Homework 01

(due day in two weeks, 4/1)

Problem 1: (20 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

(1) $\sin\frac{n\pi x}{l}$ (2) $\cos\frac{n\pi x}{l}$ (3) $f(x)=a_0\cdot e^{-x}$

(4) $f(x)=a_0+a_1\cos\frac{\pi x}{l}+b_1\sin\frac{\pi x}{l}+a_2\cos\frac{2\pi x}{l}+b_2\sin\frac{2\pi x}{l}+\ldots+a_n\cos\frac{n\pi x}{l}+b_n\sin\frac{n\pi x}{l}+\ldots$

Problem 2: (20 points)

One periodic function f(x) can be defined as the Figure 1.

(1) Please find the corresponding Fourier's Series.

(2) Please apply the result in (1) to calculate $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$

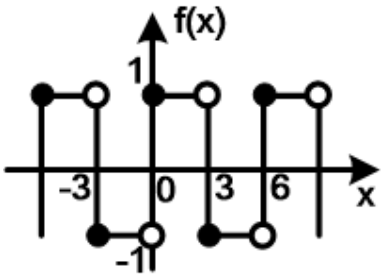

Figure 1

Problem 3: (20 points)

One periodic function f(x) can be defined as the Figure 2.

(1) Please find the corresponding Fourier's Series.

(2) Please apply the result in (1) to calculate $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+...$

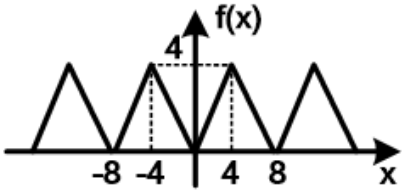

Figure 2

Problem 4: (20 points)

Given the function $f(x)=\begin{cases}0, -\pi < x < 0\\ \sin x, 0 \le x < \pi\end{cases}$

(1) Find its Fourier series

(2) Show that $\frac{1}{2}+\frac{1}{1\times 3}-\frac{1}{3\times 5}+\frac{1}{5\times 7}-\frac{1}{7\times 9}+...=\frac{\pi}{4}$

Problem 5: (20 points)

A function can be defined as $E(t)=t(\pi^2-t^2)$ when $-\pi \le t \le \pi$. Besides, $E(t+2\pi)$ is equal to $E(t)$ and $-\infty \le t \le \infty$. Please find the Fourier representation of $E(t)$.