Quiz 4 2016.05.11

1. (20%) Let X be uniform on $[0, 1]$ and $Y = \sqrt{X}$. What is the transform $M_Y(s)$?
2. (20%) Find the 4th moment of a standard normal (Gaussian) random variable.
3. (20%) Romeo and Juliet have a date at a given time, and each will arrive at the meeting place with a random delay (X for Juliet, Y for Romeo) that is exponentially distributed with parameter λ. What is the transform $M_Z(s)$ of the difference
$$Z = X - Y$$
4. (20%) Two archers shoot at a target. The distance of each shot from the center of the target is uniformly distributed from 0 to 1, independent of the other shot. What is the PDF of the distance of the winning shot from the center?
5. (20%) A remote village has 3 gas stations. Each gas station is open on any given day with probability 2/3, independent of the others. The amount of gas available in an open station is uniformly distributed between 300 and 600 gallons. What is the expectation and variance of the total amount of gas available at the gas stations that are open in one day?

The transform (moment generating function) of a random variable X is defined by
$$M_X(s) = E\left[e^{sX}\right]$$