Quiz 01 (60 mins.)

Department: 資工系 **Name:** 張碩文 **ID:** B09304007

Problem 1: (20 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

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(1) $8\sin 2x + 17\cos 10x$ (2) $\cos\frac{n\pi x}{l}$ (3) $10\cdot e^{-x} + 20\cos 5x$ (4) $\sin x \cos x$

Answer: (1) $\mathrm{LCM}(\pi, 2\pi) = 2\pi \quad \therefore T = 2\pi$ (3) 非週期函數

(2) $1\cdot x \to 2\pi$

$\frac{n\pi}{l}\cdot x \to T \quad T = 2\pi \cdot \frac{l}{n\pi} = \frac{2l}{n}$

(4) $\sin x\cos x = \frac{1}{2}\sin 2x$

$\therefore T = \pi$

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Problem 2: (20 points)

The function $f(x) = \frac{x^2}{4}$ is defined at $-\pi \le x \le \pi$, and it can be expanded to

$$f(x) = \frac{\pi^2}{12} - \cos x + \frac{1}{4}\cos 2x - \frac{1}{9}\cos 3x + \frac{1}{16}\cos 4x + \dots$$

Please solve the series solution of $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$

Answer:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{x^2}{4}dx = \frac{1}{\pi}\int_0^{\pi}\frac{x^2}{4}dx = \frac{1}{4\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{\pi^2}{12}$$

$u = x^2 \quad du = 2x\,dx$

$dv = \cos nx\,dx \quad v = \frac{\sin nx}{n}$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\underbrace{f(x)}_{\text{even}}\underbrace{\cos nx}_{\text{even}}dx = \frac{1}{\pi}\cdot 2\cdot\int_0^{\pi}\frac{x^2}{4}\cdot\cos nx\,dx = \frac{1}{2\pi}\int_0^{\pi}\underbrace{x^2}_{u}\underbrace{\cos nx\,dx}_{dv}$$

$$= \frac{1}{2\pi}\left[-\int_0^{\pi}\frac{\sin nx}{n}\cdot 2x\,dx\right] = \frac{-1}{n\pi}\left[\int_0^{\pi}\underbrace{x}_{u}\underbrace{\sin nx\,dx}_{dv}\right] = \frac{-1}{n\pi}\left[\left[\frac{-x\cos nx}{n}\right]_0^{\pi} + \int_0^{\pi}\frac{\cos nx}{n}dx\right]$$

$u = x \quad du = dx$

$dv = \sin nx\,dx \quad v = \frac{-\cos nx}{n}dx$

$$= \frac{-1}{n^2\pi}\cdot(-\pi\cdot(-1)^n) = \frac{(-1)^n}{n^2}$$

$b_n = 0$

$$\therefore f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx \quad \therefore f(\pi) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{4}$$

Problem 3: (60 points)

$$\therefore \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

Please consider a function $f(x) = \frac{3}{2} + \frac{3}{4}\cos x + \frac{2}{5}\sin x$.

(1) (30%) Please find the Fourier series of *f(x)*.

(2) (30%) If this function is defined only at $0 \le x \le \pi$, please find the Fourier cosine series expansion of *f(x)*.

Answer:

$$\frac{(-1)^n + 1}{n+1} + \frac{(-1)^{n+1} - 1}{n-1} = \frac{\quad}{n^2 - 1}$$

$-n+1 \quad +n+1$

$n-1 \quad n-1$

$$(-1)^n(n-1) + n - 1 + (-1)^{n+1}(n+1) - n - 1$$

$+ -2$

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Quiz 01 (60 mins.)

Department:__________ **Name:** 黃育翔 **ID:** B083040007

Problem 1: (20 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

(1) $8\sin 2x + 17\cos 10x$ (2) $\cos\frac{n\pi x}{l}$ (3) $10\cdot e^{-x} + 20\cos 5x$ (4) $\sin x \cos x$

Answer: (1) Yes, $T = lcm\left(\frac{2\pi}{2}, \frac{2\pi}{10}\right) = \pi$

(2) Yes, $T = \frac{2\pi}{\frac{n\pi}{l}} = \frac{2l}{n}$

(3) No

(4) Yes, $\sin x \cos x = \frac{1}{2}\sin 2x \Rightarrow T = \frac{2\pi}{2} = \pi$

Problem 2: (20 points)

A function can be defined as $f(x) = \begin{cases} -4, -\pi \le x \le 0 \\ 4\ ,0 \le x \le \pi \end{cases}$. Besides, the corresponding Fourier Series expansion is $\frac{C}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\left[(2n-1)x\right]$. Please find C.

Answer: $f(x)$ is odd func. $\Rightarrow a_0, a_n = 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin nx\cdot dx = \frac{2}{\pi}\int_0^{\pi} 4\sin nx\cdot dx$

$= \frac{8}{\pi}\cdot\frac{-\cos nx}{n}\Big|_0^{\pi} = \frac{-8}{n\pi}\cdot\left((-1)^n - 1\right) = \begin{cases} 0 & \text{if } n \in \text{even} \\ \frac{16}{n\pi} & \text{if } n \in \text{odd} \end{cases}$

$\Rightarrow f(x) = \sum_{n=1}^{\infty}\frac{16}{(2n-1)\pi}\cdot\sin[(2n-1)x] = \frac{16}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\cdot\sin[(2n-1)x]$

$\therefore C = 16$

Quiz 01 (60 mins.)

Problem 3: (30 points)

$f(x)=\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x$. Please find the Fourier series of *f(x)*.

Answer:

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x\right)\cdot dx=\frac{1}{2\pi}\left(\int_{-\pi}^{\pi}\frac{3}{2}\cdot dx+\cancel{\int_{-\pi}^{\pi}\frac{3}{4}\cos x\cdot dx}+\cancel{\int_{-\pi}^{\pi}\frac{2}{5}\sin x\cdot dx}\right)$$

$$=\frac{1}{2\pi}\cdot\frac{3}{2}x\Big|_{-\pi}^{\pi}=\frac{1}{2\pi}\cdot\left(\frac{3\pi}{2}-\left(-\frac{3\pi}{2}\right)\right)=\frac{3}{2}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x\right)\cdot\cos nx\cdot dx=\frac{1}{\pi}\left(\cancel{\int_{-\pi}^{\pi}\frac{3}{2}\cos nx\,dx}+\int_{-\pi}^{\pi}\frac{3}{4}\cos x\cos nx\,dx+\cancel{\int_{-\pi}^{\pi}\frac{2}{5}\sin x\cos nx\,dx}\right)$$

$$=\frac{3}{4\pi}\int_{-\pi}^{\pi}\cos x\cdot\cos nx\cdot dx=\begin{cases}\frac{3}{4\pi}\cdot\pi=\frac{3}{4} & \text{if } n=1\\ 0 & \text{if } n\neq 1\end{cases}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x\right)\cdot\sin nx\,dx=\frac{1}{\pi}\left(\cancel{\int_{-\pi}^{\pi}\frac{3}{2}\sin nx\cdot dx}+\cancel{\int_{-\pi}^{\pi}\frac{3}{4}\cos x\sin nx\cdot dx}+\int_{-\pi}^{\pi}\frac{2}{5}\sin x\sin nx\,dx\right)$$

$$=\frac{2}{5\pi}\int_{-\pi}^{\pi}\sin x\sin nx\,dx=\begin{cases}\frac{2}{5\pi}\pi=\frac{2}{5} & \text{if } n=1\\ 0 & \text{if } n\neq 1\end{cases}$$

$$\therefore\ f(x)=\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x$$

Problem 4: (30 points)

A function can be defined as $f(x)=|x|$ during $-2\le x\le 2$ and $f(x+4)=f(x)$ when $-\infty<x<\infty$.

Please find its Fourier Series expansion first. Then, please apply your results to calculate $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots$

Answer: $f(x)$ is even func. $\Rightarrow b_n=0$

$$a_0=\frac{1}{4}\int_{-2}^{2}f(x)\cdot dx=\frac{1}{2}\int_0^2 x\cdot dx=\frac{1}{2}\cdot\frac{1}{2}x^2\Big|_0^2=1$$

$$a_n=\frac{1}{2}\int_{-2}^{2}f(x)\cdot\cos\frac{n\pi x}{2}=\int_0^2 x\cdot\cos\frac{n\pi x}{2}\cdot dx=x\cdot\frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big|_0^2-\int_0^2\frac{2}{n\pi}\cdot\sin\frac{n\pi x}{2}\cdot dx$$

$$=\frac{2}{n\pi}\cdot\frac{2}{n\pi}\cos\frac{n\pi x}{2}\Big|_0^2=\frac{4}{(n\pi)^2}((-1)^n-1)=\begin{cases}0 & \text{if } n \text{ even}\\ \frac{-8}{(n\pi)^2} & \text{if } n \text{ odd}\end{cases}$$

$$f(x)=1-\sum_{n=1}^{\infty}\frac{8}{(2n-1)^2\pi^2}\cdot\cos(2n-1)x$$

$$x=0\Rightarrow f(x)=1-\frac{8}{\pi^2}\left(\frac{1}{1}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)=0$$

$$\Rightarrow 1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$$

A: (1) $f(x)=1-\sum_{n=1}^{\infty}\frac{8}{(2n-1)^2\pi^2}\cdot\cos\frac{(2n-1)x}{2}$

(2) $\frac{\pi^2}{8}$

Quiz 01 (60 mins.)

Department: 資工111 **Name:** 陳數揚 **ID:** B073040007

Problem 1: (15 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

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(1) $10\sin 3x + 17\sin 2x$ (2) $\cos\frac{n\pi x}{l}$ (3) $f(x) = a_0 \cdot e^{-x}$

Answer: (1) $LCM(\frac{2\pi}{3}, \frac{2\pi}{2}) = 2\pi \Rightarrow$ yes, $T = 2\pi$ ※

(2) $T = 2\pi \times \frac{l}{n\pi} = \frac{2l}{n} \Rightarrow$ yes, $T = \frac{2l}{n}$ ※

(3) No, $f(x) \in$ 非週期函數 ※

Problem 2: (30 points)

If there is one function $f(x) = \frac{x^2}{4}, -\pi < x < \pi$, please find its Fourier series first. Then, applying your

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result to find the summation value of $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots$

Answer:

$f(x) = \frac{x^2}{4} \in$ even function

$\Rightarrow b_n = 0$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{x^2}{4} dx = \frac{1}{8\pi} \cdot 2\int_0^{\pi} x^2 dx$

$= \frac{1}{4\pi} \cdot (\frac{1}{3}x^3 \Big|_0^{\pi}) = \frac{\pi^3}{12\pi} = \frac{\pi^2}{12}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{x^2}{4}\cos nx\, dx = \frac{1}{4\pi}\int_{-\pi}^{\pi} x^2 \cos nx\, dx$

$= \frac{2}{4\pi}\int_0^{\pi} x^2 \cos nx\, dx = \frac{1}{2\pi}[\cancel{x^2 \cdot \frac{\sin nx}{n}\Big|_0^{\pi}}^{\,0} - \int_0^{\pi} 2x \frac{\sin nx}{n} dx]$

$= \frac{-1}{8\pi} \cdot \frac{8}{n}\int_0^{\pi} x \sin nx\, dx = \frac{-1}{\pi n}[x \cdot \frac{-\cos nx}{n}\Big|_0^{\pi}$

$-\int_0^{\pi} \cancel{\frac{-\cos nx}{n}}^{\,0} dx]$

$= \frac{\cancel{-1}}{\cancel{\pi} n} \times \cancel{\pi} \times \frac{\cancel{-}\cos n\pi}{n} = \frac{(-1)^n}{n^2}$

$\Rightarrow f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx$ ※1

$\Rightarrow x$代π

$f(\pi) = \frac{\pi^2}{4}$

$= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos n\pi$

$= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\Rightarrow 1 + \frac{1}{4} + \frac{1}{9} + \cdots$

$= \sum_{n=1}^{\infty} \frac{1}{n^2}$

$= \frac{\pi^2}{4} - \frac{\pi^2}{12} = \frac{\pi^2}{6}$ ※2

Quiz 01 (60 mins.)

Problem 3: (30 points)

30 $f(x)=\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x$. Please find the Fourier series of *f(x)*.

Answer:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}+\frac{3}{4}\cancelto{0}{\cos x}+\frac{2}{5}\cancelto{0}{\sin x}\right)dx = \frac{1}{2\pi}\cdot\frac{3}{2}\cdot\int_{-\pi}^{\pi} 1\,dx = \frac{3}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x\right)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}\cancelto{0}{\cos nx}+\frac{3}{4}\cos x\cos nx+\frac{2}{5}\cancelto{0}{\sin x\cos nx}\right)dx$$

$$= \frac{3}{4\pi}\int_{-\pi}^{\pi}\cos x\cos nx\,dx = \begin{cases}\frac{3}{4}, & n=1\\ 0, & n\neq 1\end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{3}{2}\cancelto{0}{\sin nx}+\frac{3}{4}\cancelto{0}{\cos x\sin nx}\right.$$

$$\left.+\frac{2}{5}\sin x\sin nx\right)dx$$

$$= \frac{2}{5\pi}\int_{-\pi}^{\pi}\sin x\sin nx\,dx = \begin{cases}\frac{2}{5}, & n=1\\ 0, & n\neq 1\end{cases}$$

Problem 4: (25 points) $\Rightarrow f(x)=\frac{3}{2}+\frac{3}{4}\cos x+\frac{2}{5}\sin x$ ※

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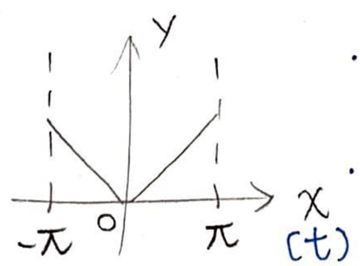
A function *f(t)* is a periodic function and its period is 2π. We define *f(t)* as $f(t)=|t|$ when $-\pi<t<\pi$.

Besides, we can represent the Fouries series of *f(t)* as $f(t)={}^{a_0}\!/_2+\sum_{n=1}^{\infty}(a_n\cos n\cancel{x}^{\,t}+b_n\sin n\cancel{x}^{\,t})$. Please find a_0, a_n, and b_n.

Answer:

$t = x$



$\because f(t) \in$ even function

$\therefore b_n = 0$ ※

$a_0 = \pi$

Ans $a_n = \frac{-4}{(2n-1)^2\pi}$ n

$b_n = 0$

$$a_0' = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} x\,dx = \frac{1}{\pi}\cdot\left(\frac{1}{2}x^2\Big|_0^{\pi}\right)$$

$$= \frac{1}{\pi}\cdot\frac{\pi^2}{2} = \frac{\pi}{2} \qquad a_0' = \frac{\pi}{2} = \frac{a_0}{2} \Rightarrow a_0 = \pi \text{ ※}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x\cdot\cancelto{0}{\frac{\sin nx}{n}}\Big|_0^{\pi} - \int_0^{\pi}\frac{\sin nx}{n}dx\right]$$

$$= \frac{-2}{n\pi}\int_0^{\pi}\sin nx\,dx = \frac{-2}{n\pi}\left(\frac{-\cos nx}{n}\Big|_0^{\pi}\right) = \frac{2}{n^2\pi}(\cos n\pi - 1)$$

$$= \frac{2}{n^2\pi}((-1)^n-1) = \begin{cases}0, & n \in \text{even}\\ \frac{-4}{n^2\pi}, & n \in \text{odd}\end{cases} = \frac{-4}{(2n-1)^2\pi} \text{ ※}$$

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Quiz 01 (60 mins.)

Department: 資工 **Name:** 廖子齊 **ID:** B063040007

Problem 1: (15 points)

Please determine whether the following functions belong to periodic function. If yes, please fine the period.

(1) $10\sin 3x + 17\sin 2x$ (2) $\cos\frac{n\pi x}{l}$ (3) $f(x) = a_0 \cdot e^{-x}$

Answer: (1) $LCM\left(\frac{2\pi}{3}, \frac{2\pi}{2}\right) = 2\pi$ Ans: yes, $T = 2\pi$

(2). $T = \frac{2\pi}{\frac{n\pi}{l}} = 2\pi \times \frac{l}{n\pi} = \frac{2l}{n}$ Ans: yes, $T = \frac{2l}{n}$

(3). 非週期函數.

Problem 2: (30 points)

If there is one function $f(x) = \frac{x^2}{4}, -\pi < x < \pi$, please find its Fourier series first. Then, applying your result to find the summation value of $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots$

Answer:

$F(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n \sin nx)$

$f(x) = f(-x) \Rightarrow$ even function $\Rightarrow b_n = 0$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{x^2}{4}dx = \frac{1}{8\pi}\int_{-\pi}^{\pi}x^2dx = \frac{1}{4\pi}\int_0^{\pi}x^2dx = \frac{1}{4\pi}\times\frac{1}{3}x^3\Big]_0^{\pi}$
$= \frac{\pi^2}{12}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{x^2}{4}\cos nx\, dx = \frac{1}{4\pi}\int_{-\pi}^{\pi}x^2\cos nx\, dx$

$= \frac{1}{2\pi}\left(\cancel{\frac{x^2\sin nx}{n}\Big]_0^{\pi}} + \frac{2}{n^2}x\cos nx\Big]_0^{\pi} - \cancel{\frac{1}{n^2}\times\frac{\sin nx}{n}\Big]_0^{\pi}}\right]$

$= \frac{1}{n^2\pi}(\pi\cos n\pi) = \frac{(-1)^n}{n^2}$

Ans① : $F(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$

$F(\pi) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}(-1)^n$

$\frac{\pi^2}{4} = \frac{\pi^2}{12} + \left(1 + \frac{1}{4} + \frac{1}{9} + \ldots\right)$

$\frac{\pi^2}{6} = 1 + \frac{1}{4} + \frac{1}{9} + \ldots$

Ans②: $\frac{\pi^2}{6}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

Quiz 01 (60 mins.)

Problem 3: (30 points)

$f(x) = \frac{3}{2} + \frac{3}{4}\cos x + \frac{2}{5}\sin 4x$. Please find the Fourier series of *f(x)*.

Answer:

$$F(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\frac{3}{2} + \frac{3}{4}\cos x + \frac{2}{5}\sin 4x\right) dx$$

$$= \frac{1}{2\pi}\left(\frac{3}{2}\times 2\pi + 0 + 0\right)$$

$$= \frac{3}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\cancel{\frac{3}{2}\cos nx} + \frac{3}{4}\cos nx \cos x + \frac{2}{5}\cancel{\cos nx \sin 4x}\right) dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{3}{4}(\cos nx \cos x)\, dx = \frac{3}{4\pi}\int_0^{\pi}(\cos(1+n)x + \cos(1-n)x)\, dx$$

$$n \neq 1 \Rightarrow \frac{3}{4\pi}\left(\left[\frac{\sin(1+n)x}{1+n}\right]_0^{\pi} + \left[\frac{\sin(1-n)x}{1-n}\right]_0^{\pi}\right) a_n = 0$$

$$n = 1 \Rightarrow \frac{3}{4\pi}(\pi) = \frac{3}{4} \quad a_1 = \frac{3}{4}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\cancel{\frac{3}{2}\sin nx} + \frac{3}{4}\cancel{\cos x \sin nx} + \frac{2}{5}\sin 4x \sin nx\right) dx$$

$$= \frac{4}{5\pi}\int_0^{\pi}\sin 4x \sin nx\, dx = \frac{2}{5\pi}\int_0^{\pi}(\cos(4-n)x - \cos(4+n)x)\, dx$$

$$n = 4 \Rightarrow b_4 = \frac{2}{5\pi}(\pi) = \frac{2}{5}$$

$$n \neq 4 \quad b_n = 0$$

Ans: $F(x) = \frac{3}{2} + \frac{3}{4}\sin x + \frac{2}{5}\cos 4x$

Problem 4: (25 points)

If one function *f* satisfies the following properties

f(-x) = -f(x) and *f(-x) = f(x)*,

please find the Fourier series of function *f*.

Answer:

因 $f(-x) = -f(x) \Rightarrow f(x)$ 為奇函數

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx = 0 \quad (\text{odd})$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = 0 \quad (\text{even} \times \text{odd} = \text{odd})$$

因 $f(-x) = f(x)$ $f(x)$ 為偶函數

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = 0 \quad (\text{even} \times \text{odd} = \text{odd})$$

Ans: $F(x) = 0$

$\cos mx \cos nx = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$

$\sin nx \sin mx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$

$\sin nx \cos mx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$

$\cos mx \sin nx = \frac{1}{2}[\sin(m+n)x - \sin(m-n)x]$

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Quiz 01 (60 mins.)

Department: 資工 109 **Name:** 趙俊諺 **ID:** B053040007

Problem 1: (15 points)

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Please determine whether the following functions belong to periodic function. If yes, please fine the period.

$\sin(x+y) = \sin x\cos y + \sin y \cos x$

$\sin(x-y) = \sin x \cos y - \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

(1) $10\sin 3x + 17\sin 2x$ (2) $\cos\frac{n\pi x}{l}$ (3) $f(x) = a_0 \cdot e^{-x}$

Answer:

(1)

$[\frac{2\pi}{3}, \pi] = 2\pi$

$T = 2\pi$

$f(0) = 0$

$f(2\pi) = 0$

(2)

$\cos x$, 2π

$T = 2\pi \times \frac{l}{n\pi} = \frac{2l}{n}$

(3)

not periodic function

Problem 2: (30 points)

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If there is one function $f(x) = \frac{x^2}{4}, -\pi < x < \pi$, please find its Fourier series first. Then, applying your result to find the summation value of $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots$

$\int u\,dv = uv - \int v\,du$

Answer:

$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi x}{T} + b_n \sin\frac{2n\pi x}{T}\right)$

$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\,dx$

$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\cos\frac{2n\pi x}{T}\,dx$

$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\sin\frac{2n\pi x}{T}\,dx$

$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n^2}\cdot\cos nx\right)$ #

$f(\pi) = \frac{\pi^2}{12} + \left(1 + \frac{1}{4} + \frac{1}{9} + \cdots\right) = \frac{\pi^2}{4}$

$\left(1 + \frac{1}{4} + \frac{1}{9} + \cdots\right) = \frac{\pi^2}{6}$ #

$\int_0^{\pi} x\sin nx\,dx$

$= x\cdot\frac{-\cos nx}{n}\Big|_0^{\pi} - \int_0^{\pi}\frac{-\cos nx}{n}\,dx$

$= x\cdot\frac{-\cos nx}{n}\Big|_0^{\pi} + \frac{\cancel{\sin nx}^{0}}{n^2}\Big|_0^{\pi}$

$= \left(\frac{\pi\cdot(-1)^n}{n}\right)$

$T = 2\pi$, even

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{x^2}{4}\,dx = \frac{1}{2\pi}\cdot\left[\frac{x^3}{12}\Big|_{-\pi}^{\pi}\right] = \frac{1}{2\pi}\left[\frac{\pi^3}{12} + \frac{\pi^3}{12}\right] = \frac{\pi^2}{12}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{x^2}{4}\cos nx\,dx = \frac{1}{2\pi}\int_0^{\pi} x^2\cos nx\,dx = \frac{1}{2\pi}\left[x^2\cdot\frac{\cancel{\sin nx}^{0}}{n}\Big|_0^{\pi} - \int_0^{\pi} 2x\cdot\frac{\sin nx}{n}\,dx\right]$

$= \frac{-1}{n\pi}\left[x\cdot\frac{-\cos nx}{n}\Big|_0^{\pi} + \int_0^{\pi}\frac{+\cos nx}{n}\,dx\right] = \frac{+1}{n^2\pi}\left[\pi\cdot(+\cos n\pi) + \frac{\sin nx}{n}\Big|_0^{\pi}\right]$

$= \frac{(-1)^n}{n^2}$

Problem 3: (30 points)

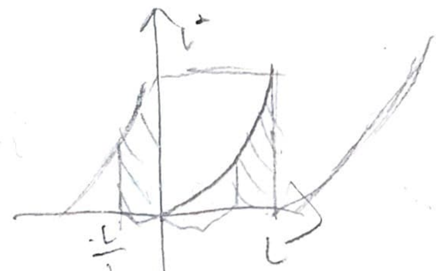
The expansion of the periodic function $f(x)=x^2$ with the period of L (0<x<L) is $f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos\frac{2n\pi x}{L}+b_n\sin\frac{2n\pi x}{L})$. Please determine the value of b_n.

Answer:

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{2n\pi x}{L}\,dx = 0$$

~~even × odd = odd~~

$I = L$



$$= \frac{2}{L}\int_0^L x^2\sin\frac{2n\pi x}{L}\,dx$$

$$= \frac{2}{L}\left\{\left(x^2\cdot\frac{-L}{2n\pi}\cos\frac{2n\pi x}{L}\right)\Big|_0^L - \int_0^L\left(-\frac{L}{2n\pi}\cos\frac{2n\pi x}{L}\right)2x\,dx\right\}$$

$(-1 + \ldots)$ $-\cos 2n\pi$

$$= \frac{2}{L}\left\{\left[-\frac{L^3}{2n\pi}\cos(2n\pi)\right]+\frac{2L}{2n\pi}\int_0^L x\cos\frac{2n\pi x}{L}\,dx\right\}$$

$\left(\frac{L}{2n\pi}\right)\left(-\cos\frac{2n\pi x}{L}\right)\Big|_0^L$

$$= \frac{2}{L}\left\{\frac{-L^3}{2n\pi}+\frac{L}{n\pi}\left[\frac{x\cdot L}{2n\pi}\sin\frac{2n\pi x}{L}\Big|_0^L - \int_0^L\sin\frac{2n\pi x}{L}\cdot\frac{L}{2n\pi}\,dx\right]\right\}$$

$-\cos 2n\pi$

$$= \frac{2}{L}\left\{\frac{-L^3}{2n\pi}+\frac{L}{n\pi}\cdot\frac{L}{2n\pi}\cdot 0\right\} = \frac{-L^2}{n\pi} \quad \#$$

$-1+1$

25

Problem 4: (25 points)

If one function f satisfies the following properties

$$f(-x) = -f(x) \text{ and } f(-x) = f(x),$$

please find the Fourier series of function f.

Answer:

$$f(-x) = -f(x) = f(x)$$

$$f(x) = \sum_{n=1}^{\infty}\left(\cos\left(2n-\tfrac{5}{4}\right)\pi + \sin\left(2n-\tfrac{5}{4}\right)\pi\right) = 0$$

$\sin\frac{\pi}{4} = \sin\frac{3}{4}\pi = \cos\frac{\pi}{4} = \cos\frac{7}{4}\pi = \frac{\sqrt{2}}{2}$

$\sin\frac{5}{4}\pi = \sin\frac{7}{4}\pi = \cos\frac{3}{4}\pi = \cos\frac{5}{4}\pi = -\frac{\sqrt{2}}{2}$