

# Modelling Nonlinear optics with the Bloch-Messiah reduction

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# Overview

Modelling  
Nonlinear  
optics with  
the  
Bloch-Messiah  
reduction

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- What is nonlinear optics?
- Why do we care about it?
- What I have been doing
- Gaussian optics
- Outlook

# Motivation quantum nonlinear optics

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## The good

### Spontaneous Parametric processes, SPDC, SFWM

- Heralded single photon sources
- Entangled photon pair generation (polarisation, spatial)

### Kerr processes

- Self-Phase modulation (SPM), generating Bannana states (CV)
- Cross-Phase modulation (XPM) for sensing

## The bad

- Generating more than two photons  $\rightarrow$  bad for quantum computing

### All Kerr nonlinear processes

- SPM  $\rightarrow$  Spectral broadening
- XPM  $\rightarrow$  Unwanted phase shifts on single photons due to propagation of the pump

# What do we mean by nonlinear optics?

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- Roughly processes that conserve energy but do not conserve photon number.

$$\vec{P} = \chi^{(1)} \vec{E}_1 + \chi^{(2)} \vec{E}_1 \vec{E}_2 + \chi^{(3)} \vec{E}_1 \vec{E}_2 \vec{E}_3 + \dots \quad (1)$$

Here we are going to talk about squeezing, i.e SPDC or SFWM, Hamiltonians are then of the form,

$$\hat{H} = A \hat{a}_S^\dagger \hat{a}_I^\dagger \hat{a}_P + h.c. \quad (2)$$

$$\hat{H} = A \hat{a}_S^\dagger \hat{a}_I^\dagger \hat{a}_P \hat{a}_P + h.c. \quad (3)$$

# Gaussian Optics

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$$\hat{U} = \exp \left[ -\frac{i}{\hbar} \left( \overset{\text{Power}}{P} \int d\omega_1 \int d\omega_2 \overset{\text{JSA}}{f(\omega_1, \omega_2)} \overset{\text{Signal \& Idler}}{\hat{a}_s^\dagger(\omega_1) \hat{a}_i^\dagger(\omega_2)} + h.c. \right) \right] \quad (4)$$

- Using the undepleted pump approximation we can write the Hamiltonians as terms which are at most quadratic in creation and annihilation operators.
- These are Gaussian transforms, they take Gaussian states to Gaussian states <sup>1</sup>

$$\begin{bmatrix} \vec{b} \\ \vec{b}^\dagger \end{bmatrix} = M \begin{bmatrix} \vec{a} \\ \vec{a}^\dagger \end{bmatrix} \quad (5)$$

<sup>1</sup>These are linear symplectic transforms which conveniently can be written as a matrix

# Hamiltonian

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We can do re-write this Hamiltonian as a Schmidt-decomposition using SVD.

$$-\frac{i}{\hbar}Pf(\omega_1, \omega_2) = \sum_k r_k \psi_k(\omega_1) \phi_k(\omega_2) \quad (6)$$

Where  $\psi$  &  $\phi$  are unitary matrices,

- with  $\psi_k(\omega_1)$  is the  $k$ -th row and  $\omega_1$ -th column of  $u_{(\omega_1, k)}$ ,
- with  $\phi_k(\omega_2)$  is the  $\omega_2$ -th row and  $k$ -th column of  $v_{(k, \omega_2)}^\dagger$

$$P'f(\omega_1, \omega_2) = \sum_k r_k u_{(\omega_1, k)} v_{(k, \omega_2)}^\dagger \quad (7)$$

Recall SVD is defined as,

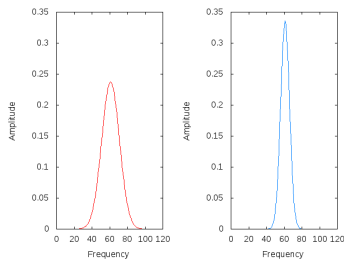
$$M = U \Sigma V^\dagger \quad (8)$$



# The Joint Spectral Amplitude (JSA)

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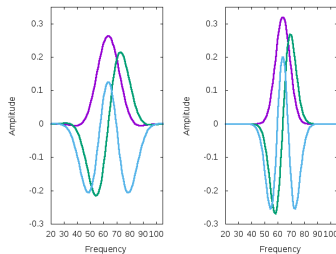


(a) Signal (red) and Idler  
(blue)

# Non-separable JSAs

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(a) Signal (red) and Idler  
(blue)

# Types of Gaussian transformations

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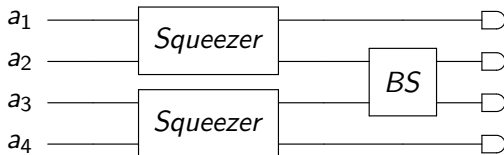


Figure: Two source HOM dip

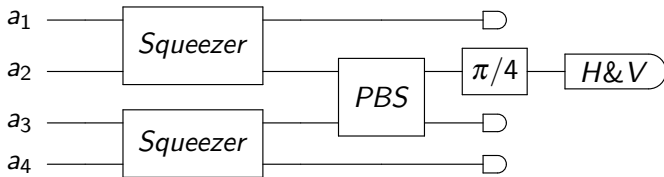
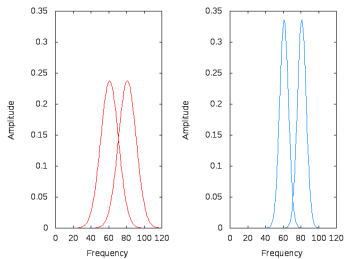


Figure: Type-1 Fusion gate

# Two squeezers JSA

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# G(4) correlation function

$$G^{(4)} = \frac{\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_1 \hat{a}_2 \hat{a}_3 \hat{a}_4 \rangle}{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle \langle \hat{a}_2^\dagger \hat{a}_2 \rangle \langle \hat{a}_3^\dagger \hat{a}_3 \rangle \langle \hat{a}_4^\dagger \hat{a}_4 \rangle} \quad (9)$$

# Outlook

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- There is much to do