

Introduction to Probability Quiz 2

1. What are the means and variances of the following random variables? One must show the reasoning process for full credit.
 - (a) (10%) $U \sim \text{uniform}[3, 8]$
 - (b) (10%) $X \sim \text{Bernoulli}(0.7)$
 - (c) (10%) $G \sim \text{geometric}(0.8)$
 - (d) (5%) $B \sim \text{binomial}(10, 0.3)$ (mean only)
 - (e) (5%) $Z \sim \text{Poisson}(0.9)$ (mean only)
2. (20%) A smoker carries one matchbox in his right pocket and one in his left pocket. Each time he wants to light a cigarette, he selects a matchbox from either pocket with probability $1/2$, independent of earlier selections. What is the PMF of the number of remaining matches at the moment when he reaches for a match and discovers that the corresponding matchbox is empty?
3. (20%) A die with r faces, numbered $1, \dots, r$, is rolled a fixed number of times n . The probability that the i th face comes up on any one roll is p_i , and the results of different rolls are assumed independent. Let X_i be the number of times that the i th face comes up. Find

$$E[X_i X_j]$$

for $i \neq j$.

4. (20%) A transmitter sends out either a 1 with probability p , or a 0 with probability $1 - p$, independent of other transmissions. If the number of transmissions within a given time interval has a Poisson PMF with parameter λ , show that the number of 1s transmitted in that same time interval has a Poisson PMF with parameter $p\lambda$.