

### Homework 01

(due day in two weeks, 3/31)

#### **Problem 1: (20 points)**

Please determine whether the following functions belong to periodic function. If yes, please find the period.

(1)  $\sin \frac{n\pi x}{l}$       (2)  $\cos \frac{n\pi x}{l}$       (3)  $f(x) = a_0 \cdot e^{-x}$

(4)  $f(x) = a_0 + a_1 \cos \frac{\pi x}{l} + b_1 \sin \frac{\pi x}{l} + a_2 \cos \frac{2\pi x}{l} + b_2 \sin \frac{2\pi x}{l} + \dots + a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} + \dots$

#### **Problem 2: (10 points)**

If  $f(x + 2\pi) = f(x)$  and  $f(x) = \begin{cases} -1, & -\pi \leq x < 0 \\ 1, & 0 < x < \pi \end{cases}$ , please find  $f(x)$ 's Fourier Series.

**Problem 3: 40 points)**

Given the function  $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 \leq x < \pi \end{cases}$

(1) Find its Fourier series

(2) Show that  $\frac{1}{2} + \frac{1}{1 \times 3} - \frac{1}{3 \times 5} + \frac{1}{5 \times 7} - \frac{1}{7 \times 9} + \dots = \frac{\pi}{4}$

**Problem 4: (30 points)**

Please find the Fourier Sine series expansion of  $\cos x$  when  $0 \leq x \leq \pi$