The 1st Quiz of Calculus 0312

1. (50 %) Find the limit (if it exists).

(a) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+3} - \sqrt{3}}{x}$

(b) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{|x-1|}$

(c) $\lim\limits_{x \to 0} \dfrac{\sin x(1 - \cos x)}{x^2}$

(d) $\lim\limits_{x \to 0} f(x)$, where $f(x) = \begin{cases} x^2, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$

(e) $\lim\limits_{x \to 1} \left(\dfrac{x^3 + x - 2}{x^2 - 1} \right)^2$

2. (30 %) True or False ? If it is true, explain why. If it is false, give a counterexample.

(1). If $\lim\limits_{x \to a} f(x)$ exists and $\lim\limits_{x \to a} g(x)$ does not exist, then $\lim\limits_{x \to a} (f(x) + g(x))$ does not exist.

(2). Suppose that both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} (f(x) \cdot g(x))$ exist, then $\lim\limits_{x \to a} g(x)$ exist.

(3). If $f(x) < g(x)$ for all $x \neq a$, then $\lim\limits_{x \to a} f(x) < \lim\limits_{x \to a} g(x)$

3. (20 %) Find an equation of the tangent line of $f(x) = \dfrac{3x - 1}{x^2 + 5x}$ at $(-3, \dfrac{5}{3})$.