機率學小考 2015.03.25 每題 20 分。作答要寫過程。

1. Let X be the sum of two rolls of a fair 6-sided die. What is the PMF of X? Let $\mathcal{A}$ be the event that X is not a prime number. What is the conditional PMF of X given $\mathcal{A}$?

2. Consider 3 independent coin tosses, each with a 1/3 probability of a head, and let X be the number of heads obtained. What are the PMF, the mean, and the variance of X?

3. Find the mean and variance of a Poisson random variable with parameter $\lambda = 2$. Note that the Poisson distribution is

$$p_Z(k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, \ldots$$

4. Find the mean and variance of a geometric random variable

$$G \sim \textbf{geometric}(p)$$

with parameter $p = \frac{1}{3}$.

5. We have a group of 6 persons. Consider clubs that consist of a special person from the group (the club leader), another person from the group (the club co-leader), and a number (possibly 0) of additional club members. How many distinct ways of composition for such a club are there?