UNIVERSITY OF BRISTOL, QUANTUM ENGINEERING CDT

Quantum Optics: Assignment 4

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PROBLEM 1: THE LANDAU-ZENER CRITERION

In this problem, we will derive the Landau-Zener criterion (perturbatively), and then make some well-founded guesses about the full-solution. Recall in class that we stated that for an atom initially in the ground state, with the laser far red-detuned, the atom will adiabatically transition to the excited state as the laser frequency is swept as long as:

$$\frac{d}{dt}\delta \ll \Omega_R^2 \tag{1.1}$$

1.1

Let's begin with the time-dependent atom-field Hamiltonian:

$$\hat{H} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \hbar\Omega_R\cos(\omega t)\hat{\sigma}_x \tag{1.2}$$

Before, we went into the rotating frame by effecting the transformation:

$$|\Psi\rangle \to \hat{R}_z(\omega t)|\Psi'\rangle \tag{1.3}$$

where $\hat{R}_z(\omega t) = \mathrm{e}^{-i\omega t\hat{\sigma}_z/2}$.
We must be careful now, since ω will now be a function of time. Instead, go into the rotating frame with $\hat{R}_z(\phi) = \mathrm{e}^{-i\phi\hat{\sigma}_z/2}$ and remember that ϕ is a function of time.
Show that

$$\hat{H}' = \frac{\hbar(\omega_0 - \dot{\phi})}{2}\sigma_z + \hbar\Omega_R\cos(\omega t)\{\hat{R}_z^\dagger(\phi)\sigma_x\hat{R}_z(\phi)\} \tag{1.4}$$

1.2

Imagine a situation where we are linearly sweeping the detuning. Think about what $\dot{\phi}$ should be (remembering the expression for ϕ from Lecture 5 stated above). Integrate this to find ϕ in terms of the atomic transition frequency, ω_0, and the rate at which we sweep the detuning, which we can call α.

1.3

Show that, using the rotating wave approximation, the second term (which we call $\hat{V}$ here) in the expression for $\hat{H}'$ evaluates to:

$$\hat{V} = \frac{\hbar\Omega_R}{2}\begin{pmatrix} 0 & \mathrm{e}^{i(\phi-\omega t)} \\ \mathrm{e}^{-i(\phi-\omega t)} & 0 \end{pmatrix} \tag{1.5}$$

This is not particularly easy, but you can use the tricks we learned in lecture 5.

1.4

Now, treat $\hat{V}$ like a perturbation and (going all the way back to lecture 3), calculate the probability for ending up in the excited state at $t = +\infty$ given that you began in the ground state at time $t = -\infty$ (HINT1: You can go back to lecture 3 and copy the formula that we used directly, no need to derive it again. You will notice that a term that depends on the energy level splitting cancels out nicely! HINT2:you have to integrate some expression, and you will have to look up the solution to this integral unless it happens to be fresh in your mind).

1.5

It is customary to write this probability:

$$P_e = 2\pi\Gamma \tag{1.6}$$

What is Γ? Remember our earlier guess for the adiabatic criterion. Was it correct?

PROBLEM 2: RABI REVIVAL

2.1

We derived an expression for the state of an atom, initially in the excited state, interacting with a fully quantized field. In class, we assumed the field was very intense, and thus the coherent state amplitude $\alpha \gg 1$. Let's not make this assumption here. Instead, go back to the equation we derived for the state of the field and atom as a function of time, and calculate the probability of the atom being in the excited state *regardless* of the state of the field.

2.2

Plot this, for $\Omega_R = 1$ and $\alpha = 4$, up to time $t = 20$. Please note that this must be done numerically (and approximately) since the series expression will not have an analytic solution. Think carefully about how many terms in the expansion you should keep to get a reasonably accurate result.

2.3

The behavior you see is the real behavior of the system, no semi-classical approximations made. There are a range of frequencies (the dominant one being the Rabi frequency). These different frequency terms will eventually be out of phase, causing the collapse behavior. Let's estimate the time it takes for the system to decay ie. the "dephasing time".
Classically, oscillators with a range of frequencies $\Delta\Omega$ will dephase after a time $(\Delta\Omega)^{-1}$. What is the range of frequencies that contribute to the dynamics here? ie. look at the equation for the wavefunction and note that the expansion coefficients are all small, except for some values of n. Calculate the range of important frequencies in this problem (Hint: they are a direct result of the terms in the coherent state with $n = \bar{n} \pm \sqrt{\bar{n}}$ where $\bar{n} = |\alpha|^2$ is the average number of photons) and thus calculate the dephasing time.

2.4

Your answer for the dephasing time should not depend on $\bar{n}$. But from our treatment in class, when $\bar{n} \gg 1$ we see the semi-classical Rabi oscillations (with no collapse). So what's going on here?! (Hint: if you plot the bahavior for different values of $\bar{n}$, the relevant time scales will also be a function of $\bar{n}$ so the x-axis scale should change!) (BIG Hint: Think if you were actually trying to do an experiment and observe Rabi oscillations. If all you knew was a semi-classical world, would you care what happens after a few Rabi oscillations?)