UNIVERSITY OF BRISTOL, QUANTUM ENGINEERING CDT

Quantum Optics: Assignment 2

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PROBLEM 1: SPONTANEOUS EMISSION

1.1

Consider a two-level atom initially prepared in the excited state. Semi-classically, this is an eigenstate of the system (since there is no electromagnetic field to interact with the atom) and so the atom will remain in the excited state for all time. We will later learn that in the full quantum mechanical description of this problem, the electromagnetic vacuum field causes the atom to transition to the ground state. Recall that in lecture, we learned about two possible behaviours: Rabi oscillation and exponential decay. Which of these behaviours do you expect the atom to exhibit and why? Please justify with energy level diagrams. Finally, state an expression for the probability to find the atom in the excited state as a function of time (for short times only).

PROBLEM 2: THE QUANTUM WATCHED POT (SOMETIMES) NEVER BOILS

2.1

There is a set of well known philosophical paradoxes (called Zeno's paradoxes) that can be summarized by the rather simplified "a watched pot never boils". Stated more eloquently:

> If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless.
>
> —as recounted by Aristotle, Physics VI:9, 239b5

Now, consider the atom from problem 1 and the expression you derived for finding the atom in the excited state. Imagine, in time T, performing a measurement of which state the atom is in L times. Calculate the probability that the atom is still in the excited state at the end of the time T. Does this differ from the case if you had just left the atom alone?

2.2

Now, consider an atom making a transition to the ground state due to the presence of an electromagnetic field. We know from lecture that at small times the probability of being in the excited state goes something like

$$P_e = 1 - \alpha T^2 \tag{2.1}$$

Again, imagine measuring the state of the atom L times in time T and calculate the probability of remaining in the excited state. What happens as $L \to \infty$?

2.3

This effect of measurement-induced halting of evolution is called the Quantum Zeno Effect. The case of a quadratic Hamiltonian was demonstrated quite some time ago (1989, Wineland). The case of a halting the evolution of an *unstable* quantum system was not observed until 2001 (Raizan), due to the difficulty of engineering an unstable system that was governed by a quadratic evolution.

PROBLEM 3: TWO PHOTON ABSORPTION

Consider a three level atom, with energy levels E_1, E_2, and E_3. Each of the levels are coupled by a sinusoidal perturbation, that is gradually turned on, given by $H_I = \hat{W} e^{\epsilon t} \cos \omega t$, where $\omega = (E_2 - E_1)/\hbar + \delta = (E_3 - E_2)/\hbar - \delta$, and $\langle i|\hat{W}|j\rangle = W$ for $i \neq j$. ϵ is the time it takes for the interaction to turn on, and should be very small, such that $T\epsilon \sim 1$ (ϵ multiplied by the final time is near unity). δ is called the "detuning".

3.1

Draw an energy level diagram, indicating the energy levels, $\hbar\omega$, and the detuning.

3.2

Calculate the first order transition probability from state 1 to state 3, as a function of time T and starting at time $t_0 = -\infty$. Evaluate this for realistic values of ω_{12}, ω_{32}, and the detuning.

3.3

Calculate the second order transition probability from state 1 to state 3, as a function of time T and starting at time $t_0 = -\infty$. Again, evaluate this for realistic values of ω_{12}, ω_{32}, and the detuning.