REMAINDERS:

A number M when divided by N leaves remainder R , and quotient is Q can be represented by M=NQ + R. where M is dividend, N is divisor,Q is quotient and R is remainder. The above rule is what is commonly called as the Division algorithm.

The concepts required to solve the questions of remainders are enumerated below

- Reducing remainders
- Negative remainders
- Fermat's little theorem
- Chinese remainders
- Wilson's rule.

Reducing remainders

Some basic rules are given below:

Remainders (axb)/c= remainder(a/c) x remainder(b/c)
Remainder (a+b)/c= remainder (a/c) + remainder (b/c)
Remainder (a-b)/c=remainder (a/c) –remainder (b/c)

Examples:
a)(142+143+145)/7. what is the remainder.?
(2 + 3+ 5)/7 =remainder is 5
b) (142x143x145)/7 What is the remainder .?
(2 x 3 x 5)/7 =2
c) (142x142x142x-------------------100 times) /7.what is the remainder?
= (2x2x2x………………..100 times)/7.
= (8x8x8……33 times x2)/7
= 2.
Keep on dividing the remainders till the final remainder is less than divisor.

Concept of negative remainder:
Remainder 27/7 =6 or its conjugate -1
Remainder 26/7 =5 or its conjugate -2
What is the remainder $15^{97}/8$?
= (15x15x15x……………97 times)/8
= (-1x-1x-1x………………… 97 times)/8
= -1 or its conjugate7

Fermat's little theorem:
Remainder (M^{N-1})/N =1
Where M and N are coprime and N is a prime number.
Example: $(2^{100})/101 = 1$, $(3^{96})/97 = 1$.
Find the remainder when (5^{1000}) is divided by 77?
$=(5^{1000})/(7x11)$

using fermat's rule 5^6 /7 = remainder is 1 so 5^{30}/7 (remainder) =1
using fermat's rule 5^{10}/11= remainder 1 so 5^{30}/11 (remainder) =1
5^{30}/77 remainder =1
$((5^{30})^{33} \times 5^{10})$ /77 = remainder 23 . where(5^{10}/77) remainder is 23 has to be dealt separately by reducing remainders theory.

Chinese remainders:
Remainder N/(axb) =apr_1+bqr_2: where remainder of N/a = r_2 and N/b = r_1 and ap+ bq =1.
=(5^{1000}) /(7x11)
= remainder (5^{1000})/7 = $((5^6)^{166} \times 5^4)$ /7 = 2 (using fermat's rule 5^6 /7 = mainder is 1.)
= remainder (5^{1000})/ 11 =$(5^{10})^{100}$ /11 =1 (using fermat's rule 5^{10}/11=remainder 1
7p+11q=1 for p= -3,q= 2
So the final remainder is = 7x-3x1 + 11x2x2 =23.

Wilson's rule:
Remainder ((N-1)!+1) when divided by N has a remainder of 0
Example: (4! + 1) / 5 = remainder is 0, (6! + 1) /7 =remainder is 0.
Find the remainder for (96!+1000)/97 :
(96!+1) is divisible by 97
So final remainder is remainder 999/97 =29.

Important result:
Theorem: $a^n + b^n$ is divisible by a + b when n is ODD.
Theorem 2: $a^n – b^n$ is divisible by a + b when n is EVEN.
Theorem 3: $a^n – b^n$ is ALWAYS divisible by a – b.