學號：　　姓名：

國立中山大學 97 學年度第 2 學期資訊工程學系資工數學

Quiz #4

2009/04/15

Find the Laplace transforms of the following functions. Show the details of your work. (ω, θ are constants.)

1. $\cos(\omega t + \theta)$ (20%)

$\cos(\omega t + \theta) = \cos\omega t \cdot \cos\theta - \sin\omega t \cdot \sin\theta$

$\mathcal{L}(\cos(\omega t + \theta)) = \cos\theta\,\mathcal{L}(\cos\omega t) - \sin\theta\,\mathcal{L}(\sin\omega t)$

$$= \cos\theta\frac{s}{s^2+\omega^2} - \sin\theta\frac{\omega}{s^2+\omega^2}$$

$$= \frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$$

2. $e^{-t}\sinh 5t$ (20%)

$$e^{-t}\sinh 5t = e^{-t}\cdot\frac{1}{2}(e^{5t} - e^{-5t}) = \frac{1}{2}(e^{4t} - e^{-6t})$$

$$\mathcal{L}(e^{-t}\sinh 5t) = \frac{1}{2}\left(\frac{1}{s-4} - \frac{1}{s+6}\right) = \frac{5}{(s+1)^2 - 25}$$

（背面尚有試題）

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Find the inverse transform. Show the details. (*L*, *n* are constraints)

3. $\dfrac{n\pi L}{L^2 s^2 + n^2\pi^2}$ (20%)

$$\mathcal{L}^{-1}\left(\frac{n\pi L}{L^2 s^2 + n^2\pi^2}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{n\pi / L}{s^2 + (n\pi / L)^2}\right)$$

$$= \sin\frac{n\pi t}{L}$$

4. $\dfrac{18s-12}{9s^2-1}$ (20%)

$$\mathcal{L}^{-1}\left(\frac{18s-12}{9s^2-1}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{2s-4/3}{s^2-1/9}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{2s}{s^2-(1/3)^2} - \frac{4/3}{s^2-(1/3)^2}\right)$$

$$= 2\cosh\frac{1}{3}t - 4\sinh\frac{1}{3}t$$

5. $\dfrac{\sqrt{8}}{(s+\sqrt{2})^3}$ (20%)

$$\mathcal{L}^{-1}\left(\frac{2}{s^3}\right) = t^2$$

$$\mathcal{L}^{-1}\left(\frac{\sqrt{8}}{(s+\sqrt{2})^3}\right) = t^2 e^{-t\sqrt{2}} \cdot \frac{\sqrt{8}}{2} = \sqrt{2}\, t^2 e^{-t\sqrt{2}}$$