# 機率學期中考 2015.04.15

1. We toss a fair coin 3 times. Define events

$$\begin{aligned} \mathcal{A} &= \{\text{more heads then tails come up}\}, \\ \mathcal{B} &= \{\text{1st toss is a head}\}. \end{aligned}$$

   Find the conditional probability $\mathbf{P}(\mathcal{A}|\mathcal{B})$.

2. You roll a fair 4-sided die. Whenever the result is 1, you can roll once more, otherwise you stop. What is the probability that the sum total of your rolls is at least 4?

3. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present, the radar generates a false alarm with probability 0.05. Suppose that an aircraft is present with probability 0.01. Given that an alarm has been generated by the radar, what is the probability that an aircraft is present?

4. Find the mean and variance of

   (a) a Poisson random variable with parameter $\lambda = \frac{1}{2}$. Hint:

$$p_Z(k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, \ldots$$

   (b) a geometric random variable

$$G \sim \textbf{geometric}\left(p = \frac{1}{5}\right).$$

5. A student will take a certain test repeatedly, up to a maximum of 3 times, each time with a probability 0.8 of passing, independent of the number of previous attempts. What is the conditional PMF of the number of attempts $N$, given that the student passes the test?

6. If the weather is good (with probability 0.7), Alice walks the 3 miles to school at a speed of 3 miles per hour. If it is cloudy (with probability 0.2), Alice rides her bicycle to school at a speed of 8 miles per hour. Otherwise, she drives to school at a speed of 20 miles per hour. What is the mean time for Alice to get to school?

7. Alvin's walking time to work is between 10 and 20 minutes in a sunny day, between 15 and 25 minutes in a cloudy day, and between 20 to 30 minutes in a rainy day, with all times being equally likely in each case. A day is sunny with probability 1/2, cloudy with probability 1/3, and rainy with probability 1/6. What is the PDF of the walking time $X$?

8. You are allowed to take a certain test 3 times, and your final score will be the maximum of the test scores. Assume that your score in each test takes one of the values from 1 to 10 with equal probability 1/10, independently of the scores in other tests. What is the probability that the final score is 5?

9. The yearly rainfall at Mountain Rainier is modeled as a normal random variable with a mean of 50 inches and a standard deviation of 15 inches. What is the probability that this year's rainfall will be at least 60 inches? Please express the probability by $\Phi(y)$.

10. A surface is ruled with parallel lines, which are at distance 8 cm from each other. Suppose that we throw a needle of length 6 cm on the surface at random. What is the probability that the needle will intersect one of the lines?