

線性代數期中考 II 12.12.12

1. (10%) Let \mathbf{A} be a 4-by-4 matrix with $|\mathbf{A}| = 1/2$. Find $|2\mathbf{A}|$ and $|\mathbf{A}^2|$.
2. (10%) Find the determinant of the matrix

$$\begin{bmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{bmatrix}$$

via the elimination method.

3. (10%) What is the projection of $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ onto $\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$?
4. (15%) Find an orthonormal basis for the plane $x - y + z = 0$ in \mathbb{R}^3 , and find the projection matrix onto the plane.
5. (20%) If \mathbf{u} is a unit vector, show that the matrix $\mathbf{Q} \equiv \mathbf{I} - 2\mathbf{u}\mathbf{u}^T$ is symmetric and orthogonal. Compute \mathbf{Q} with $\mathbf{u} = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ 1/2 \end{bmatrix}$.
6. (20%) Find the best straight-line fit (least-squares) to the measurements

$$\begin{aligned} b = 4 \text{ at } t = -2, & \quad b = 3 \text{ at } t = -1, \\ b = 1 \text{ at } t = 0, & \quad b = 0 \text{ at } t = 2. \end{aligned}$$

Then find the projection of $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ onto the column space of

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

7. (10%) Let \mathbf{p} be the projection vector of \mathbf{b} onto the line through \mathbf{a} with

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find \mathbf{p} , and show that $\mathbf{e} \equiv \mathbf{b} - \mathbf{a}$ is perpendicular to \mathbf{a} .

8. (10%) Suppose that \mathbf{A} is 3-by-4, \mathbf{B} is 4-by-5, and $\mathbf{AB} = \mathbf{0}$. Prove that

$$\text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B}) \leq 4.$$