

## UNIT 2

### CHAPTER 3

### SIMPLE & COMPOUND INTEREST

#### BASIC CONCEPT BUILDER

When a person has to borrow some money as a loan from his friends, relatives, bank etc. He promises to return it after a specified time period along with some extra money for using the money of the lender. The money borrowed is called the Principal, usually denoted by P, and the extra money paid is called the Interest, usually denoted by I. The total money paid back, that is, the sum of Principal and the Interest is called the Amount, and is usually denoted by A.

Thus,  $A = P + I$

The interest is mostly expressed as a rate percent per year (per annum). Interest depends on, how much money (P) has been borrowed and the duration of time (T) for which it is used. Interest is calculated according to a mutually agreed rate percent, per annum (R).

[i.e.  $R = r\% = 100$ ]

When the interest is calculated on the Principal for the entire period of loan, the interest is called simple interest.

Thus,  $\text{Interest} = \frac{(\text{Principal}) \times (\text{Rate \% per annum}) \times \text{time}}{100}$

Relationship between principal and amount is:

$$P = \frac{100 \times \text{Amount}}{100 + RT}$$

**Example 1:** What is the interest on Rs.400 for 3 years at the rate of 6% per annum?

Sol: Interest on Rs 100 for 1 year = 6RS

$$\text{Interest on Re 1 for 1 year} = \frac{6}{100}$$

$$\text{Interest on Rs 400 for 1 year} = \frac{400 \times 6}{100}$$

$$\text{Interest on Rs 400 for 3 years} = \frac{400 \times 6 \times 3}{100} = \text{Rs. 72}$$

**Example 2:** What sum of money will yield Rs. 60 as simple interest at 6% per annum in 5 years?

Sol:  $P = \frac{100 \times 60}{6 \times 5} = \text{Rs 200}$

**Example 3:** Find the rate percent annum at which the interest on Rs.600 be Rs. 156 in two years?

Sol:  $R = \frac{100 \times 156}{600 \times 2} = 13\%$

**Example 4:** In how many years will a sum of money double itself at 12% per annum?

Sol:

Suppose principal is Rs. P. As it doubles itself, the simple interest is also equal to Rs.P.

$$\text{Rate} = 12\%, \text{Time} = \frac{100 \times P}{P \times 12} = 8 \text{ year 4 months}$$

Interest for a number of days: In counting the number of days between two given dates the first day is not included.

**Example 5:** The simple interest on a sum of money is  $\frac{1}{16}$  of the principal and the number of years is equal to the rate percent per annum. What is the rate of percent?

Sol: Suppose, principal = P ; Time = n years ; Rate = n%

$$\text{S.I.} = \frac{P}{16} = \frac{P \times n \times n}{100} \text{ or } \frac{P}{16} = \frac{P \times n^2}{100}$$

$$n^2 = \frac{100}{16} = 2\frac{1}{2} \%$$

**Example 6:** A man took a loan from a bank at the rate of 12% p.a. simple interest. After 3 years he had to pay Rs.5400 interest only for the period. The principal amount borrowed by him was?

$$\text{Sol: Principal} = \frac{100 \times 5400}{12 \times 3} = \text{Rs } 15,000$$

**Example 7:** A lent Rs. 5000 to B for 2 years and Rs 3000 to C for 4 years on simple interest at the same rate of interest and received Rs. 2200 in all from both of them as interest. The rate of interest per annum is?

Sol: Let R% is the S.I

$$\frac{5000 \times 2 \times R}{100} + \frac{3000 \times 4 \times R}{100} = 2200$$

$$100R + 120R = 2200 \quad 220R = 2200$$

$$R = 10\%$$

**Example 8:** A sum of money amounts to Rs.9800 after 5 years and Rs.12005 after 8 years at the same rate of simple interest. Then find the rate of interest per annum?

Sol: SI for 3 years = 12005 – 9800 = Rs. 2205

$$\text{Si for 5 years} = \frac{2205}{3} \times 5 = \text{Rs. } 3675$$

$$\text{Principle} = 9800 - 3675 = \text{Rs. } 6125$$

$$\text{Hence Rate} = \frac{100 \times 3675}{6125 \times 5} = 12\%$$

## Compound Interest

If this interest is due (not paid) after the decided time period, then it becomes a part of the principal and so is added to the principal for the next time period, and the interest is calculated for the next time period on this new principal. Interest calculated, this way is called compound interest.

The time period after which the interest is added to the principal for the next time period is called the Conversion Period.

The conversion period may be one year, six months or three months and the interest is said to be compounded, annually, semi-annually or quarterly, respectively

$$\text{Compound Interest} = P \left( 1 + \frac{R}{100} \right)^n - P = P \left[ \left( 1 + \frac{R}{100} \right)^n - 1 \right]$$

If the rate of interest differs from year to year i.e.  $R_1$  in the first year,  $R_2$  in the second year,  $R_3$  in the third year. Then

$$A = P \left( 1 + \frac{R_1}{100} \right) \left( 1 + \frac{R_2}{100} \right) \left( 1 + \frac{R_3}{100} \right)$$

When the principal changes every year, we say that the interest is compounded annually. Then,

$$A = P \left( 1 + \frac{R}{100} \right)^n$$

When the principal changes every six months, we say that the interest is compounded half yearly or semi-annually. Then,

$$A = P \left( 1 + \frac{R/2}{100} \right)^{2n}$$

When the principal changes every three months, we say that the interest is compounded quarterly. Then,

$$A = P \left( 1 + \frac{R/4}{100} \right)^{4n}$$

When the interest is compounded annually but time is in fraction say  $2\frac{3}{4}$  year. Then

$$A = P \left( 1 + \frac{R}{100} \right)^2 \left( 1 + \frac{\frac{3R}{4}}{100} \right)$$

The difference between the simple interest and compound interest for 2 years (or terms) is given by the formula

$$D = P \left( \frac{R}{100} \right)^2$$

The above calculations will be clear if we take the Principal to be 1000 and Rate to be 10%.

Year	Simple Interest			Compound Interest		
	Principal	Interest	Amount	Principal	Interest	Amount
1st	1000	100	1100	1000	100	1100
2nd	1000	100	1200	1100	110	1210
3rd	1000	100	1300	1210	121	1331

Deductions from the above table:

- Simple Interest earned in any year is always the same.
- The SI and CI is same for the first year.
- CI earned in a year keeps increasing every year.

At CI,  $A = P (1 + (r/100))^n$

For CI, the formula is of Amount and for SI the formula is of Interest.

In the formulae of SI, make sure to highlight the fact that SI is directly proportional to P, r and n. Thus if an amount becomes 3 times in 7 years, in how many years will it become 9 times? Amount is not directly proportional to the number of years but for the Simple Interest, it is.

In this case the SI earned is 2P in 7 years and hence to earn a SI of 8P it will take 28 years. However had it been the case of CI, the amount would have become 9 times in 14 years itself because in every 7 years the amount triples, thus in next 7 years 3P will become 9P.