109-2 Solution

Quiz 04 (60 mins.)

Department:__________ Name:___________ ID:___________

Problem 1: (30 points)

Using the Fourier transform to solve the following differential equation.

$$y''(x)+4y'(x)+3y(x)=\frac{-1}{2i}[\delta(x-3)-\delta(x+3)]$$

Answer

$$\mathcal{F}\left[y''+4y'+3y=\frac{-1}{2i}(\delta(x-3)-\delta(x+3))\right]$$

$$(iw)^2Y(w)+4(iw)Y(w)+3Y(w)=\sin 3w$$

$$\therefore [(iw)^2+4(iw)+3]Y(w)=\sin 3w$$

$$Y(w)=\frac{\sin 3w}{(iw)^2+4(iw)+3}=\frac{\frac{1}{2}\sin 3w}{1+iw}+\frac{-\frac{1}{2}\sin 3w}{3+iw}$$

$\mathcal{F}^{-1}$:

$$\frac{1}{2}\cdot\frac{-1}{2i}[\delta(x-3)-\delta(x+3)]*e^{-x}H(x)$$

$$=\frac{-1}{4i}[e^{-(x-3)}H(x-3)-e^{-(x+3)}H(x+3)]$$

$\mathcal{F}^{-1}$:

$$-\frac{1}{2}\cdot\frac{-1}{2i}[\delta(x-3)-\delta(x+3)]*e^{-3x}H(x)$$

$$=\frac{1}{4i}[e^{-3(x-3)}H(x-3)-e^{-3(x+3)}H(x+3)]$$

$$\therefore y(x)=\mathcal{F}^{-1}[Y(w)]$$

$$=\frac{1}{4i}[e^{-3(x-3)}-e^{-(x-3)}]H(x-3)+$$

$$\frac{1}{4i}[e^{-(x+3)}-e^{-3(x+3)}]H(x+3)$$ #

Problem 2: (20 points)

You decide to transfer one signal *s(x)* (*i.e.*, $s(x)=e^{-2|x|}$)and make the *f(x)=cos3x* as the carrier wave.

Based on the modulation technique, you can create one new signal *r(x)* containing the *s(x)* and *f(x)*. Please describe the formulation of the *r(x)* in the frequency domain.

Answer:

$$s(x)=e^{-2|x|}\xrightarrow{\mathcal{F}}S(w)=\frac{4}{4+w^2}$$

$$f(x)=\cos 3x\xrightarrow{\mathcal{F}}F(w)=\pi[\delta(w-3)+\delta(w+3)]$$

$$\mathcal{F}[s(x)f(x)]=\frac{1}{2\pi}S(w)*F(w)=\frac{1}{2\pi}\left[\frac{4}{4+w^2}*(\pi(\delta(w-3)+\delta(w+3))\right]$$

$$=\frac{1}{2}\left[\frac{4}{4+w^2}*\delta(w-3)+\frac{4}{4+w^2}*\delta(w+3)\right]$$

$$=\frac{2}{4+(w-3)^2}+\frac{2}{4+(w+3)^2}$$ #

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Problem 3: (30 points)

One function *f(x)* is defined as $f(x)=e^{-2x}H(x)$. Please determine how many adds and multiplications should be involved if you use 4-point DFT and 4-point FFT respectively.

Answer:

	DFT	FFT
MUL	16	4
ADD	12	8

Problem 4: (20 points)

Please use the following property $F(\omega)=\Im[f(x)]=\sqrt{\frac{\pi}{a}}e^{-\frac{\omega^2}{4a}}$ to find the Fourier Transform of

$f(t)=4e^{-3t^2}\sin(2t)$

Answer:

$\because \mathcal{F}[4e^{-3t^2}]=4\sqrt{\frac{\pi}{3}}e^{-\frac{w^2}{4\cdot 3}}=4\sqrt{\frac{\pi}{3}}e^{-\frac{w^2}{12}}$, $\mathcal{F}[\sin 2t]=\mathcal{F}[\frac{1}{2i}(e^{i2t}-e^{-i2t})]=\frac{1}{2i}[2\pi\delta(w-2)-2\pi\delta(w+2)]$

$=\frac{\pi}{i}[\delta(w-2)-\delta(w+2)]$

By using Modulation process,

$\mathcal{F}[h(t)\cdot g(t)]=\frac{1}{2\pi}H(w)*G(w)$

$\therefore \mathcal{F}[f(x)]=F(w)=\mathcal{F}[4e^{-3t^2}\sin 2t]=\frac{1}{2\pi}[4\sqrt{\frac{\pi}{3}}e^{-\frac{w^2}{12}}*\frac{\pi}{i}(\delta(w-2)-\delta(w+2)]=\frac{2}{i}\sqrt{\frac{\pi}{3}}(e^{-\frac{(w-2)^2}{12}}-e^{-\frac{(w+2)^2}{12}}$