Hong-Ou-Mandel Dip Experiment

Literature Review

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Abstract ?

What, precisely, does this mean?

1 Introduction

This intro is a bit clunky. Reword.

The Hong-Ou-Mandel (HOM) effect is an interference phenomenon concerning the interaction between two indistinguishable photons. The effect can be seen when two identical photons generated from parametric down-conversion enter a beam splitter at right angles. After the photons travel through the beam splitter they are sent into detectors at the output ports. The probability of detecting one photon at each output port depends on how distinguishable the two photons are. The detector count drops significantly when the photons are indistinguishable; this decrease in coincidence detection is known as the 'HOM dip'.

The HOM experiment was initially devised by Hong, Ou and Mandel in their 1987 paper *Measurement of Subpicosecond Time Intervals between Two Photons by Interference* [1] where they indirectly measured the length of the photon wave packet. It can however be used for a wide range of applications, including measuring the indistinguishablity of two photons [2].

You have mentioned just 2.

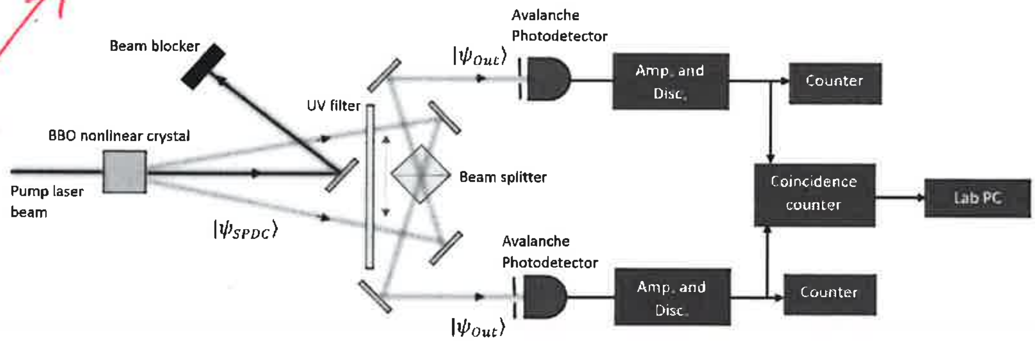

Figure 1: A diagram illustrating the experimental setup used in order to achieve a HOM dip. Both signals from each avalanche photo-detector (APD) are combined in the coincidence counter and sent to a laboratory computer for collection. Adapted from [1].

We will examine the theory behind the experiment, including spontaneous parametric down-conversion and the mathematics of beam splitters. We will then move on to describing different experimental realisations, ending with some applications of the HOM effect.

Motivation

2 Spontaneous parametric down-conversion

Spontaneous parametric down-conversion (SPDC) is the process of generating a pair of correlated photons from a single photon. The phenomenon of "parametric luminescence", where one photon splits into two in a nonlinear medium, was studied theoretically in 1967 [3], and experimentally in 1970 [4]. Burnham and Weinberg [5] studied the time between photon emission in nonlinear media, which they referred to as "parametric flourescence".

SPDC can be achieved using a monochromatic pump laser, which is directed into a nonlinear crystal (NLC), producing two lower frequency photons. The process is very inefficient with an emission rate of the order of 10^{-12} [6]. The pair of photons are constrained by the conservation of energy (Eq. 1) and momentum (Eq. 2):

$$\omega_p = \omega_a + \omega_b, \tag{1}$$

$$k_p = k_a + k_b, \tag{2}$$

where subscript p denotes the pump laser, a and b denote the signal and idler photons. Eq. 1 does not guarantee that the signal and idler photons have equal frequencies.

The two correlated photons produced do not have well defined frequencies or phase, so classically there would be no interference between the pair [7]. The state after SPDC is given by [8]:

for a CW pump.

$$|\psi_{SPDC}\rangle = \int d\omega_a \int d\omega_b f(\omega_a,\omega_b)\delta(\omega_a+\omega_b-\omega_p)\hat{a}_1^\dagger(\omega_a)\hat{a}_2^\dagger(\omega_b)|0\rangle\,, \tag{3}$$

where $\omega_{a,b,p}$ are the frequencies from Eq. 1, $\hat{a}_1^\dagger(\omega_a),\hat{a}_2^\dagger(\omega_b)$ are the creation operators for photons with frequencies ω_a,ω_b. The frequency distribution for the two photons is given by $f(\omega_a,\omega_b)$ and $|0\rangle$ is the vacuum state.

Defining $\omega' = \omega_a - \frac{\omega_p}{2}$ and using the Dirac delta function, δ, to simplify the integration we have:

$$|\psi_{SPDC}\rangle = \int d\omega' f'(\omega')\hat{a}_1^\dagger\left(\frac{\omega_p}{2}+\omega'\right)\hat{a}_2^\dagger\left(\frac{\omega_p}{2}-\omega'\right)|0\rangle\,, \tag{4}$$

Where did this specific form come from? You have made assumptions

$$f'(\omega') = \left(\frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{\sigma}}\right)e^{-\frac{(\omega'-\omega_p)^2}{4\sigma^2}}, \tag{5}$$

where we have chosen ω' as half of the difference between ω_a,ω_b. The frequency distribution is a Gaussian function with standard deviation σ.

There are several types of NLC resulting in different SPDC effects. Type-I produces photons with the same parallel polarization whereas Type-II produces photons with orthogonal polarizations [9]. It is necessary for the photons to have the same polarization to observe the HOM dip, meaning only Type-I crystals are used in experiments generating indistinguishable photons [10]. Not true. Manipulation of polarization is simple.

3 Interference

The photons after SPDC are considered to be indistinguishable. In order to control their distiguishability we can add a delay in one of the photon paths. A delay in path 2 can be written as $\hat{a}_2^\dagger(\omega) \longrightarrow \hat{a}_2^\dagger(\omega)\, e^{-i\omega\tau}$, where τ represents the distance added in path 2. The state in Eq. 4 then changes to [8]:

$$|\psi_{PD}(\tau)\rangle = \int d\omega' f'(\omega')\hat{a}_1^\dagger\left(\frac{\omega_p}{2}+\omega'\right)\hat{a}_2^\dagger\left(\frac{\omega_p}{2}-\omega'\right)e^{-i(\frac{\omega_p}{2}-\omega')\tau}\,|0\rangle\,. \tag{6}$$

We consider the four possible outcomes when two photons enter a 50:50 beam splitter.

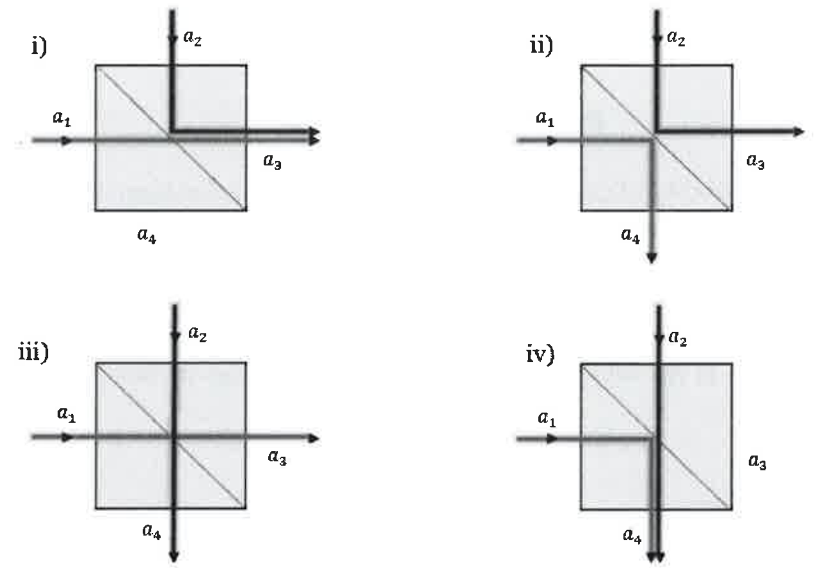

Figure 2: The possible outcomes of two photons (red and green) interacting with a 50:50 beam splitter. i) Red beam reflects while green transmits, ii) Both red and green beams reflect, iii) Both beams transmit and iv) Red transmits while green reflects.

Placing detectors at a_3 and a_4, we are able to determine where the photons have exited. When the two photons are indistinguishable, only case (i) and (iv) are observed as the other two cases cancel out. This observation is due to a fourth-order interference effect [11] between photon 1 and photon 2. Outcomes (ii) and (iii) can only be observed when the photons are distinguishable from each other. As a consequence of this, the HOM dip is used as a measurement of the distinguishability of photons. This is not quite true. HOM measures exchange symmetry

Coming back to our state after the SPDC (Eq. 4), we now want to consider it as the input on a beam splitter (BS) which acts as:

$$\hat{a}_1^\dagger(\omega) \to \frac{1}{\sqrt{2}}\left(i\hat{a}_3^\dagger(\omega)+\hat{a}_4^\dagger(\omega)\right), \qquad \hat{a}_2^\dagger(\omega) \to \frac{1}{\sqrt{2}}\left(\hat{a}_3^\dagger(\omega)+i\hat{a}_4^\dagger(\omega)\right). \tag{7}$$

The state after the beam splitter is then:

$$\begin{aligned}|\psi_{out}(\tau)\rangle = \left(\frac{1}{2}\right)\int d\omega' f'(\omega')&\left[i\hat{a}_3^\dagger\left(\frac{\omega_p}{2}+\omega'\right)+\hat{a}_4^\dagger\left(\frac{\omega_p}{2}+\omega'\right)\right] \\ &\times\left[\hat{a}_3^\dagger\left(\frac{\omega_p}{2}-\omega'\right)+i\hat{a}_4^\dagger\left(\frac{\omega_p}{2}-\omega'\right)\right]e^{-i(\frac{\omega_p}{2}-\omega')\tau}\,|0\rangle\end{aligned} \tag{8}$$

The detectors in paths a_3 and a_4 are modelled as measurements acting as [12]:

$$M_3 = \int d\omega_a \hat{a}_3^\dagger(\omega_a)\,|0\rangle\langle 0|\,\hat{a}_3(\omega_a), \qquad M_4 = \int d\omega_b \hat{a}_4^\dagger(\omega_b)\,|0\rangle\langle 0|\,\hat{a}_4(\omega_b). \tag{9}$$

The coincidence probability is given by:

$$p_{34}(\tau) = \langle \psi_{out}(\tau)|\,(M_3 \otimes M_4)\,|\psi_{out}(\tau)\rangle\,. \tag{10}$$

Putting the final state (Eq. 8) and measurements (Eq. 9) into Eq. 10 we get:

$$p_{34}(\tau) = \frac{1}{4}\int d\omega' |f'(\omega')|^2 \left(2 - e^{-i2\omega'\tau} - e^{i2\omega'\tau}\right). \tag{11}$$

Replacing the spectral amplitude function (Eq. 5) and performing the integration gives:

$$p_{34}(\tau) = \frac{1}{2}\left(1 - e^{-2\sigma^2\tau^2}\right). \tag{12}$$

Eq. 12 gives the probability of coincidence at the detectors as a function of the path difference, τ, between the two correlated photons. In particular, we highlight two cases. On the one hand, when considering a path delay between the two photons, they become distinguishable and the probability of coincidences is non-zero. The probability tends to 0.5 for large values of τ. On the other hand, when considering indistinguishable photons, the probability of coincidences becomes zero which is the HOM dip. The general behaviour of this function is shown in Fig. 3.

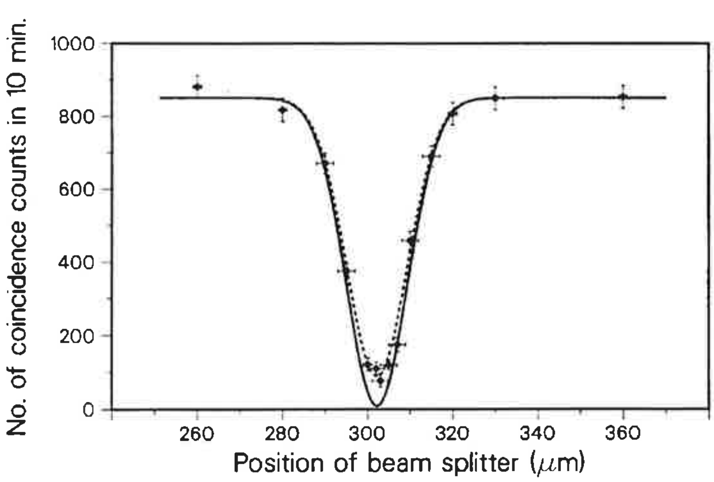

Figure 3: HOM experiment results taken from [1]. Probability of coincidences at detectors in paths 3 and 4 as a function of the path delay τ. The points represent the experimental data (the dotted line being experimental best fit), and the continuous curve represents the theoretical prediction.

4 Experimental Realisations

The experimental setup presented by Hong, Ou and Mandel in their original paper is just one possible way to observe the HOM effect. Nearly all aspects of the experiment may be modified, from the source through to the beam splitter and even the types of particle involved in the experiment. For example, Maunz et al. [13] observe the HOM effect using an arrangement which replaces the SPDC source with two remotely trapped ions, which each provide one photon. In a similar vein, Patel et al. [14] use tunable quantum dots to provide the photons. Li et al. [15] observed HOM interference between a single photon source and a weak classical source, demonstrating that the interference may be obtained between two different types of sources.

A wide variety of experimental setups have been used to realise path difference before the beam splitter. The original paper [1] used free space bulk optics, displacing the beam splitter from the central point to investigate the effect of path difference; whereas Rarity and Tapster [16] displaced prisms to achieve the same thing. However, it is not necessary to use a beam splitter at all to achieve the interference effect. Politi et al. use the evanescent fields of two sufficiently close silica waveguides to recreate the interference effect, in a device known as a directional coupler [17]. In the field of plasmonics, Dutta Gupta and Agarwal [18] realise HOM interference effects using plasmons in a gap-plasmon guide. In another direction, Lim and Beige [19] propose a generalised HOM experiment in which they use an $N \times N$ multi-port beam splitter to investigate the HOM-like behaviour of multiple bosons and fermions.

Recently, the HOM effect has been used as a practical tool in undergraduate laboratories to introduce students to quantum effects. In 2012, Carvioto-Lagos et al. conducted the experiment using bulk optics in a setting suitable for undergraduates. However, "The HOM experiment itself is notoriously difficult to recreate, especially in a teaching, nonresearch setting because of the challenge of making the momentum modes of the photons overlap." [20]. In 2015 Ourjoumtsev et al. [10] proposed using coupled fibres for the interaction to simplify the alignment in the context of an undergraduate laboratory.

This section doesn't explain why people do these experiments. HOM effect is an important fundamental quantum effect with practical uses!

5 Applications

In 1988, shortly after their first paper, Ou and Mandel published a paper [21] showing how HOM interference could be used to demonstrate a violation of Bell's inequalities. Their method involved placing linear polarizers at the input ports of the beam splitter, and measuring the resulting coincidence counts at the output ports. Use of the HOM interferometer in violating Bell's inequalities has also appeared in relatively recent research [22], which uses a different method where the photons entering the beam splitter come from separate SPDC processes.

One of the clearest applications of the HOM effect is to test the indistinguishably of photons. For example, Thoma et al. [23] investigate the indistinguishably of consecutive photons emitted from a quantum dot, as a function of the time separation between photon emission.

HOM interference has also been used in a variety of quantum technologies; computation, communication, and metrology. In quantum computation and quantum information, the HOM interferometer in the form of a directional coupler has been used to realise integrated photonic circuits [24]. Lang et al. [2] have used the HOM effect in the microwave range for use in quantum communication and information processing. HOM interference has also been proposed as the basis for an attack on the quantum key distribution protocol [25]. In the field of quantum metrology, a quantum photonic sensor has been proposed which is based on an 'effective beam splitter' [26]. The parameter to be measured changes the properties of the beam splitter, which changes the observed HOM interference.

There is very little in the way of application here, and virtually no detail.

6 Conclusion

The Hong-Ou-Mandel dip experiment was initially proposed to investigate the theoretically proposed fourth-order interference effect between photon pairs generated in SPDC. Since the experiment, modified setups of the HOM interferometer have been used as a measure of indistinguishability of photons, to demonstrate a violation of Bell's inequalities and as an introduction to quantum effects in undergraduate labs. The HOM dip is a valuable quantum optics tool which demonstrates a very definite non-classical effect. The observed fourth-order interference between two photons highlights the mysterious nature of quantum mechanics.

References

[1] C. K. Hong, Z. Y. Ou, and L. Mandel. Measurement of subpicosecond time intervals between two photons by interference. *Phys. Rev. Lett.*, 59:2044–2046, Nov 1987.

[2] C Lang, C Eichler, L Steffen, J M Fink, M J Woolley, A Blais, and A Wallraff. Correlations, indistinguishability and entanglement in Hong-Ou-Mandel experiments at microwave frequencies. *Nature Physics*, 9(6):345–348, 2013.

[3] D. N. Klyshko. Coherent Photon Decay in a Nonlinear Medium. *Soviet Journal of Experimental and Theoretical Physics Letters*, 6:23, July 1967.

[4] D. N. Klyshko, A. N. Penin, and B. F. Polkovnikov. Parametric Luminescence and Light Scattering by Polaritons. *ZhETF Pisma Redaktsiiu*, 11:11, January 1970.

[5] David C. Burnham and Donald L. Weinberg. Observation of simultaneity in parametric production of optical photon pairs. *Phys. Rev. Lett.*, 25:84–87, Jul 1970.

[6] Alexander Ling, Antia Lamas-Linares, and Christian Kurtsiefer. Absolute emission rates of spontaneous parametric down-conversion into single transverse gaussian modes. *Physical Review A*, 77(4):043834, 2008.

[7] R. Ghosh, C. K. Hong, Z. Y. Ou, and L. Mandel. Interference of two photons in parametric down conversion. *Phys. Rev. A*, 34:3962–3968, Nov 1986.

[8] Agata M. Brańczyk, Alessandro Fedrizzi, Thomas M. Stace, Tim C. Ralph, and Andrew G. White. Engineered optical nonlinearity for quantum light sources. *Opt. Express*, 19(1):55–65, Jan 2011.

[9] Morton H. Rubin, David N. Klyshko, Y. H. Shih, and A. V. Sergienko. Theory of two-photon entanglement in type-II optical parametric down-conversion. *Physical Review A*, 50(6):5122–5133, dec 1994.

[10] A Ourjoumtsev, M-C Dheur, T Avignon, and L Jacubowiez. Two-photon quantum interference for an undergraduate lab. *European Journal of Physics*, 36(6):065034, 2015.

[11] Z. Y. Ou. Quantum theory of fourth-order interference. *Phys. Rev. A*, 37:1607–1619, Mar 1988.

[12] Agata M Brańczyk, T C Ralph, Wolfram Helwig, and Christine Silberhorn. Optimized generation of heralded fock states using parametric down-conversion. *New Journal of Physics*, 12(6):063001, 2010.

[13] P Maunz, D L Moehring, S Olmschenk, K C Younge, D N Matsukevich, and C Monroe. Quantum interference of photon pairs from two remote trapped atomic ions. *Nature Physics*, 3(8):538–541, 2007.

[14] Raj B Patel, Anthony J Bennett, Ian Farrer, Christine A Nicoll, David A Ritchie, and Andrew J Shields. Two-photon interference of the emission from electrically tunable remote quantum dots. *Nature Photonics*, 4(9):632–635, 2010.

[15] Xiaoying Li, Lei Yang, Liang Cui, Zhe Yu Ou, and Daoyin Yu. Observation of quantum interference between a single-photon state and a thermal state generated in optical fibers. *Optics Express*, 16(17):12505, 2008.

[16] John G Rarity and PR Tapster. Fourth-order interference in parametric downconversion. *JOSA B*, 6(6):1221–1226, 1989.

[17] Alberto Politi, Martin J Cryan, John G Rarity, Siyuan Yu, and Jeremy L O'brien. Silica-on-Silicon Waveguide Quantum Circuits. *Science*, 320(43):646–649, 2008.

[18] S Dutta Gupta and G S Agarwal. Two-photon quantum interference in plasmonics: theory and applications. *Optics Letters*, 39(2):390, 2014.

[19] Yuan Liang Lim and Almut Beige. Generalized Hong-Ou-Mandel experiments with bosons and fermions. *New Journal of Physics*, 7, 2005.

[20] J Carvioto-Lagos, G Armendariz, V Velázquez, E López-Moreno, M Grether, and E J Galvez. The hong-ou-mandel interferometer in the undergraduate laboratory. *European Journal of Physics*, 33(6):1843, 2012.

[21] Z Y Ou and L Mandel. Violation of Bell's Inequality and Classical Probability in a Two-Photon Correlation Experiment. *Phys. Rev. Lett.*, 61(1), 1988.

[22] Daniel Cavalcanti, Nicolas Brunner, Paul Skrzypczyk, Alejo Salles, and Valerio Scarani. Large violation of Bell inequalities using both particle and wave measurements. *Physical Review A*, 84(2):022105, 2011.

[23] A Thoma, P Schnauber, M Gschrey, M Seifried, J Wolters, J. H. Schulze, A Strittmatter, S Rodt, A Carmele, A Knorr, T Heindel, and S Reitzenstein. Exploring Dephasing of a Solid-State Quantum Emitter via Time- and Temperature-Dependent Hong-Ou-Mandel Experiments. *Physical Review Letters*, 116(3), 2016.

[24] Jonathan C F Matthews, Alberto Politi, André Stefanov, and Jeremy L. O'Brien. Manipulation of multiphoton entanglement in waveguide quantum circuits. *Nature Photonics*, 3(6):346–350, 2009.

[25] Chil Min Kim, Yong Wan Kim, and Young Jai Park. Attack with Hong-Ou-Mandel interferometer to quantum key distribution. *Current Applied Physics*, 11(4):1006–1009, jul 2011.

[26] Sahar Basiri-Esfahani, Casey R Myers, Ardalan Armin, Joshua Combes, and Gerard J Milburn. Integrated quantum photonic sensor based on Hong-Ou-Mandel interference. *Opt. Express*, 23(12), 2015.

A good lit. review should (IMO):

1. Present an overview of field
2. Give good sense of impact + importance.
3. Present recent progress
4. Present likely future progress.

You have made an attempt at 1+2 but it could use more detail (and pictures!). 3+4 are practically non-existent here.

There are some inconsistencies in the analysis as well. Assumptions have been made without stating them, and this has led to incomplete conclusions (the HOM effect is only a measure of distinguishability for separable photons)..