

## Quiz 2 2018.6.6

1. The time  $X$  from home to subway Station A,  $Y$  from Station A to Station B, and  $Z$  from Station B to school are random variables, with  $X \sim \mathcal{N}(10, 4)$ ,  $Y \sim \mathcal{N}(20, 3)$ ,  $Z \sim \mathcal{N}(5, 2)$ . What is the probability that it takes  $\leq 30$  minutes to go from home to school by subway?
2.  $(X, Y)$  takes each pair of values  $(1, 0)$ ,  $(0, 1)$ ,  $(-1, 0)$ ,  $(0, -1)$  with probability  $1/4$ .
  - What is the covariance  $\text{cov}(X, Y)$ ?
  - What is the correlation coefficient  $\rho(X, Y)$ ?
3. Two archers shoot at a target. The shot distances from the center are independent, and uniform over  $(0, 1)$ . What is the PDF of the distance of the winning shot from the center?
4.  $X \perp\!\!\!\perp Y$  are continuous and uniform over  $(0, 1)$ , and  $Z = XY$ . What is  $\mathbf{P}(Z \leq 0.5)$ ?
5. Random variables  $X \sim \mathbf{exponential}(2)$  and  $Y \sim \mathbf{Laplace}(2)$  (2-sided exponential).
  - (a) What are  $E[X]$  and  $\text{var}(Y)$ ?
  - (b) By Markov or Chebyshev inequality, find bound for  $\mathbf{P}(X > 2)$  and  $\mathbf{P}(|Y| > 2)$ .
6. In a standard normal table we find  $\Phi^{-1}(0.97) = 1.88$ ,  $\Phi^{-1}(0.98) = 2.05$ ,  $\Phi^{-1}(0.99) = 2.33$ . In a poll, we require 98% confidence and  $\pm 0.5\%$  accuracy. What is the minimum sample size  $n$ 
  - according to Chebyshev inequality
  - according to central limit theorem (Gaussian approximation)