

Modelling Nonlinear optics with the Bloch-Messiah reduction

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August 24, 2018

Overview

Modelling
Nonlinear
optics with
the
Bloch-Messiah
reduction

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References

- What is nonlinear optics?
- Why do we care about it?
- Gaussian optics
- What I have been doing
- Outlook

Motivation quantum nonlinear optics

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The good

Spontaneous Parametric processes, SPDC, SFWM

- Heralded single photon sources
- Entangled photon pair generation (polarisation, spatial)

Kerr processes

- Self-Phase modulation (SPM), generating Bannana states (CV)
- Cross-Phase modulation (XPM) for sensing

The bad

Spontaneous parametric processes

- Generating more than two photons \rightarrow bad for quantum computing
- Understanding filtering

All Kerr nonlinear processes

- SPM \rightarrow Spectral broadening
- XPM \rightarrow Unwanted phase shifts on single photons due to propagation of the pump

What do we mean by nonlinear optics?

- Roughly processes that conserve energy but do not conserve photon number.

$$\vec{P} = \chi^{(1)} \vec{E}_1 + \chi^{(2)} \vec{E}_1 \vec{E}_2 + \chi^{(3)} \vec{E}_1 \vec{E}_2 \vec{E}_3 + \dots \quad (1)$$

Here we are going to talk about squeezing, i.e SPDC or SFWM, Hamiltonians are then of the form,

$$\hat{H} = A \hat{a}_S^\dagger \hat{a}_I^\dagger \hat{a}_P + h.c. \quad (2)$$

$$\hat{H} = A \hat{a}_S^\dagger \hat{a}_I^\dagger \hat{a}_P \hat{a}_P + h.c. \quad (3)$$

Gaussian Optics

- Using the undepleted pump approximation we can write the Hamiltonians as terms which are at most quadratic in creation and annihilation operators.

$$\hat{U} = \exp \left[-\frac{i}{\hbar} \left(\overset{\text{Power}}{P} \int d\omega_1 \int d\omega_2 \underset{\text{JSA}}{f(\omega_1, \omega_2)} \overset{\text{Signal \& Idler}}{\hat{a}_s^\dagger(\omega_1) \hat{a}_i^\dagger(\omega_2)} + h.c. \right) \right] \quad (4)$$

- Just like Beamsplitters can be written as unitary matrices,

$$\begin{bmatrix} \vec{b} \end{bmatrix} = \mathbf{U} \begin{bmatrix} \vec{a} \end{bmatrix} \quad (5)$$

- We want to extend the type of transforms to all Gaussian transforms¹

$$\begin{bmatrix} \vec{b} \\ \vec{b}^\dagger \end{bmatrix} = \mathbf{M} \begin{bmatrix} \vec{a} \\ \vec{a}^\dagger \end{bmatrix} \quad (6)$$

¹These are linear symplectic transforms which conveniently can be written as a matrix [1]

Types of Gaussian transformations

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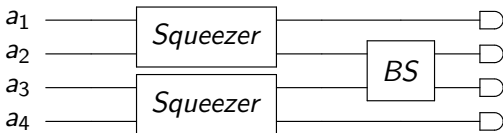


Figure: Two source HOM dip

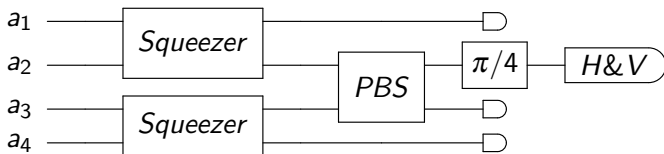


Figure: Type-1 Fusion gate

¹These are two-mode squeezers

Schmidt decomposition

We can re-write the Hamiltonian using a Schmidt-decomposition [2] as,

$$P'F(\omega_1, \omega_2) = \sum_k r_k \psi_k(\omega_1) \phi_k(\omega_2) \quad (7)$$

Where r_k is the Schmidt number, ψ & ϕ are unitaries.

To solve this numerically we discretize the function and the Schmidt-decomposition is then the Singular value decomposition (SVD) of the JSA (F).

$$P'F(\omega_1, \omega_2) = \sum_k r_k \mathbf{U}_{(\omega_1, k)} \mathbf{V}_{(k, \omega_2)}^\dagger \quad (8)$$

- with $\psi_k(\omega_1)$ is the k -th row and ω_1 -th column of $\mathbf{U}_{(\omega_1, k)}$,
- with $\phi_k(\omega_2)$ is the ω_2 -th row and k -th column of $\mathbf{V}_{(k, \omega_2)}^\dagger$

Joint Spectral Amplitudes (JSAs)

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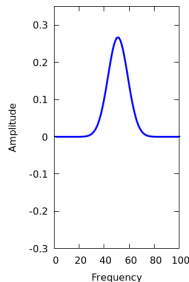
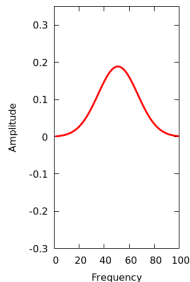
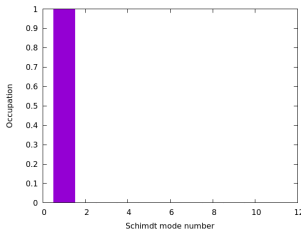
¹Moving to the rotating frame...

Seperable JSAs Schmidt modes

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(a) Signal (red) and Idler (blue)

$$F(\omega_1, \omega_2) = \exp \left[-0.2 \left(\left(\frac{\omega_1}{\sigma_1} \right)^2 + \left(\frac{\omega_2}{\sigma_2} \right)^2 \right) \right] \quad (9)$$

normalised so that,

$$\int d\omega_1 \int d\omega_2 F(\omega_1, \omega_2) = 1 \quad (10)$$

Non-separable JSAs

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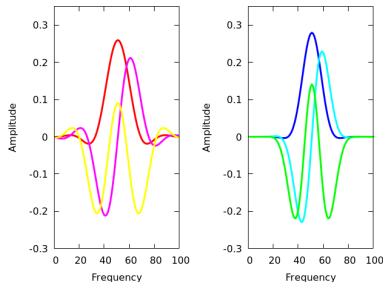
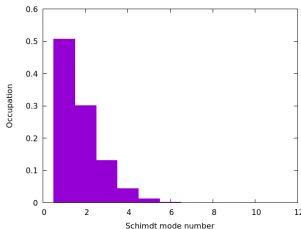
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Non-separable JSAs Schimdt modes

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(a) Signal (red) and Idler (blue)

$$F(\omega_1, \omega_2) = \text{sinc}(2(\omega_1 - \omega_2)) \exp \left[-0.1 \left(\left(\frac{\omega_1}{\sigma_1} \right)^2 + \left(\frac{\omega_2}{\sigma_2} \right)^2 \right) \right] \quad (11)$$

normalised so that,

$$\int d\omega_1 \int d\omega_2 F(\omega_1, \omega_2) = 1 \quad (12)$$

Reducing the size of the state-space

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- The Schmidt decomposition lets us represent the system in a finite number of broadband modes $(\psi_k(\omega_1), \phi_k(\omega_2))$ [3]
- Defining new mode operators for signals, \hat{A}_k and idlers, \hat{B}_k

$$\hat{A}_k = \int d\omega_s \psi_k(\omega_s) \hat{a}_s \quad (13)$$

$$\hat{B}_k = \int d\omega_i \phi_k(\omega_i) \hat{a}_i \quad (14)$$

Correlations in a HOM dip

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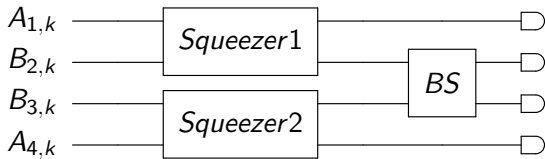


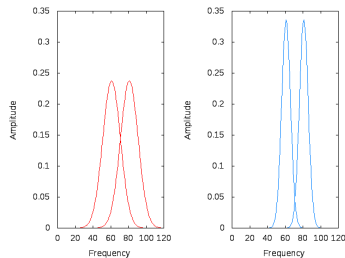
Figure: Two source HOM dip

Two squeezers JSA

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(a) The signals A_k (red) and idlers B_k (blue)

G(4) correlation function

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$$G^{(4)} = \frac{\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_1 \hat{a}_2 \hat{a}_3 \hat{a}_4 \rangle}{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle \langle \hat{a}_2^\dagger \hat{a}_2 \rangle \langle \hat{a}_3^\dagger \hat{a}_3 \rangle \langle \hat{a}_4^\dagger \hat{a}_4 \rangle} \quad (15)$$

Where,

$$a_i = \sum_j a_i(\omega_j) \quad (16)$$

Meaning we sum over all of the spectral modes of the spatial modes (1,2,3,4) separately. We end up with,

$$G^{(4)} = 1 - \left(\frac{2 | \cosh(r) |^2}{| \cosh(r) |^2 + | \sinh(r) |^2} \sin(\theta) \cos(\theta) \right)^2 \quad (17)$$

$G(4)$ correlation function

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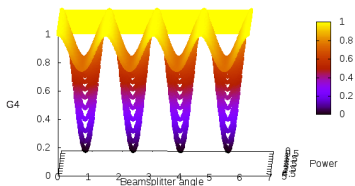
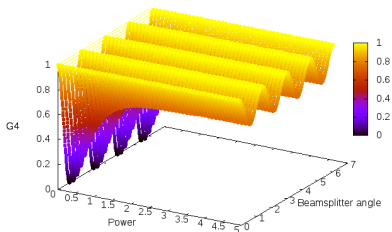
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$G(4)$ correlation function

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Summary

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- The Schmidt decomposition is a useful technique to exactly reduce the continuum of spectral modes in a JSA to a discrete set of broadband modes.
- We have derived a closed form expression for the $G^{(4)}$ and it agrees with computational simulation for two identical, separable squeezers.

Outlook,

- Currently, the method creating the symplectic matrix \mathbf{M} involves using the Bloch-Messiah reduction which fully diagonalises it.
- This flattens all of the degrees of freedom and puts them on equal footing which is not the case experimentally!
- We want to generalise the method so that it respects spatial and spectral degrees of freedom separately.

References

- [1] Gerardo Adesso, Sammy Ragy, and Antony R Lee. Continuous variable quantum information: Gaussian states and beyond. *Open Systems & Information Dynamics*, 21(01n02):1440001, 2014.
- [2] Al Lvovsky, Wojciech Wasilewski, and Konrad Banaszek. Decomposing a pulsed optical parametric amplifier into independent squeezers. *Journal of Modern Optics*, 54(5):721–733, 2007.
- [3] Wojciech Wasilewski, Al Lvovsky, Konrad Banaszek, and Czesław Radzewicz. Pulsed squeezed light: Simultaneous squeezing of multiple modes. *Physical Review A*, 73(6):063819, 2006.