Homework 03

(due day in two weeks, 5/20)

Problem 1: (30 points)

(1) If *f(x)* is an even function, find the Fourier transform of *f(x)*, $F(\alpha) = ?$

(2) Based on (1), if $\int_0^\infty f(x)\cos\alpha x\,dx = \begin{cases} 1-\alpha, 0 \le \alpha \le 1 \\ 0, \alpha > 1 \end{cases}$, find *f(x)* = ?

Answer

(1) $f(x) \in$ even function.

$$F(\alpha) = F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx$$

$$= \int_{-\infty}^{\infty} f(x)(\cos\alpha x - i\sin\alpha x) dx$$

$$= 2\int_0^{\infty} f(x)\cos\alpha x\, dx, \quad F(\alpha) \in \text{even} \quad \#$$

(2) $F(\alpha) = \begin{cases} 2(1-\alpha), & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$

$$f(x) = \frac{1}{\pi}\int_0^1 2(1-\alpha)\cos\alpha x\, d\alpha$$

$$= \frac{2}{\pi}\cdot\frac{1}{x}(1-\alpha)\sin\alpha x\Big|_0^1 + \frac{2}{\pi x}\int_0^1 \sin\alpha x\, d\alpha$$

$$= \frac{2}{\pi x^2}[1-\cos x] = \frac{2}{\pi x^2}\cdot 2\sin^2\frac{x}{2}$$

$$= \frac{4}{\pi x^2}\sin^2\frac{x}{2} \quad \#$$

Problem 2: (30 points)

Please find the 4th Maclaurin series of $\sqrt{x+1}$ and find its value at $\sqrt{0.9}$. Afterward, please show the approximation error in this case.

Answer

①

$$f(x) = (x+1)^{\frac{1}{2}} \Rightarrow f(0) = 1$$

$$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}} \Rightarrow f'(0) = \frac{1}{2}$$

$$f''(x) = -\frac{1}{4}(x+1)^{-\frac{3}{2}} \Rightarrow f''(0) = -\frac{1}{4}$$

$$f'''(x) = \frac{3}{8}(x+1)^{-\frac{5}{2}} \Rightarrow f'''(0) = \frac{3}{8}$$

$$f^{(4)}(x) = -\frac{5}{16}(x+1)^{-\frac{7}{2}} \Rightarrow f^{(4)}(0) = -\frac{15}{16}$$

$$\therefore f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 \quad \#$$

② Assume $x = -0.1$

$$\therefore f(-0.1) = 1 + \frac{1}{2}(-0.1) - \frac{1}{8}(-0.1)^2 + \frac{1}{16}(-0.1)^3 - \frac{5}{128}(-0.1)^4$$

$$\doteqdot 0.948683594 \quad \#$$

③ Approximation error $= \dfrac{f^{(5)}(x)}{5!}x^5$

$$\because f^{(5)}(x) = \frac{35}{32}(x+1) \Rightarrow f^{(5)}(0) = \frac{35}{32}$$

$$\therefore \text{Approximation error} = \frac{\frac{35}{32}}{5!}(-0.1)^5 \doteqdot -9.11\times 10^{-8} \quad \#$$

Problem 3: (20 points)

Find the Fourier Transform of $f(x) = e^{-|x+3|} - 2e^{-|x|}$.

Answer

$$\mathcal{F}[e^{-|x+3|} - 2e^{-|x|}] = \mathcal{F}[e^{-|x+3|}] - 2\,\mathcal{F}[e^{-|x|}]$$

$$= e^{i3w}\frac{2}{1+w^2} - 2\cdot\frac{2}{1+w^2}$$

$$= e^{i3w}\frac{2}{1+w^2} - \frac{4}{1+w^2}$$

Problem 4: (20 points)

In order to move one cannon, two group of soldiers try to tug it, as shown in Fig.1. As the commander knows, the vector of one tugging group is (3,4) and the two tugging groups angle 45°. Please determine the vector of the cannon moving direction.

Answer:

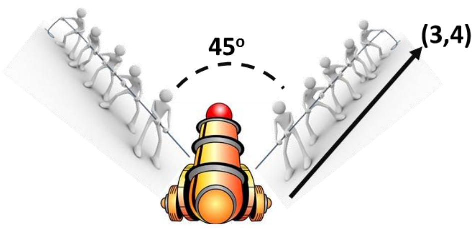
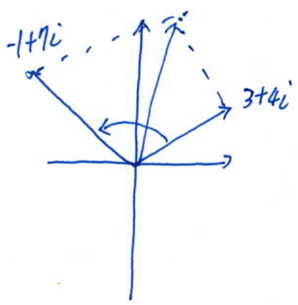


Fig.1



$(3+4i)*(1+i) = -1+7i$

$(-1+7i)+(3+4i) = 2+11i$

$\therefore$ Ans. $(2, 11)$