HCF & LCM

Factors and Multiples:

If number *a* divided another number *b* exactly, we say that *a* is a **factor** of *b*. In this case, *b* is called a **multiple** of *a*.

Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.):

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.There are two methods of finding the H.C.F. of a given set of numbers:

- **Factorization Method:** Express the each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.
- **Division Method:** Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number.

Similarly, the H.C.F. of more than three numbers may be obtained.

Least Common Multiple (L.C.M.):

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

There are two methods of finding the L.C.M. of a given set of numbers:

- **Factorization Method:** Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.

  Example: Find the H.C.F. and LCM of 72, 126 and 270.

  Solution:.Using Prime factorisation
  $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
  $126 = 2 \times 3 \times 3 \times 7 = 2^1 \times 3^2 \times 7^1$
  $270 = 2 \times 3 \times 3 \times 3 \times 5 = 2^1 \times 3^3 \times 5^1$

  H.C.F. of the given numbers = the product of common factors with least index= $2^1 \times 3^2$
  L.C. M of the given numbers = the product of common factors with highest index and

the non-common terms = $2^3 \times 3^3 \times 5^1 \times 7^1$

- **Division Method (short-cut):** Arrange the given numbers in a row in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

Some General rules for LCM/HCF

- **H.C.F. and L.C.M. of Fractions:**

1. $\text{H.C.F.} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$

2. $\text{L.C.M.} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

- **H.C.F. and L.C.M. of Decimal Fractions:**

In a given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

- **Comparison of Fractions:**

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Applications of HCF & LCM

Example: Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

Solution:-
L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

So, the bells will toll together after every 120 seconds (2 minutes).
In 30 minutes, they will toll together 30/2 + 1 = 16 times. One is added as for first time at t = 0 they tolled together.

Example:
Six racers take 2, 4, 6, 8 10 and 12 seconds respectively to run a circular field. In 30 minutes, how many times will they meet together at the point from which they started ?

The Logic is same. They will meet 16 times if we take this fact that at t=0, they started together.

Some important Facts:

1. If a, b and c give remainders p, q and r respectively, when divided by the same number H, then H is HCF of (a-p), (b-q), (c-r).

2. If the HCF of two numbers 'a' and 'b' is H, then, the numbers (a+b) and (a-b) are also divisible by H.

3. If a number N always leaves a remainder R when divided by the numbers a, b and c, then N = LCM (or a multiple of LCM) of a, b and c + R.

4. If a Number when divided by a,b,c leaves a remainders of x,y,z respectively and a-x= b-y= c-z= P, then the smallest number satisfying this condition is L.C.M(a,b,c)- P.

Example: Which is the smallest numbers which leaves a common remainder of 4 when divided by 6, 7, and 9?
Solution: Here you should remember that the smallest number is 4.The next such number will be (LCM of 6,7,9) + 4 i.e. 126+4 or 130