

Midterm 2017.04.18

1. You are told that \$100 is equally likely to be in any one of 5 closed envelopes. You first pick one envelop. The host opens for you one of the remaining envelopes, after making sure that the money is not in it. At this point, you can stick with the original choice, or switch to another unopened envelop. Is it worth paying \$25 for a game? Why?
2. The test result for a certain rare disease is correct 90% of the time. Any person has a probability of 0.01 of having the disease. Given that John Doe tests positive **twice**, what is the probability that he actually has the disease?
3. A train arrives at a station every quarter hour starting at 6:00 a.m. You walk into the station every morning between 7:10 a.m. and 7:40 a.m., and your arrival time is a discrete uniform random variable distributed uniformly over the minutes. What is the expectation of the time you have to wait for the first train to arrive?
4. Every school day, Adam flips a fair coin to decide how to get to school. If a head shows up, he walks, taking 10-20 minutes. If a tail shows up and is followed by a head, he rides on his motorcycle, 2-5 minutes. If 2 tails show up in a row, he bikes, 5-10 minutes. All random times are discrete and uniformly distributed. What is the expectation of the time T for Adam to get to school?
5. Consider a game where a person is given 2 questions and can select which one to answer first. Question A is answered correctly with probability 0.8, and the prize money is 100. Question B is answered correctly with probability 0.6, and the prize money is 200. If the first question attempted is answered incorrectly, the quiz terminates. If it is answered correctly, the person is allowed to attempt the second question to earn more money. What is the expected total prize money if
 - (a) Question A is answered first?
 - (b) Question B is answered first?
6. At every plate appearance, Chen has a hit with probability 0.05, independent of other appearances. In one game, he attempts 1-3 times with equal probability $1/3$. What is the probability that he has exactly a hit in a game?
7. Consider a discrete random variable X taking $n \geq 2$ values uniformly over the interval $[1, 6]$. What is the expectation and variance of X as a function of n ?
8. The number of e-mails Bill gets in one hour is a Poisson random variable with parameter $\lambda = 0.5$, independent of other times. Suppose he gets N new messages in a 2-hour interval.
 - (a) What is the probability of $N = 1$?
 - (b) What is the variance of N ?