

### Homework 04

(due day in two weeks, 6/16)

#### Problem 1: (30 points)

You decide to transfer one signal  $s(x)$  (i.e.,  $s(x) = e^{-2|x|}$ ) and make the  $f(x) = \cos 3x$  as the carrier wave.

Based on the modulation technique, you can create one new signal  $r(x)$  containing the  $s(x)$  and  $f(x)$ .

Please describe the formulation of the  $r(x)$  in the frequency domain.

Answer

$$S(x) = e^{-2|x|} \xrightarrow{\mathcal{F}} S(\omega) = \frac{4}{4+\omega^2}$$

$$f(x) = \cos 3x \xrightarrow{\mathcal{F}} F(\omega) = \pi [\delta(\omega-3) + \delta(\omega+3)]$$

$$\mathcal{F}[S(x)f(x)] = \frac{1}{2\pi} S(\omega) * F(\omega) = \frac{1}{2\pi} \left[ \frac{4}{4+\omega^2} * (\pi [\delta(\omega-3) + \delta(\omega+3)]) \right]$$

$$= \frac{1}{2} \left[ \frac{4}{4+\omega^2} * \delta(\omega-3) + \frac{4}{4+\omega^2} * \delta(\omega+3) \right] \quad \text{--- Note.}$$

$$= \frac{2}{4+(\omega-3)^2} + \frac{2}{4+(\omega+3)^2}$$

$$\text{Note: } f(x) * \delta(x) = f(x) \Rightarrow f(x) * \delta(x-a) = f(x-a)$$

$$\text{P.F. } \mathcal{F}^{-1}(\mathcal{F}[f(x) * \delta(x)] = \mathcal{F}[f(x)] \cdot \mathcal{F}[\delta(x)] = \mathcal{F}[f(x)] \cdot 1) \Rightarrow f(x) * \delta(x)$$

$$\mathcal{F}^{-1}(\mathcal{F}[f(x) * \delta(x-a)] = \mathcal{F}[f(x)] \cdot \mathcal{F}[\delta(x-a)] = \mathcal{F}[f(x)] \cdot e^{-i\omega a}$$

$$\Rightarrow f(x) * \delta(x-a) = f(x-a) \quad \text{u.z.p.}$$

#### Problem 2: (30 points)

Please compare how many add and how many multiplication are involved to compute the 4-point DFT and 4-point FFT respectively.

Answer

|     | DFT | FFT |
|-----|-----|-----|
| MUL | 16  | 4   |
| ADD | 12  | 8   |

**Problem 3: (30 points)**

Using the Fourier transform to solve the following differential equation.

$$y''(x) + 4y'(x) + 3y(x) = 3\delta(x)$$

**Answer**

The same as the solution of the Problem 4-(1)

**Problem 4: (10 points)**

Imagine that you need to design a problem of the final exam to your friends. Please design a problem or problem group (題組) with detail solutions. Your designed problem must be related to our teaching scope (*i.e.*, Fourier Integral, Fourier Transform, DTFT, DFT, and FFT), which is introduced after the midterm exam. You are allowed to design your problem with either Chinese or English.

**Answer**