECHNOLOGY

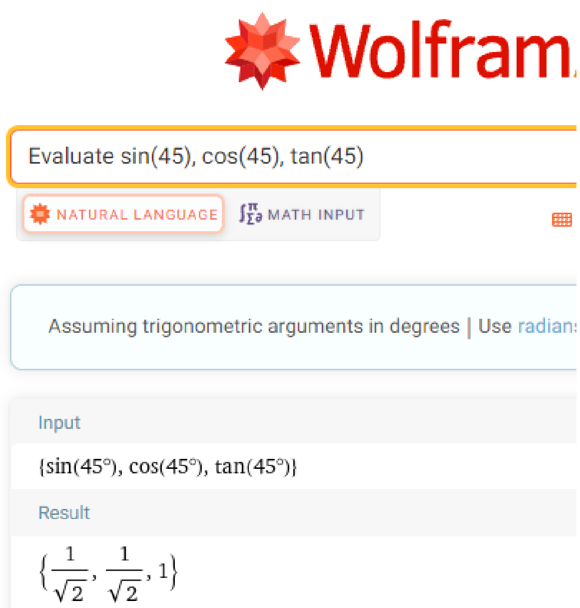
WolframAlpha evaluates the sines, cosines, and tangents of angles for us.

Go to www.wolframalpha.com.

Example (3)

Find $\sin 45°$, $\cos 45°$ and $\tan 45°$.

To compute these ratios, enter Evaluate sin(45), cos(45), tan(45) into the entry field.
Separate the entries with commas. W|A does not see spaces.
WolframAlpha tells you what it thinks you entered, then tells you its answers.



We conclude that $\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, and $\tan 45° = 1$.

W|A also provides us with decimal approximations to these ratios.

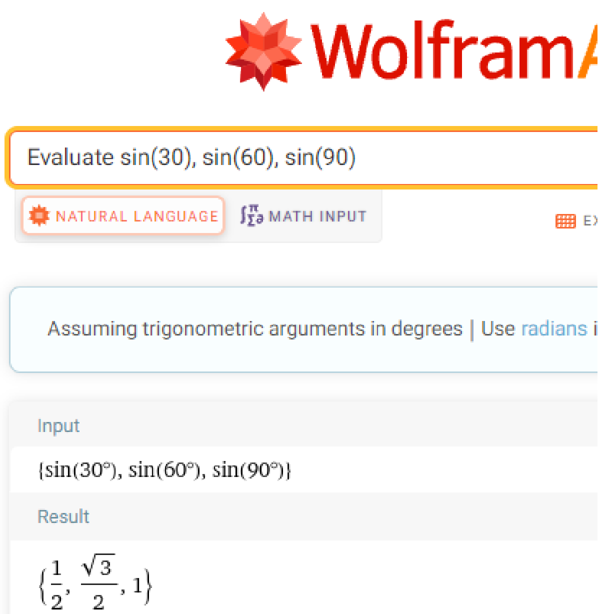
$\sin 45° = 0.7070107,\ \cos 45° = 0.7070107$, and $\tan 45° = 1$

Notice that these are the same values we got in Example 2.

Example (4)

Find $\sin 30°$, $\sin 60°$, $\sin 90°$.

To compute these ratios, enter Evaluate sin(30), sin(60), sin(90) into the entry field.
Separate the entries with commas. W|A does not see spaces.
WolframAlpha tells you what it thinks you entered, then tells you its answers.



We conclude that $\sin 30° = \frac{1}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$, and $\sin 90° = 1$.

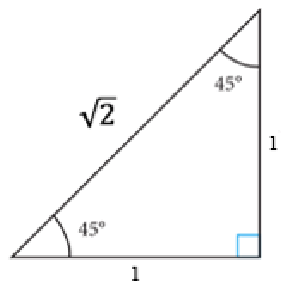
W|A also provides us with decimal approximations to these ratios.

$\sin 30° = 0.5,\ \sin 60° = 0.866025$, and $\sin 90° = 1$.

TRY THESE

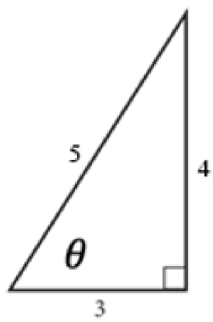
Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ for each triangle. Write your answers as decimal numbers rounded to 4 places.

1.



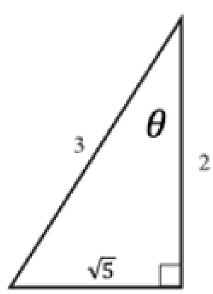
ANS: $\sin 45° = \frac{1}{\sqrt{2}} = 0.7071, \cos 45° = \frac{1}{\sqrt{2}} = 0.7071, \tan 45° = 1$

2.



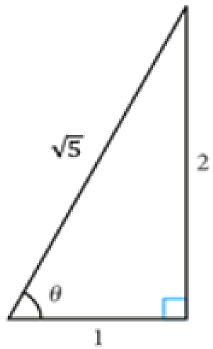
ANS: $\sin\theta = \frac{4}{5} = 0.8, \cos\theta = \frac{3}{5} = 0.6, \tan\theta = \frac{4}{3} = 1.3333$

3.



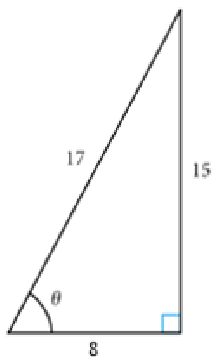
ANS: $\sin\theta = \frac{\sqrt{5}}{3} = 0.7454, \cos\theta = \frac{2}{3} = 0.6666, \tan\theta = \frac{\sqrt{5}}{2} = 1.1180$

4.



ANS: $\sin\theta = \frac{2}{\sqrt{5}} = 0.8944, \cos\theta = \frac{1}{\sqrt{5}} = 0.4472, \tan\theta = \frac{2}{1} = 2$

5.



ANS: $\sin\theta = \frac{15}{17} = 0.8834, \cos\theta = \frac{8}{17} = 0.4705, \tan\theta = \frac{15}{8} = 1.875$

Find each value. Write your answers as decimal numbers rounded to 4 places.

6. $\sin 30°,\ \cos 30°, \tan 30°$ ANS: $\sin 30° = 0.5,\ \cos 30° = 0.8661, \tan 30° = 0.5774$

7. $\sin 90°,\ \cos 90°$ ANS: $\sin 90° = 1,\ \cos 90° = 0$

8. $\sin 0°,\ \cos 0°, \tan 0°$ ANS: $\sin 0° = 0, \cos 0° = 1, \tan 0° = 0$

9. $\sin 180°,\ \cos 180°$ ANS: $\sin 180° = 0, \cos 180° = -1$

10. $\sin 120°, \cos 120°$ ANS: $\sin 120° = 0.8660, \cos 120° = -0.5$

NOTE TO INSTRUCTOR

This section is intended as a brief introduction to right angle trigonometry. It is not intended as even a brief course in trig. We present only the sine and cosine functions since they are the functions used in controlling vertical and horizontal motion of an object. They are two functions that best serve the needs of gaming programmers at an introductory level. We present the sine and cosine first using right triangles as they give a visual understanding of both functions.

The next section will present circular trigonometry. In computer games, vertical and horizontal motion of objects takes place over time, and the graphs of sine and cosine as time increases helps us to see how these two functions control vertical and horizontal motion.

The following section presents how the amplitude and period of the sine and cosine functions determine the height of an object and the speed at which an object changes its height.

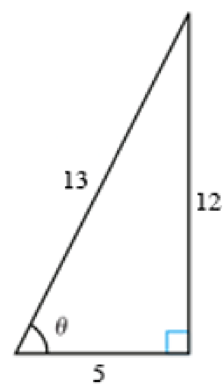
Consider demonstrating these examples:

Example (1)

Find both $\sin\theta$ and $\cos\theta$ for the 5-12-13 triangle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13} = 0.9231$$

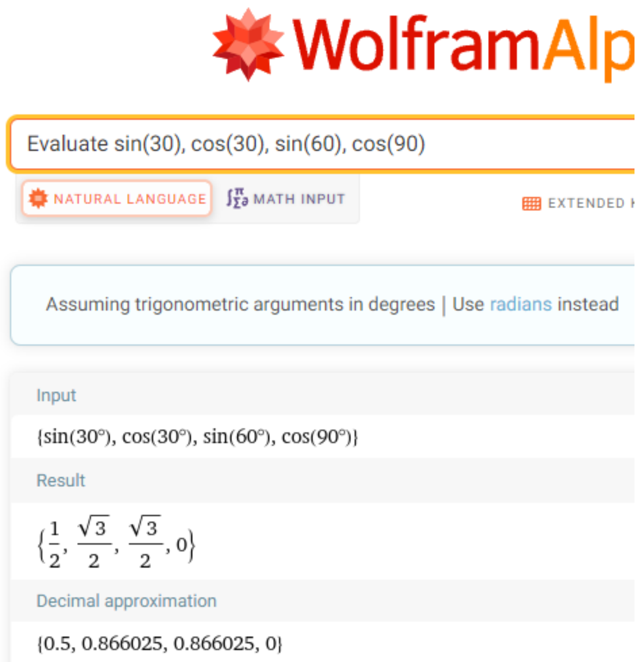
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{13} = 0.3846$$



Example (2)

Find $\sin 30°$, $\cos 30°$, $\sin 60°$, $\cos 90°$.

To compute these ratios, enter Evaluate sin(30), cos(30), sin(60), cos(90) into the entry field.
Separate the entries with commas. W|A does not see spaces.
WolframAlpha tells you what it thinks you entered, then tells you its answers.



We conclude that $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$, and $\cos 60° = \frac{1}{2}$

W|A also provides us with decimal approximations to these ratios.

$\sin 30° = 0.5$, $\cos 30° = 0.8660$, $\sin 60° = 0.8660$, and $\cos 60° = 0.5$.