Final exam 2019.06.12

1. You and friends go to the gym to play badminton. There are 4 courts, and only your group is waiting. Suppose each group on court plays an exponential random time with mean 20 minutes. What is the probability that your group is the last to hit the shower?
2. John Fast is driving from Seattle to Portland, a distance of 210 miles at a constant speed, whose value is uniformly distributed between 60 and 70 miles per hour. What is the PDF of the duration of the trip?
3. The time T_1 from home to office and T_2 from office back to home are normal random variables, with $T_1 \sim \mathcal{N}(20, 4)$ and $T_2 \sim \mathcal{N}(25, 5)$. What is the probability that the commuting time in one day is more than 50 minutes?
4. $X \sim$ **uniform**$(0, 1)$ and $Y \sim$ **uniform**$(0, 2)$. Find the PDF of $Z = X + Y$.
5. City-tour buses arrive on time on the hour and quarter past the hour. A tourist shows up at a random time, any time being equally likely, and wait for a city-tour bus. What is the expected waiting time of the tourist?
6. $G \sim$ **geometric**(0.5). Apply Chebyshev inequality to bound $\mathbf{P}(G \geq 5)$.
7. $U \sim$ **uniform** $\left(0, \frac{\pi}{2}\right)$. Find the expectation of $V = \sin U$.
8. Let $P \sim$ **Poisson**(3). Find $f(s) = \mathbb{E}\left[e^{sP}\right]$.
9. $\Theta \sim$ **uniform**$(0, 2\pi)$. Find the variance of Θ.
10. You are the 36th waiting in IRS hotline, excluding the one being served. Assume that the departures of the callers is a Poisson process with the rate of 2 arrivals per minute. Estimate the probability that you wait for at least 20 minutes in terms of $\Phi(\cdot)$, the CDF of a standard normal, to be serviced.

$$X \sim \textbf{Poisson}(\beta), \;\; p_X(k) = e^{-\beta}\frac{\beta^k}{k!}, \; k = 0, 1, \ldots \quad (E[X] = \beta, \text{var}(X) = \beta)$$

$$X \sim \textbf{exponential}(\lambda), \;\; f_X(x) = \lambda e^{-\lambda x}, \; x \geq 0 \quad \left(E[X] = \frac{1}{\lambda}, \text{var}(X) = \frac{1}{\lambda^2}\right)$$

$$X \sim \mathcal{N}(\mu, \sigma^2), \;\; f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \; x \in \mathbb{R} \quad \left(E[X] = \mu, \text{var}(X) = \sigma^2\right)$$