Introduction to Calculus and Analysis

Volume II

Richard Courant Fritz John

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With the assistance of
Albert A. Blank and Alan Solomon

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Preface

Richard Courant's Differential and Integral Calculus, Vols. I and II, has been tremendously successful in introducing several generations of mathematicians to higher mathematics. Throughout, those volumes presented the important lesson that meaningful mathematics is created from a union of intuitive imagination and deductive reasoning. In preparing this revision the authors have endeavored to maintain the healthy balance between these two modes of thinking which characterized the original work. Although Richard Courant did not live to see the publication of this revision of Volume II, all major changes had been agreed upon and drafted by the authors before Dr. Courant's death in January 1972.

From the outset, the authors realized that Volume II, which deals with functions of several variables, would have to be revised more drastically than Volume I. In particular, it seemed desirable to treat the fundamental theorems on integration in higher dimensions with the same degree of rigor and generality applied to integration in one dimension. In addition, there were a number of new concepts and topics of basic importance, which, in the opinion of the authors, belong to an introduction to analysis.

Only minor changes were made in the short chapters (6, 7, and 8) dealing, respectively, with Differential Equations, Calculus of Variations, and Functions of a Complex Variable. In the core of the book, Chapters 1–5, we retained as much as possible the original scheme of two roughly parallel developments of each subject at different levels: an informal introduction based on more intuitive arguments together with a discussion of applications laying the groundwork for the subsequent rigorous proofs.

The material from linear algebra contained in the original Chapter 1 seemed inadequate as a foundation for the expanded calculus structure. Thus, this chapter (now Chapter 2) was completely rewritten and now presents all the required properties of nth order determinants and matrices, multilinear forms, Gram determinants, and linear manifolds.

The new Chapter 1 contains all the fundamental properties of linear differential forms and their integrals. These prepare the reader for the introduction to higher-order exterior differential forms added to Chapter 3. Also found now in Chapter 3 are a new proof of the implicit function theorem by successive approximations and a discussion of numbers of critical points and of indices of vector fields in two dimensions.

Extensive additions were made to the fundamental properties of multiple integrals in Chapters 4 and 5. Here one is faced with a familiar difficulty: integrals over a manifold M, defined easily enough by subdividing M into convenient pieces, must be shown to be independent of the particular subdivision. This is resolved by the systematic use of the family of Jordan measurable sets with its finite intersection property and of partitions of unity. In order to minimize topological complications, only manifolds imbedded smoothly into Euclidean space are considered. The notion of "orientation" of a manifold is studied in the detail needed for the discussion of integrals of exterior differential forms and of their additivity properties. On this basis, proofs are given for the divergence theorem and for Stokes's theorem in n dimensions. To the section on Fourier integrals in Chapter 4 there has been added a discussion of Parseval's identity and of multiple Fourier integrals.

Invaluable in the preparation of this book was the continued generous help extended by two friends of the authors, Professors Albert A. Blank of Carnegie-Mellon University, and Alan Solomon of the University of the Negev. Almost every page bears the imprint of their criticisms, corrections, and suggestions. In addition, they prepared the problems and exercises for this volume.[1]

Thanks are due also to our colleagues, Professors K. O. Friedrichs and Donald Ludwig for constructive and valuable suggestions, and to John Wiley and Sons and their editorial staff for their continuing encouragement and assistance.

FRITZ JOHN
New York
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[1]In contrast to Volume I, these have been incorporated completely into the text; their solutions can be found at the end of the volume.

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