

FACTORIAL

Factorial is an important topic in quantitative aptitude preparation. The factorial of a non negative integer n is denoted as $n!$. The notation was introduced by Christian Kramp in 1808. $n!$ is calculated as the product of all positive integers less than or equal to n .

$$\text{i.e } 6! = 1 * 2 * 3 * 4 * 5 * 6 = 720$$

$$n! = 1 \text{ when } n = 0, \text{ and } n! = (n-1)! * n \text{ if } n > 0$$

$n!$ is the number of ways we can arrange n distinct objects into a sequence.

$2! = 2$ means numbers 1, 2 can be arranged in 2 sequences (1, 2) and (2, 1).

We can arrange 0 in one way. So $0! = 1$, not zero. Now we know why, and no need to say “its like that” if someone asks ;-)

Find the highest power of a prime number in a given factorial

The highest power of prime number p in $n! = [n/p^1] + [n/p^2] + [n/p^3] + [n/p^4] + \dots$ where $[n/p^1]$ denotes the quotient when n is divided by p

Solved Example:

The maximum power of 5 in $60!$

Sol: $60! = 1 \times 2 \times 3 \dots \dots \dots 60$ so every fifth number is a multiple of 5. So there must be $60/5 = 12$

In addition to this 25 and 50 contribute another two 5's. so total number is $12 + 2 = 14$

Short cut: $[60/5] + [60/5^2] = 12 + 2 = 14$

Here $[]$ Indicates greatest integer function.

Solved Example:

How many zero's are there at the end of $100!$

Sol: A zero can be formed by the multiplication of 5 and 2. Since $100!$ contains more 2's than 5's, we can find the maximum power of 5 contained in $100!$

For your understanding:

$$\Rightarrow 100/2 + 100/4 + 100/8 + 100/16 + 100/32 + 100/64 = 50 + 25 + 12 + 6 + 3 + 1 = 97$$

$$\Rightarrow 100/5 + 100/25 = 20 + 4 = 24$$