

## Midterm 2018.04.25

1. (20%) In a move, three cups are switched left-right, left-middle, middle-right with probability  $1/2$ ,  $1/3$ ,  $1/6$ , respectively. A marble is initially in the middle cup. After 2 moves, what is the probability that the marble is in the middle cup? In the left cup?
2. (10%) Describe 2 sample spaces (sets of possible outcomes) for Blackjack.
3. (20%) Romeo and Juliet are having a date. Romeo will arrive with a delay of no more than 30 minutes, and Juliet will arrive with a delay of no more than 60 minutes. All pairs of delays are equally likely. The first to arrive will wait for 10 minutes. What is the probability that they meet?
4. (10%) A conservative team and an innovative team are asked to separately design a new product within a month. From past experience we know that
  - the conservative team is successful with probability  $2/3$
  - the innovative team is successful with probability  $1/2$
  - at least one team is successful with probability  $3/4$

Draw a **tree diagram** for this probability model. It should have 4 leaf nodes, with each edge labeled by a conditional probability.

5. (20%) In a trip to Canada, Tom asked his travel agent to book an aisle seat for him. At the gate, he sees 10 passengers (including Tom) are waiting in Zone 2, which includes 3 rows of 2-3-2 seats. Assume the seat assignment of the other passengers is uniform. What is the probability that he sits next to no other passengers? What would be the probability had he not asked for an aisle seat?
6. (20%) A player is given 3 questions, A, B, C to answer. Question A is answered correctly with probability 0.7, and the prize money is \$2. Question B is answered correctly with probability 0.6, and the prize money is \$3. Question C is answered correctly with probability 0.5, and the prize money is \$4. Until an attempted question is answered incorrectly, the player is allowed to answer more questions to earn more money, without penalty. To maximize the expected prize money, which question should be attempted first? If the first attempted question is correctly answered, which question should be attempted next?
7. (Extra 10%) What is the sum of all 4-digit numbers formed by 1, 2, 3, 4, 5 without repetition? With repetition?