University of Bristol, Quantum Engineering CDT

Quantum Optics: Assignment 1

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Problem 1: The Light-bulb Laser

Read Howard Wiseman's paper: "How many principles does it take to change a lightbulb...into a laser?"

1.1

Derive an expression for the average number of photons at frequency ω in a blackbody oven at temperature t. Plot it for some reasonable value of T.

1.2

Derive Howard's equation for the power per unit length of a lightbulb-laser: $P_{coll} = \frac{\pi}{12}\frac{(k_B T^2)^2}{\hbar}$. Make sure you understand how this derivation works, Howard skips quite a few conceptual steps in his derivation.

1.3

What does the approximation in equation 15 mean? Show all the steps between equation 17 and equation 18. Note that you will have to use the approximation you just justified!

1.4

Look at equation 20 and 21. How does the fraction of collected power (collimated and filtered) scale with the temperature? There are two parameters we are trying to control here:

1. The power per "laser-like" mode

2. The fraction of power collected from the total power

Do you see why these are at odds with each other? Explain!

PROBLEM 2: THE MODE-LOCKED LASER

Here we will study pulsed lasers.

2.1

Consider a very multimode laser, that has N modes. These modes are equally spaced and have frequencies $\omega_k = \omega_0 + k\Delta$ and phases ϕ_k. Derive an expression for the electric field of these N modes (you will need to express it as a sum of the electric field due to each mode separately). You may assume the modes are monochromatic. Don't forget that each mode will have its own phase!

2.2

Show that the instantaneous intensity is the sum of two terms, one which osciallates very fast and one which oscillates with a frequency Δ.

2.3

Using the argument that your photodetector will be fast enough to see the oscillations at frequency Δ, but not fast enough to see the oscillations at the faster frequency, derive the expression for the time-averaged intensity seen by the detector:

$$I(t) = \frac{NE_0^2}{2} + E_0^2 \sum_{k,j>k} \cos[(j-k)\Delta t + \phi_j - \phi_k] \tag{2.1}$$

Plot this for $E_0 = 1$, $N = 10$, $\Delta = 1kHz$, and $\Phi = [0, 0.32, 0.54, 1.35, -0.07, 2.15, -1.89, -0.76, -1.33, 0.44]$.

2.4

Finally, imagine all of the phases have some fixed relationship (they are not random, but determined by some physical process). To make the math easy, let's assume all the phases are zero.

What does the expression for the intensity become? Hint: Equation 2.1 can be greatly simplified in this case by seeing it as a geometric series.

Plot this and compare this to the random phase case.

2.5

We know from lecture that each mode in a laser does not oscillate perfectly monochromatically. Technical broadening (as well as the Autler-Townes limit) broadens each mode. Does this effect the previous treatment? If so, how? If not, why?

2.6

The case where all the phases have a fixed relationship is called a *mode-locked laser.* There are numerous methods for obtaining this type of laser operation such as saturable absorbers, amplitude modulation, phase modulation, Kerr-lens mode-locking, etc. Read about one to tell us in the next lecture.