In ←

/100

Quiz ~~02~~ 03 (60 mins.)

Department: CSE Name: 艾偉丞 ID: B083040004

Problem 1: (30 points)

The complex number z_1 and z_2 are given by $z_1 = p+2i$ and $z_2 = 1-2i$ where p is an integer.

(1) Find $\frac{z_1}{z_2}$ in the form $a+bi$ where a and b are both real. Give your answer in its simplest form in terms of p.

(2) Given that $\left|\frac{z_1}{z_2}\right| = 13$. Please find the possible value of p.

Answer:

(1) $\frac{(p+2i)(1+2i)}{(1-2i)(1+2i)} = \frac{(p-4)+(2p+2)i}{1+4} = \frac{p-4}{5} + \frac{2p+2}{5}i$ #

(2) $\left(\frac{p-4}{5}\right)^2 + \left(\frac{2p+2}{5}\right)^2 = 13^2$

$\frac{p^2-8p+16}{25} + \frac{4p^2+8p+4}{25} = 169$

$\frac{5p^2+20}{25} = 169$

$p^2+4 = 169\times5$

$p^2 = 845-4 = 841$

$p = \pm\sqrt{841}$

$= \pm 29$ #

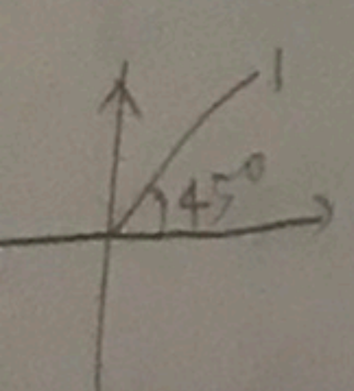
Problem 2: (30 points)

In the first quadrant, there is a vector at a 45-degree angle to the origin and the vector length is 1.

(1) Please define the vector on a complex plane.

(2) If we rotate this vector eight times counterclockwise and each rotation angle is 45 degree as well, please find the final length of this vector.

Answer:



(1) $\cos45^\circ + \sin45^\circ i$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ #

(2) 只有轉動,長度不變

→ the final length = 1

(2) $\left[1\cdot(\cos45^\circ + i\sin45^\circ)\right]^8$

$= (1)^8\left(\cos(\underbrace{45^\circ+45^\circ+45^\circ+\cdots+45^\circ}_{360^\circ}) + i\sin(\underbrace{45^\circ+45^\circ+\cdots+45^\circ}_{360^\circ})\right)$

$= 1$ #

Quiz 02 (60 mins.)

Problem 3: (20 points)

In the practical engineering way, we can use $\boxed{\text{Taylor series}}$ to approximate arbitrary functions. Please consider a function $f(x) = 1 + x + x^2$ and determine the Taylor polynomial with zero approximation error when x is equal to a.

Answer:

$f(x) = x^2 + x + 1 \rightarrow f(0) = 1$

$f'(x) = 2x + 1 \rightarrow f'(0) = 1$

$f''(x) = 2 \rightarrow f''(0) = 2$

從這開始均為0. $\leftarrow$ $f'''(x) = 0$

後面就不會有error

$f(x) = 1 + \frac{1}{1}x + \frac{2}{2!}x^2$

$= 1 + x + x^2$ #

$f(x) = f(a) + (2a+1)(x-a) + \frac{2}{2!}(x-a)^2$

$= (1+a+a^2) + (2a+1)(x-a) + (x-a)^2$

$= 1 + x + x^2$ #

Problem 4: (20 points)

Find the Fourier Transform of $f(x) = e^{-|x+3|} - 2e^{-|x|}$.

《hint: $f(x) = e^{-a|x|}, a > 0$. Then, the fourier transform of $f(x)$ is $\frac{2a}{a^2 + w^2}$.》

Answer:

$F\{e^{-|x+3|} - 2e^{-|x|}\} = F\{e^{-|x+3|}\} - 2F\{e^{-|x|}\}$

$= e^{i3w}\frac{2}{1+w^2} - 2\frac{2}{1+w^2}$

$= e^{i3w}\frac{2}{1+w^2} - \frac{4}{1+w^2}$ #