University of Colorado - Boulder

ASEN 2802: Aerospace Sciences Lab I

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Lab 2 Submission 2: Wing Loading Lab

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I. Milestone 3

The process for determining the lift coefficient as a function of the spanwise location involves multiple integrations across the span of the wing. Using a method similar to the one employed in Milestone 2 (III.F) to calculate the coefficient of pressure across the different sensors along the chord of the wing, we can evaluate the coefficient of pressure at each of the 11 spanwise locations at an angle of attack of 10 degrees.

Once the coefficients of pressure are determined for each spanwise port, the axial and normal forces experienced by the wing at these locations can be calculated. These forces are then used to compute the coefficients of lift and drag at each spanwise position. The resulting graph (Fig. 1) illustrates the variation of the coefficient of lift along the span of the wing.

The coefficient of lift versus spanwise location provides insight into the distribution of lift generation along the wing. A higher coefficient of lift indicates greater lift generation at that particular spanwise position. As shown in (Fig. 1), the maximum lift is generated at span location 3, which is situated exactly 3 inches from the point where the wing is mounted to the wind tunnel wall.

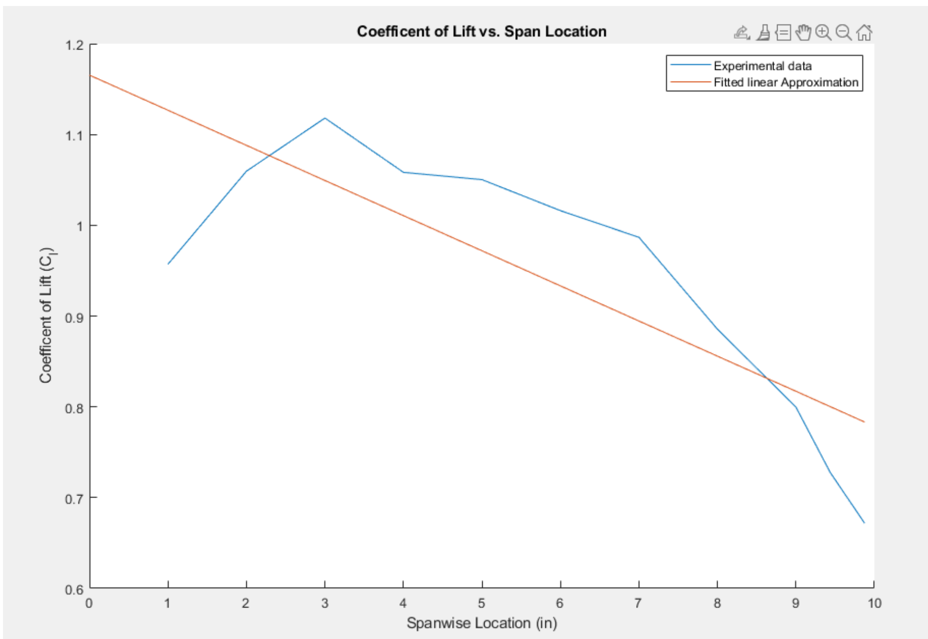


Fig. 1 This graph shows sectional lift coefficient against the spanwise position

Expression for the deflection due to a triangular loading (derivation in III.D):

$$v(x) = \frac{1}{EI}\left(-\frac{\omega_0}{120L}x^5 + \frac{\omega_0}{24}x^4 - \frac{\omega_0 L}{12}x^3 + \frac{\omega_0 L^2}{12}x^2\right) \tag{1}$$

Expression for the deflection due to trapezoidal loading (derivation in III.D):

$$v(x) = \frac{1}{EI}\left(-\frac{(\omega_0 - \omega_1)}{120L}x^5 + \frac{\omega_0}{24}x^4 - \frac{(\omega_0 + \omega_1)L}{12}x^3 + \frac{(\omega_0 + 2\omega_1)L^2}{12}x^2\right) \tag{2}$$

We determined $\omega(x)$ to most closely follow a trapezoidal loading, as the force is not uniform across the span of the wing so it can't be rectangular, and it doesn't reduce to zero at the end of the wing, so it can't be triangular. The loading starts at higher force and ends at a lower force, and while in reality it's not linear, for our purposes we can treat it as such to give us a trapezoidal loading.

To approximate $\omega(x)$, we used the sectional coefficient of lift calculated by the aerodynamics team at the base and the tip of the wing, and multiplied that by the the sectional dynamic pressure (also determined by the aerodynamics team) and the chord of the wing. From there, we plugged the ω values that resulted from these calculations into the $\omega(x)$ for at trapezoidal loading, which is

$$\omega(x) = \frac{-(\omega_0 - \omega_1)}{L} x + \omega_0 \tag{3}$$

In the end, with our discretized force function in III.C we got a $\omega(x)$ of 2.1523 pounds, from a ω_0 of 0 .2581 pounds and a ω_1 of 0.1724 pounds. If we use these in our derived deflections equations, we should get that a rectangular loading has the most deflection, as it has the most force along the wing, and more importantly more force acting on the end of wing, where the most deflection occurs in a cantilever beam. Both the trapezoidal and the triangular loadings decrease in force towards the tip of the wing, so there will be less deflection than the rectangular loading. The trapezoidal loading will be closer to the rectangular, with the triangular being the smallest of the deflections.

Sure enough, when coded into Matlab, the deflection comes out to be 0.4481 inches for the rectangular loading, 0.3389 inches for the trapezoidal loading, and 0.1195 inches for the triangular loading, which follows what we expected.

For our whiffletree, we calculated our loading force to act on the beam at the points of 0.83 (point a), 3.61 (point b), 6.17 (point c), and 8.69 inches (point d) from the base of the wing. From there, if we subtract point a from point b, and point c from point d, we get that the middle rods, ideally, should be 2.78 and 2.52 inches long, respectively. The points at which forces act on those beams are calculated through moment equations, and come out to act 1.33 inches from point a on rod ab, and 1.19 inches from point c on rod cd. That allows us to calculate the ideal distance of the bottom rod, at 5.2 inches. The total force on the whiffle tree acts directly in the middle of that bottom rod.

As stated above, the analytical deflection, based on the derived formula for a trapezoidal loading, is .3389 inches (between 2/8 and 3/8 of an inch). In the wind tunnel, our deflection came out to be a about 3/8 of an inch, and our whiffletree defection came out to be about 7/16 of an inch. It makes sense that the wind tunnel came out to be more than the analytical value of deflection because the trapezoidal loading pattern we used is not exact. The wind tunnel has the real-life loading pattern, which is slightly skewed so that the max force isn't at the base of the wing, as we used with a linear loading in our code. As for the whiffletree, there was most likely human error there, and the measurements for where the forces act and what that force is were not perfect due to the materials we had.

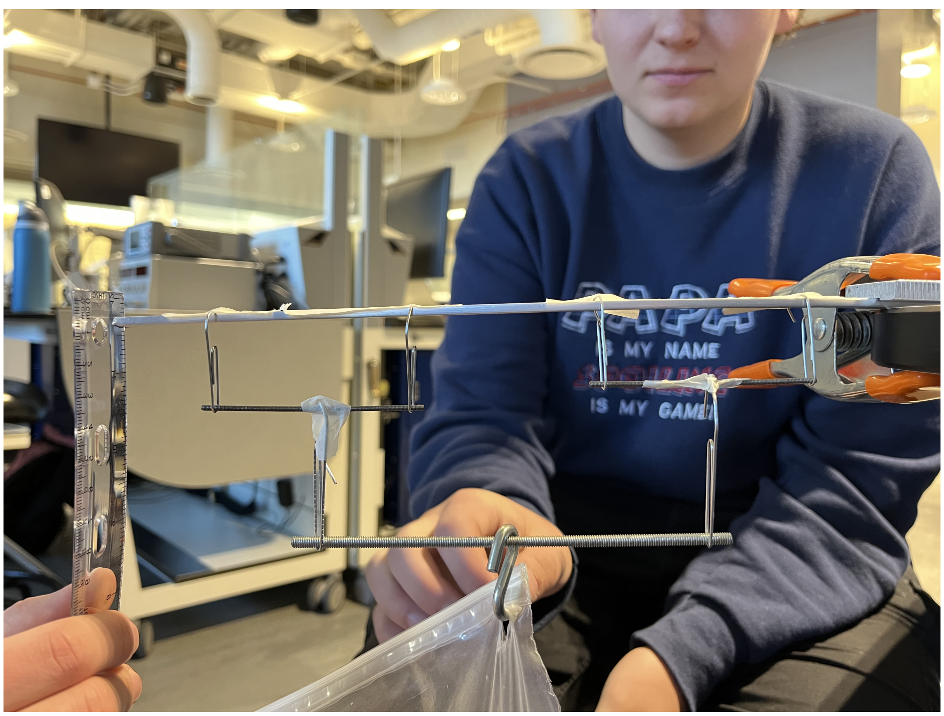


Fig. 2 This picture shows the entire whiffletree design

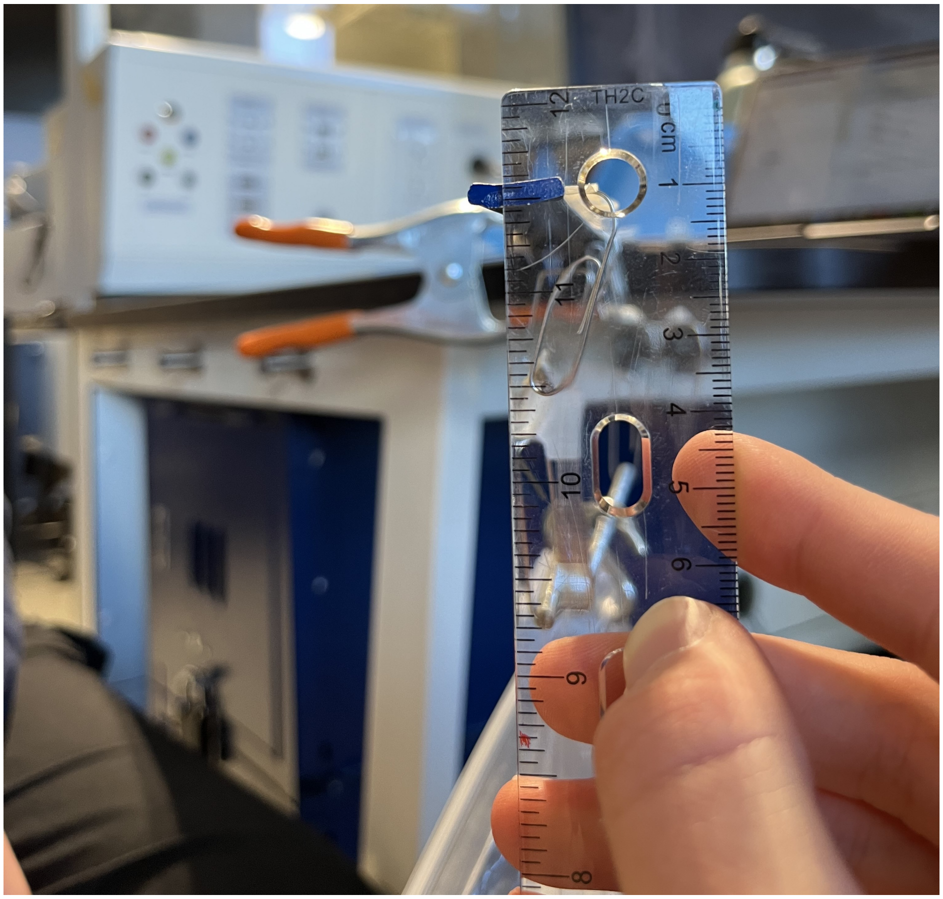


Fig. 3 This picture shows the amount of deflection on the beam

II. Acknowledgments

Our group would like to acknowledge the help of the entire teaching team including the professor, Professor Robert Hodgkinson. Furthermore, we would like thank the University of Colorado Boulder and the Smead Aerospace Engineering Sciences programs for providing phenomenal facilities to further our education.

III. Appendix

A. Contributions

Milestone 3, Aero (lift coefficient vs distance):
Primary - Robert Reynoso, Mark Anifowose, and Daniel Jeung
Milestone 3, Structures:
Primary - Colby Muchlinski, Aja Kimsey, Luke McCord
Milestone 3, Compare and Contrast:
Primary: Aja Kimsey, Colby Muchlinski, Robert Reynoso
Secondary: Daniel Jeung, Mark Anifowose, Luke McCord
Formatting into Overleaf:
Primary - Aja Kimsey, Colby Muchlinski
Secondary - Robert Reynoso

B. MATLAB Code for Sectional Lift Coefficient

```matlab
% Authors: Mark Anifowose, Robert Reynoso, Daniel Jueng
clc;
clear;
close all;

%%
% Load data
data = readmatrix('FullSpanTest.csv');
[spanLocation, I] = unique(data(:,9));
for i = 1 : 11
    temp = data((i-1)*20+1 : (i)*20, 1 : end-1);
    data(i, 1 : end-1) = mean(temp);
end
data = data(1 : 11, :);


port_data = readmatrix('ClarkY14_PortLocations.xlsx');

y = port_data(1:17, 3); % y-coordinates

% Constants
rho = 1.225; % Air density (kg/m^3)
angle_of_attack = 10;
 y_TE = 3.5031; % Y_traling edge

%% preallocate space
Cp_values = zeros(11, 17);
C_n = zeros(11,1);
C_a = zeros(11,1);
C_l = zeros(11,1);
C_d = zeros(11,1);

%calculate everything
for i = 1 : 11
    [Cp_values_temp, chord_norm] = calculateCp(data, port_data, y, rho, y_TE, i);
    Cp_values(i,1:17) = Cp_values_temp;

    z = port_data(1:17, 4); % z - coordinates
    chord_normZ = z/ y_TE;
    chord_normY = y / y_TE;
    % Integrate Cp over the chord to get Cl and Cd
    C_n(i) = -trapz(chord_normY, Cp_values_temp .* cosd(angle_of_attack));
    C_a(i) = trapz(chord_normZ, Cp_values_temp .* sind(angle_of_attack));

    C_l(i) = C_n(i) .* cosd(angle_of_attack) - C_a(i) .* sind(angle_of_attack);
    C_d(i) = C_n(i) .* sind(angle_of_attack) + C_a(i) .* cosd(angle_of_attack);
end

function [Cp_values, chord_norm] = calculateCp(data, port_data, y, rho, y_TE, spanNum) % Extrapolate pressure
     at trailing edge

    port_8 = data(spanNum, 22); % Port 8
    port_9 = data(spanNum, 23); % Port 9
    port_10 = data(spanNum, 24); % Port 10
    port_11 = data(spanNum, 25); % Port 11

    y_coords_upper = [port_data(8, 3), port_data(9, 3)];
    pressure_upper = [port_8, port_9];

    y_coords_lower = [port_data(11, 3), port_data(12, 3)];
    pressure_lower = [port_10, port_11];


    % Linear regression for upper and lower surface extrapolations
    p_upper = polyfit(y_coords_upper, pressure_upper, 1); % Linear fit
    pressure_upper_TE = polyval(p_upper, y_TE);

```

```
    p_lower = polyfit(y_coords_lower, pressure_lower, 1); % Linear fit
    pressure_lower_TE = polyval(p_lower, y_TE);

    % Average extrapolated pressures at TE
    pressure_TE = (pressure_upper_TE + pressure_lower_TE) / 2;

    % Freestream conditions
    freestream_velocity = data(spanNum, 4); % Freestream velocity

    % Calculate dynamic pressure
    dynamic_pressure = 0.5 * rho * freestream_velocity^2;

    % Collect pressure data at each port, including extrapolated trailing edge
    diff_pressure = [data(spanNum, 15:23), pressure_TE, data(spanNum, 24:end - 1)];

    % Calculate Cp values at each port
    Cp_values = (diff_pressure) / dynamic_pressure;

    % Normalize chord length
    chord_norm = y / y_TE;

end


%% plots of coefficents with respect to span location

figure;
hold on;
plot(spanLocation, C_l);
P = polyfit(spanLocation, C_l, 1);
x = linspace(0, 9.875, 100);
yFit = polyval(P, x);
plot(x, yFit);
title("Coefficent of Lift vs. Span Location")
legend("Experimental data", "Fitted linear Approximation")
xlabel("Spanwise Location (in)")
ylabel("Coefficent of Lift (C_l)")


figure;
plot(spanLocation, C_d);
title("Coefficent of Drag vs. Span Location")
xlabel("Spanwise Location (in)")
ylabel("Coefficent of Drag (C_d)")
```

C. MATLAB Code for Structures Whiffletree

```
% Author: Colby Muchlinski
clc;
close all;
clear;
%% Force and Location
[spar,eqn] = getConst(); %Call constant and initial structures
[int.force,int.loc] = trapezoidDis(eqn.func,eqn.length,4); %Call integration function
%% Rods
f.tot = int.force(1,1) + int.force(2,1) + int.force(3,1) + int.force(4,1); %this sums the forces to find the total force
f.r1 = int.force(1,1); %this is the forces for the two beams in the middle of the whiffletree
f.r2 = int.force(2,1);
f.r3 = int.force(3,1);
f.r4 = int.force(4,1);
f.f12 = f.r1 + f.r2; %this is the forces for the beam at the bottome of the whiffletree
f.f13 = f.r3 + f.r4;
rod.a = int.loc(2)-int.loc(1); %Finds the length of the middle beams
rod.b = int.loc(4) - int.loc(3);
```

```matlab
%system finds the distances relative to the whiffle tree beams
matrix.A = [1 1 0 0;...
         0 0 1 1;...
         f.r1 -f.r2 0 0;...
         0 0 f.r3 -f.r4;];
matrix.B = [rod.a;rod.b;0;0];
matrix.x = matrix.A\matrix.B;
dist.a = matrix.x(1,1);
dist.b = matrix.x(2,1);
dist.c = matrix.x(3,1);
dist.d = matrix.x(4,1);
dist.x = spar.length;
rod.c = (int.loc(3) + dist.c) - (int.loc(1) + dist.a); %finds the length of the bottom rod
f.sumf = f.r1 + f.r2 + f.r3 + f.r4;
%% Deflection Equations
recDef = (eqn.w0*(dist.x^2)*(6*(spar.length^2) - 4*spar.length*dist.x + dist.x^2))/(24*spar.elastic*spar.moI);
triDef = (1/(spar.elastic*spar.moI))*(((eqn.w0/24)*dist.x^4) - (eqn.w0/(120*spar.length))*(dist.x^5)-...
      (((eqn.w0)*spar.length)/(12))*(dist.x^3) + (((eqn.w0)*spar.length^2)/(12))*(dist.x^2));
trapDef = (1/(spar.elastic*spar.moI))*(((eqn.w0/24)*dist.x^4) - ((eqn.w0 - eqn.w1)/(120*spar.length))*(dist.x^5)
    -...
      (((eqn.w0 + eqn.w1)*spar.length)/(12))*(dist.x^3) + (((eqn.w0 + 2*eqn.w1)*spar.length^2)/(12))*(dist.x^2))
      ;
%% Functions
%integration function
function [force,location] = trapezoidDis(func,length,nSections)
dx = length/nSections;
force = zeros(nSections,1);
location = zeros(nSections,1);
row = 1;
for i = 0:dx:length-1
area = ((func(i)+func(i+dx))/2) * dx;
centroid = (((i+dx) + (2*i))/(3*(i+(i+dx)))) * dx;
force(row,1) = area;
location(row,1) = centroid+i;
row = row +1;
end
end
%constants function
function [spar,eqn] = getConst()
spar.width = .5;
spar.length = 10;
spar.height = .12;
spar.elastic = 1e7;%psi
spar.moI = (spar.width * spar.height^3)/12;
spar.sa = spar.length * spar.width;
eqn.chord = 3.5;
eqn.length = 10;
eqn.cl0 = 1.165;
eqn.clL = .7832;
eqn.conv = 1/6895; %Pa to PSI
eqn.q0 = 436.46 *eqn.conv; %Pa
eqn.qL = 433.52 *eqn.conv; %Pa
eqn.conv = 1/6895;
eqn.w0 = eqn.cl0*eqn.q0*eqn.chord; %put in the load value
%{
for rectangle: w0 = w1
for trapezoid: W0 > w1
for triangle: w0 is a values, w1 = 0
%}
eqn.w1 = eqn.clL*eqn.qL*eqn.chord;
eqn.func = @(x) (-(eqn.w0 - eqn.w1) / (eqn.length)) * x + eqn.w0; %area function
end
```

D. Deflection Derivation

The following is the full derivation for the triangular loading:

Triangular Loading

$$\omega(x) = -\frac{\omega_0}{L}x + \omega_0$$

$$V(x) = \int \omega(x)\,\mathrm{d}x = \int -\frac{\omega_0}{L}x + \omega_0\,\mathrm{d}x = -\frac{\omega_0}{2L}x^2 + \omega_0 x + C$$

$$V(L) = 0$$

$$0 = -\frac{\omega_0}{2L}L^2 + \omega_0 L + C$$

$$C = -\frac{\omega_0 L}{2}$$

$$V(x) = -\frac{\omega_0}{2L}x^2 + \omega_0 x - \frac{\omega_0 L}{2}$$

$$M(x) = \int V(x)\,\mathrm{d}x = \int -\frac{\omega_0}{2L}x^2 + \omega_0 x - \frac{\omega_0 L}{2}\mathrm{d}x$$

$$= -\frac{\omega_0}{6L}x^3 + \frac{\omega_0}{2}x^2 - \frac{\omega_0 L}{2}x + C$$

$$M(L) = 0$$

$$0 = -\frac{\omega_0}{6L}L^3 + \frac{\omega_0}{2}L^2 - \frac{\omega_0 L}{2}L + C$$

$$C = \frac{\omega_0 L^2}{6}$$

$$M(x) = -\frac{\omega_0}{6L}x^3 + \frac{\omega_0}{2}x^2 - \frac{\omega_0 L}{2}x + \frac{\omega_0 L^2}{6}$$

$$\theta(x) = \frac{\int M(x)dx}{EI} = \frac{1}{EI}\int -\frac{\omega_0}{6L}x^3 + \frac{\omega_0}{2}x^2 - \frac{\omega_0 L}{2}x + \frac{\omega_0 L^2}{6}\mathrm{d}x$$

$$= \frac{1}{EI}\left(-\frac{\omega_0}{24L}x^4 + \frac{\omega_0}{6}x^3 - \frac{\omega_0 L}{4}x^2 + \frac{\omega_0 L^2}{6}x + C\right)$$

$$\theta(0) = 0$$

$$C = 0$$

$$\theta(x) = \frac{1}{EI}\left(-\frac{\omega_0}{24L}x^4 + \frac{\omega_0}{6}x^3 - \frac{\omega_0 L}{4}x^2 + \frac{\omega_0 L^2}{6}x\right)$$

$$\mathrm{v}(x) = \int \theta(x)\,\mathrm{d}x = \frac{1}{EI}\int -\frac{\omega_0}{24L}x^4 + \frac{\omega_0}{6}x^3 - \frac{\omega_0 L}{4}x^2 + \frac{\omega_0 L^2}{6}x\,\mathrm{d}x$$

$$= \frac{1}{EI}\left(-\frac{\omega_0}{120L}x^5 + \frac{\omega_0}{24}x^4 - \frac{\omega_0 L}{12}x^3 + \frac{\omega_0 L^2}{12}x^2 + C\right)$$

$$v(0) = 0$$

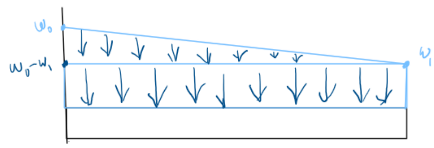
$$C = 0$$

$$\boxed{\mathrm{v}(x) = \frac{1}{EI}\left(-\frac{\omega_0}{120L}x^5 + \frac{\omega_0}{24}x^4 - \frac{\omega_0 L}{12}x^3 + \frac{\omega_0 L^2}{12}x^2\right)}$$

The following is the full derivation for the trapezoidal loading:

Trapezoidal Loading (using superposition)

Deflection from Rectangular Loading (from Milestone 2) : $\nu(x) = \frac{1}{EI}\left(\frac{\omega_0}{24}x^4 - \frac{\omega_0 L}{6}x^3 + \frac{\omega_0 L^2}{4}x^2\right)$



Deflection from Trapezoidal Loading = (Deflection from Rectangular Loading with ω(x) = ω1) + (Deflection from Triangular Loading with ω0 = (ω0 - ω1))

$$\nu(x) = \frac{1}{EI}\left(\frac{\omega_1}{24}x^4 - \frac{\omega_1 L}{6}x^3 + \frac{\omega_1 L^2}{4}x^2\right) + \frac{1}{EI}\left(-\frac{(\omega_0 - \omega_1)}{120L}x^5 + \frac{(\omega_0 - \omega_1)}{24}x^4 - \frac{(\omega_0 - \omega_1)L}{12}x^3 + \frac{(\omega_0 - \omega_1)L^2}{12}x^2\right)$$

$$\nu(x) = \frac{1}{EI}\left(-\frac{(\omega_0 - \omega_1)}{120L}x^5 + \frac{(\omega_0 - \omega_1)}{24}x^4 + \frac{\omega_1}{24}x^4 - \frac{(\omega_0 - \omega_1)L}{12}x^3 - \frac{\omega_1 L}{6}x^3 + \frac{(\omega_0 - \omega_1)L^2}{12}x^2 + \frac{\omega_1 L^2}{4}x^2\right)$$

$$\nu(x) = \frac{1}{EI}\left(-\frac{(\omega_0 - \omega_1)}{120L}x^5 + \frac{\omega_0}{24}x^4 - \frac{(\omega_0 L - \omega_1 L)}{12}x^3 - \frac{2\omega_1 L}{12}x^3 + \frac{(\omega_0 L^2 - \omega_1 L^2)}{12}x^2 + \frac{3\omega_1 L^2}{12}x^2\right)$$

$$\boxed{\nu(x) = \frac{1}{EI}\left(-\frac{(\omega_0 - \omega_1)}{120L}x^5 + \frac{\omega_0}{24}x^4 - \frac{(\omega_0 + \omega_1)L}{12}x^3 + \frac{(\omega_0 + 2\omega_1)L^2}{12}x^2\right)}$$

E. Milestone 1

The calculated velocities presented in Fig. 4 were derived from sensor data collected during a desktop wind tunnel experiment conducted in class. In this experiment, a pitot-static probe was used to measure the dynamic pressure within the test section. The recorded dynamic pressure data was imported into MATLAB, where the velocity equation outlined in Equation (1) from the Milestone1.pdf was applied.

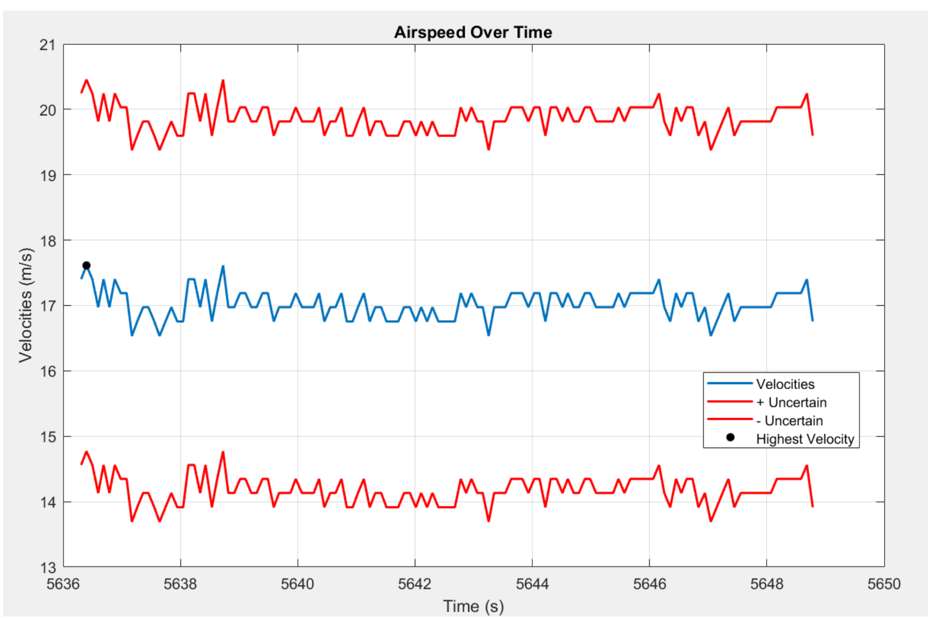


Fig. 4 This graph shows the velocity estimate and uncertainty of the desktop wind tunnel

It is important to note that the pitot-static probe provided data exclusively on dynamic pressure. By utilizing the second equality in Equation (1) from the Milestone1.pdf, the airflow velocities at each time point captured by the sensor were computed. The analysis determined that the maximum airflow velocity reached 17.61 m/s.

However, this calculation comes with uncertainties due to the limitations of the pitot-static probe. To account for these uncertainties, the general uncertainty propagation method was employed, based on the third equality in Equation (1) from the Milestone1.pdf. This approach was chosen because it incorporates the primary sources of measurement uncertainty: dynamic pressure, air temperature, and static (atmospheric) pressure in the room during the experiment.

By including these uncertainties, the maximum velocity of 17.61 m/s was estimated with an associated uncertainty of ±2.84 m/s. This uncertainty range reflects the possible variation in the measured airflow velocity due to the combined effects of sensor and environmental factors.

Table for Forces and Locations

	Rectangular (ω_1 = 10 lb, ω_2 = 10 lb)	Triangular (ω_1 = 10 lb, ω_2 = 0 lb)
Force 1	25 lb	21.88 lb
Force 2	25 lb	15.63 lb
Force 3	25 lb	9.38 lb
Force 4	25 lb	3.13 lb
x_1	0.83 in	0.83 in
x_2	3.61 in	3.61 in
x_3	6.17 in	6.17 in
x_4	8.69 in	8.69 in

Fig. 5 This table shows the discretized forces and the location of each force on the 10-in spar beam, for both rectangular and triangular loadings

Each load (rectangular, triangular, trapezoidal) was discretized using the same function. A function for the force load in the form of $\frac{-(a-b)}{L}x + a$ where $a = \omega_1$ and $b = \omega_2$ in the code from Submission 1, representing the maximum and minimum sides of the force loading, respectively, and L is the length of the spar. For a rectangular load, $a = b$; for a triangular load, a is the upper force value, and $b = 0$; and for a trapezoidal load, a is the upper force value and b is the lower force value, both non-zero values. From this function, the trapezoidal method of integration was used to calculate the force in each discretized section and determine the centroid of each discretized trapezoid to locate where that force is applied. The whiffletree design was derived from these force locations, with the two middle rods (labeled as A and B in Submission 1) representing the distance from the location where Force 1 acts to the location where Force 2 acts, and the distance from the location where Force 3 acts to the location where Force 4 acts, respectively. Using the calculated lengths and forces, a system of equations was set up to identify where the forces act on rods A and B (arranged as a matrix in Submission 1), with the distance between these points determining the length of the bottom rod, designated as rod C. To verify the accuracy of the code, sample values from the slide deck were utilized to ensure that the computed distance and location values matched the example rod lengths.

F. Milestone 2

This was done based on numerical data analysis, aerodynamic theory, and visualization in order to get the results under observation. Firstly, we imported the experimental pressure data and airfoil geometry from the supplied datasets. From the pressure measurements over the surface ports and their chordwise location, the coefficient of pressure (c_p) at each port was calculated at different angles of attack. To handle missing trailing-edge pressure data, linear extrapolations were carried out using the nearest upstream pressures on both the upper and lower surfaces.We normalized chordwise locations and plotted c_p versus (y/c) at different angles of attack to observe how pressure distribution changes under various aerodynamic conditions. Then, we integrated the c_p distributions to get the normal and axial force coefficients that we converted into lift and drag, coefficients at each angle of attack. Results Comparisons are made with benchmark NACA data for that airfoil. We combined the NACA data in one plot showing the variation of lift, and drag with angle of attack, and superimposed our infinite wing results onto those. The coefficient of lift relative to the angle of attack for our infinite wing, at a velocity of 30 m/s, begins to decline after reaching an angle of attack of 10 degrees indicating a possible flow separation. In contrast, the NACA data, which was measured at a velocity of 24.45 m/s and for angles of attack ranging from -8 to 28 degrees, indicates that lift can be maintained up to an angle of 20 degrees before it starts to decrease. This difference is understandable, as pressure is inversely proportional to velocity in a low-speed wind tunnel.

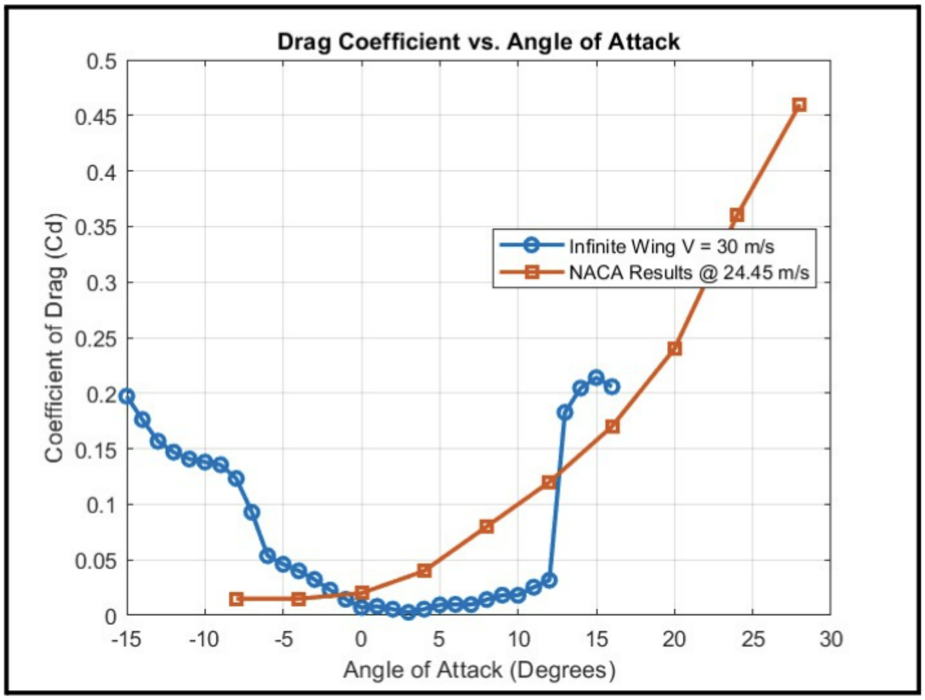


Fig. 6 This plot shows The drag coefficient as the angle of attack increases.

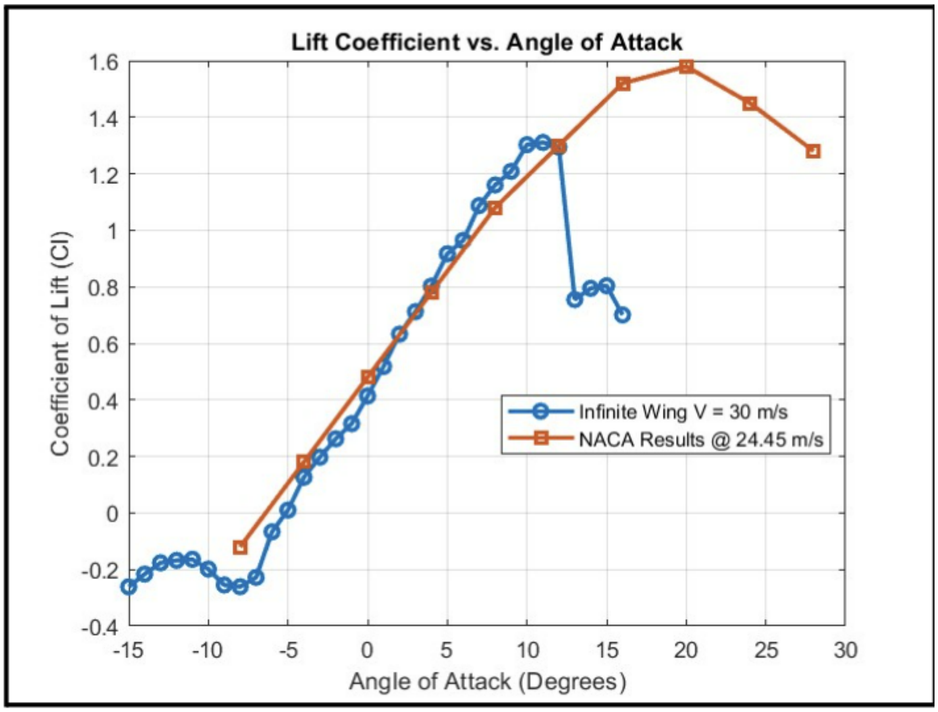


Fig. 7 This plot shows the lift coefficient as the angle of attack increases.

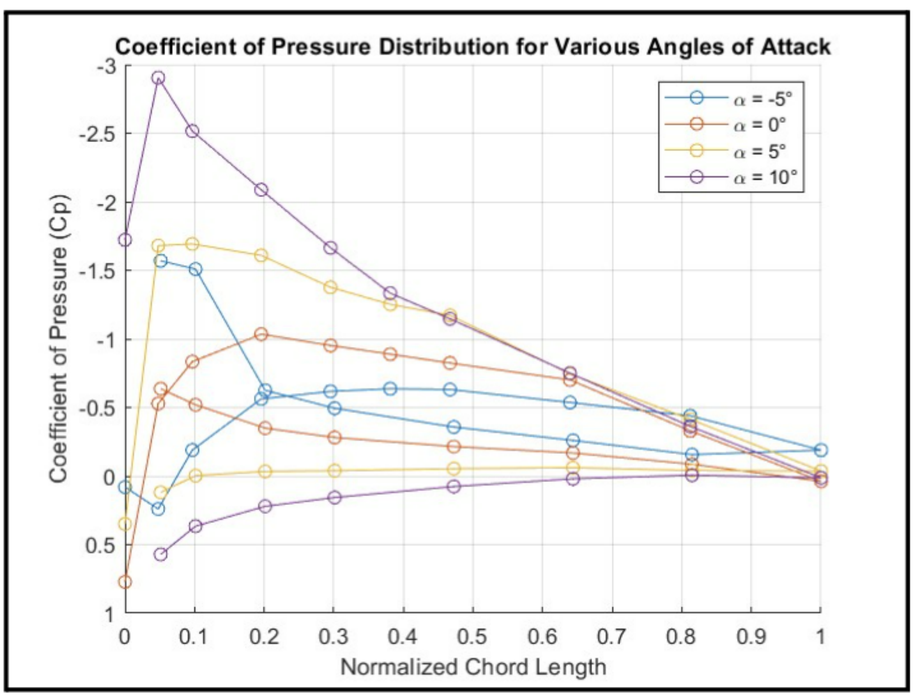


Fig. 8 This plot shows the coefficient of pressure as the angle of attack increases.

The equations used are as follows, in order of internal moment, stress, strain, and deflection:

$$M = \frac{\omega_0}{2}L^2 + \frac{\omega_0}{2}x^2 - \omega_0 L x \tag{4}$$

$$\sigma(x, z) = -\frac{M(x) * z}{I} \tag{5}$$

$$\epsilon(x, z) = \frac{\sigma(x, z)}{E} \tag{6}$$

$$v(x) = \frac{\omega_0 x^2(6L^2 - 4Lx + x^2)}{24EI} \tag{7}$$

The maximum cross-sectional stress, strain, and internal moment is going to occur at the base of the wing, where it meets the body, and the maximum deflection will occur at the wing tip.

The load intensity (ω_0) will come from the sectional coefficient of lift, multiplied by the pressure and the chordwise length of each particular section. We have the sectional coefficient of lift, coming out to be about 1.5636, but we were not able to get the pressure and chordwise length to actually calculate ω_0 yet. We know it is expected to be about 0.34 pounds per inch. Using that expected force value, if we plug in $x = 0$ then we get that the maximum internal moment is 17 pound-inches. From there, if we also plug in $z = .06$ (the max stress occurs at furthest point from the neutral axis, which in this case is in the middle of the 0.12 inch spar, so the furthest point is 0.06 inches away), and the moment of inertia I, found using equation (12) in the Milestone2.pdf, we get that the maximum stress is -14166.7 pounds per square inch, and the strain is -0.001417 inch per inch. In this case, the negative sign just denotes direction into the beam. Finally, the maximum deflection comes out to be about .59 inches.

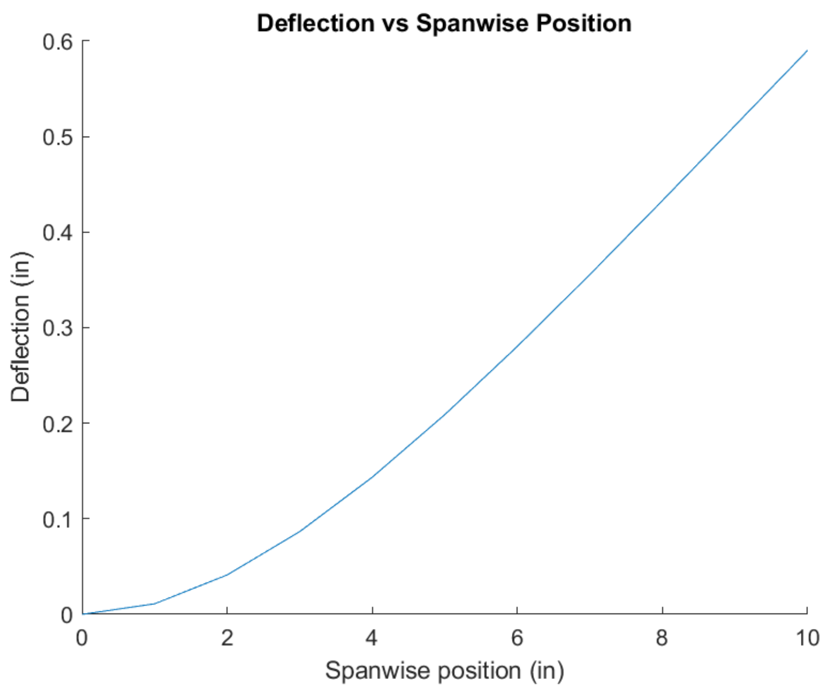


Fig. 9 This plot shows deflection as a function of spanwise position for a rectangular load.

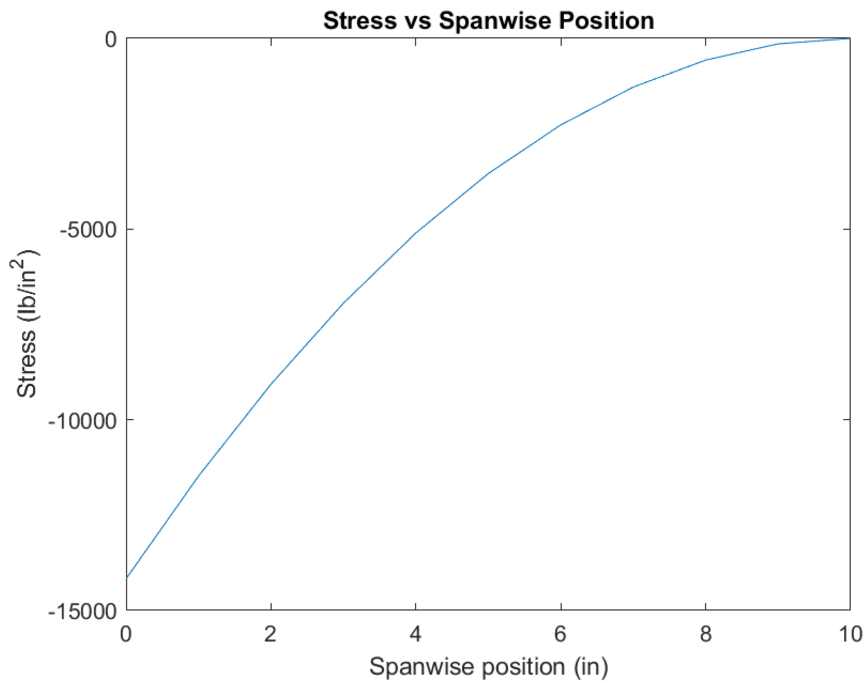


Fig. 10 This plot shows internal stress as a function of spanwise position for a rectangular load.

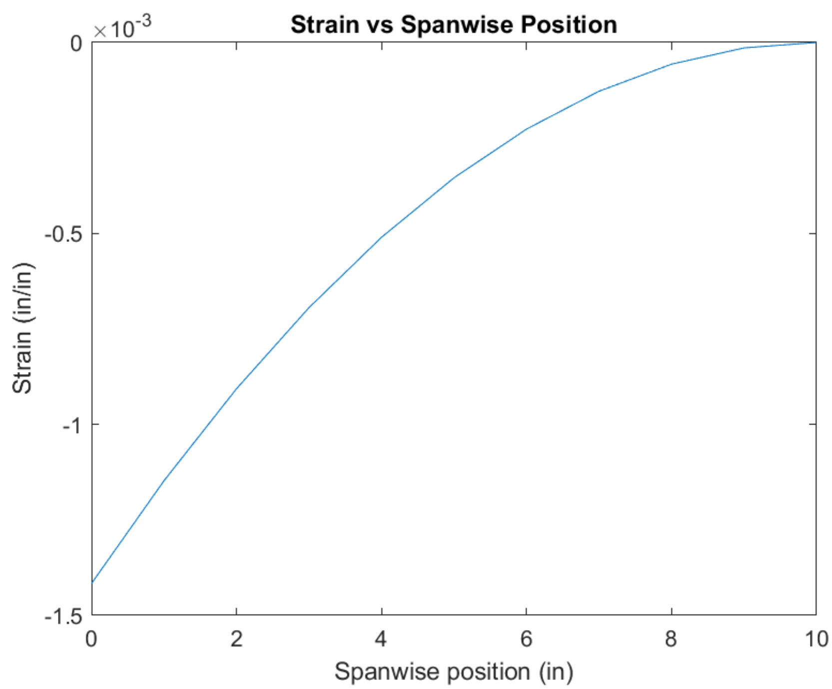


Fig. 11 This plot shows strain as a function of spanwise position for a rectangular load.