Modelling Nonlinear optics with the Bloch-Messiah reduction

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Overview

- What is nonlinear optics?
- Why do we care about it?
- Gaussian optics
- What I have been doing
- Outlook

Motivation quantum nonlinear optics

The good

Spontaneous Parametric processes, SPDC, SFWM

- Heralded single photon sources
- Entangled photon pair generation (polarisation, spatial)

Kerr processes

- Self-Phase modulation (SPM), generating Bannana states (CV)
- Cross-Phase modulation (XPM) for sensing

The bad

Spontaneous parametric processes

- Generating more than two photons -> bad for quantum computing
- Understanding filtering

All Kerr nonlinear processes

- SPM -> Spectral broadening
- XPM -> Unwanted phase shifts on single photons due to propagation of the pump

What do we mean by nonlinear optics?

- Roughly processes that conserve energy but do not conserve photon number.

$$\vec{P} = \chi^{(1)}\vec{E}_1 + \chi^{(2)}\vec{E}_1\vec{E}_2 + \chi^{(3)}\vec{E}_1\vec{E}_2\vec{E}_3 + \dots \tag{1}$$

Here we are going to talk about squeezing, i.e SPDC or SFWM, Hamiltonians are then of the form,

$$\hat{H} = A\hat{a}_S^\dagger\hat{a}_I^\dagger\hat{a}_P + h.c. \tag{2}$$

$$\hat{H} = A\hat{a}_S^\dagger\hat{a}_I^\dagger\hat{a}_P\hat{a}_P + h.c. \tag{3}$$

Gaussian Optics

- Using the undepleted pump approximation we can write the Hamiltonians as terms which are at most quadratic in creation and annihilation operators.

$$\hat{U} = \exp\left[-\frac{i}{\hbar}\left(\underbrace{P}_{\text{Power}} \int d\omega_1 \int d\omega_2 \; \underbrace{f(\omega_1, \omega_2)}_{\text{JSA}} \; \underbrace{\hat{a}_s^\dagger(\omega_1)\hat{a}_i^\dagger(\omega_2)}_{\text{Signal \& Idler}} + h.c.\right)\right] \tag{4}$$

- Just like Beamsplitters can be written as unitary matrices,

$$\left[\vec{b}\right] = \mathbf{U}\left[\vec{a}\right] \tag{5}$$

- We want to extend the type of transforms to all Gaussian transforms[1]

$$\begin{bmatrix} \vec{b} \\ \vec{b}^\dagger \end{bmatrix} = \mathbf{M} \begin{bmatrix} \vec{a} \\ \vec{a}^\dagger \end{bmatrix} \tag{6}$$

[1] These are linear symplectic transforms which conviently can be written as a matrix [1]

Types of Gaussian transformations

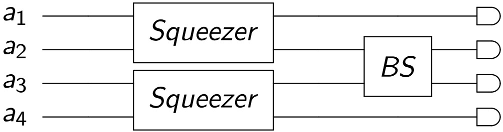


Figure: Two source HOM dip

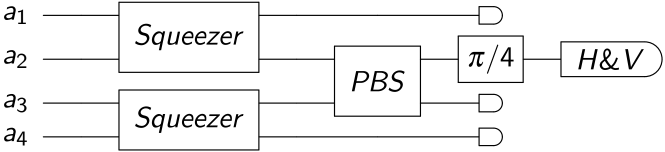


Figure: Type-1 Fusion gate

[1] These are two-mode squeezers

Schmidt decomposition

We can re-write the Hamiltonian using a Schmidt-decomposition [2]as,

$$P' F(\omega_1, \omega_2) = \sum_k r_k \psi_k(\omega_1) \phi_k(\omega_2) \tag{7}$$

Where r_k is the Schmidt number, ψ & ϕ are unitaries.

To solve this numerically we discretize the function and the Schmidt-decomposition is then the Singular value decomposition (SVD) of the JSA (F).

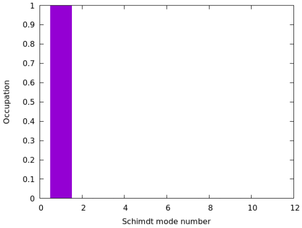
$$P' \mathbf{F}_{(\omega_1,\omega_2)} = \sum_k r_k \mathbf{U}_{(\omega_1,k)} \mathbf{V}^{\dagger}_{(k,\omega_2)} \tag{8}$$

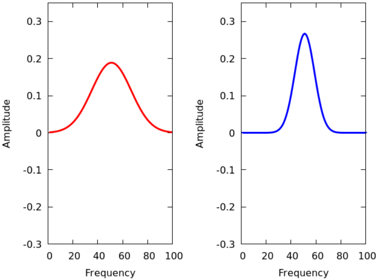
- with $\psi_k(\omega_1)$ is the k-th row and ω_1-th column of $\mathbf{U}_{(\omega_1,k)}$,
- with $\phi_k(\omega_2)$ is the ω_2-th row and k-th column of $\mathbf{V}^{\dagger}_{(k,\omega_2)}$

Joint Spectral Amplitudes (JSAs)

[1] Moving to the rotating frame...

Seperable JSAs Schmidt modes





(a) Signal (red) and Idler (blue)

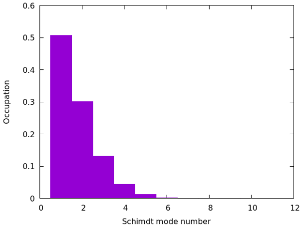
$$F(\omega_1,\omega_2) = exp\left[-0.2\left(\left(\frac{\omega_1}{\sigma_1}\right)^2 + \left(\frac{\omega_2}{\sigma_2}\right)^2\right)\right] \tag{9}$$

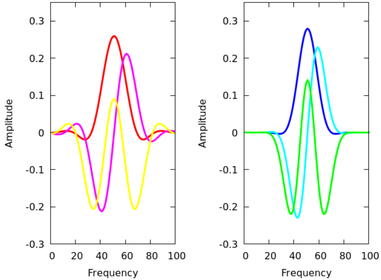
normalised so that,

$$\int d\omega_1 \int d\omega_2 F(\omega_1,\omega_2) = 1 \tag{10}$$

Non-separable JSAs

Non-separable JSAs Schimdt modes





(a) Signal (red) and Idler (blue)

$$F(\omega_1,\omega_2) = sinc(2(\omega_1-\omega_2))exp\left[-0.1\left(\left(\frac{\omega_1}{\sigma_1}\right)^2+\left(\frac{\omega_2}{\sigma_2}\right)^2\right)\right] \tag{11}$$

normalised so that,

$$\int d\omega_1 \int d\omega_2 F(\omega_1,\omega_2) = 1 \tag{12}$$

Reducing the size of the state-space

- The Schmidt decomposition lets us represent the system in a finite number of broadband modes $(\psi_k(\omega_1), \phi_k(\omega_2))$ [3]
- Defining new mode operators for signals, $\hat{A}_k$ and idlers, $\hat{B}_k$

$$\hat{A}_k = \int d\omega_s \psi_k(\omega_s) \hat{a}_s \tag{13}$$

$$\hat{B}_k = \int d\omega_i \phi_k(\omega_i) \hat{a}_i \tag{14}$$

Correlations in a HOM dip

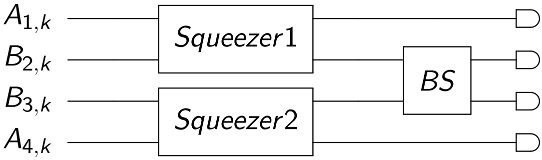
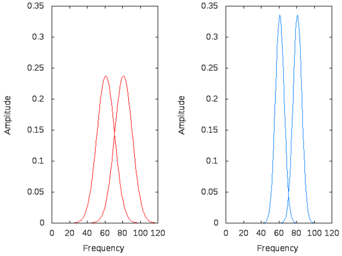


Figure: Two source HOM dip

Two squeezers JSA



(a) The signals A_k (red) and idlers B_k (blue)

G(4) correlation function

$$G^{(4)} = \frac{\left\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_1 \hat{a}_2 \hat{a}_3 \hat{a}_4 \right\rangle}{\left\langle \hat{a}_1^\dagger \hat{a}_1 \right\rangle \left\langle \hat{a}_2^\dagger \hat{a}_2 \right\rangle \left\langle \hat{a}_3^\dagger \hat{a}_3 \right\rangle \left\langle \hat{a}_4^\dagger \hat{a}_4 \right\rangle} \tag{15}$$

Where,

$$a_i = \sum_j a_i(\omega_j) \tag{16}$$

Meaning we sum over all of the spectral modes of the spatial modes (1,2,3,4) separately. We end up with,

$$G^{(4)} = 1 - \left(\frac{2 \mid cosh(r) \mid^2}{\mid cosh(r) \mid^2 + \mid sinh(r) \mid^2} sin(\theta) cos(\theta) \right)^2 \tag{17}$$

G(4) correlation function

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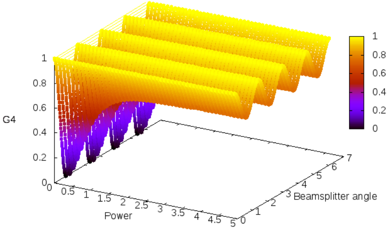

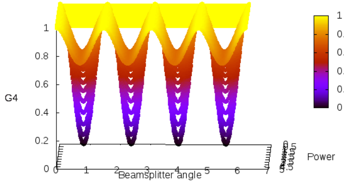

Summary

- The Schmidt decomposition is a useful technique to exactly reduce the continuum of spectral modes in a JSA to a discrete set of broadband modes.
- We have derived a closed form expression for the $G^{(4)}$ and it agrees with computational simulation for two identical, seperable squeezers.

Outlook,

- Currently, the method creating the sympectic matrix $\mathbf{M}$ involves using the Bloch-Messiah reduction which fully diagonalises it.
- This flattens all of the degrees of freedom and puts them on equal footing which is not the case experimentally!
- We want to generalise the method so that it respects spatial and spectral degrees of freedom separately.

References

[1] Gerardo Adesso, Sammy Ragy, and Antony R Lee. Continuous variable quantum information: Gaussian states and beyond. *Open Systems & Information Dynamics*, 21(01n02):1440001, 2014.

[2] AI Lvovsky, Wojciech Wasilewski, and Konrad Banaszek. Decomposing a pulsed optical parametric amplifier into independent squeezers. *Journal of Modern Optics*, 54(5):721–733, 2007.

[3] Wojciech Wasilewski, AI Lvovsky, Konrad Banaszek, and Czesław Radzewicz. Pulsed squeezed light: Simultaneous squeezing of multiple modes. *Physical Review A*, 73(6):063819, 2006.