UNIVERSITY OF BRISTOL, QUANTUM ENGINEERING CDT

Quantum Optics: Assignment 3

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PROBLEM 1: SATURATION (AGAIN....)

1.1

Recall the optical bloch equations:

$$\frac{d}{dt}\sigma_x = \delta\sigma_y - \frac{\Gamma}{2}\sigma_x \tag{1.1}$$

$$\frac{d}{dt}\sigma_y = -\delta\sigma_x - \Omega_R\sigma_z - \frac{\Gamma}{2}\sigma_y \tag{1.2}$$

$$\frac{d}{dt}\sigma_z = \Omega_R\sigma_y - \Gamma(\sigma_z + 1) \tag{1.3}$$

Derive an explicit expression for the expectation values of the three Pauli operators in steady state.

1.2

Now, show that the probability of finding the atom in the excited state can be expressed as:

$$P_{ee} = \frac{S/2}{1+S} \tag{1.4}$$

What is S? Plot this function? Look familiar?

1.3

Now, consider the special case $\delta = 0$. Solve the OBE's exactly (hint: the equations are a set of inhomogeneous first-order differential equations), for an atom initially in the ground state. Plot the probability of being in the excited state as a function of time for several different values of the dimensionless parameter Γ/Ω_R.
Now answer the question we asked in class: which wins, the drive or the damping?

PROBLEM 2: FUN WITH THE OBE'S

2.1

Often in quantum optics, it is desirable to know the frequency of a laser very precisely. There are many different ways of achieving this, depending on how presicely the laser frequency needs to be known. For example, simply using the absorption profile of a warm gas with a known electronic level structure is enough to determine the frequency of a laser to some MHz. Using more complicated techniques (for a REALLY interesting read, look up "Pound-Drever-Hall") this can be brought down to something like the linewidth of the laser. But, as we know, atomic clocks achieve precision much greater than this. Here, the frequency reference is again atoms, but we use a slightly different method.
Recall the OBE's for an atomic subject to a constant drive (see question 1), but set $\Gamma = 0$ for simplicity. Imagine turning this drive on at time $t = 0$, and off again at time $t = \pi/\Omega_R$.
Solve these equations exactly (its much easier to solve these now, since there is no damping).

2.2

Plot the probability of being in the excited state after at time t as a function of the detuning δ, in units of Ω_R.

2.3

What is the width of this function? (or: using P_e as your observable, what is the precision with which you can determine δ)

PROBLEM 3: MORE FUN WITH THE OBE'S

2.1

We learned in the last question that Rabi oscillations can be used as a frequency reference. This suffers from a few problems, however. Hopefully, you realized that your ability to determine the frequency of the drive is limited by the time that the atom interacts with the laser. This time cannot be arbitrarily extended, however, because the drive has a finite linewidth and thus the beam is only phase coherent for a certain amount of time. Instead, consider a different situation:

- Apply a drive for time $\tau = \pi/2\Omega_R$
- Wait for a time T.
- Apply a drive for time $\tau = \pi/2\Omega_R$

Again, solve the OBE's and calculate the probability of being in the excited state. NB: this is a complicated series of solutions. It is highly suggested that you solve it step by step, making simplifications as you go along. Finally, express this in units of Ω_R, as you did in problem 2, and plot it as a function of δ for different values of T. What is the width of this function? (this might be difficult to determine, but by plotting for several different values of T you should be ableto eyeball it)

2.2

This is called "Ramsey Interferometry" and is used in atomic clocks. The probability of being in the excited state is used as a measurement of the detuning, and can be used to establish a very accurate frequency reference. By increasing T, the precision of this measurement can be increased nearly arbitrarily.