QO2 - Lecture 7

The Adiabatic Basis

So far, we have started in $|g\rangle$ or $|e\rangle$, applied a field, and saw time evolution (Rabi flopping). Clearly, these are not the energy eigenstates of the system.

Neglecting spontaneous emission, what are the eigenstates and what is their significance?

$$\hat{H} = \underset{\substack{\uparrow \\ \text{Atomic}}}{\hat{H}_A} + \underset{\substack{\uparrow \\ \text{interaction}}}{H_I}$$

In the rotating frame, after the RWA,

$$\hat{H} \xrightarrow[\{|e\rangle, |g\rangle\}]{} \frac{\hbar}{2}\begin{pmatrix} -\delta & \Omega_R \\ \Omega_R & \delta \end{pmatrix} \quad \text{where} \quad \Omega_R = -E_0 \langle e | q \vec{\varepsilon} \cdot r | g \rangle / \hbar$$

$$\delta = \underset{\substack{\uparrow \\ \text{laser frequency}}}{\omega} - \underset{\substack{\uparrow \\ \hbar\omega_0 \text{ is atomic splitting}}}{\omega_0}$$

Straightforward way to do this is to solve:

$$\hat{H}|\psi\rangle = E|\psi\rangle \quad \text{for} \quad |\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

Another way is to remember our geometrical (Bloch) picture!

$$\hat{H} = -\delta \hat{S}_z + \Omega_R \hat{S}_x$$

$$= \Omega(\hat{\vec{S}} \cdot \vec{n}), \quad \text{where} \quad \hat{\vec{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$$

$$\vec{n} = \left(\frac{\Omega_R}{\Omega}, 0, \frac{-\delta}{\Omega}\right)$$

ie) the dynamics are a rotation about the $\vec{n}$ axis.

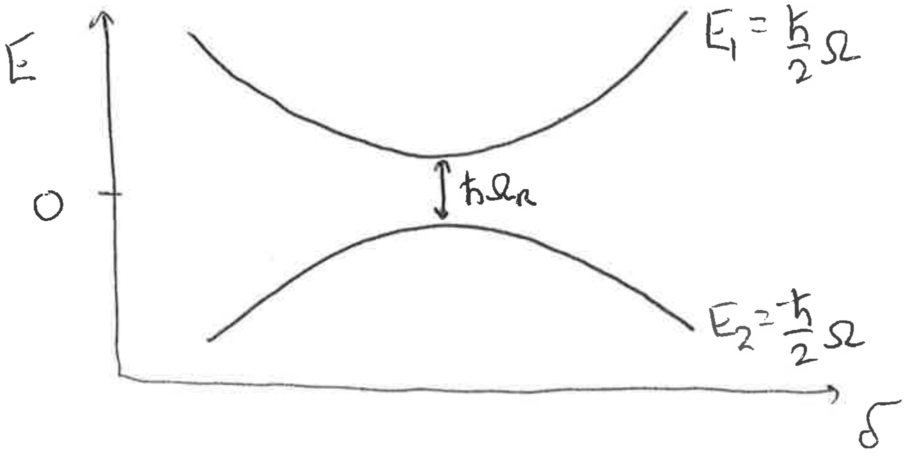
If you are familiar with Spin formalism, the rest is standard...

The solution is, as we have seen,

$$|1\rangle = \cos\theta\,|e\rangle + \sin\theta\,|g\rangle$$
$$|2\rangle = \sin\theta\,|e\rangle + \cos\theta\,|g\rangle \quad , \text{ where } \cos 2\theta = -\frac{\delta}{\Omega_R}$$

$$E_{1,2} = \pm\frac{\hbar}{2}\Omega$$
$$= \pm\frac{\hbar}{2}\sqrt{\delta^2+\Omega_R^2}$$

So let's draw this plot once more!



Note 1

- At $\delta = 0$, $\Omega = \Omega_R$, so the splitting is $\hbar\Omega_R$.
- Since energy of atomic excitation is ω_0 and quantum of field is ω, when $\delta = \omega - \omega_0 = 0$ the exchange of energy is resonant.
- for $\Omega_R = 0$, there are two degenerate states: $\hat{H} = 0$
- for $\Omega_R > 0$, coupling SPLITS THE STATES. This is common in physics, sometimes called an "avoided crossing"
- at this point $(\delta = 0, \Omega_R > 0)$

$$|1\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$$
$$|2\rangle = \frac{1}{\sqrt{2}}(-|e\rangle + |g\rangle)$$

50/50 superposition.

Note 2:

Limits far from $\delta = 0$

$\underline{\delta < 0}$

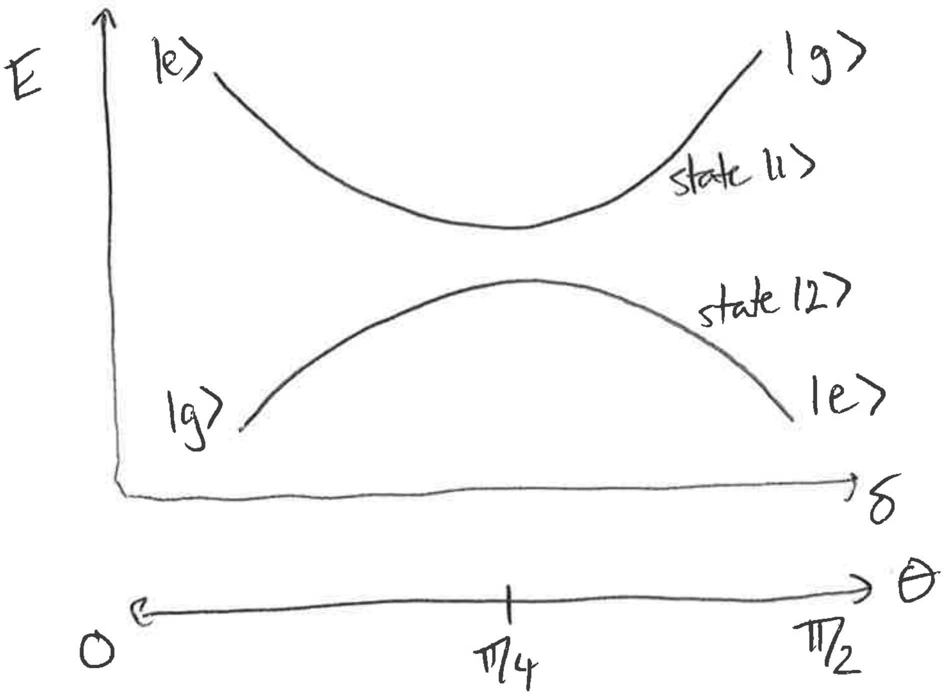
$|\delta| \gg \Omega_R$

$$\left.\begin{aligned} 2\theta &\to 0 \\ \cos\theta &\to 1 \\ \sin\theta &\to 0 \end{aligned}\right\} \begin{aligned} |1\rangle &\to |e\rangle \\ |2\rangle &\to |g\rangle \end{aligned}$$

$\underline{\delta > 0}$

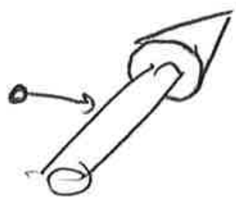
$|\delta| \gg \Omega_R$

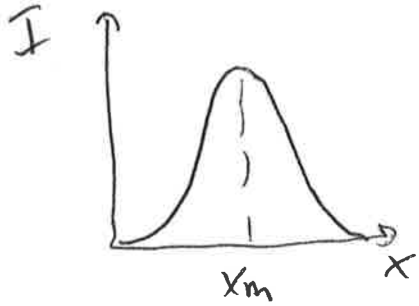
$$\left.\begin{aligned} 2\theta &\to \pi \\ \cos\theta &\to 0 \\ \sin\theta &\to 1 \end{aligned}\right\} \begin{aligned} |1\rangle &\to |g\rangle \\ |2\rangle &\to |e\rangle \end{aligned}$$



Note 3: Strong and weak-field seekers

Consider an atom passing through a laser beam @ frequency ω



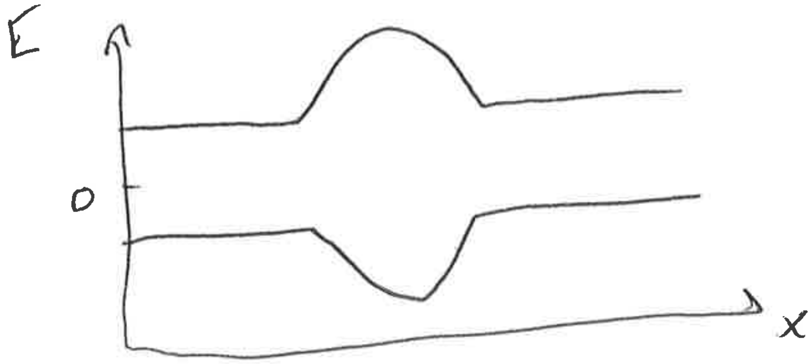


$\Rightarrow$

because $\Omega_R \propto \sqrt{I}$

Consider the eigenstates at every position:

$$E_{1,2} = \pm \frac{\hbar \Omega}{2} = \pm \frac{\hbar}{2} \sqrt{\delta^2 + \Omega_R^2(x)}$$



Assume the atom is moving slowly so the entire process is adiabatic and the atom stays in the same state.

Atom in state $|1\rangle$

REPELLED from max intensity...

"weak-field seeker"

Atom in state $|2\rangle$

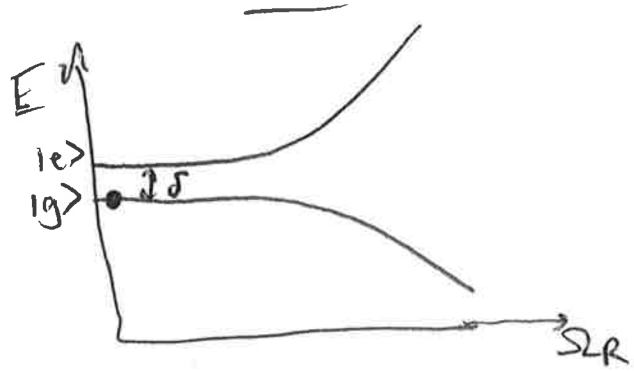
ATTRACTED to max intensity...

"strong-field seeker"

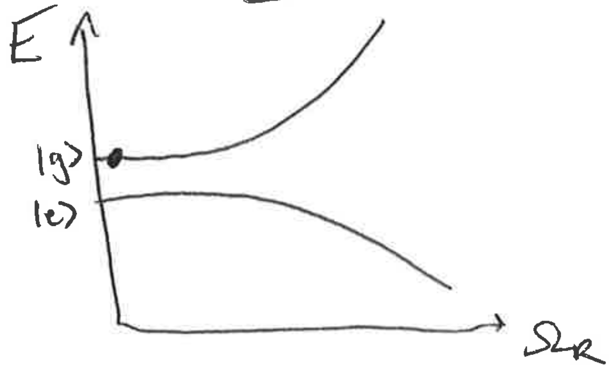
Note 4: Ground State Atoms

Imagine slowly ramping the field on:

$\delta < 0$



$\delta > 0$



- ground state is $|2\rangle$
- $|2\rangle$ is "strong-field seeking"

$$\boxed{\text{RED DETUNED BEAMS ATTRACT (ground-state) ATOMS}}$$

- ground state is $|1\rangle$
- $|1\rangle$ is "weak-field seeking"

$$\boxed{\text{BLUE DETUNED BEAMS REPEL (ground-state) ATOMS}}$$

Also, remember that, in equilibrium, $N_g > N_e$. This means that this rule should work for atoms in equilibrium as well.

Note 5: Light Shift.

What is the energy shift on the atom due to the applied field?

Consider $\delta > 0$

As discussed, $|g\rangle \Rightarrow |1\rangle$

$$E_1 = \frac{1}{2}\hbar\sqrt{\delta^2 + \Omega_R^2}$$

So moving from $\Omega_R = 0$ to $\Omega_R \neq 0$ gives:

$$\Delta E = \frac{1}{2}\hbar\sqrt{\delta^2 + \Omega_R^2} - \frac{1}{2}\hbar\delta$$

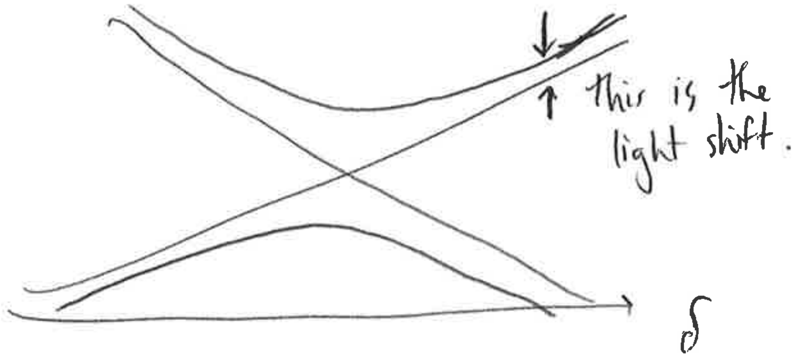
In the far-detuned limit, we can expand the first term:

$$\Delta E \approx \frac{1}{2}\hbar\delta\left(1 + \frac{1}{2}\frac{\Omega_R^2}{\delta^2}\right) - \frac{1}{2}\hbar\delta$$

$$= \frac{1}{2}\hbar\frac{\Omega_R^2}{2\delta}$$

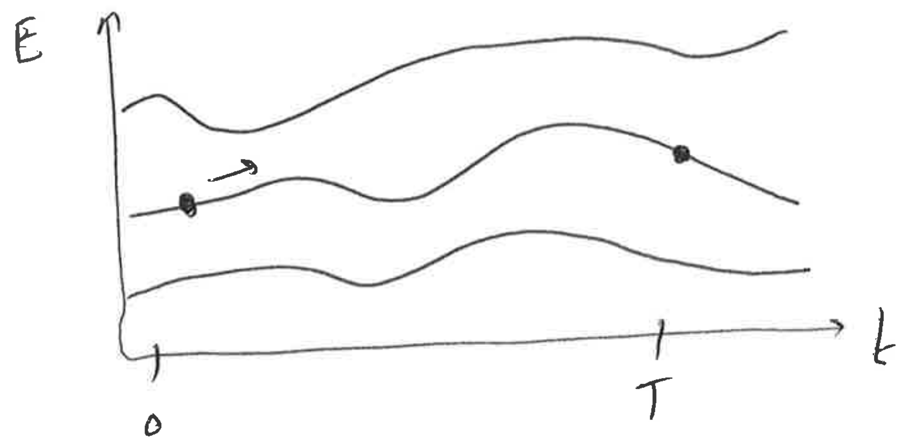
thus: $\boxed{\Delta E = \dfrac{\hbar\Omega_R^2}{4\delta}}$ is the light shift on ground-state atom. (also works for $\delta < 0$)

Graphically,



The Adiabatic Theorem

If $\hat{H}(t)$ changes adiabatically from $t=0$ to $t=T$, a system in an energy eigenstate with $E_n(0)$ at $t=0$ evolves to an energy eigenstate with eigenvalue $E_n(T)$.

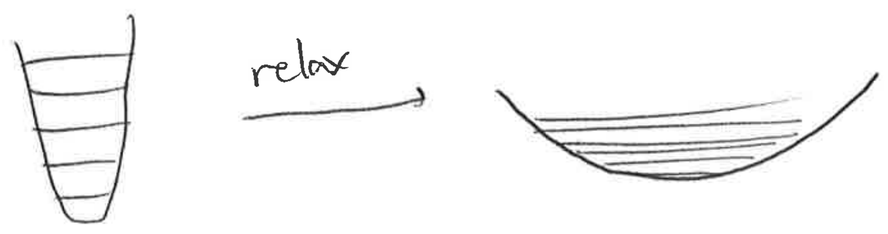


But How SLOW IS SLOW?

If it's too fast, state may jump from one eigenstate to the next.

Example: Ramp down ω for a SHO, $\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$

$E = \hbar\omega(n+\frac{1}{2})$



What is the "adiabatic criterion" for $\frac{d}{dt}\omega$?

From dimensional analysis, there is only one thing in the problem with units $[\text{rate}]^2$, so we could make a guess!

$$\frac{d}{dt}\omega \ll \omega^2$$

which turns out to be correct!

Typically, the time derivative of a <u>dimensionless parameter</u> (eg. in state space) must be less than the <u>energy splitting</u> between levels. Trick is to find the correct dimensionless parameter!

ie) $\hbar\dot{\Theta} \ll \Delta E$

Wider spaced energy levels always helps.

You will solve this problem for a two-level atom in an EM field sweeping the detuning in the homework!

The resulting criterion:

$$\frac{d}{dt}\delta \ll \Omega_R^2$$

is called the

$$\boxed{\text{Landau-Zener Criterion}}$$