UNIT-3

CHAPTER 2

RATIO and its APPLICATIONS (PROPORTION, VARIATION AND PARTNERSHIP)

CONTENT AS PER IBPS PATTERN

 Ratio:

The ratio of two quantities *a* and *b* in the same units, is the fraction $\frac{a}{b}$ and we write it as *a* : *b*.

In the ratio *a* : *b*, we call *a* as the first term or *antecedent* and b, the second term or *consequent*.

Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

 Proportion:

The equality of two ratios is called proportion.

If *a* : *b* = *c* : *d*, we write *a* : *b* **::** *c* : *d* and we say that *a, b, c, d* are in proportion.

Here *a* and *d* are called *extremes*, while *b* and *c* are called *mean terms*.

Product of means = Product of extremes.

Thus, *a* : *b* **::** *c* : *d* $\Leftrightarrow$(*b* x *c*) = (*a* x *d*).

 Fourth Proportional:

If *a* : *b* = *c* : *d*, then *d* is called the fourth proportional to *a, b, c*.

Third Proportional:

a : *b* = *c* : *d*, then *c* is called the third proportion to *a* and *b*.

Mean Proportional:

Mean proportional between *a* and *b* is *ab*.

Compounded Ratio:

The compounded ratio of the ratios: (*a* : *b*), (*c* : *d*), (*e* : *f*) is (*ace* : *bdf*).

- *Duplicate Ratios:*

Duplicate ratio of (*a* : *b*) is (a^2 : b^2).

Sub-duplicate ratio of (*a* : *b*) is (*a* : *b*).

Triplicate ratio of (*a* : *b*) is (a^3 : b^3).

Sub-triplicate ratio of (*a* : *b*) is ($a^{1/3}$: $b^{1/3}$).

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

- *Variations:*

We say that *x* is directly proportional to *y*, if *x* = *ky* for some constant *k* and we write, $x \propto y$.

We say that *x* is inversely proportional to *y*, if *xy* = k for some constant *k and*

we write, $x \propto \frac{1}{y}$.

Examples:

1. a : b = 3: 4; b : c = 6 : 7. Find a : b : c.
Sol: a b c
3 4
6 7
a : b : c = 3 × 6 : 6 × 4 : 4 × 7 = 9 : 12 : 14

2. A sum of Rs.4960 has been divided among A, B and C in the ratio of 5:4:7. Find the share of B.
Sol: B's share = $\frac{4}{5+4+7} \times 4960$
= Rs.1240

3. 36% of first number is 28% of the second number. What is the respective ratio of the first number to the second number?
Sol: Let the numbers be x and y.

36% of x = 28% of y

$\frac{x}{y}=\frac{28}{36}=\frac{7}{9}$

$\therefore$ x : y = 7 : 9

4. Two numbers are in 4:7 ratio. The difference between them is 27. What is the bigger number?

Sol: Let the numbers be 4x and 7x.

7x − 4x = 27

$\Rightarrow$ 3x = 27 $\Rightarrow$ x = 9

$\therefore$ Bigger number is 7x = 7 × 9 = 63

Short cut: The difference of the terms of the ratio = 7 − 4 = 3.

But the actual difference between the numbers is 27

$\therefore$ 3 parts is equal to 27

7 parts (Bigger number) = $\frac{7}{3}\times$ 27 = 63

5. The ratio of the ages of a man and his son is 7: 3. The average of their ages is 30 years. What will be the ratio of their ages after 4 years?

Sol: Average age = 30 years

Total age = 2 × 30 = 60 years.

Let their present ages be 7x and 3x years

$\therefore$ 7x + 3x = 60 $\Rightarrow$ x = $\frac{60}{10}$ = 6

$\therefore$ Their present ages are

7 × 6 and 3 × 6 = 42 and 18.

$\therefore$ Their ages after 4 years

= 42 + 4 and 18 + 4 = 46 and 22 years

$\therefore$ ratio = 46 : 22 = 23 : 11

6. Two numbers are in the ratio of 3:4. If 4 is subtracted from each, the remainders are in the ratio of 5:7. What are the numbers?

Sol: Let the numbers be 3x and 4x.

If 4 is subtracted from each, the numbers will be (3x −4) and (4x −4).

$\therefore$ (3x −4) : (4x −4) = 5: 7

Product of means = Product of extremes

(3x –4) 7 = (4x – 4) 5

$\Rightarrow$ 21x – 28 = 20x – 20

$\Rightarrow$ x = 8

$\therefore$ The numbers are 3 × 8 and 4 × 8

= 24 and 32

7. In a bowl there is 30 litre mixture of milk and water. The ratio of milk and water is 7:3. How much water must be added to it so that the ratio of milk to the water be 3:7?

Sol : Milk quantity in the mixture

$= \frac{7}{10} \times 30 = 21$ litres

$\therefore$ Water = 30 - 21 = 9 litres

New ratio = 3 : 7

$\therefore$ 3 parts of milk is 21 litres (There is no difference in the milk quantity of new mixture)

$\therefore$ Water quantity in the mixture

$= \frac{7}{3} \times 21 = 49$ litres

$\therefore$ 49 - 9 = 40 litres water is to be added in the new mixture

8. A bag contains of one rupee, 50 paise and 25 paise coins. if these coins are in the ratio of 5 : 6 : 8, and the total amount of coins is Rs. 210, find the number of 50 paise coins in the bag.

Sol : Let the number of one rupee, 50 paise, 25 paise coins be 5, 6 and 8 respectively

The value of one rupee coins

= Rs. 1×5 = Rs. 5

The value of fifty paise coins

= Rs. 0.50×6 = Rs. 3

The value of twenty five paise coins

= Rs. 0.25×8 = Rs. 2

Total value = 5 + 3 + 2 = Rs. 10

If the total value is Rs. 10

there are 6 coins of fifty paise

if the total value is Rs. 210, then the number of 50 coins = $\frac{210}{10} \times 6 = 126$

9. If a sum of Rs.3150 were distributed among Ravi, Vijay and Suresh in the ratio of 12:9:14 respectively, then find the share of Vijay.

Ans: Rs.810

Sol: Vijay's Share = $\frac{9}{35} \times 3150$ = Rs.810

10. The total number of students in a school is 2850. If the number of boys in the school is 1650, then what is the respective ratio of the total number of boys to the total number of girls in the school?

Ans: 11:8

Sol: Total number of students = 2850

Number of boys = 1650

Number of girls = 2850-1650 = 1200

Ratio between boys and girls

=1650 : 1200 = 11 : 8

11. A sum of money is divided among A, B, C and D in the ratio of 5 : 6 : 12 : 15 respectively. If the share of C is Rs. 4092, then what is the total amount of money?

Ans: Rs. 12958

Sol: Let the share of A, B, C and D be Rs. 5x, 6x, 12x and 15x respectively. C's share is Rs.4092

$\Rightarrow 12x = 4092 \Rightarrow x = \frac{4092}{12} = 341$

$\therefore$ Total money = $38x = 38 \times 341$= Rs.12958

PARTNERSHIP

When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

Ratio of Division of Gains:

1.When the investments of all the partners are do the same time, the gain or loss is distributed among the partners in the ratio of their investments.

Suppose A and B invest Rs x and Rs y respectively for a year in a business, then at the end of the year:
(A's share of profit):(B's share of profit)=x:y

2.When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital*number of units of time). Now gain or loss is divided in the ratio of these capitals. Suppose A invests Rs x for p months and B invests Rs y for q months,then (A's share of profit):(B's share of profit)=xp:yq

3. Working and sleeping partners: A partner who manages the business is known as working partner and the one who simply invests the money is a sleeping partner.

Formulae

1.When investments of A and B are Rs x and Rs y for a year in a business ,then at the end of the year A's share of profit):(B's share of profit)=x:y

2.When A invests Rs x for p months and B invests Rs y for q months, then A's share profit:B's share of profit=xp:yq

EXAMPLES:

1. P and Q started a business investing Rs 85,000 and Rs 15,000 respectively. In what ratio the profit earned after 2 years be divided between P and Q respectively?

Sol: 85,000*2 : 15,000*2 = 17*2 : 3*2=34:6

2. A,B and C started a business by investing Rs 1,20,000, Rs 1,35,000 and Rs 1,50,000.Find the share of each ,out of an annual profit of Rs 56,700?

Sol: Ratio of shares of A,B and C=Ratio of their investments = 120,000:135,000:150,000 =8:9:10
A's share=Rs 56,700*(8/27)=Rs 16,800
B's share =Rs 56,700*(9/27)=Rs 18,900
C's share =Rs 56,700*(10/27)=Rs 21,000

3. 3 milkman A,B,C rented a pasture A grazed his 45 cows for 12 days B grazed his 36 cows for 15 days and c 60 cows for 10 days. If B's share of rent was Rs 540 What is the total rent?

Sol: 45*12:36*15:60*10=9:9:10
9 parts is equal to Rs 540 then one part is equal to Rs 60 total rent=60*28=Rs 1680

4. Ramu and Krishna entered into a partnership with Rs 50,000 and Rs 60,000, after 4 months Ramu invested Rs 25,000 more while Krishna withdraw Rs 20,000 . Find the share of Ramu in the annual profit of Rs 289,000.

Sol: Ramu : Krishna=50,000*4+75,000*8: 60,000*4+40,000*8 =10:7
Ramu's annual profit=289000*(10/17)=Rs 170,000

5. A,B,C enter into partnership .A invests 3 times as much as B invests and B invests two third of what C invests. At the end of the year ,the profit earned is Rs 6600. what is the share of B?

Sol: let C's capital =Rs x B's capital=Rs (2/3)*x A's capital =3*(2/3)*x=Rs 2x
ratio of their capitals=2x:(2/3)*x:x=6x:2x;3x
B's share =Rs 6600(2/11)=Rs 1200

6. A,B and C enter into a partnership by investing in the ratio of 3:2:4.After one year ,B invests another Rs 2,70,000 and C,at the end of 2 years, also invests Rs 2,70,000.At the end of 3 years ,profit are shared in the ratio of 3:4:5.Find the initial investment of each?

Sol: Initial investments of A,B,c be Rs 3x, Rs 2x, Rs 4x then for 3 years
(3x*36):[(2x*12)+(2x+270000)*24]:[(4x*24)+(4x+270000)*12]=3:4:5
108x: (72x+640,000): (144x+3240000)=3:4:5
108x:72x+6480000:144x+3240000=3:4:5
(108x)/(72x+6480000)=3/4
432x=216x+19440000
216x=19440000

x=Rs 90000
A's initial investment=3x=3*90,000=Rs 2,70,000
B's initial investment=2x=2*90,000=Rs 1,80,000
C's initial investment=4x=4*90,000=Rs 3,60,000

EXERCISE -RATIO AND PROPORTION, VARIATION AND PARTNERSHIP

CONTENT AS PER IBPS/SSC LEVEL

1. A, B and C enter into partnership with capitals in the ratio of 2 : 5 : 3. B joins a few months later than A and C joins 2 months further than B and withdraws from the business some time before the year ended. If the ratio in which profits are distributed is 8 : 10 : 3, how many months later does B join?
A] 5 months B] 4 months C] 6 months D] 7 months

2.. The ratio of the amount of money with A & B is 3 : 1. If we add amount of money with C that is Rs.20 to A's amount and subtract amount of money held by D that is Rs.10 from B's amount, we get another ratio which is 1/15 more than the original ratio. What is the sum of money held by A & B?
A] Rs.2441 B] Rs.3045 C] Rs.3040 D] Rs.3042

3. The ratio of marks obtained by Tom and Julia in English are 3 : 4. From the data on marks obtained, it can be inferred that Tom and Julia have got 3 and 6 marks respectively in English per 5 marks obtained in Mathematics. If the marks obtained by Julia in English are 60, then what is the sum total of marks obtained by Tom in both the subjects?
A] 75 B] 120 C] 60 D] 105

4. The monthly incomes of A and B are in the ratio 4 : 3 and the expenses are in the ratio 6 : 5. If their savings are Rs.5000 and Rs.3000 respectively, find their respective monthly incomes.
A] Rs.14000, Rs.10500 B] Rs.24000, Rs.18000
C] Rs.16000, Rs.12000 D] None of these

5. Rohit and Mohit started a business in partnership. They earned a profit of Rs.40000 as a whole at the end of first year. If Rohit took two-fifths of the share of the profit and his investment was Rs.50000, find the investment done by Mohit.
A] Rs.40000 B] Rs.60000 C] Rs.75000 D] Rs.100000

6. Ram, Shyam and Mohan start a business by investing Rs.12000, Rs.16000 and Rs.10000 respectively. Ram is a working partner and gets one-fifth of the total profit for his services while the remaining profit is divided amongst the three in proportion to their investments. If Ram gets Rs.2000 for his services, then what are the shares of profit of Ram, Shyam and Mohan respectively?
A] Rs.4526, Rs.3369, Rs.2105 B] Rs.2526, Rs.3369, Rs.2105
C] Rs.3158, Rs.4210, Rs.2632 D] None of these

7. Rs.9700 has been divided among X, Y and Z such that if their shares are reduced by Rs.30, Rs.20 and Rs.50 the balance is in the ratio of 3 : 4 : 5. What is Y's share?
A] Rs.3180 B] Rs.3220 C] Rs.3253.33 D] Rs.3200

8. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then $\frac{2a^4b^2+3a^2e^2-5e^4f}{2b^6+3b^2f^2-5f^5}$ =
A] $\frac{a}{b}$ B] $\frac{a^2}{b^2}$ C] $\frac{a^4}{b^4}$ D] Cannot be determined

9. If $\frac{y}{x-z}=\frac{y+x}{z}=\frac{x}{y}$, find x : y : z.
A] 2 : 1 : 3 B] 4 : 2 : 3 C] 2 : 1 : 2 D] 4 : 1 : 2

10. Solve the equation: $\frac{2x^3-3x^2+x+1}{2x^3-3x^2-x-1}=\frac{3x^3-x^2+5x-13}{3x^3-x^2-5x+13}$.

A] 0, 5 B] 0, 7, $\frac{8}{7}$ C] 0, 5, $\frac{8}{7}$ D] 5, $\frac{8}{7}$

11. The price of Computer and CD player are in the ratio 6:5. If the computer costs Rs. 5000 more than CD player. What is the price of the computer?
A) 25,000
B) 15,000
C) 50,000
D) 30,000

12. If Rs. 582 be divided into three parts, proportional to 1/2:2/3:3/4, then the first part is?
A) 161
B) 151.8
C) 142
D) 153

13. The price of mixer, grinder and washing machine are in the ratio 3:4:5. After one year the price of the items are increased 20%,15%,25% respectively. Then what will be ratio after one year?
A) 18:23:25
B) 17:22:23
C) 16:10:22
D) 18:22:24

14. A person has 25p, 10p and 5p in the ration 2:3:4 in his pocket. If the person has Rs. 45 in all, how many 5 paise cons are there?
A) 110
B) 100
C) 130
D) 120

15. A shopkeeper contains apples, oranges and bananas in the ratio 5:7:8. There is a demand to increase their quantity by 50%, 60% and 70% respectively. What will be ratio of the increased quantity?
A) 25:75:100
B) 26:72:112
C) 75:112:136
D) 76:100:201

16 A packet of sweets is distributed among A,B,C,D in the proportion of 6:8:5:4. If B gets 10 sweets more than D then what is A's share?
A) 16
B) 17
C) 15
D) 18

17. If A's 60% of salary is equal to two-third of B's salary. Now find the ratio of A's salary to B's salary.
A) 9:10
B) 10:9

C) 11:12
D) 13:11

18. If 15% of x is the same as 20% of y, then x : y is :.
a) 3 : 4
b) 17 : 16
c) 4 : 3
d) 16 : 17

19.. In a college, the ratio of the number of boys to girls is 8 : 5. If there are 160 girls, the total number of students in the college is:.
a) 100
b) 260
c) 250
d) 416

20. The ratio of income of A to that of B is 5 : 4 and the expenditure of A to that of B is 3: 2. If at the end of the year, each saves Rs, 800, the income of A is : .
a) Rs. 1600
b) Rs. 2000
c) Rs. 1800
d) Rs. 2200

21. If p : q = 3 : 4 and q : r= 8 : 9, then p : r is :.
a) 1: 3
b) 2 : 3
c) 3 : 2
d) 1: 2

22. If a+b : b+c : c+a = 6 : 7 : 8 and a + b + c = 14, then the value of c is : .
a) 6
b) 8
c) 7
d) 2

23 Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:?.
a) 5:4
b)3:2
c) 4:5
d) 2:3

24. If three numbers in the ratio 3 : 2: 5 be such that the sum of their squares is 1862, the middle number will be:.
a) 7
b) 21
c) 14
d) 35

25. A certain amount was divided between Salim and Rahim in the ratio of 4 : 3. If Rahim's share was Rs. 2400, the total amount was.
a) Rs. 5600
b) Rs. 9600

c) Rs. 3200
d) Rs. 16800

26. A profit of Rs. 30000 is to be distributed among A, B, C in the proportion 3 : 5 : 7. What will be the difference between B's and C's shares?.
a) Rs. 2000
b) Rs. 10000
c) Rs. 4000
d) Rs. 14000

27. An amount of money is to be distributed among F, Q and R in the ratio 3 : 5 : 7. If Qs share is Rs. 1500, what is the difference between Ps and Rs shares? .
a) Rs. 1200
b) Rs. 1600
c) Rs, 1500
d) Rs. 1900

28. Rs. 120 are divided among A, B, C such that A's share is Rs. 20 more than B's and Rs. 20 less than C's. What is B's share..
a) Rs. 10
b) Rs. 20
c) Rs. 15
d) Rs. 25

29. The compounded ratio of (2 : 3), (6: 11) and (11 :2) is.
a) 1: 2
b) 11: 24
c) 2: 1
d) 36: 121

30. What number should be added to each of the numbers 8, 21, 13 and 31 so that the resulting numbers, in this order form a proportion?.
a) 2
b) 5
c) 3
d) 7

31. An alloy is to contain copper and zinc in the ratio 9 :4. The zinc required (in kg) to be melted with 24 kg of copper, is 7
a) 10.66
b) 9.66
c) 10.33
d) 9

32. What number should be subtracted from both the terms of the ratio 15 : 19 so as to make it as 3 : 4 ?

a) 3
b) 6
c) 5
d) 9

33. Rs.432 is divided amongst three workers A, B and C such that 8 times A’s share is equal to 12 times B’s share which is equal to 6 times C’s share. How much did A get?

a) Rs.192
b) Rs.133
c) Rs.144
d) Rs.128

34. If 20 men or 24 women or 40 boys can do a job in 12 days working for 8 hours a day, how many men working with 6 women and 2 boys take to do a job four times as big working for 5 hours a day for 12 days?

a) 2 men
b) 12 men
c) 8 men
d) 24 men

35. P,Q and R enter into a partnership with capitals in the ratio 3:2:1. After 4 months, Pleaves the business and after 4 more months Q also leaves the business and R continues till the end of the year. If R takes 10 of the profit for managing the business, then what part of the profit does R get?

a) 37%
b) 36%
c) 27%
d) 30%

36. An outgoing batch of students wants to gift a PA system worth Rs 4,200 to their school. If the teachers, offer to pay 50 more than the students and an external benefactor gives three times the teacher's contribution, then how much should the teachers donate?

a) Rs 600
b) Rs 840
c) Rs 900
d) Rs 1,200

37.The monthly incomes of A and B are in the ratio 4:5, their expenses are in the ratio 5 : 6. If 'A' saves Rs.25 per month and 'B' saves Rs.50 per month, what are their respective incomes?

a) Rs.400 and Rs.500
b) Rs.240 and Rs.300
c) Rs.320 and Rs.400
d) Rs.440 and Rs.550