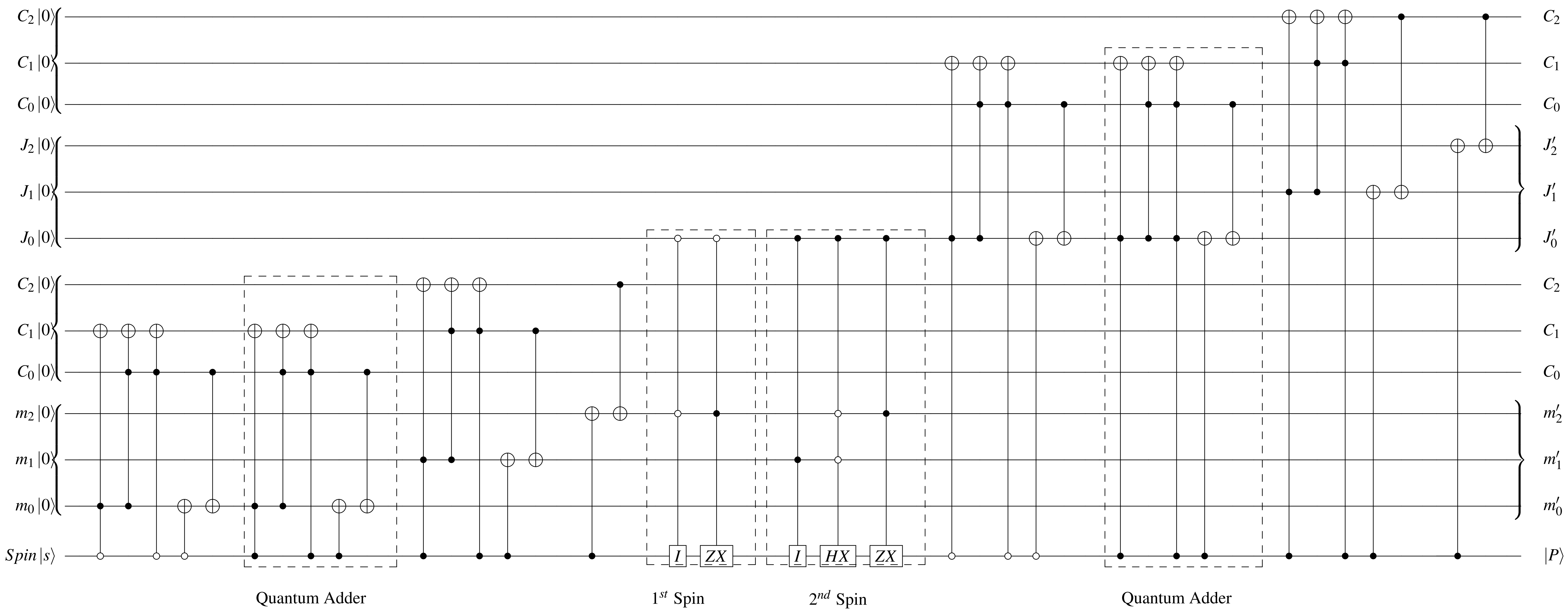
1 Circuit for the Quantum Schur transform for up to 2 qubits ($|S\rangle$)



J_2	J_1	J_0	J
0	0	0	0
0	0	1	$\frac{1}{2}$
0	1	0	1
0	1	1	$\frac{3}{2}$
1	0	0	-2
1	0	1	-$\frac{3}{2}$
1	1	0	-1
1	1	1	-$\frac{1}{2}$

m_2	m_1	m_0	M’
0	0	0	0
0	0	1	$\frac{1}{2}$
0	1	0	1
0	1	1	$\frac{3}{2}$
1	0	0	-2
1	0	1	-$\frac{3}{2}$
1	1	0	-1
1	1	1	-$\frac{1}{2}$

Figure 1: Tables giving binary Two’s complement encoding to spin values of the M and J registers

Circuit uses the encoding for $|S\rangle : |0\rangle \mapsto Spin = +\frac{1}{2}, |1\rangle \mapsto Spin = -\frac{1}{2}$ and the same for $|P\rangle$.

The circuit adds the value of the spin to be added, $|S\rangle$, to the M register to calculate the M’ register value. This is done by implementing the quantum reversible equivalent to the digital full adder.

The case where $|S\rangle = |0\rangle$ means the spin is $+\frac{1}{2}$ so to add $\frac{1}{2}$ to M one is added to the m_0 bit. The very first Quantum Adder (QAdd) uses Toffoli gates controlled on $|0\rangle$ on $|s\rangle$ (denoted by the white control circle) with the current m_0 value and C_0 (an ancilla carry) so that in the case $m_0 = 1$ and we try and add 1 to it, m_0 goes to 0 and m_1 is increased using the carry as $001 + 1 = 010$. The rest of the QAdd stages then just check the carry of the previous qubit to complete to M $+\frac{1}{2}$ addition as $|S\rangle = |0\rangle$ does not trigger any of the rest of the control gates.

The case where $|S\rangle = |1\rangle$ means the spin is $-\frac{1}{2}$ we do M $-\frac{1}{2}$ which is done by adding the binary string for $-\frac{1}{2}$ which is the all 1’s string, 111. This time the very first Quantum Adder does not trigger and $|s\rangle$ is then added to all of the bits of M using C-NOT gates with carries to check for overflow.

The Unitary is then performed on $|S\rangle$ depending on the values of the newly calculated M’ and J registers. The Identity is shown in the circuit for completeness on all the J and M’ values. The J register is then updated to J’ by adding the value of $|P\rangle$ to J using the QAdd sequence of gates.

To add the second qubit in the values of J’ and M’ are passed in as the initial register values. It is easy to extend this to many qubits being streamed in one at a time by carefully conditioning the controls on the unitaries, I think in the general case you need at most N controls for coupling up to N qubits in one at a time. The circuit written here has redundancy in the Identity and ZX gates appearing twice.