

## The 5th Quiz of Calculus 0430

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1. (60%) Find the following integral

(1).  $\int e^x \sin 2x dx$

(2).  $\int \sin 5x \cos 4x dx$

(3).  $\int \frac{1}{\sqrt{1+4x^2}} dx$

(4).  $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

2. (40%) Use L'Hopital's rule to find the following limit

(1).  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

(2).  $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

3.(10%) Determine all values of  $p$  for which the improper integral

$$\int_1^{\infty} \frac{1}{x^p} dx \text{ converges.}$$

公式:

$$\sin(m+n)x + \sin(m-n)x = 2 \sin mx \cos nx$$

$$1. \quad \sin(m+n)x - \sin(m-n)x = 2 \cos mx \sin nx$$

$$\cos(m+n)x + \cos(m-n)x = 2 \cos mx \cos nx$$

$$\cos(m+n)x - \cos(m-n)x = -2 \sin mx \sin nx$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left( a^2 \sin^{-1} \left( \frac{x}{a} \right) + x \sqrt{a^2 - x^2} \right) + C$$

$$2. \quad \int \sqrt{x^2 - a^2} dx = \frac{1}{2} \left( x \sqrt{x^2 - a^2} - a^2 \ln |x + \sqrt{x^2 - a^2}| \right) + C, x > a$$

$$\int \sqrt{a^2 + x^2} dx = \frac{1}{2} \left( x \sqrt{a^2 + x^2} + a^2 \ln |x + \sqrt{a^2 + x^2}| \right) + C$$