Nonlinear Optics and Continuous Variable Computing

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June 18, 2018

Abstract

We briefly review nonlinear optics, notably focusing on the use of nonlinear effects to generate squeezing and how squeezing can be used as a single photon source. The third-order nonlinear Kerr effects are discussed along with current methods to minimise unwanted effects such as self phase modulation and two photon absorption. Lastly, we mention all of the necessary components for continuous variable computing and comment on its progress.

1 Introduction

Nonlinear optics is used extensively and has a wide range of applications over many fields. Some of the main uses include, frequency mixing for frequency doubling, optical parametric amplification (OPA), using the Kerr effect for self-focusing of high intensity beams, Kerr-lens mode-locking to make ultra-fast pulsed lasers and using spontaneous parametric down-conversion (SPDC) for generating entangled photons.

All current schemes for linear optical quantum computing rely on nonlinear optical effects (mainly in photon generation), these are referred to as *active* as they do not conserve photon number. Discrete variable computation requires single photons, the most popular current methods use nonlinear effects such as heralding on a parametric process, SPDC or spontaneous four wave mixing (SFWM). Non-parametric nonlinear processes such as the light-matter interaction in quantum dots are also used as single photon sources.

The nonlinear effects mentioned above depend on the susceptibility of the material which is given by, $\chi^{(x)}$ where x denotes the order of nonlinearity. The second order susceptibility, $\chi^{(2)}$ is only nonzero (meaning second order nonlinear effects only occur) in noncentrosymmetric materials e.g. birefringent crystals.

The third order susceptibility, $\chi^{(3)}$ the most relevant is silicon which only has a third order nonlinearity (due to its symmetry $\chi^{(2)} = 0$) this third order nonlinearity is also called the optical Kerr effect. There are undesirable nonlinear effects which occur in materials with a third order susceptibility such as self- and cross- phase modulation (SPM & XPM) or two-photon absorption (TPA).

Continuous variable (CV) quantum computation can be built up from nonlinear optics, most CV experiments use optical setups as photons allow easy access to extremely high dimension Hilbert spaces. CV computation is a much younger field when compared to discrete variable computation [1] and uses squeezing as the irreducible resource for computation [2]. Continuous variables are also being used in quantum cryptography [3] and QKD [4].

A current overview of the most recent experimental records for squeezing is given, along with a comparison of the devices used. Nonlinear interferometers [5] and their uses are then mentioned. We conclude with a review of current experimental progress on CV computing.

2 Quantum nonlinear optics

One of the most used nonlinear effects is squeezing for single photon production (heralding SPDC) or squeezing of quadratures for CV applications. Bulk nonlinear crystals such as BBO and PPKTP are the most popular $\chi^{(2)}$ materials used for squeezing.

Recently there has been a shift of moving towards integrated nonlinear optics to generate squeezing, this is for a number of reasons but mainly for the stability integrated setups offer and scalability for quantum information processing applications. Periodically poled lithium niobate (PPLN) is one of the most popular integrated $\chi^{(2)}$ materials allowing waveguides to be fabricated out of it.

Most of the integrated focus is on CMOS compatible structures, such as silicon which has a moderate $\chi^{(3)}$ nonlinearity. Silicon photonics used in quantum information processing makes use of spirals and micro-ring resonators to increase the interaction length to make up for the weaker nonlinearity of silicon.

2.1 Squeezing

Squeezing refers to reducing the variance of the quadrature to below that of that of the vacuum. For example, if light is amplitude squeezed the variance in measuring the amplitude will be less than the vacuum fluctuations.

There are two distinct uses for squeezing, using squeezing for single photon generation which is discussed above. The reason for using squeezing is that a squeezed vacuum state will only generate even Fock states so it is possible with post selection to ensure you only have one photon present after heralding. The second use, is using squeezing to reduce the variance of either amplitude or phase to do below shot noise measurements. Shot noise is the standard noise due to vacuum fluctuations that happens with a measurement.

The squeezed light used for single photon sources removes all of the spectral correlations between the signal and idler which then removes most of the other properties of squeezing from the state. This is normally done using a large amount of filtering which reduces the total brightness by discarding a lot of the squeezed light.

Bulk optical setups currently have the highest records for squeezing are $\chi^{(2)}$ processes, the squeezing record for a bulk cavity setup is 15.3dB using PPKTP [6] and 10.12 ± 0.15 dB using a 7% MgO:LiNbO$_3$ crystal [7]. The record for an integrated Lithium niobate waveguide resonator 2.9 ± 0.1dB (4.9 ± 0.1 dB after detection losses) for a continuous-wave [8].

The values for $\chi^{(3)}$ processes tend to be much lower, 1.1 ± 0.08dB (1.95 ± 0.17dB after detection losses) in dispersion shifted fibre [9]. The largest squeezing using an integrated silicon nitride micro-ring is reported as 1.7dB [10].

If both of the daughter photons from squeezing are used it is possible to take advantage of the entanglement between them. This allows for reduced noise in phase sensitive measurements if they are phase squeezed or reduced noise in photon number counts if they are amplitude which has a wide number of uses for sensing below the shot noise limit. Quadrature squeezing can be used to entangle different degrees of freedom such as polarisation with position [11], which may be useful for hybrid optical systems. The reduction in variance of quadrature is used in CV computation.

2.2 Nonlinear optics for single photons

Down-conversion is a 3-wave mixing process (function of the $\chi^{(2)}$ of the material) which converts a pump photon into a signal and an idler photons. SFWM is the 4 wave mixing equivalent (function of the $\chi^{(3)}$ of the material) which converts two pump photons into a signal and idler photons.

Depending on the specific hardware and required properties of the signal and idler one of the down-conversion processes is used in linear optical quantum computing. A necessary property of the daughter photons involves removing all of the spectral correlations between the signal and idler photons. CV computing uses down-conversion to generate squeezed or EPR entangled states.

A current measure of the quality of the down-conversion is as a single photon source is the spectral purity of the state. The joint spectral amplitude (JSA) gives the spectral purity of the signal and idlers. In the best case there are no spectral correlations between the signal and idler so that when one is heralded we are left with a completely spectrally pure single photon.

Depending on the application tunable sources may be desirable, work has been done on tunable Telecoms range photons. They used KKTP as a single photon source at 1460 - 1675 nm wavelengths and reported a purity > 0.81 over the wavelength range [12].

Tunable sources can also be used for sensing, in this case tunable refers to the signal photon being at a wavelength that highest detection efficiency whilst the idler is tuned to a wavelength suited for the sample. This lets you interact with the sample at the correct wavelength with the idler and use the correlations between the two photons and measure the signal photon at a wavelength with high detector efficiency. The signal photon will have gained phase information when the idler interacts, therefore you have gained information about the sample even though the signal photon never directly interacted with it.

Structures	Silicon			Hydex	Si_3N_4
Parameters	Nanowire[95]	Ring[77]	PhC[96]	Ring[97]	Ring
Nonlinear coefficient ($W^{-1}m^{-1}$)	300	-	4000	0.22[98]	-
Q-factor	-	37 500	-	1 375 000	2 000 000
Coupled pump average power (mW)	0.18	0.019	0.055	21	3
Collected photon bandwidth (GHz)	25	5.2	50	0.11	0.09
Brightness (pairs s^{-1} mW^{-2} GHz^{-1})	1.6×10^5	4.4×10^8	1.5×10^6	6.2×10^3	4.3×10^8
CAR	320	602	330	11	-
$g^{(2)}(0)$	-	-	0.09	0.14	-
Number of entangled photons	2[99]	2[100,101]	2[102]	4[92]	2[103]

Table 1: Current experimental results for integrated single photon sources, taken from [13].

Table 1 contains a review on integrated photon sources using nonlinear optics [13]. Cavities and ring resonators (the integrated equivalent) are currently the best for producing spectrally narrow single photons, note the collected photon bandwidth is smallest for the ring resonators. The rings also have the largest number of entangled photons produced and the silicon & silicon nitride both require an order of magnitude less of pump power which is good as it reduces the unwanted third order effects such as SPM. Cavities enables tuning the resonances to favour a particular frequency band for the signal and idlers.

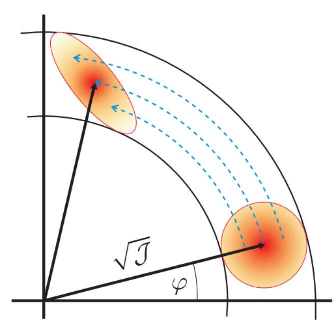

Figure 1: Kerr squeezing where the phase shift is intensity dependent, taken from [19].

2.3 Kerr-processes

The optical Kerr effect is when a bright beam of light changes the refractive index of the medium it is passing through by an amount proportional to its own intensity [14]. Kerr processes include SPM, XPM, TPA which are all problematic in silicon which is used for CMOS integrated photonics. Mode-locked lasers take advantage of the Kerr effect to provide access to ultra fast pulses used in femptosecond lasers.

Self phase modulation limits silicon waveguides in pump power to $\sim$ 10 mWs [15] before spectral broadening occurs. However, this intensity dependent refractive index shift can also be used to make self focusing beams which reduces the mode profile inside the waveguide leading the better confinement.

Cross phase modulation is when a bright channel can impart a phase shift in a nearby channel, such two waveguides, the pump can change the phase of the signal or idler photons if the pump is sufficiently bright.

TPA is reduced at larger wavelengths [16], moving to 2.1μm wavelengths means that TPA stops and only higher order effects (such as 3-photon absorption) occur.

The Kerr effect is also used to generate squeezing shown in Fig. 1 which are sometimes referred to as banana states. Kerr squeezing has been investigated considerably [17],[18] and is a different form to the squeezing seen in down-conversion as it is intensity dependent.

Another proposed use of squeezing is for nonlinear interferometers. These have applications mainly for amplifying experiments in the low photon number regime such as quantum sensing or imaging. Quadrature squeezing has been theoretically proposed to increase the Kerr cross-phase modulation effect [20] which is one method of performing deterministic two-photon interactions.

3 Continuous variables

CV computing can be split into two parts, a Gaussian optics part and a non-Gaussian part. Gaussian optics contains single mode operations, such as displacement operators, rotations and multimode operations that only interact at most two modes, such as squeezing and a two-mode beamsplitter. The Gaussian states are the coherent (displaced states) and squeezed-coherent states. The Gaussian detection is homodyne to measure squeezing and heterodyne to measure α for coherent states.

The non-Gaussian optics is the single mode cubic phase gate which is greater than or equal to degree 3 in mode operators ($\hat{a}\&\hat{a}^{\dagger}$). The final part is a non-Gaussian measurement which is commonly chosen as a number resolving detector to measure in the Fock basis.

There have been theoretical studies in resource estimation for CV computing [21], shown in Fig. 2 which is the Block-Messiah reduction also given in [2]. This Illustrates that only single mode squeezing and then passive linear optics are needed to build up any CV cluster state. This is reassuring as single mode squeezing is a relatively well understood and active area of research.

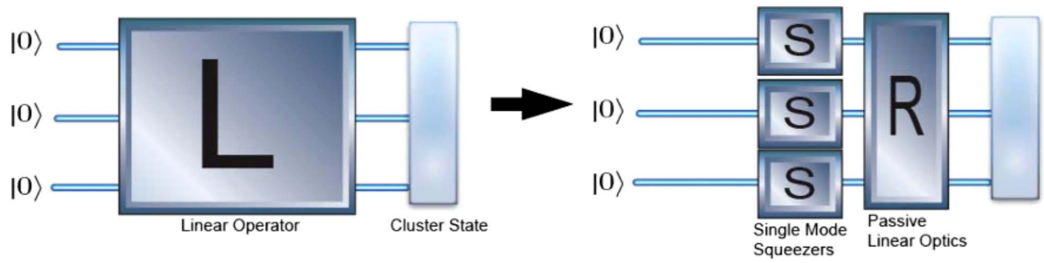

Figure 2: CV cluster state equivalence between an arbitrary multimode squeezed state and a series of single mode squeezers with passive linear optics such as beamsplitters and phaseshifters, taken from [21]

3.1 Detection

We now mention how to measure squeezing and cover the types of detection needed for continuous variable information processing, these are optical homodyne and heterodyne detection which involve mixing the weak optical signal with a known local oscillator using a beamsplitter and then subtracting the two signals to measure the weak signal. The last type of detection is photon number resolving detectors (PNRDs).

Homodyne detection has a number of nice properties, firstly as photo-diode/photo-current detectors can be used this means the detectors do not need to be cooled down to cryogenic temperatures, unlike single photon detectors. Secondly, homodyne detection is now so good the detectors available are approaching unit detection efficiency.

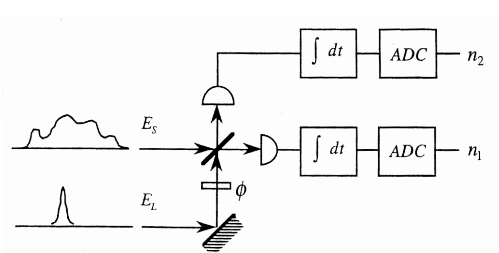

Figure 3: The local oscillator E_L is mixed with the signal E_S, taken from [22].

The general scheme for homodyne detection is given in Fig. 3, by mixing the signals and subtracting the two photo-currents it enables measurement of the phase or amplitude of a weak signal.

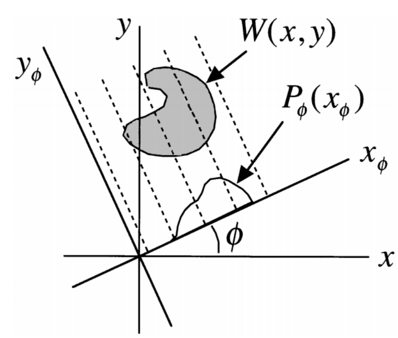

Figure 4: Reconstructing the Wigner function for the state by measuring the quadrature values, taken from [22].

Fig. 4 illustrates how scanning through the phase of the local oscillator it is possible to reconstruct the quadrature values and measure the amount of squeezing present.

The amount of squeezing can be measured using homodyne detection, this is done by sweeping through the phase angle on the local oscillator. Homodyne detection values are over 99.6% detection efficiency [23]. Heterodyne detection uses the same principles and equipment as homodyne but in heterodyne detection the signal and local oscillator are not from the same source this lets you measure the amplitude of the weak signal.

PNRDs are required as being able to perform a photon counting measurement is requirement for CV computing to be universal [24]. A quantum non demolition measurement [25] is also needed for CV computation which can be implemented using the Kerr effect and a PNRDs.

Photon number resolving detectors are currently being implemented using a number of different techniques. The main difference being spatially multiplexing existing bucket type detectors [26]. The other approach is to use specifically designed photo-detectors that respond linearly which makes it possible to measure the amount of photons in each detection event. Examples of the former include, an array of superconducting nanowire detectors [27]. However, this would mean CV computing would lose one of the big advantages it has over discrete linear optics which that homodyne detection works at room temperature. Another approach is to use transition edge sensors as PNRDs [28] but these are very slow and would limit the clock rate of the computer severely.

3.2 Integrated CV

Continuous variable entanglement on a chip was achieved in 2015 [29] and since then much effort has been on integrating the platform, similar to the linear optical community. For CV computing to be realised fully showing that the platform can be scaled up is crucial.

Fig. 5 is an integrated photonic chip for quantum information using continuous variables [30]. In most CV experiments the larger the squeezing the better, lithium niobate is used over silicon because it has a $\chi^{(2)}$ nonlinearity which leads to much larger squeezing.

Universal quantum computing using CV cluster states requires infinite squeezed vacuum states [31] so progress is needed on increasing the squeezing achievable. One of the main difficulties with CV computing currently is number resolving detectors.

4 Conclusion

Nonlinear optics is an important tool with the many applications not just limited to quantum photons and quantum information processing. Here we focused on using nonlinear effects to generate squeezing.

Squeezing can then be either used as a heralded single photon source which is popular in integrated LOQC or directly for the correlated squeezed states vacuum squeezing produces. The correlations present in the squeezed state can be used for correlated photon measurements using the quadrature variance reduction from squeezing to make below shot noise limited measurement. Squeezed states can also be used for continuous variable information processing such as CV computation discussed here or CV QKD as well as below shot noise sensing and imaging.

Currently the squeezing record is with second order nonlinear materials and bulk systems have achieved the largest squeezing reported. This is one the experimental difficulties for CV computing as very large squeezing is needed and for it to be a serious platform it will need to be scalable which will require even better integrated nonlinear components to generate high squeezing on chip. The photon number resolving detectors also will need to be improved upon for use in CV computing.

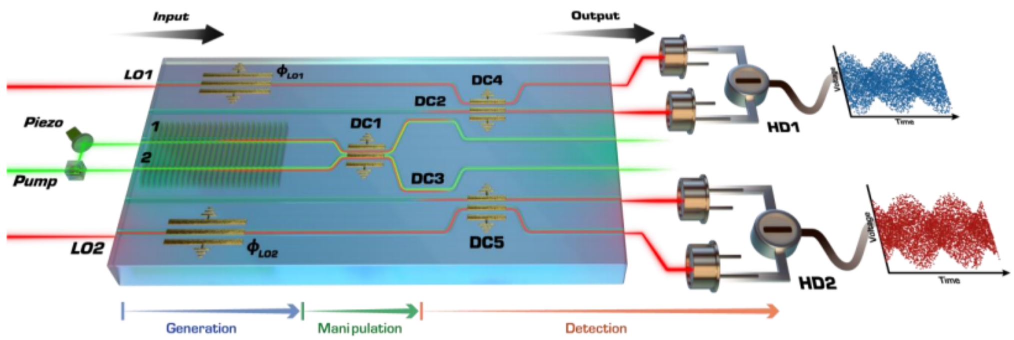

Figure 5: Integrated CV quantum information processor the chip containing Periodically poled lithium niobate waveguides for squeezing, directional couplers DC1 to mixed the squeezed states, DC2,3 for pump filtering and DC4,5 & $\phi_{LO1,2}$ are for the homodyne detection, taken from [30].

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