

機率學期末考

1. What is the score you need to achieve in the final exam in order to pass? If you need to achieve more than 60 to pass and your answer to the above question is within ± 5 , your final-exam score will be transformed by

$$y = x > 60 ? \min(100, x + 0.2(x - 60)) : x$$

2. (20%) Decide appropriate sample spaces for 2 alternative games as follows, both involving 5 successive coin tosses.
 - (a) Game 1: We receive \$1 each time a head comes up.
 - (b) Game 2: We receive \$1 for each coin toss, up to and including the first time a head comes up. Then we receive \$2 for each coin toss, up to and including the second time a head comes up. More generally, the dollar amount per toss is doubled each time a head comes up.
3. (10%) If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.9. If an aircraft is not present, the radar generates a false alarm with probability 0.1. Suppose that an aircraft is present with probability 0.2. Given that an alarm has been generated by the radar, what is the probability that an aircraft is present?
4. (10%) Professor May B. Right answers each of her students' questions incorrectly with probability $1/3$, independent of other questions. In each lecture, she is asked 1 or 2 questions with equal probability $1/2$. Let X and Y be respectively the number of questions she is asked and the number of questions she answers wrong in a given lecture. Find $p_{XY}(x, y)$.
5. (10%) Consider a continuous random variable X uniformly distributed over the interval $[1, 3]$. What is the expectation and variance of X ?
6. (10%) Bill gets e-mails according to a Poisson process at a rate of $\lambda = 0.5$ messages per hour. He checks email every 6 hours. What is the probability of finding 1 new message?
7. (20%) **Alice** is taking a probability course. In each week, she can be **up-to-date** or **fallen-behind**. If she is up-to-date in a given week, the probability that she will be up-to-date the next week is 0.7. If she is fallen-behind in a given week, the probability that she will be up-to-date the next week is 0.5.
 - (a) Construct a Markov chain using transition probability graph.
 - (b) What are the steady-state probabilities?

8. (20%) A computer executes 2 types of jobs, **priority** and **non-priority**, and operates in **slots**. A slot is **busy** if the computer executes a priority job within the slot, and is **idle** otherwise. We call a string of idle (or busy) slots, flanked by busy (or idle) slots, an idle (or busy) **period**.

A priority job arrives with probability 0.2 at the beginning of each slot, independent of other slots, and requires one full slot to execute. A non-priority job is always available and is executed at a given slot if no priority job is available.

What are the means of the following random variables?

- (a) X : the time index of the first idle slot
- (b) Y : the length of the first busy period

9. (20%) A gambler wins \$1 at each round with probability p , and loses \$1 with probability $1 - p$. He plays continuously until he either accumulates a target amount of \$10, or loses all his money. Suppose he begins with \$6. What is the probability that he wins

- (a) if $p = \frac{1}{2}$?
- (b) if $p = \frac{1}{3}$?