Linear Algebra Midterm 2011.10.26

1. (10%) Solve the following system of linear equations

$$\begin{aligned} u + v + w &= 2 \\ u + 3v + 3w &= 0 \\ u + 3v + 5w &= 2 \end{aligned}$$

2. (10%) True or False? Give a counterexample when false.

   (a) If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB.

   (b) If rows 1 and 3 of B are the same, so are rows 1 and 3 of AB.

   (c) If rows 1 and 3 of A are the same, so are rows 1 and 3 of AB.

   (d) $(AB)^2 = A^2B^2$.

3. (10%) Show that

$$(A^{-1} + B^{-1})^{-1} = A(A + B)^{-1}B.$$

4. (10%) Show that

$$\text{rank}(AB) \leq \text{rank}(A)$$

5. (10%) If A is a 64×17 matrix of rank 11, how many linearly independent vectors satisfy $Ax = 0$? How many linearly independent vectors satisfy $A^T y = 0$?

6. (10%) If you exchange the first two rows of A, which of the four subspaces stay the same? If $y = (1, 2, 3, 4)$ is in the left nullspace of A, write down a vector in the left nullspace of the new matrix.

7. (10%) If A is a 12×17 incidence matrix of a connected graph, what is its rank? How may free variables are there in the solution to $Ax = b$?

8. (10%) What is the axis and the rotation angle for the linear transformation that takes (x_1, x_2, x_3) into (x_2, x_3, x_1)?

9. (20%) Find the set of general solutions for the following system of linear equations

$$\begin{aligned} 1u + 2v + 3w + 5x &= 0 \\ 2u + 4v + 8w + 12x &= 6 \\ 3u + 6v + 7w + 13x &= -6 \end{aligned}$$