The Midterm of Calculus 0409

(題目卷)

1. (20 %) Find the limit, if it exists. If the limit does not exist, carefully explain at each step which limit law you have used.

(a) $\lim_{x\to 0}\dfrac{\left(\dfrac{1}{x+4}\right)-\dfrac{1}{4}}{x}$ (b) $\lim_{x\to 0}\dfrac{\sin 2x}{\sin 3x}$ (c) $\lim_{x\to 1}\dfrac{x^3-1}{x^2+x+1}$

(d) $\lim_{x\to\infty}\dfrac{\sin x}{x}$

2. (20 %) Find $\dfrac{dy}{dx}$.

(a) $y=x^3-e^{4x}-\cos 2x+\ln x$ (b) $y=\dfrac{\tan x}{\sqrt{x}+x}$ (c) $y=\dfrac{\sec x}{x}$

(d) $y^3+y^2-5y-x^2=4$

3. (10 %) Let $f(x)=\begin{cases} ax^3 & if\ x\le 2 \\ x^2+b & if\ x>2 \end{cases}$, find the values of a and b that make f differentiable everywhere.

4. (10%) Let $f(x)=\begin{cases} x^2\sin\left(\dfrac{1}{x}\right) & ,\ \text{if}\ x\neq 0 \\ 0 & ,\ \text{if}\ x=0 \end{cases}$.

Determine whether $f'(0)$ exists, justify your answer.

5. (10%) Let $f(x)=\dfrac{x^3}{4}+x-1$, find the value of $\left(f^{-1}\right)'(3)$

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6. (20%) Sketch the graph of $f(x) = x^4 - 4x^3$ and find the critical numbers, the points of inflections, and the relative extrema.

7. (10%) $\dfrac{d}{dx}\left(\int_{e^x}^{x^2+2} \ln(t)\, dt \right)$

8. (20 %) Find the following integrals.

(a) $\int \dfrac{e^x}{1+e^x} dx$ (b) $\int xe^x dx$ (d) $\int \sin^3 x \cos^2 x dx$

(e) $\int \dfrac{x+2}{\sqrt{4-x^2}}\, dx$

公式表：

1. $\left(x^n\right)' = nx^{n-1}$ 2. $\left(e^x\right)' = e^x$ 3. $\left(\ln x\right)' = \dfrac{1}{x}$ 4. $\left(\sin x\right)' = \cos x$

5. $\left(\cos x\right)' = -\sin x$ 6. $\left(\tan x\right)' = \sec^2 x$ 7. $\left(\cot x\right)' = -\csc^2 x$

8. $\left(\sec x\right)' = \sec x \tan x$ 9. $\left(\csc x\right)' = -\csc x \cot x$ 10. $\left(\sin^{-1} x\right)' = \dfrac{1}{\sqrt{1-x^2}}$

11. $\left(\cos^{-1} x\right)' = \dfrac{-1}{\sqrt{1-x^2}}$ 12. $\left(\tan^{-1} x\right)' = \dfrac{1}{1+x^2}$

13. $(uv)' = u'v + uv'$ 14. $\left(\dfrac{v}{u}\right)' = \dfrac{u'v - uv'}{u^2}$

GOOD LUCK