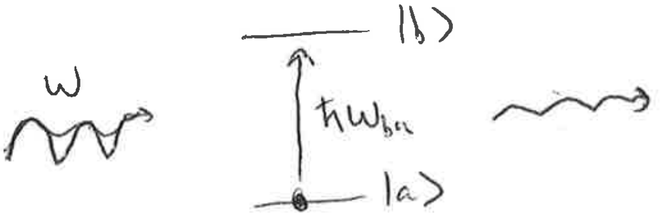
QO2 - Lecture 1

Welcome! In the second half of QO, we will mostly be studying the interaction of light and matter. As it turns out, this will mostly involve a semi-classical treatment: the atom will be treated QM'ly and the light will be treated classically.

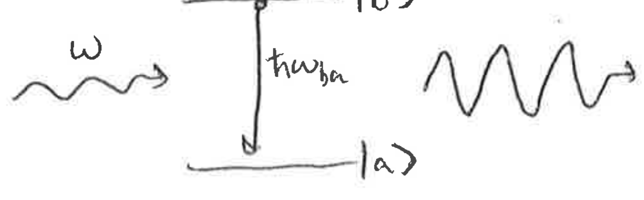
What can happen when light interacts with an atom?

1. Absorption



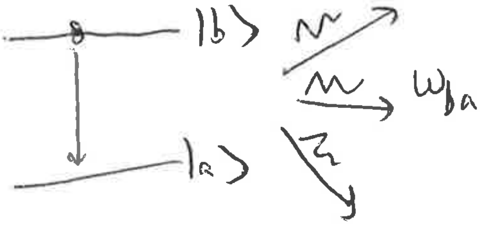
- Atom moves from state $|a\rangle$ to $|b\rangle$
- Amplitude of incident light decreases
- Only occurs (significantly) when $\omega \sim \omega_{ba}$ ("quasi-resonant")

2. Stimulated Emission



- atom moves from state $|b\rangle$ to $|a\rangle$
- amplitude of incident light increases
- proposed by Einstein, not observed until much later (we'll see why later!)
- emitted light is in same direction, with same pol. and phase as incident light.

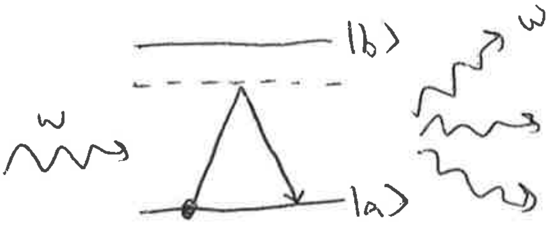
3. Spontaneous Emission



- atom moves from state $|a\rangle$ to $|b\rangle$
- light is emitted with energy $\hbar\omega_{ba}$ but in a random dir$^{\underline{n}}$ with a random phase.
- this cannot be explained (semi-) classically!

Remember: $|b\rangle$ is an eigenstate of the atom hamiltonian, so the atom should remain in this state <u>forever</u>! It is only through interaction with the electromagnetic vacuum that $|b\rangle$ ceases to be an eigenstate.

4. Elastic Scattering



- atom stays in the same state
- amplitude of light decreases (in one direction)
- light is scattered in a spherical wave
- frequency of scattered light is ω, but the amplitude of scattered light varies:

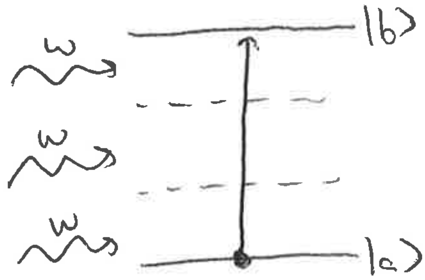
$\propto \omega^4$ when $\omega \ll \omega_0$ (Rayleigh Scattering)

$\propto C$ when $\omega \gg \omega_0$ (Thomson Scattering)

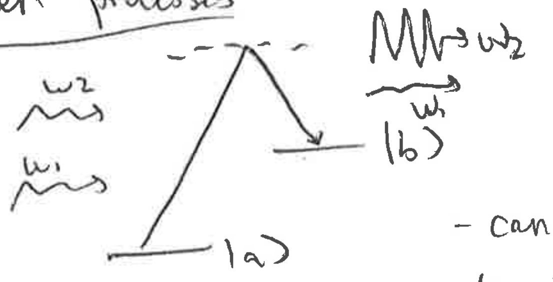
5. Nonlinear processes

eg. Multiphoton absorption

$\omega_{ba} \simeq 3\omega$

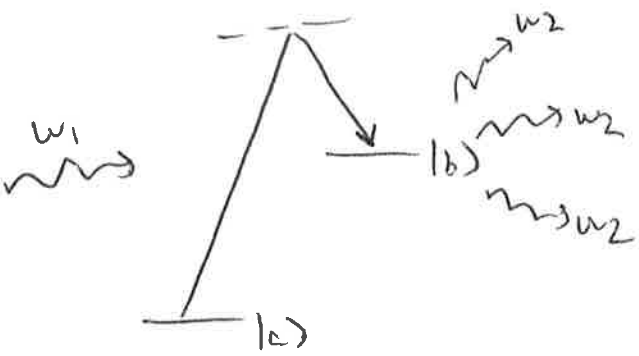


Raman processes:



$\omega_1 - \omega_2 \sim \omega_{ba}$

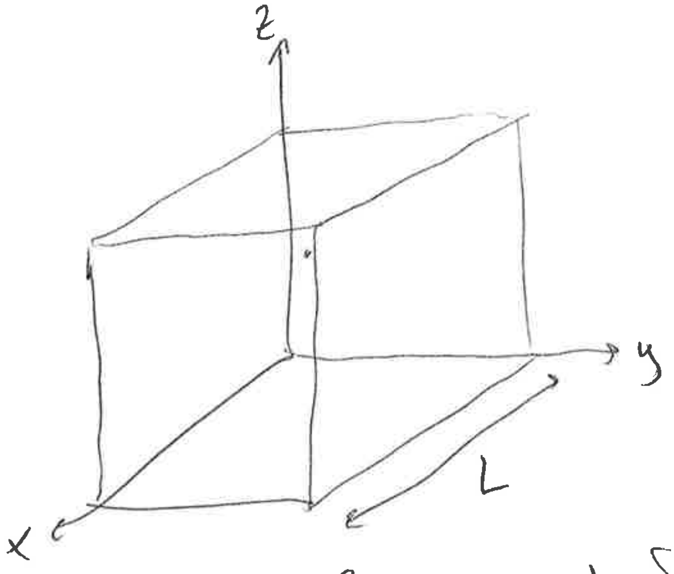
- can think of this as stim. absorption of ω_1 followed by stim. emission of ω_2.



- absorption of ω_1 followed by spontaneous emission of ω_2.

Density of Field Modes in a Cavity

Before we study how light interacts with matter, we will begin with a description of light in a cavity. Once we have done this we are free to take $L \to \infty$ to describe free space.



Given the wave equation $\nabla^2 E(r,t) = \frac{1}{c^2} \frac{\delta^2 E(r,t)}{\delta t^2}$

and the boundary conditions (field = 0 at boundary)

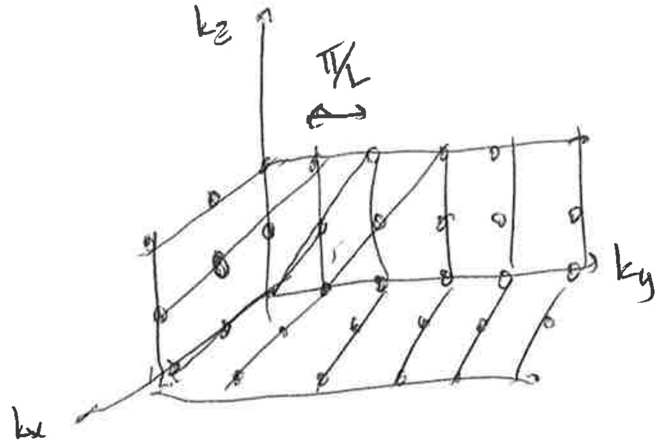
you may recall:

$$E_x(r,t) = E_x(t)\cos(k_x x)\sin(k_y y)\sin(k_z z)$$
$$E_y(r,t) = E_y(t)\sin(k_x x)\cos(k_y y)\sin(k_z z)$$
$$E_z(r,t) = E_z(t)\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

with $k_x = \frac{\pi V_x}{L}$, $k_y = \frac{\pi V_y}{L}$, $k_z = \frac{\pi V_z}{L}$

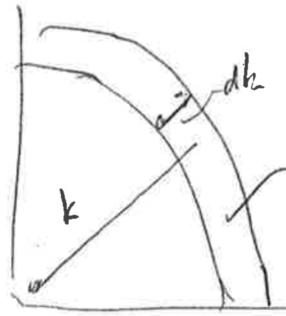
$$V_x, V_y, V_z = 0, 1, 2 \ldots$$

ie)



A grid with spacing π/L

Then the number of field moded between k & $k+dk$ is just the volume of the shell divided by the density of the lattice.



$\frac{1}{8}(4\pi k^2 dk)$ ← volume

density of lattice points $= \left(\frac{\pi}{L}\right)^3$

Then the number of field modes between k & dk is:

$$\frac{1}{8}(4\pi k^2 dk)(\pi/L)^{-3} \times 2$$

where the factor of 2 comes from the 2 polarizations of light.

We want the <u>density of modes</u> which is the number of modes per unit volume of space.

ie) $\frac{d(\text{modes})}{dk} = \rho(k) \Rightarrow d(\text{modes}) = \rho(k)\,dk$

$$= \frac{k^2 dk}{\pi^2}$$

Then $\rho(k) = \frac{k^2}{\pi^2}$ and subbing in $\omega = ck$ we have

$$\boxed{\rho(\omega) = \frac{\omega^2}{\pi^2 c^3}} \; * \; **$$

Recall (from the first half of the course) that the electromagnetic field behaves like a quantum harmonic oscillator with energies: $E_n = \hbar\omega(n+\frac{1}{2})$

In thermal equilibrium (at temperature T) the distribution of excitations is given by the Boltzmann distribution:

$$P(n) = \frac{\exp(-E_n/k_BT)}{\sum_n \exp(-E_n/k_BT)} \qquad \text{sub in } U = \exp(-\hbar\omega/k_BT)$$

$$= \frac{U^n}{\sum_n U^n} = (1-U)U^n$$

Then $\langle n\rangle = \sum_n nP(n) = (1-U)\sum_n nU^n = (1-U)U\frac{\partial}{\partial U}\sum_n U^n$

$$\boxed{\langle n\rangle = \frac{U}{1-U} = \frac{1}{\exp\left(\frac{\hbar\omega}{k_BT}\right)-1}}$$

Now we know the mode density, and the mean energy $(\hbar\omega\langle n\rangle)$.
Then the mean energy density is just:

$$\boxed{\langle W(\omega)\rangle d\omega = \langle n\rangle\hbar\omega\, p(\omega)\, d\omega = \frac{\hbar\omega^3}{\pi^2c^3}\frac{d\omega}{\exp(\hbar\omega/k_BT)-1}}$$ **

Then the total energy density is:

$$\boxed{\int_0^\infty d\omega\langle W(\omega)\rangle = \frac{k_B^4T^4}{\pi^2c^3\hbar^3}\int_0^\infty \frac{x^3dx}{e^x-1} = \frac{\pi^2k_B^4T^4}{15c^3\hbar^3}}$$

(Stefan-Boltzmann Law)

By measuring the spectrum of radiation coming from the cosmic microwave background, one can show $T_{universe} = 2.728K$.

This is really neat!

Note that this is not possible to show unless you assume light is absorbed and emitted in "discrete chunks" of size $\hbar\omega$.

Einstein Coefficients

- simple phenomenological theory introduced by Einstein to understand interaction of light with matter.
- short-term goal: reproduce Stefan-Boltzmann Law (qualitatively)
- long-term goal: learn about LASERS.

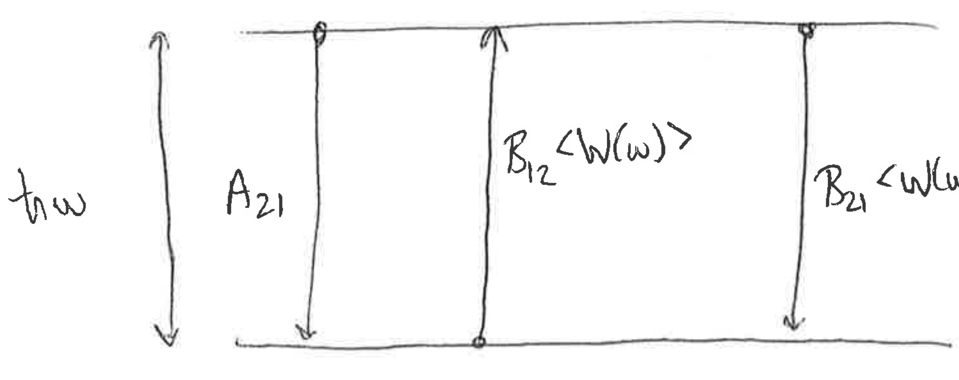
Consider: Some atoms in a cavity in thermal equilibrium with electromagnetic radiation.

Two bound-state energy levels with energy E_1/E_2

$$\boxed{\hbar\omega \equiv E_2 - E_1}$$

Populations in ground and excited state are N_1/N_2

$$\boxed{N_1 + N_2 = N}$$



Spontaneous emission occurs at a rate of A_{21}

Absorption occurs at a rate $B_{12}\langle W(\omega)\rangle$, proportional to the radiative energy density.

Similarly, stimulated emission occurs at a rate $B_{21}\langle W(\omega)\rangle$

This third process was postulated by Einstein!

Let's come up with a rate equation for the atomic population:

$$\frac{dN_1}{dt} = \frac{-dN_2}{dt} = \underset{\text{spont. emission}}{N_2 A_{21}} - \underset{\text{absorption}}{N_1 B_{12}\langle W(\omega)\rangle} + \underset{\text{stim. emission}}{N_2 B_{21}\langle W(\omega)\rangle}$$

In steady state (thermal equilibrium):

$$\frac{dN_1}{dt} = 0 = N_2 A_{21} - N_1 B_{12}\langle W(\omega)\rangle + N_2 B_{21}\langle W(\omega)\rangle$$

and
$$\langle W(\omega)\rangle = \frac{A_{21}}{\left(\frac{N_1}{N_2}\right) B_{12} - B_{21}}$$

Now use the Boltzmann distribution:

$$\frac{N_1}{N_2} = \frac{\exp(-E_1/k_BT)}{\exp(-E_2/k_BT)} = \exp\left(\frac{\hbar\omega}{k_BT}\right)$$

and
$$\boxed{\langle W(\omega)\rangle = \frac{A_{21}}{\exp\left(\frac{\hbar\omega}{k_BT}\right) B_{12} - B_{21}}}$$

This looks just like Planck's Law!

Comparing the two gives us:

$B_{12} = B_{21}$: The rate of absorption and stimulated emission is the same (all other things, like N_1 & N_2, being equal)

$$\left(\frac{\hbar\omega^3}{\pi^2 c^3}\right) B_{21} = A_{21} \Rightarrow \frac{A_{21}}{B_{21}} = \frac{\hbar\omega^3}{\pi^2 c^3} = W_s \quad \left(\text{"saturation energy density"}\right)$$

$$B_{21}\langle W(\omega)\rangle + A_{21} = A_{21}(\langle n\rangle + 1)$$

(the sum of the stimulated and spontaneous emission rates)

Also $\langle W(\omega)\rangle = \dfrac{A_{21}\langle n\rangle}{B_{21}}$

or more conveniently, $\dfrac{B_{21}\langle W(\omega)\rangle}{A_{21}} = \langle n\rangle$

Stimulated emission only wins over spontaneous emission when $\langle n\rangle \gg 1$. We will see that this is important to make a laser.

At room temperature, $\langle n\rangle \sim 1$ for $\lambda \,\hat{=}\, 50\,\mu m$
This is not really "light", so clearly we need to do something differently!

Hint:
What are the atomic populations doing?

Recall: $$\frac{dN_1}{dt} = N_2 A + (N_2 - N_1) B\langle W\rangle \qquad \begin{pmatrix} \text{where } A_{21} = A \\ B_{21} = B_{12} = B \end{pmatrix}$$

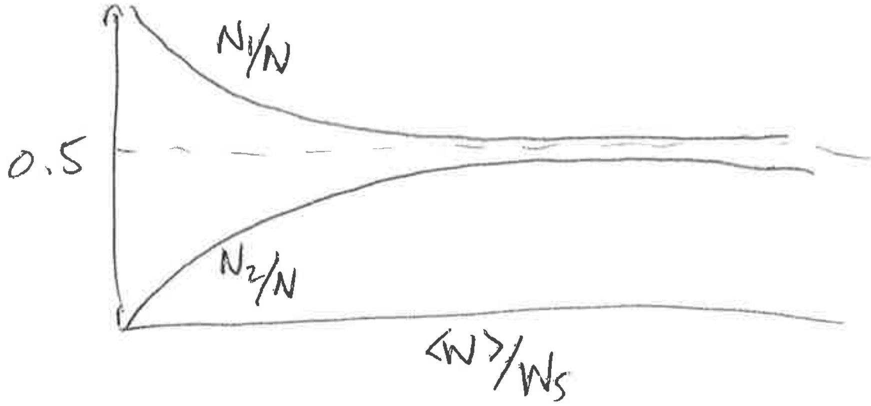
In steady state:

$$N_2 A + (N_2 - N_1) B\langle W\rangle = 0$$

or $$N_2 W_s + (N_2 - N_1)\langle W\rangle = 0$$

Sub in $N = N_1 + N_2$ to get

$$\boxed{N_1 = \left(\frac{W_s + \langle W \rangle}{W_s + 2\langle W \rangle}\right) N \qquad N_2 = \left(\frac{\langle W \rangle}{W_s + 2\langle W \rangle}\right) N}$$



- by increasing the energy density, you drive the atoms upwards (via absorption) exactly as hard (asymptotically) as you drive them downwards (via stimulated emission).

↳ You have "SATURATED" the transition, and yet you have not achieved "POPULATION INVERSION".

So what?

Consider one mode (or group of modes). The scattering out of this mode (and into other modes) is given by the rate of spontaneous emission since stimulated emission, by definition, emits into the same mode.

In a volume V, energy is $E = \dfrac{\langle W(\omega)\rangle \hbar\omega}{V}$

Then $\dfrac{\partial \langle W \rangle}{\partial t} = \dfrac{(N_2 - N_1) B \langle W(\omega) \rangle \hbar\omega}{V}$

The RHS is <u>ALWAYS</u> $-ve$ because, as we showed before, there is no population inversion!

This means that, over time, any specific cavity mode will lose energy!

$\hookrightarrow$ NO GAIN!