

UNIT-3

CHAPTER-1

AVERAGE AND ALLIGATION

BASIC CONCEPT BUILDER

The problems on A-M-A(Average,Mixture and Alligation) can be broadly classified into the following categories.

1. Simple average
2. Weighted average
3. Mixtures
4. Alligations
5. Replacement of a solution with another

SIMPLE AVERAGE

The term Average refers to the sum of all observations divided by the total number of observations. Average is used quite regular in our day to day life. For example to calculate the average marks of the students, Average height of a particular group etc. The term average is also referred to as 'Mean'. Basic formula to calculate the average is as follows

$$\text{Average} = \frac{\text{Sum of Observations}}{\text{Number of Observations}}$$

Example 1. What is the average of First 10 Prime numbers?

Solution: First 10 Prime number are 2,3,5,7,11,13,17,19,23,29.

Hence, Average = $\{2+3+5+7+11+13+17+19+23+29\} / 10$

$$= 129 / 10$$

$$= 12.90$$

From the above we can make the following important observation regarding average.

- (1) Average always lies between the least and greatest of given set of observations.
- (2) If each variable in a group increases by k, average also increases by k.
- (3) The same can be observed/extended for subtraction, multiplication as well as division.

Also the following are some important deductions we can have for finding the average depending on the number of observations.

(1) If the number of observations are evenly spread (i.e., they are in arithmetic progression), hence the average, is the middle observation, if the number of observations is odd.

And it is the average of the middle two observations is even.

Take the following two examples to make them clear with the above concepts.

Example 2: Seven persons have 20, 30, 40, 50, 60, 70 and 80 chocolates with them. What is the average number of chocolates with them?

$$\begin{aligned}\text{Sol: Average} &= (20 + 30 + 40 + 50 + 60 + 70 + 80) / 7 \\ &= 350/7 = 50\end{aligned}$$

Using the above point, the middle observation is 50.

Example 3: Four children have 30, 40, 50 and 60 toffees with them. What is the average number of toffees with them?

$$\text{Sol: Average} = (30 + 40 + 50 + 60) / 4 = 45$$

Using the above point, the average is $(40+50)/2 = 45$

2. Deviation Method for Calculation of average

Now take a set of values which are typically cumbersome to deal with.

Example can be 213, 227, 233, 223 and 229.

First calculate the traditional method and then show the application of the deviation method of calculating the average.

In the above case take the arbitrary number to be 220.

Then the average can be found as $\{220 + (-7+7+13+3+9)\}/5 = 225$.

Let us consider another example:

Example 4: Average of 1093, 1103, 1109, 1089 and 1096?

This can be done as

$$(1093 + 1103 + 1109 + 1089 + 1096) / 5$$

Or we observe that the numbers are close to 1100. Let us say, the numbers are

$$1093 = 1100 - 7$$

$$1103 = 1100 + 3$$

$$1109 = 1100 + 9 \quad 1089 = 1100 - 11 \quad 1096 = 1100 - 4 \quad \text{Sum} = 1100 \times 5 - 10$$

$$\text{So average} = (1100 \times 5 - 10) / 5 = (1100/5) - 2 = 1098$$

This can be written as

$$P = P + \frac{\sum(P_i - P)}{n}$$

Here it doesn't really make a difference whether we assume the mean as 1100 or 1095 etc.

This method would come in handy while dealing with weighted averages or in DI.

Consider the following example which is useful for showing both deviations approach and progressions approaches.

The sales of a company in January 2012 was Rs. 348 crores. In February and March the sales of the company were Rs. 364 crores and Rs. 380 crores. It continued the same trend till December of that year. What is the average sales of the company in 2012? (in Rs. crores)

Observing the given data the sales of the company for January, February and March were Rs. 348 crores, Rs. 364 crores and 380 crores, we can say that the sales are following arithmetic progression (i.e., having a constant increase)

Hence the average

$$= 348 + 364 + 380 + \dots + (348 + 11 \times 16) / 12$$

$$= 350 \times 12 + (-2 + 14 + 30 + \dots + 174) / 12$$

$$= 350 + 86 = \text{Rs. 436 crores}$$

A very common variety of questions on averages involves a person leaving or joining a group. Let us take the following examples to understand this

Example 5: The average weight of a group of 15 friends increases by 1 kg, when a person joins the group. Find the weight of the person who joins the group, if the initial average weight of the group is 48 kg.

Sol: Let the weight of the person joining the group be x kg.

$$\text{Given, } (15 \times 48 + x) / 16 = 49$$

$$\Rightarrow x = 16 \times 49 - 15 \times 48 = 784 - 720 = 64$$

Hence the weight of the new person is 64 kg.

Alternate Method:

When the new person joins group as the average weight of the group increases by 1 kg, we can understand that this person is bringing 1 kg additionally not only for himself but even for others. Hence his weight should be $48 + 1(16)$ i.e., 64 kg

Example 6: The average age of a group of 10 students decreased by 1 year when a new boy of age 23 joined the group and an existing boy left. What is the age of the boy who left?

Sol: Let the original average of the group be a and the age of the boy who left be y .

As average = Sum of numbers/number of Numbers

$$\Rightarrow \text{Sum} = \text{Average} \times \text{Nos.}$$

$$\Rightarrow a.10 + 23 - y = (a - 1).10 \Rightarrow y = 33$$

This basic method of forming an equation goes a long way in solving questions based on average.

Alternate solution:

Since the average of 10 boys decreased by 1, the sum of the ages of the 10 boys should have decreased by 10 years. Since subtraction of 23 years happened due to one boy leaving the group, the new boy must add that number of years which makes it effectively +10.

Thus $-23 + \text{new boy} = 10$. So the age of the new boy should be equal to 33 years.

WEIGHTED AVERAGE

In the concept of weighted average we emphasise the concept of “group” instead of “individual” values.

Example 7: In a class there are two sections – A and B. Section A contains 50 students with an average of 30 marks in Maths. Section B contains 75 students with an average of 40 marks in Maths. What is the average mark of the whole class?

Sol: For finding out the average mark of the whole class we will need the sum of marks of the whole class.

The sum of marks of all students in section A is $50 \times 30 = 1500$

The sum of marks of all students in section B is $75 \times 40 = 3000$

Thus total sum of marks for the class is $(1500 + 3000) = 4500$

So the average mark of the class is $4500 / (50 + 75) = 36$

Let us take another example.

Example 8: What is the average mark of a class having three sections - A, B, C with 34, 51 and 85 students respectively if the average marks of the three sections are 55, 65 and 75 respectively?

Sol: Obviously the question can be solved using the basic rule of weighted average.

$$\text{Average Marks} = \frac{34 \times 55 + 51 \times 65 + 85 \times 75}{34 + 51 + 85} = 11560/170 = 68$$

Let us now look at the application of two very important concepts to weighted average.

(i) The number of items can be broken into ratios

$$\text{So } 34 : 51 : 85 = 2 : 3 : 5$$

(ii) The concept of deviation method of calculation can be used in calculating weighted average

⇒ Let us say the WA is 65.

⇒ Deviations are (− 10), (0) and (+ 10)

So the calculation simplifies to

$$\text{Weighted Average} = 65 + \frac{(-10) \cdot 2 + 0 \cdot 3 + 10 \cdot 5}{2 + 3 + 5} = 68$$

Additional Examples

1: Twelve years ago, the average of the ages of the members of a joint family having ten members was 25 years. Four years later a member aged 50 years died and a child was born in the family that year. Four years after that, another member aged 50 years died and another child was born. Find the present average age of the members of the family (in years).

Explanation:

Had there been no alterations, the current average would have been $25 + 12 = 37$.

The first person who goes out takes with him 50 years, Similarly, the second person who goes out takes with him another 50 years.

$$\begin{aligned} \text{Present average} &= 37 - (50 + 50)/10 = 37 - 10 \\ &= 27 \text{ years.} \end{aligned}$$

2: A class has 31 students. If the top scorer's score is not considered, the average score of the remaining students falls by 2 marks. Instead, if the lowest scorer's score is not considered, the average score of the remaining students increases by 1 mark. If the top scorer's score and the lowest scorer's score are in the ratio of 10 : 1, what is the score of the top scorer?

Explanation:

Let the average score be n , the score of the topper $10x$ and the score of the least scorer x .

As we can see $(31n - 10x)/30 = n - 2$

$(31n - x)/30 = n + 1$

Taking the difference of the two equations, we get

$$9x = 90 \text{ or } x = 10$$

The score of the topper is 100.

3. The average of five positive numbers is 213. The average of the first two numbers is 233.5 and the average of last two numbers is 271. What is the third number?

Explanation:

The sum of the five numbers $= 5 \times 213 = 1065$

The sum of the first two numbers $= 2 \times 233.5 = 467$

The sum of the last two numbers $= 542$

Then the sum of the four numbers $= 467 + 542 = 1009$

So, the third number will be $= 1065 - 1009 = 56$. **Ans.**

4. The average age of a woman and her daughter is 16 years. The ratio of their ages is 7:1 respectively. What is the woman's age?

Explanation:

Sum of their ages $= 2 \times 16 = 32$

Let $7x$ and x be their respective ages, then, $8x = 32$ and $x = 4$

So, the age of the woman $= 7x = 7 \times 4 = 28$ years. **Ans.**

5 The average age of 54 girls in a class was calculated as 14 years. It was later realized that the actual age of one of the girls in the class was calculated as 13 years. What is the actual average age of the girls in the class? (Rounded off to two digits after decimal).

Explanation:

The sum of the ages of the 54 girls entered as error $= 54 \times 14 = 756$.

Now, deduct the error i.e. $13 - 10.5 = 2.5$

Then the actual sum of the ages $= 756 - 2.5 = 753.5$

So, the actual average $= 753.5/54 = 13.95$ years. **Ans.**

6. The average age of a man and his son is 40.5 years. The ratio of their ages is 2 : 1 respectively. What is the man's age?

Explanation:

Let the age of the son = x

Then, $2x + 1x$ i.e. $3x = 40.5 \times 2 = 81$

Therefore, $x = 27$ and so the man's age = $2x = 2 \times 27 = 54$ years. **Ans.**

7. The average marks of 65 students in a class was calculated as 150. It was later realized that the marks of one of the students was calculated as 142, whereas his actual average marks were 152. What is the actual average marks of the group of 65 students? (Rounded off to two digits after decimal)

Explanation:

Increase in total marks = $152 - 142 = 10$

Therefore the New average = $150 + 10/65 = 150.15$. **Ans.**

OR

Sum of the total average marks of 65 students = $65 \times 150 = 9750$

Here, add the difference of 10 marks = 9760.

Therefore the New average = $9760/65 = 150.15$. **Ans.**

8. In a class there are 32 boys and 28 girls. The average age of the boys in the class is 14 years and the average age of the girls in the class is 13 years. What is the average age of the whole class? (Rounded off to two digits after decimal)

Explanation:

The sum of the ages of 32 boys = $32 \times 14 = 448$

The sum of the ages of 28 girls = $28 \times 13 = 364$

Therefore, the sum of the ages of the whole class of 60 students = 812

The average age of the whole class of 60 students = $812/60 = 13.53$. **Ans.**

9. The average of 5 consecutive even numbers A, B, C, D and E respectively is 74. What is the product of C and E?

Explanation:

Let the sum of the 5 consecutive even numbers = $x + x + 2 + x + 4 + x + 6 + x + 8 = 5 \times 74 = 370$

$5x + 20 = 370$; $5x = 370 - 20 = 350$

Therefore, $x = 350/5 = 70$, then the nos. A, B, C, D and E are 70, 72, 74, 76 and 78 respectively.

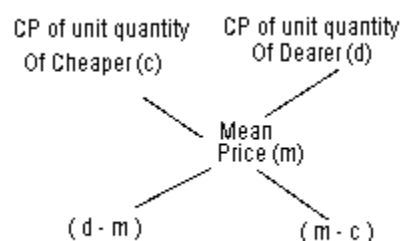
Product of C and E = $74 \times 78 = 5772$. **Ans.**

ALLIGATION –

In this we find the proportion in which ingredients at given prices must be mixed to produce a mixture at a given price, this is termed as alligation. Alligation method can not be applied for absolute values, but it can be applied for Question related to Ratio, Rate, Percentage, Speed etc (Means it can be applied for comparable values like per hour, per km , per kg etc.)

Rule of Alligation : If gradients are mixed in a ratio then we can write

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{CP of dearer} - \text{Mean Price}}{\text{Mean Price} - \text{CP of cheaper}}$$



Example 1: In what proportion must wheat at Price 4.10 per kg must be mixed with wheat at Price 4.60 per kg, so that the mixture be worth Rs 4.30 a Kg ?

Solution : Here, CP of unit quantity of Dearer = 460 per kg , CP of unit quantity of Cheaper = 410 per kg
So we can use above formula :

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{460 - 430}{430 - 410} = \frac{30}{20} = \frac{3}{2}$$

So, the required ratio be 3 : 2

Example 2: How many kg of rice at Rs. 60 per kg, must be mixed with 30 kg of rice at Rs 25 per kg, so that he may on selling the mixture at Rs 50 per kg gain 25 % on the outlay ?

Solution : First we have to find cost price of mixture, as seller is gaining 25 % profit on mixture so its cost price will be

$$50 = \text{CP of mixture} \times \frac{125}{100} \Leftrightarrow \text{CP of mixture} = 40$$

Now use the formula of alligation to find to find out quantity of dearer rice,

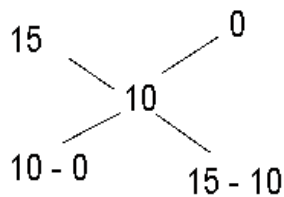
(Note : Here mean price will be CP of mixture, do not get confused by Selling price Rs 50 per kg)

$$\frac{30}{\text{Quantity of dearer}} = \frac{60 - 40}{40 - 25} = \frac{20}{15} = \frac{4}{3}$$

$$\text{Quantity of dearer} = \frac{30 \times 3}{4} = 22.5 \text{ kgs}$$

Example 3: A mixture of certain quantity of milk with 20 liters of water of worth Rs 10 per liter. If pure milk is of worth Rs 15 per liter, how much milk is there in the mixture ?

Solution : By rule of alligation



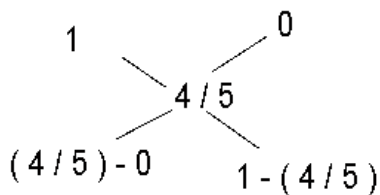
$$\frac{20}{\text{Quantity of Milk}} = \frac{15 - 10}{10 - 0}$$

So, Quantity of milk will be 40 liters

Example 4: In what proportion must water be mixed with milk to gain 20 % by selling it at cost price ?

Solution : Let cost price of milk be Rs 1 per liter, then S.P of mixture is also Rs 1 per liter

Now CP of mixture be = $1 - (20 \% \text{ of Rs } 1) = 1 - (20 * 1 / 100) = 80 / 100 = \text{Rs } 4 / 5$



Required ratio = 1 : 4

Question based on Mixture from two vessels (Note : Frequently asked in MBA entrance exams)

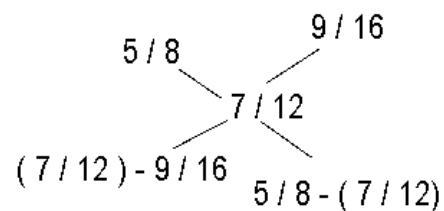
Example 5: Milk and water are mixed in a vessel A in the ratio 5 : 3 and in vessel B in ratio 9 : 7 . In what ratio should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 7 : 5 ?

Solution : In vessel A, milk = $5 / 5 + 3 = 5 / 8$ of the weight of mixture

In vessel B milk = $9 / 9 + 7 = 9 / 16$ of the weight of mixture

Now, we have to form a mixture in which milk be $7 / 12$ of the weight of the mixture

Now according to rule of alligation :



$$\frac{7}{12} - \frac{9}{16} = \frac{1}{48} \quad \frac{5}{8} - \frac{7}{12} = \frac{1}{24}$$

Required Ratio = $1/24 : 1/48 = 2:1$

Example 6: A goldsmith has two qualities of gold, one of 10 carats and another of 15 carates purity. In what proportion should he mix both to make an ornament of 12 carats purity ?

Solution : By applying rule of alligations

$$\begin{array}{ccc}
 10 & & 15 \\
 & \diagdown \quad \diagup & \\
 & 12 & \\
 & \diagup \quad \diagdown & \\
 15 - 12 = 3 & & 12 - 10 = 2
 \end{array}$$

So both qualities of gold should be mixed in the ratio of 3 : 2

Example7: 400 gm spirit solution has 40 % spirit in it , How may grams of spirit should be added to make it 60 % in the solution ?

Solution : By applying rule of alligations and mixtures

$$\begin{array}{ccc}
 40\% & & 100\% \\
 & \diagdown \quad \diagup & \\
 & 60\% & \\
 & \diagup \quad \diagdown & \\
 100 - 60 = 40\% & & 60 - 40 = 20\%
 \end{array}$$

So, the two mixtures should be added in ratio 2 : 1

Concept of Replacement:

Suppose a container contains a solution from which some quantity of solution is taken out and replaced with one of the ingredients. This process is repeated n times then,

Final Amount of ingredient that is not replaced=

$$\text{Initial Amount} \times (\text{Vol. after removal}/\text{Vol. after replacing})^n$$

Above formula is not only true for absolute amounts but for ratios as well. So following formula is also valid:

Final ratio of ingredient not replaced to total=

$$\text{Initial ratio} \times (\text{Vol. after removal}/\text{Vol. after replacing})^n$$

EXERCISE
AVERAGE AND ALLIGATION
CONTENT AS PER IBPS/SSC LEVEL

Q 1. Average of four consecutive odd numbers is 106. What is the third number in ascending order?

- (a) 107 (b) 111 (c) 113 (d) Cannot be determined (e) None of these

Q 2. OF the three numbers, the average of the first and the second is greater than the average of the second and the third by 15. What is the difference between the first and the third of the three numbers?

- (a) 15 (b) 45 (c) 60 (d) Data inadequate (e) None of these

Q 3. The ratio of roses and lilies in a garden is 3:2 respectively. The average number of roses and lilies is 180. What is the number of lilies in the garden?

- (a) 144 (b) 182 (c) 216 (d) 360 (e) None of these

Q 4. The average monthly income of a family of four earning members was Rs.15130. One of the daughters in the family got married and left home, so the average monthly income of the family came down to Rs.14660. What is the monthly income of the married daughter?

- (a) Rs.15350 (b) Rs.12000 (c) Rs.16540 (d) Cannot be determined (e) None of these

Q 5. The average temperature of Monday, Tuesday, Wednesday and Thursday was 36.5°C and for Tuesday, Wednesday, Thursday and Friday was 34.5°C . If the temperature on Monday was 38°C , find the temperature on Friday.

- (a) 34°C (b) 36°C (c) 37.4°C (d) 32°C (e) 30°C

Q 6. The average age of a man and his two sons born on the same day is 30 years. The ratio of the ages of father and one of his sons is 5 : 2 respectively. What is the father's age?

- (a) 50 years (b) 30 years (c) 45 years (d) 20 years (e) None of these

Q 7. A cricketer has an average score of 49 runs in 24 innings. How many runs must he score in the 25th innings to make his average 50?

- (a) 94 (b) 84 (c) 74 (d) 76 (e) None of these

Q 8. The average age of 14 boys in a class is 13 years. If the class teacher's age is included, the average age is increased by one year. What is the class teacher's age?

- (a) 31 years (b) 27.5 years (c) 24 years (d) 28 years (e) None of these

Q 9. The average of four positive integers is 73.5. The highest integer is 108 and the lowest integer is 29. The difference between the remaining two integers is 15. Which of the following is the smaller of the remaining two integers?

- (a) 80 (b) 86 (c) 73 (d) Cannot be determined (e) None of these

Q 10. The average age of a woman and her daughter is 46 years. The ratio of their ages is 15 : 8 respectively. What will be the respective ratio of their ages after 8 years?

- (a) 8 : 5 (b) 10 : 17 (c) 17 : 10 (d) 5 : 8 (e) None of these

Q 11. In a class of 75 students, the average age is 23 years. The average age of male students is 25 years and that of female students is 20 years. Then the ratio of male to female students is

(a) 3:2 (b) 7:3 (c) 8:7 (d) 6:9 (e) 9:6

Q 12. The average age of 3 friends is 32 years. If the age of the fourth friend is added, their average age comes to 31 years. What is the age of the fourth friend?

(a) 32 years (b) 28 years (c) 24 years (d) 26 years (e) None of these

Q 13. Average salary of 19 workers in an industry Rs.2500. The salary of supervisor is Rs.5550. Find the average salary of all the 20 employees.

(a) Rs.3355.5 (b) Rs.4500.00 (c) Rs.4642.5 (d) Rs.2652.5 (e) None of these

Q 14. The average of three even consecutive numbers is 24. What is the summation of the three numbers?

(a) 24 (b) 72 (c) 26 (d) 80 (e) None of these

Q 15. The sum of seven consecutive even numbers of a set is 532. What is the average of first four consecutive even numbers of the same set?

(a) 76 (b) 75 (c) 74 (d) 73 (e) None of these

Q 16. There are 50 boys in a class. One boy weighing 40 kg goes away and at the same time another boy joins the class. If the average weight of the class is thus decreased by 100 g, find the weight of the new boy.

(a) 35kg (b) 43kg (c) 36kg (d) 30 kg (e) None of these

Q 17. Kamlesh bought 65 books for Rs.1050 from one shop and 50 books for Rs.1020 from another. What is the average price he paid per book?

(a) Rs. 36.40 (b) Rs.18.20 (c) Rs. 24 (d) Rs.18 (e) None of these

Q 18. A car covers the first 39 kms. of its journey in 45 minutes and covers the remaining 25 kms. in 35 minutes. What is the average speed of the car?

(a) 40 kms/hr (b) 64 kms./hr (c) 49 kms./hr (d) 48 kms./hr (e) none of these

Q19. The sum of five numbers 260. The average of the first two numbers is 30 and the average of the last two numbers is 70. What is the third number?

(a) 33 (b) 60 (c) 75 (d) Cannot be determined (e) None of these

Q 20. The average weight of 12 boys is 35 kgs. If the weight of an adult is added, the average becomes 37 kgm. What is the weight of the adult?

(a) 65kgs (b) 68 kgs (c) 62 kgs (d) 63 kgs (e) None of these

Q 21. The average marks in Science subject of a class of 20 students is 68. If the marks of two students were misread as 48 and 65 of the actual marks 72 and 61 respectively, then what would be the correct average?

(a) 68.5 (b) 69 (c) 69.5 (d) 70 (e) 66

Q 22. The average speed of a car is 75 kms/hr. What will be the average speed of the car if the driver decreases the average speed of the car by 40 percent?

(a) 50 kms/hr (b) 45 kms/hr (c) 40 kms/hr (d) 55 kms/hr (e) None of these

Q 23. The average marks of a student in seven subjects is 41. After re-evaluation in one subject the marks were changed to 42 from 14 and in remaining subjects the marks remained unchanged. What are the new average marks?

(a) 45 (b) 44 (c) 46 (d) 47 (e) None of these

Q 24. The average marks of nine students in a group is 63. Three of them scored 78, 69 and 48 marks. What are the average marks of remaining six students?

(a) 63.5 (b) 64 (c) 63 (d) 62.5 (e) None of these

Q 25 . Out of the three given numbers, the first number is twice the second and thrice the third. If the average of the three numbers is 154. What is the difference between the first and the third number?

(a) 126 (b) 42 (c) 166 (d) 52 (e) None of these

Q 26 . The average of four positive integers is 124.5. The highest integer is 251 and the lowest integer is 65. The difference between the remaining two integers is 26. Which of the following integers is higher of the remaining two integers?

(a) 78 (b) 102 (c) 100 (d) Cannot be determined (e) None of these

Q 27. The average height of 21 girls was recorded as 148 cms. If the teacher's height was added, the average increased by one. What was the teacher's height?

(a) 156 cms. (b) 168 cms. (c) 170 cms. (d) 162 cms. (e) none of these

Q28. If average of 11 consecutive odd numbers is 17, what is the difference between the smallest and the largest number?

(a) 18 (b) 20 (c) 22 (d) 24 (e) None of these

Q 29. The average of marks obtained in 120 students was 35. If the average of passed candidates was 39 and that of failed candidates is 15, the number of candidates who passed the examination is:

(a) 100 (b) 110 (c) 120 (d) 80

Q 30. The average marks in Hindi subject of a class of 54 students is 76. If the marks of two students are misread as 60 and 77 of the actual marks 36 and 47 respectively, then what would be the correct average?

(a) 75.5 (b) 77 (c) 75 (d) 76.5 (e) None of these

Q 31. The average salary of all the workers in a workshop is Rs.8000. The average salary of 7 technicians is Rs.12000 and the average salary of the rest is Rs.6000. How many workers are there in the workshop?

(a) 21 (b) 22 (c) 23 (d) 24

Q 32. In an examination, a student's average marks were 63. If he had obtained 20 more marks for his Geography and 2 more marks for his history, his average would have been 65. How many subjects were there in the examination?

(a) 12 (b) 11 (c) 13 (d) 14

Q 33 :The average age of a husband and his wife was 23 years at the time of their marriage. After five years they have a one year old child. What is the average age of the family ?