Quiz 2018.05.23

1. (10%) May P answers each of her students' questions incorrectly with probability 1/4, independent of other questions. In each lecture, she is asked 0, 1, or 2 questions with equal probability 1/3. Given that she answered only one question incorrectly in the last session, what is the probability that she was asked exactly a question in that session?

2. (10%) Alvin's driving time to work is between 15 and 25 minutes in a sunny day, and between 20 and 30 minutes in a rainy day, with all times being equally likely in each case. Assume that a day is sunny with probability 2/3, and rainy with probability 1/3. What is the PDF of the driving time, viewed as a random variable X?

3. (30%) In a championship series, the first team to win 4 games clinches the title and ends the series. Games 1, 2, 5, and 7 are held in the city of Team H, and Games 3, 4, and 6 are held in the city of Team G. Assume that a game is won by the home team with probability 2/3.

   - What is the probability of Game 7?
   - What is the probability that Team H clinches the title at home?
   - What is the probability that Team G clinches the title away from home?

4. (30%) Discrete random variables X, Y, Z have joint PMF

$$p_{XYZ}(x, y, z) = c(x + y + z), \quad (x, y, z) \in \{1, 2, 3\} \times \{1, 2, 3\} \times \{1, 2, 3\}$$

   Find $p_X(1), p_{YZ}(1, 1), p_{X|Y}(1|1), p_{X|YZ}(1|1, 1), p_{YZ|X}(1, 1|1)$ and c.