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Quiz 02 (60 mins.)

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Problem 1: (30 points)

A function $f(x) = e^{-a|x|}$, $a > 0$, please find 《hint: $\int_0^\infty e^{-ax}\cos\omega x dx = \frac{a}{a^2+\omega^2}$》

(1) Fourier integral of $f(x)$

(2) Calculate $\int_0^\infty \frac{\cos(2x)}{x^2+4}dx$

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Answer:

$f(x) = \frac{1}{\pi}\int_0^\infty (A(w)\cos wx + B(w)\sin wx)dw$

(1) $\because f(x) = f(-x)$

$\therefore f(x) \in$ even function

$\Rightarrow B(w) = 0$

$A(w) = \int_{-\infty}^{\infty} f(x)\cos wx\, dx$

$= \int_{-\infty}^{\infty} e^{-a|x|}\cos wx\, dx$

$= \int_{-0}^{\infty} e^{ax}\cos wx\, dx$

$= \frac{a}{w^2+a^2}$

$\therefore f(x) = \frac{2}{\pi}\int_0^\infty \left(\frac{a}{w^2+a^2}\cos wx\right)dw$ #

(2) $a = 2$

$f(x) = \frac{1}{\pi}\int_0^\infty \left(\frac{2}{w^2+4}\cos wx\right)dw$

$f(2) = \frac{1}{\pi}\int_0^\infty \left(\frac{2}{w^2+4}\cos 2w\right)dw$

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$e^{-4} = \frac{1}{\pi}\int_0^\infty \left(\frac{2}{x^2+4}\cos 2x\right)dx$

$\therefore \int_0^\infty \frac{\cos 2x}{x^2+4}dx = e^{-4} \times \frac{\pi}{2}$ #

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Problem 2: (30 points)

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Please use Fourier Integral representation to show that $\int_0^\infty \frac{\cos(\pi\omega/2)\cos\pi\omega}{1-\omega^2} = \begin{cases} \frac{\pi}{2} & ,|x| < \frac{\pi}{2} \\ 0 & ,|x| > \frac{\pi}{2} \end{cases}$

Answer:

$\frac{1}{\pi}\int_0^\infty \frac{\cos\left(\frac{wx}{2}\right)\cos(wx)}{1-w^2}dw = \begin{cases} \frac{1}{2} & ,|x| < \frac{\pi}{2} \\ 0 & ,|x| > \frac{\pi}{2} \end{cases}$

$\int_0^\infty \frac{\cos\left(\frac{wx}{2}\right)\cos(wx)}{1-w^2}dw = \begin{cases} \frac{\pi}{2} & |x| < \frac{\pi}{2} \\ 0 & |x| > \frac{\pi}{2} \end{cases}$

$f(x) = \frac{1}{\pi}\int_0^\infty (A(w)\cos wx + B(w)\sin wx)dw$

$f(x) \in$ even function

$\rightarrow B(w) = 0$.

$\frac{\cos\frac{wx}{2}}{1-w^2}$

$A(w) = \int_{-\infty}^{\infty} f(x)\cos wx\, dx$

$= 2\int_0^{\frac{\pi}{2}} \frac{1}{2}\cos wx\, dx$

$= \left[\frac{1}{w}\sin wx \Big|_0^{\frac{\pi}{2}}\right]$

$= \frac{1}{w}\sin\frac{\pi}{2}w$

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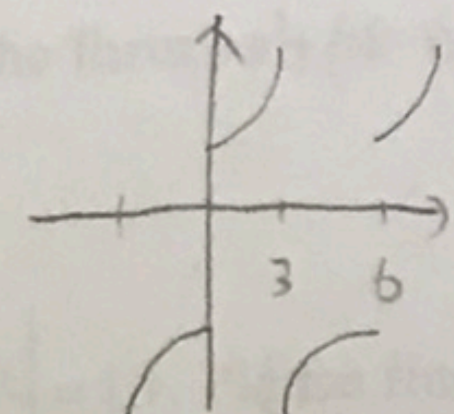
Problem 3: (20 points)

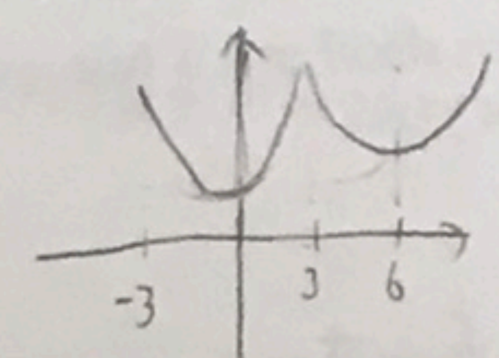
You can expand the function defined by $f(x) = x^2 + 3,\ 0 < x < 3$ in a Fourier series, a cosine series, or a sine series. Please choose the correct answers.

(a) $f(6) = 3$ for Fourier sine series; (b) $f(3) = 12$ for Fourier cosine series; (c) $f(0) = 3$ for Fourier series; (d) $f(-1) = 4$ for Fourier sine series; (e) $f(-3) = 12$ for Fourier cosine series

Answer: be.



sine Series



cosine series

$\frac{2n\pi}{T}$

Fourier cosine Series

⇒ $f(x) \in$ even function

⇒ $b_n = 0$

$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\,dx = \frac{1}{6}\times 2\times\int_0^3 (x^2+3)\,dx = \frac{1}{3}\times\left[\frac{1}{3}x^3+3x\Big|_0^3\right] = \frac{1}{3}\times(9+9) = 6.$

$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\cos\frac{n\pi}{3}x\,dx = \frac{1}{3}\times 2\int_0^3 (x^2+3)\cos\frac{n\pi}{3}x\,dx = \frac{2}{3}\left[(x^2+3)\frac{3}{n\pi}\sin\frac{n\pi}{3}x\Big|_0^3 - \int\frac{3}{n\pi}\sin\frac{n\pi}{3}x\cdot 2x\,dx\right] = \frac{-2}{3}\left[\int\frac{3}{n\pi}\sin\frac{n\pi}{3}x\cdot 2x\cdot dx\right]$

Fourier Sine Series

→ odd function

→ $a_0 = a_n = 0$

$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\sin\frac{2n\pi}{T}x\,dx = \frac{1}{3}\int_{-3}^{3} f(x)\sin\frac{n\pi}{2}x\,dx = \frac{2}{3}\int_0^3 (x^2+3)\sin\frac{n\pi}{3}x\,dx$

$= \frac{2}{3}\left[\underbrace{(x^2+3)}_{u}\frac{-3}{n\pi}\cos\frac{n\pi}{3}x\Big|_0^3 - \int_0^3\frac{-3}{n\pi}\cos\frac{n\pi}{3}x\cdot(2x)\,dx\right]$

$= \frac{2}{3}\left[\left(12\cdot\frac{-3}{n\pi}\cos n\pi - 3\cdot\frac{-3}{n\pi}\right) + \frac{3}{n\pi}\int_0^3\cos\frac{n\pi}{3}x\cdot 2x\,dx\right]$

Problem 4: (20 points)

You can expand the function defined by $f(x) = x,\ 0 < x < \pi$ in a Fourier sine series and Fourier cosine series. Give a set of criteria, and there by judge which series is the better. Please explain. -10

Answer:

Fourier Sine Series is better because Fourier cosine series is only for all the integer n, (n ∈ odd)

Fourier Sine Series

→ $f(x) \in$ odd function

→ $a_0 = a_n = 0$.

$T = 2l = 2\times\pi = 2\pi$

$b_n = \int_{-\pi}^{\pi} f(x)\sin nx\,dx = 2\int_0^\pi x\cdot\sin nx\,dx = 2\left[x\cdot\frac{-1}{n}\cos nx\Big|_0^\pi - \int_0^\pi\frac{-1}{n}\cancel{\cos nx}\,dx\right] = 2\left[\pi\cdot\frac{-1}{n}\cos n\pi\right] = \frac{-2\pi}{n}\cos n\pi$

∴ $f(x) = \sum_{n=1}^{\infty}\left(\frac{-2\pi}{n}(-1)^n\sin nx\right)$

Fourier Cosine Series

→ $f(x) \in$ even function

→ $b_n = 0$

→ $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\cdot dx = \frac{2}{2\pi}\int_0^\pi x\,dx = \frac{1}{\pi}\left(\frac{1}{2}x^2\Big|_0^\pi\right) = \frac{1}{\pi}\times\frac{1}{2}\times\pi^2 = \frac{\pi}{2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cdot\cos nx\,dx = \frac{2}{\pi}\int_0^\pi x\cos nx\,dx = \frac{2}{\pi}\left[\left(x\cdot\frac{1}{n}\cancel{\sin nx}\Big|_0^\pi\right) - \int_0^\pi\frac{1}{n}\sin nx\cdot dx\right] = \frac{-2}{n\pi}\int_0^\pi\sin nx\,dx = \frac{-2}{n\pi}\left(\frac{-1}{n}\cos nx\Big|_0^\pi\right) = \frac{-2}{n^2\pi}(\cos n\pi - 1) = \begin{cases}\frac{4}{n^2\pi}, & n \in \text{odd}\\ 0, & n \in \text{even}\end{cases}$

∴ $f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\left(\frac{4}{(2n-1)^2\pi}\cos(2n-1)x\right)$