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Quiz 01 (60 mins.)

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Problem 1: (20 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

(1) $8\sin 2x + 17\cos 10x$ (2) $\cos\frac{n\pi x}{l}$ (3) $10\cdot e^{-x} + 20\cos 5x$ (4) $\sin x\cos x$

非週期函數

Answer:

(1)

$LCM\left(\frac{2\pi}{2}, \frac{2\pi}{10}\right)$

$= LCM(\pi, \frac{\pi}{5})$

$= \pi$

(2)

$\frac{2\pi}{\frac{n\pi}{l}}$

$= \frac{2l}{n}$

(3)

It is not periodic function.

(4)

2π π

$\sin x\cos x$

$= \frac{1}{2}[\sin(x+x) + \sin(x-x)]$

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$= \frac{1}{2}\sin 2x$

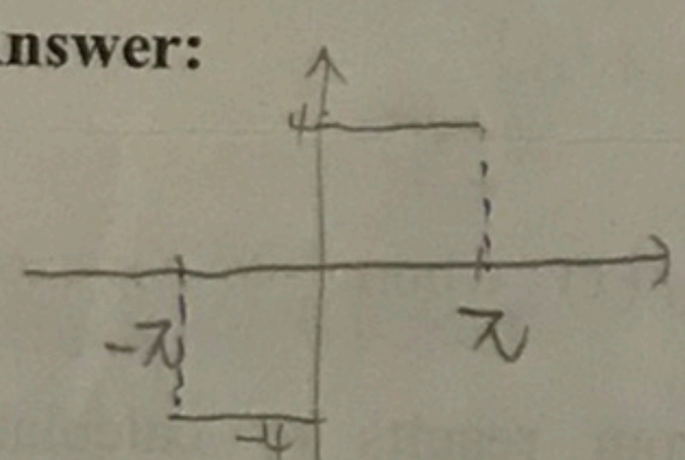
$\rightarrow \frac{2\pi}{2} = \pi$

Problem 2: (20 points)

A function can be defined as $f(x) = \begin{cases} -4, -\pi \le x \le 0 \\ 4\ ,0 \le x \le \pi \end{cases}$. Besides, the corresponding Fourier Series expansion is $\frac{C}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin[(2n-1)x]$. Please find C.

C = 16.

Answer:



T = 2π

∵ f(x) is odd function

∴ $a_0 = a_n = 0$.

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

odd × odd = even

$= \frac{2}{\pi}\int_0^{\pi} 4\cdot\sin nx\,dx$

$dv = 4$ $v = 4x$

$u = \sin nx$ $du = \cos$

$= \frac{8}{\pi}\int_0^{\pi}\sin nx\,dx$

$= \frac{8}{\pi}\left[\frac{\cos nx}{-n}\Big|_0^{\pi}\right]$

$= \frac{-8}{n\pi}(\cos n\pi - 1)$

$= \frac{-8}{n\pi}[(-1)^n - 1]$

$= \begin{cases} 0 & n \in \text{even} \\ \frac{16}{n\pi} & n \in \text{odd} \end{cases}$

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$\therefore f(x) = \frac{16}{\pi}\sum_{\substack{n=1 \\ n\in odd}}^{\infty}\frac{1}{n}\sin nx$

Odd function

$\rightarrow a_0 = a_n = 0$

$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\sin\frac{2n\pi}{T}x\,dx$

$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$= \frac{2}{\pi}\int_0^{\pi} 4\sin nx\,dx$

$= \frac{8}{\pi}\left[\frac{-1}{n}\cos nx\Big|_0^{\pi}\right]$

$= \frac{-8}{n\pi}((-1)^n - 1)$

$= \begin{cases} 0 & n\in even \\ \frac{16}{n\pi} & n \in odd \end{cases}$

$\therefore f(x) = \sum_{\substack{n=1 \\ n\in odd}}^{\infty}\left(\frac{16}{n\pi}\sin nx\right)$

$a_0\ \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n\ \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$b_n\ \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

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Problem 3: (30 points)

$f(x)=\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x$. Please find the Fourier series of *f(x)*.

Answer: $T=2\pi$

$f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)dx$

$=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}+\cancel{\frac{3}{4}\cos x}+\cancel{\frac{2}{5}\sin x}\right)dx$

$=\frac{1}{2\pi}\left[\frac{3}{2}x\Big|_{-\pi}^{\pi}\right]$

$=\frac{1}{2\pi}\left[\frac{3}{2}\pi-\left(-\frac{3}{2}\pi\right)\right]$

$=\frac{1}{2\pi}\times 3\pi$

$=\frac{3}{2}$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx$

$=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\cancel{\frac{3}{2}}+\frac{3}{4}\cos x+\cancel{\frac{2}{5}\sin x}\right)\cos nx\,dx$

$=\frac{1}{\pi}\cdot\frac{3}{4}\pi=\frac{3}{4}$ $\begin{cases}\frac{3}{4} & n=1\\ 0, & n\neq 1\end{cases}$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx$ odd

$=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\cancel{\frac{3}{2}}+\cancel{\frac{3}{4}\cos x}+\frac{2}{5}\sin x\right)\sin nx\,dx$

$=\frac{1}{\pi}\times\frac{2}{5}\pi=\frac{2}{5}$ $\begin{cases}\frac{2}{5} & n=1\\ 0 & n\neq 1\end{cases}$

$f(x)=\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x$

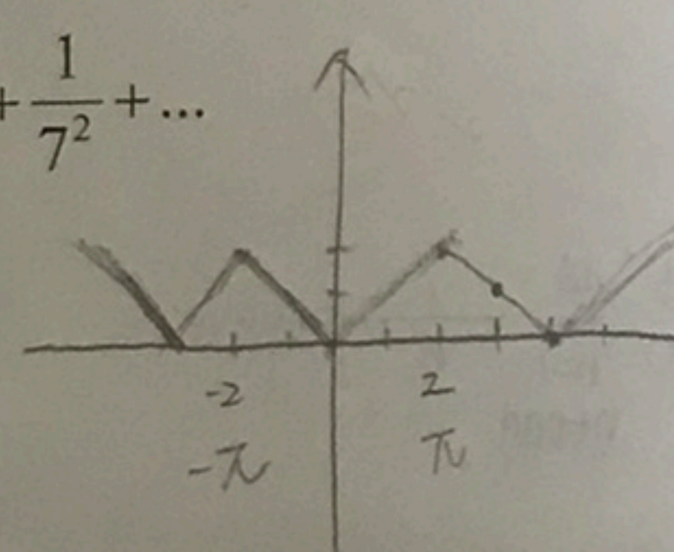
$f(x)=\frac{3}{2}+\sum_{n=1}^{\infty}\left(\frac{3}{4}\cos nx+\frac{2}{5}\sin nx\right)$ #

Problem 4: (30 points)

A function can be defined as $f(x)=|x|$ during $-2\le x\le 2$ and $f(x+4)=f(x)$ when $-\infty<x<\infty$.

① Please find its Fourier Series expansion first. ② Then, please apply your results to calculate $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots$

Answer:



$f(3)=f(-1)$

$f(4)=f(0)$

$f(5)=f(1)$

$f(6)=f(2)$

$f(7)=f(3)$

$T=2\pi$

$f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$

$\because f(x)$ even function

$\therefore \boxed{b_0=0}$

$\boxed{a_0}=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)dx$

$=\frac{2}{2\pi}\int_0^{\pi}x\,dx$

$=\frac{2}{2\pi}\left[\frac{1}{2}x^2\Big|_0^{\pi}\right]$

$=\frac{1}{\pi}\left(\frac{1}{2}\pi^2\right)$

$=\frac{\pi}{2}$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx$ even even

$=\frac{2}{\pi}\int_0^{\pi}\underset{u}{x}\cdot\underset{dv}{\cos nx\,dx}$

$u=x$ $du=1dx$

$dv=\cos nx\,dx$

$V=\frac{\sin nx}{n}$

$=\frac{2}{\pi}\left[x\cdot\frac{\sin nx}{n}-\int_0^{\pi}\frac{\sin nx}{n}\cdot dx\right]$

$=\frac{2}{\pi}\left[x\cdot\frac{\sin nx}{n}-\frac{1}{n}\right]$

$=\frac{2}{n\pi}\left[x\cdot\sin x-1\right]$

$f(x)=1+\sum_{\substack{n=1\\ n\in odd}}^{\infty}\frac{-8}{(n\pi)^2}\cos\frac{n\pi}{2}x$

$\frac{\pi^2}{8}$

$T=4$

even function

$\to b_n=0$

$a_0=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(x)dx$

$=\frac{1}{4}\int_{-2}^{2}f(x)dx$

$=\frac{1}{2}\int_0^2 x\,dx$

$=\frac{1}{2}\left[\frac{1}{2}x^2\Big|_0^2\right]=1$

$a_n=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(x)\cos\frac{2n\pi}{T}x\,dx$

$=\frac{1}{2}\int_{-2}^{2}x\cos\frac{n\pi}{2}x\,dx$

$=\frac{4}{2}\left(x\cdot\frac{2}{n\pi}\sin\frac{n\pi}{2}x+\frac{4}{n^2\pi^2}\cos\frac{n\pi}{2}x\Big|_0^2\right)$

$=\frac{4}{(n\pi)^2}(\cos n\pi-1)$

$=\frac{4}{n^2\pi^2}((-1)^n-1)$

$=\begin{cases}\frac{-8}{n^2\pi^2} & n\in odd\\ 0 & n\in even\end{cases}$