Introduction to Probability Quiz 1

1. (20%) Design team C and design team N are separately asked to design a new product within a month. The probability that team C is successful is 2/3, the probability that team N is successful is 1/2, and the probability that at least one team is successful is 3/4. Assuming that exactly one successful design is produced, what is the probability that it was designed by team N?

2. (20%) You enter a chess tournament where your probability of winning is 0.3 against half the players, 0.4 against a quarter of the players and 0.5 against the remaining players. You play a game against a randomly chosen opponent. What is your probability of winning?

3. (20%) Prove that

$$\mathbf{P}(\mathcal{A}_1 \cap \mathcal{A}_2 \cap \cdots \cap \mathcal{A}_n) \geq \mathbf{P}(\mathcal{A}_1) + \mathbf{P}(\mathcal{A}_2) + \cdots + \mathbf{P}(\mathcal{A}_n) - (n-1)$$

4. (20%) Suppose that $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ for every $n > 1$, and $\mathcal{A} = \cup_{n=1}^{\infty} \mathcal{A}_n$. Show that

$$\mathbf{P}(\mathcal{A}) = \lim_{n \to \infty} \mathbf{P}(\mathcal{A}_n).$$

5. (20%) Let $\mathcal{A}$ and $\mathcal{B}$ be independent events. Show that

$$\mathcal{A}^c \perp \mathcal{B}^c.$$