Modelling Nonlinear optics with the Bloch-Messiah reduction

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August 23, 2018

Overview

- What is nonlinear optics?
- Why do we care about it?
- Gaussian optics
- What I have been doing
- Outlook

Motivation quantum nonlinear optics

The good

Spontaneous Parametric processes, SPDC, SFWM

- Heralded single photon sources
- Entangled photon pair generation (polarisation, spatial)

Kerr processes

- Self-Phase modulation (SPM), generating Bannana states (CV)
- Cross-Phase modulation (XPM) for sensing

The bad

Spontaneous parametric processes

- Generating more than two photons -> bad for quantum computing
- Understanding filtering

All Kerr nonlinear processes

- SPM -> Spectral broadening
- XPM -> Unwanted phase shifts on single photons due to propagation of the pump

What do we mean by nonlinear optics?

- Roughly processes that conserve energy but do not conserve photon number.

$$\vec{P} = \chi^{(1)}\vec{E}_1 + \chi^{(2)}\vec{E}_1\vec{E}_2 + \chi^{(3)}\vec{E}_1\vec{E}_2\vec{E}_3 + \dots \tag{1}$$

Here we are going to talk about squeezing, i.e SPDC or SFWM, Hamiltonians are then of the form,

$$\hat{H} = A\hat{a}_S^\dagger\hat{a}_I^\dagger\hat{a}_P + h.c. \tag{2}$$

$$\hat{H} = A\hat{a}_S^\dagger\hat{a}_I^\dagger\hat{a}_P\hat{a}_P + h.c. \tag{3}$$

Gaussian Optics

- Using the undepleted pump approximation we can write the Hamiltonians as terms which are at most quadratic in creation and annihilation operators.

$$\hat{U} = \exp\left[-\frac{i}{\hbar}\left(\underbrace{P}_{\text{Power}} \int d\omega_1 \int d\omega_2 \; \underbrace{f(\omega_1,\omega_2)}_{\text{JSA}} \;\; \underbrace{\hat{a}_s^\dagger(\omega_1)\hat{a}_i^\dagger(\omega_2)}_{\text{Signal \& Idler}} + h.c.\right)\right] \tag{4}$$

- These are Gaussian transforms, they take Gaussian states to Gaussian states [1]

$$\begin{bmatrix} \vec{b} \\ \vec{b}^\dagger \end{bmatrix} = \mathbf{M} \begin{bmatrix} \vec{a} \\ \vec{a}^\dagger \end{bmatrix} \tag{5}$$

[1] These are linear symplectic transforms which conviently can be written as a matrix

Types of Gaussian transformations

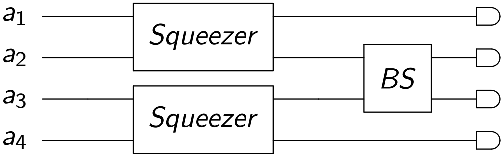


Figure: Two source HOM dip

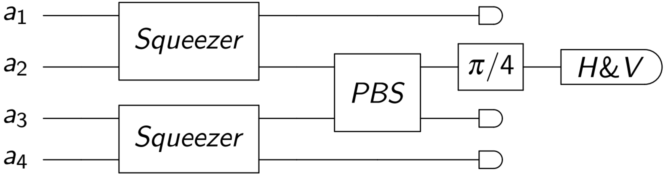


Figure: Type-1 Fusion gate

Schmidt decomposition

We can re-write the Hamiltonian using a Schmidt-decomposition as,

$$P' F(\omega_1, \omega_2) = \sum_k r_k \psi_k(\omega_1)\phi_k(\omega_2) \tag{6}$$

Where r_k is the Schmidt number, ψ & ϕ are unitaries.

To solve this numerically we discretize the function and the Schmidt-decomposition is then the Singular value decomposition (SVD) of the JSA (F).

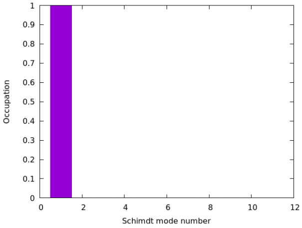
$$P'\mathbf{F}_{(\omega_1,\omega_2)} = \sum_k r_k \mathbf{U}_{(\omega_1,k)} \mathbf{V}^{\dagger}_{(k,\omega_2)} \tag{7}$$

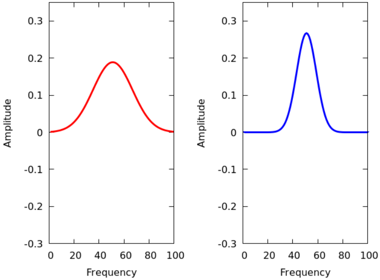
- with $\psi_k(\omega_1)$ is the k-th row and ω_1-th column of $\mathbf{U}_{(\omega_1,k)}$,
- with $\phi_k(\omega_2)$ is the ω_2-th row and k-th column of $\mathbf{V}^{\dagger}_{(k,\omega_2)}$

Joint Spectral Amplitudes (JSAs)

[1] Moving to the rotating frame...

Seperable JSAs Schmidt modes

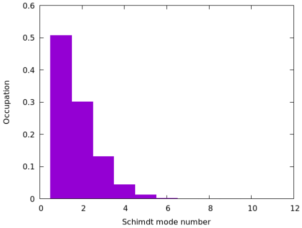


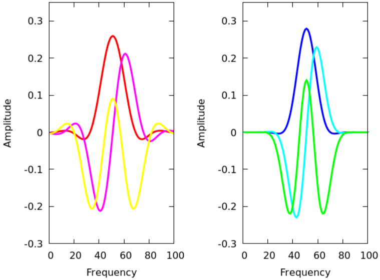


(a) Signal (red) and Idler (blue)

Non-separable JSAs

Non-separable JSAs Schimdt modes





(a) Signal (red) and Idler (blue)

Reducing the size of the state-space

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- The Schmidt decomposition lets us represent the system in a finite number of broadband modes $(\psi_k(\omega_1), \phi_k(\omega_2))$
-

Correlations in a HOM dip

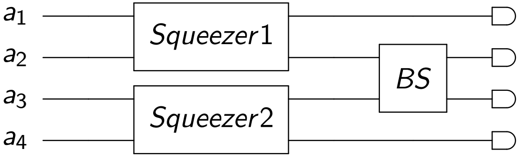
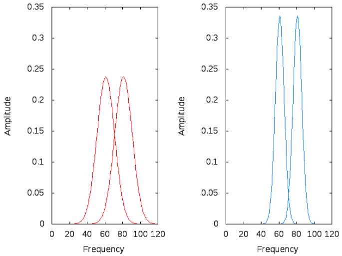


Figure: Two source HOM dip

Two squeezers JSA



G(4) correlation function

$$G^{(4)} = \frac{\left\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_1 \hat{a}_2 \hat{a}_3 \hat{a}_4 \right\rangle}{\left\langle \hat{a}_1^\dagger \hat{a}_1 \right\rangle \left\langle \hat{a}_2^\dagger \hat{a}_2 \right\rangle \left\langle \hat{a}_3^\dagger \hat{a}_3 \right\rangle \left\langle \hat{a}_4^\dagger \hat{a}_4 \right\rangle} \tag{8}$$

Where,

$$a_i = \sum_j a_i(\omega_j) \tag{9}$$

Meaning we sum over all of the spectral modes of the spatial modes (1,2,3,4) separately. We end up with,

$$G^{(4)} = 1 - \left(\frac{2 \mid cosh(r) \mid^2}{\mid cosh(r) \mid^2 + \mid sinh(r) \mid^2} sin(\theta) cos(\theta) \right)^2 \tag{10}$$

G(4) correlation function

Summary

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