Final exam 2017.06.13

1. (10%) Given that the NBA finals 2017 is 3:1 after 4 games, give a reasonable estimate of the probability that the leading Warriors win the title.

2. (10%) Based on any observations available in one CPBL game, estimate the probability for a spectator with a baseball glove to snatch a baseball during a game. How about for a spectator without a glove?

3. (10%) What does it mean when the weather forecast says that the probability of raining tomorrow in Kaohsiung is 60%?

4. (10%) A Markov chain is defined by its state space and state transition probabilities. Describe how to model the board game of Go as a discrete-time Markov chain.

5. (10%) Show that the expectation of a random variable R, which takes the values of 0 and 1, is the probability of $R = 1$. This is used in developing AlphaGo, where maximizing the probability of winning is equivalent to maximizing the expected return of a Markov decision process.

6. (20%) A surface contains repeating rectangles with the side lengths of w and d. Suppose that a needle of length l is dropped on the surface at random. Assume that $l < w < d$. What is the probability that the needle intersects 2 sides of a square?

7. (15%) Suppose there are 3 light bulbs with **iid** exponentially distributed lifetimes with the mean of 1000 hours. Find the expected time for the light bulbs to burn out with the following plans.

   (a) Turn all 3 bulbs on at the same time.

   (b) Turn 2 bulbs on. As soon as 1 bulb burns out, turn on the third.

   (c) Turn 1 bulb on. When it burns out, turn on the others at once.

8. (25%) When you are being serviced in a bank, all 3 tellers are busy, and two persons wait in a common queue. No more customers are allowed to enter that day. Assume that the service times for the customers are **iid** exponential random variables. What are the respective probabilities that you are the first, second, third, fourth, and the last to finish service?