QO2 - Lecture 10

What have we learned so far?

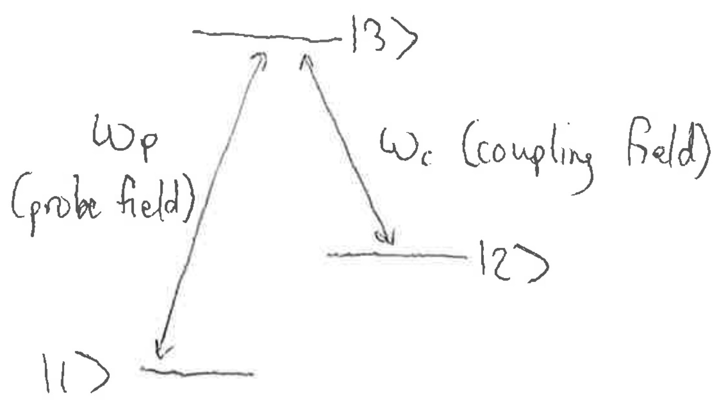
- "Phenomenological Models"
  - ↳ Einstein rate equations
  - ↳ Theory of a laser
- Study of perturbations
  - ↳ Finite (discrete) levels ⟹ Rabi oscillations
  - ↳ Quasi-continuum ⟹ exponential decay, Fermi's Golden Rule.
- Semiclassical interaction between atom & field.
  - ↳ dipole approx., RWA
  - ↳ optical Bloch equations
  - ↳ adiabatic basis / Landau-Zener
- Fully quantum
  - ↳ Jayne's - Cummings
  - ↳ Rabi revival
- Light effects
  - ↳ saturation
  - ↳ broadening

Phew!

Lot's of things to study, mostly two-level atoms!

THE LAST LECTURE

We have studied 2-level systems in great detail. Do any new phenomena emerge with 3-level systems?



To understand this system, we must go through some math (which will look familiar). However, before we do this, consider $\Omega_c \gg 1$ & $\delta_c = 0$.

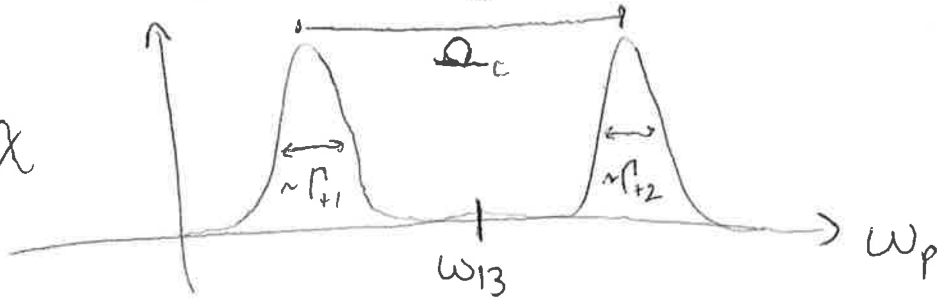
We can treat the $2 \to 3$ system as a two level system.

What are the eigenenergies?

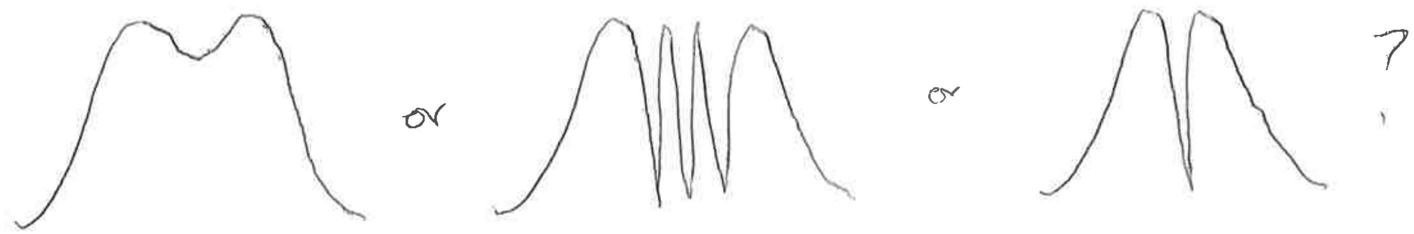
In the rotating frame:

This means that the $|1\rangle$ state will experience two coupled levels, whose energies differ by $\hbar\Omega$.

ie) $\mathrm{Im}\,\chi$



What happens when Ω_c is not very big?



Before we get the answer, let's learn about

Optical Pumping and Coherent Population Trapping

So far, we have considered an idealized 2-level atom. Real atoms, though they might have isolated 2-level-like states, have angular momentum! For a hydrogenic atom, the good quantum numbers are $J = L+S$, and m_J.

The transition matrix elements

$$\langle n_1, J_1, m_1 | \vec{\varepsilon} \cdot \vec{d} | n_2, J_2, m_2 \rangle$$

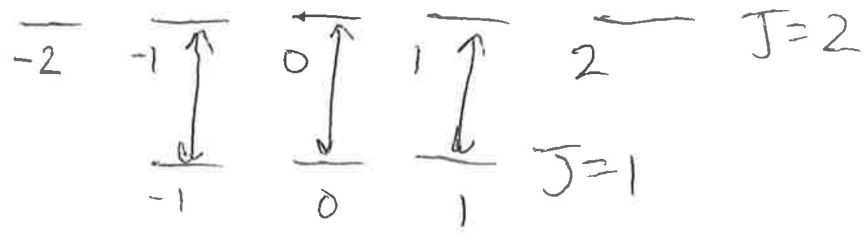
determine the rate at which an atom moves between levels when illuminated with $\vec{E} = \vec{\varepsilon} E_0 \cos \omega t$.

When $\vec{\varepsilon} = \vec{\varepsilon}_z$ (linearly polarized), the transition matrix elements are zero unless:

$$\boxed{\begin{array}{l} J_1 - J_2 = \pm 1 \text{ or } 0 \\ m_1 - m_2 = 0 \end{array}}$$

These selection rules determine which states are coupled.

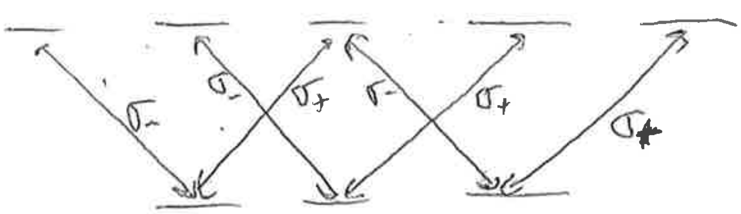
So consider the following level scheme:



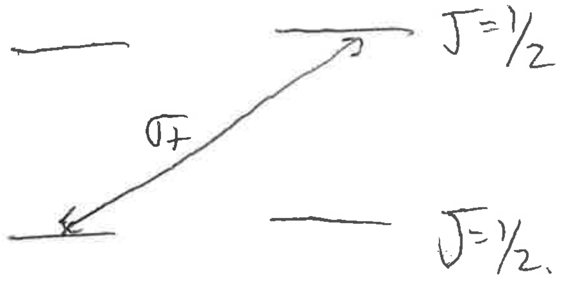
Similarly, for circularly polarized light, we can derive:

$$J_1 - J_2 = 0, \pm 1$$

$$m_1 - m_2 = \pm 1 \quad (+ \text{ sign for } \sigma_+, - \text{ sign for } \sigma_-)$$



Finally, consider the $|n_1, J=1/2, m_J\rangle \Rightarrow |n_2, J=1/2, m_J\rangle$ transition:

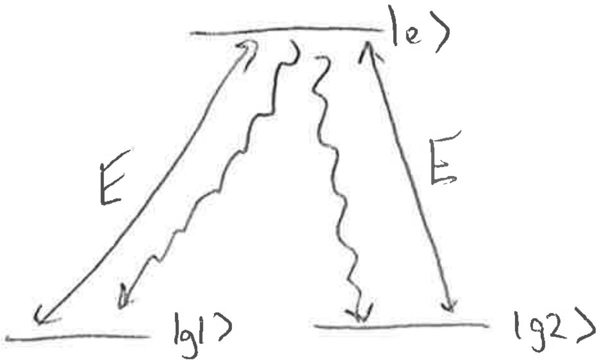


If illuminated by σ_+ light, atom eventually gets stuck in $|n_1, J=1/2, m_J=1/2\rangle$ (since there is nowhere for it to go once it lands there).

This is called OPTICAL POMPING.

Coherent Population Trapping

Consider:



Define the dipole moments:

$$d_1 = \langle g_1 | \varepsilon \cdot D | e \rangle$$
$$d_2 = \langle g_2 | \varepsilon \cdot D | e \rangle$$

and the two states:

$$|\psi_+\rangle = \frac{d_1|a_1\rangle + d_2|a_2\rangle}{\sqrt{d_1^2 + d_2^2}}$$

$$|\psi_-\rangle = \frac{d_2|a_1\rangle - d_1|a_2\rangle}{\sqrt{d_1^2 + d_2^2}}$$

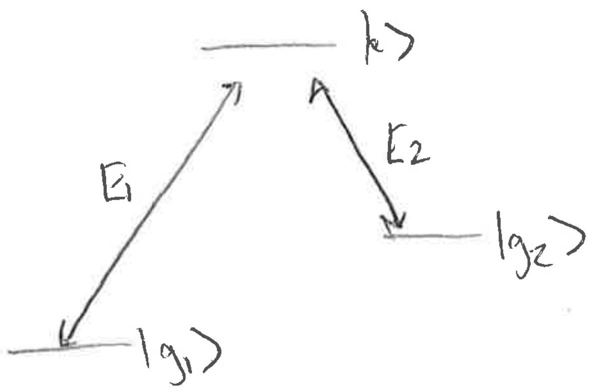
Note that:

$$\langle \psi_+ | D \cdot \varepsilon | e \rangle = \sqrt{d_1^2 + d_2^2}$$
$$\langle \psi_- | D \cdot \varepsilon | e \rangle = 0$$

$|\psi_-\rangle$ is called a "dark state": once an atom finds itself in the dark state, it remains there (as there is no coupling to other levels).

Since spontaneous emission from the excited state populates $|\psi_{+/-}\rangle$, but no process is capable of exciting the system out of $|\psi_-\rangle$, the system will rapidly populate $|\psi_-\rangle$.

Non-degenerate Ground States



$$\vec{E}_1 = E_1 \vec{\varepsilon}_1 \cos(\omega_1 t)$$
$$\vec{E}_2 = E_2 \vec{\varepsilon}_2 \cos(\omega_2 t)$$
$$\Omega_1 = -\langle g_1 | D\cdot\varepsilon_1 | e\rangle E_1/\hbar$$
$$\Omega_2 = -\langle g_2 | D\cdot\varepsilon | e\rangle E_2/\hbar$$

Consider

$$|\psi_-\rangle = \frac{1}{\sqrt{\Omega_1^2+\Omega_2^2}}\left(\Omega_2 e^{-iE_{g_1}t/\hbar}|g_1\rangle - \Omega_1 e^{-iE_{g_2}t/\hbar}|g_2\rangle\right)$$

We can write the interaction Hamiltonian as:

$$H_I = \begin{bmatrix} 0 & 0 & \Omega_1\cos\omega_1 t \\ 0 & 0 & \Omega_2\cos\omega_2 t \\ \Omega_1\cos\omega_1 t & \Omega_2\cos\omega_2 t & 0 \end{bmatrix}$$

And show that

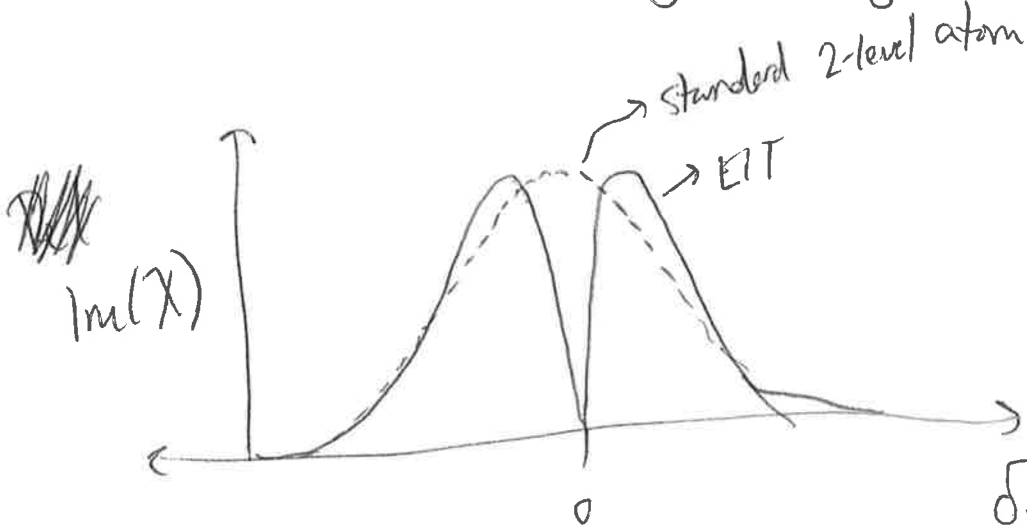
$$\langle\psi_-|H_I|e\rangle = \frac{\Omega_1\Omega_2 e^{-iE_e t/\hbar}}{2\sqrt{\Omega_1^2+\Omega_2^2}}\left(e^{+i(E_{g_1}+\hbar\omega_1)t/\hbar} - e^{+i(E_{g_2}+\hbar\omega_2)t/\hbar}\right)$$

Which is $\emptyset$ when

$$\hbar(\omega_1-\omega_2) = E_{g_2} - E_{g_1}$$

This is called the EIT condition (electromagnetically induced transparency).
When it is satisfied, the absorption goes to zero because the system is trapped in the dark state!

The full treatment (see assignment) yields:

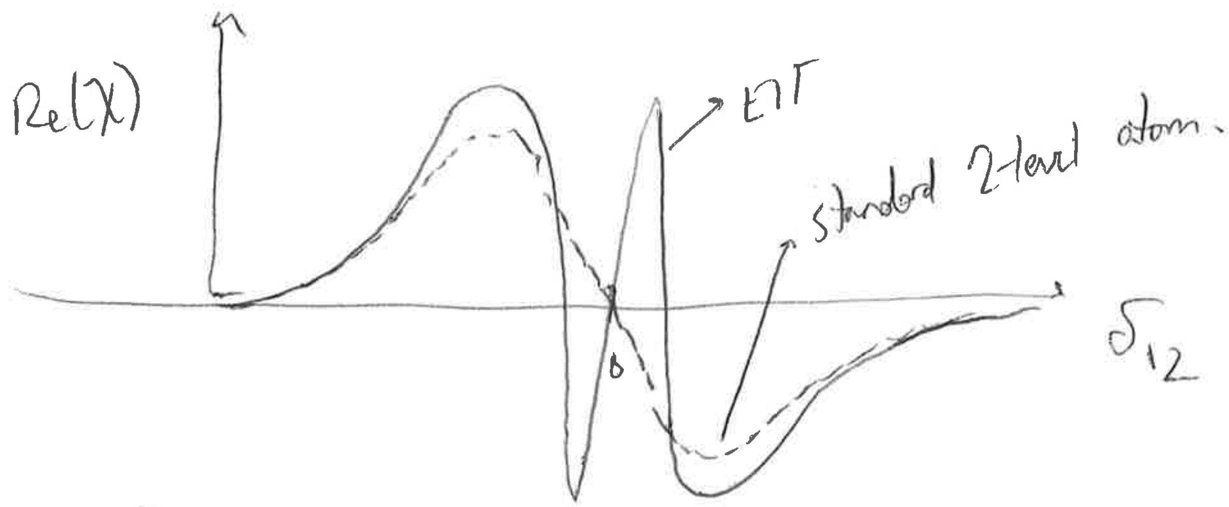


$$\delta_{12} = \delta_1 - \delta_2$$
$$= (\omega_1 - \omega_{g_1}) - (\omega_2 - \omega_{g_2})$$

This has other consequences!
Recall (from undergrad) that the real part of χ is related to the imaginary part (Kramers-Kronig) by:

$$\chi'(\omega) = \frac{2}{\pi}\int_0^{\infty} \omega' d\omega' \left(\frac{\chi''(\omega')}{\omega'^2 - \omega^2} \right)$$

Then we get:



When $\delta_{12} = 0$, group velocity

$$V_g = \frac{c}{n + \omega \, dn/d\omega} \ll c$$

SLOW LIGHT!

World record: ~ 100 km/h!

In summary,

small Ω_1 $\Rightarrow$ EIT

large Ω_1 $\Rightarrow$ Autler-Townes Effect