Modelling Nonlinear optics with the Bloch-Messiah reduction

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Overview

- What is nonlinear optics?
- Why do we care about it?
- What I have been doing
- Gaussian optics
- Outlook

Motivation

The good

Spontaneous Parametric processes, SPDC, SFWM

- Heralded single photon sources
- Entangled photon pair generation (polarisation, spatial)

Kerr processes

- Self-Phase modulation (SPM) for generating Bannana states (CV)
- Cross-Phase modulation (XPM) for sensing

The bad

- Generating more than two photons -> bad for quantum computing

All Kerr nonlinear processes

- SPM -> Spectral broadening
- XPM -> Unwanted phase shifts on single photons due to propagation of the pump

What do we mean by nonlinear optics?

- Roughly processes that conserve energy but do not conserve photon number.

$$\vec{P} = \chi^{(1)}\vec{E}_1 + \chi^{(2)}\vec{E}_1\vec{E}_2 + \chi^{(3)}\vec{E}_1\vec{E}_2\vec{E}_3 + \dots \tag{1}$$

Here we are going to talk about squeezing, i.e SPDC or SFWM, Hamiltonians are then of the form,

$$\hat{H} = A\hat{a}_S^\dagger\hat{a}_I^\dagger\hat{a}_P + h.c. \tag{2}$$

$$\hat{H} = A\hat{a}_S^\dagger\hat{a}_I^\dagger\hat{a}_P\hat{a}_P + h.c. \tag{3}$$

Note for the rest of this presentation I will drop the hat notatiaion and using the convention a, b are annihilation operators in modes a & b

Hamiltonian

$$\hat{U} = \exp\left[-\frac{i}{\hbar}\left(P\int d\omega_1 \int d\omega_2 f(\omega_1,\omega_2)\hat{a}_1^\dagger(\omega_1)\hat{a}_2^\dagger(\omega_2) + h.c.\right)\right] \tag{4}$$

We can do re-write this Hamiltonian as a Schmidt-decomposition using SVD.

$$-\frac{i}{\hbar} P f(\omega_1, \omega_2) = \sum_k r_k \psi_k(\omega_1) \phi_k(\omega_2) \tag{5}$$

Where ψ & ϕ are unitary matrices,

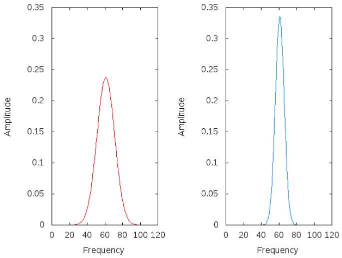
- with $\psi_k(\omega_1)$ is the k-th row and ω_1-th column of $u_{(\omega_1,k)}$,
- with $\phi_k(\omega_2)$ is the ω_2-th row and k-th column of $v^\dagger_{(k,\omega_2)}$

$$P' f(\omega_1, \omega_2) = \sum_k r_k u_{(\omega_1,k)} v^\dagger_{(k,\omega_2)} \tag{6}$$

Recall SVD is defined as,

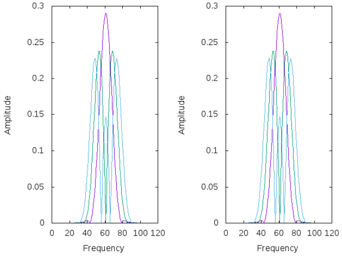
$$M = U\Sigma V^{\dagger} \tag{7}$$

The Joint Spectral Amplitude (JSA)



(a) Signal (red) and Idler (blue)

Non-separable JSAs



(a) Signal (red) and Idler (blue)

Gaussian Optics

- Using the undelpeted pump approximation we can write the Hamiltonians as terms which are at most quadratic in creation and annihilation operators.
- These are Gaussian transforms, they take Gaussian states to Gaussian states

$$\begin{bmatrix} \vec{b} \\ \vec{b}^{\dagger} \end{bmatrix} = M \begin{bmatrix} \vec{a} \\ \vec{a}^{\dagger} \end{bmatrix} \tag{8}$$

Types of Gaussian transformations

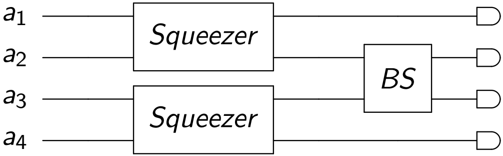


Figure: Two source HOM dip

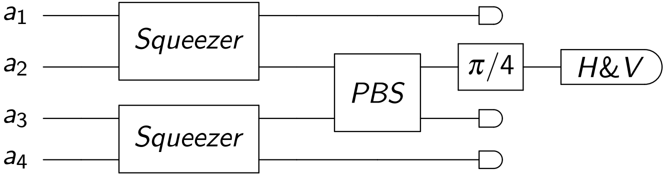
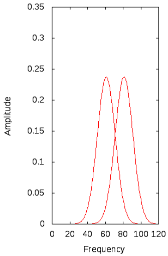
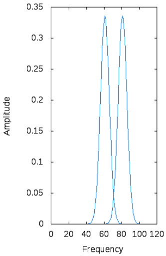


Figure: Type-1 Fusion gate

Two squeezers JSA





G(4) correlation function

$$G^{(4)} = \frac{\left\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_1 \hat{a}_2 \hat{a}_3 \hat{a}_4 \right\rangle}{\left\langle \hat{a}_1^\dagger \hat{a}_1 \right\rangle \left\langle \hat{a}_2^\dagger \hat{a}_2 \right\rangle \left\langle \hat{a}_3^\dagger \hat{a}_3 \right\rangle \left\langle \hat{a}_4^\dagger \hat{a}_4 \right\rangle} \tag{9}$$

Outlook

- There is much to do