UNIT 2
CHAPTER 1
PERCENTAGE
BASIC CONCEPT BUILDER

What is percentage?

In mathematics, a **percentage** is a number or ratio expressed as a fraction of 100. It is often denoted using the percent sign, "%", or the abbreviations "pct.", "pct"; sometimes the abbreviation "pc" is also used.[1] A percentage is a dimensionless number (pure number).

For example:

1- If a goalie saves 96 out of 100 shots, his save *percentage* is 96 percent.

2- If you score 60 marks Information technology does that mean you score 60%. The answer is NO. Why? Because you don't know the base. Percentage is basically a game of base. If you don't know the base than you can't calculate the percentage.

Percentage and Fraction values

We list the reciprocals of all natural numbers from 1 to 12, simultaneously with their multiples.
Reciprocal **of 2** (i.e 1/2) is 50%, that **of 4** will be half of 50% i.e 25%.
Similarly, reciprocal **of 8** will be half of 25% = 12.5% and that of 16 will be 6.25%

Reciprocal **of 3** is 33.33%. Thus reciprocal **of 6** will be half of 33.33% i.e 16.66%
and that **of 12** will be 8.33%

Reciprocal **of 9** is 11.11% and reciprocal **of 11** is 9.0909%. Reciprocal **of 9** is composed of 11's and reciprocal **of 11** is composed of 09's. If any calculation has 9 in the denominator, the decimal part will be only 1111 or 2222 or 3333 or 4444... ex. 95/9 will be 10.5555
Reciprocal **of 20** is 5% ; Reciprocal **of 21** is 4.76% and **of 19** is 5.26%.
Thus we can easily remember reciprocals of 19, 20, 21 as 5.25%, 5 , 4.75% i.e 0.25% more and less than 5%
Similarly, reciprocal **of 25** is 4 % Reciprocal **of 24** is 4.16% and **of 26** is 3.84%.
Thus, we can easily remember reciprocals of 24, 25, 26 as 4.15%, 4, 3.85% i.e 0.15% more and less than 4%.
Reciprocal **of 29** is 3.45% (i.e 345 in order) and reciprocal **of 23** is 4.35% (same digits but order is different. If 1/29 = 3.45% than definitely 1/23 will be more than 3.45%. Reverse the digits and the answer comes to 4.35%)
Reciprocal **of 18** is half of 11.1111% i.e 5.5555% i.e it consists of only 5's. Reciprocal **of 22** is half of 09.0909%. i.e 4.5454% i.e consists of 45's.
One can remember 1/8 = 12.5% and tables of 8, and one can easily remember fractions
such as 2/8, 3/8, 5/8, 7/8 which are used very regularly.

1/8 is 12.5%, 2/8 is 25% (12.5*2), 3/8 is 37.5% (12.5*3), 5/8 is 62.5% (12.5*5),
7/8 = 87.5% Ex: 37.5% of 880 = 3/8 × 880 = 330

Shortcut to Calculating Percentages

We use the approximation techniques to calculate percentage values

A. how to calculate the value of 11% of 1264

Here, the concept of 10% and 1%. i.e, for any value, say 1264, 10% of the value is obtained by simply shifting the decimal point by one place (or digit) to the left. $\therefore$ 10% of 1264.0 = 126.40

(i.e. the decimal point moves to the left by one place (or digit)). Similarly, 1% of 1264.0 will be obtained by shifting the decimal point by two places to the left. Hence, 1% of 1264.0 = 12.640.

Again 36% of 1325 = (40% – 4%) of 1325 = (4 × 10% – 4 × 1%) of 1325

= (4 × 132.5 – 4 × 13.25) = 530 – 53 = 477.

B. Similarly consider another example, say, 18% of 3250 = (20% – 2%) of 3250

= (2 × 10% – 2 × 1%) of 3250 = (2 × 325 – 2 × 32.5) = 585.

Hence if there is a 10% increase then the new value will become 1.1 times the old value and in general if there is an increase of p%, the new value will become 1.p times the old value. Here we should know, when to use this concept and when not to use i.e. if there is an increase of 33.33% then the new value will become 4/3 times the old value. Calculating in this way i.e. converting $33^1/_3$ into a fraction and simplifying is faster.

Whenever percentage increase cannot easily be converted into a convenient fraction, then the approximate percentage increase p, in integer form, must be found and then 1.p has to be used. The same logic holds for percentage decrease.

Change of base

We explain this concept with a simple examples given below

- ✓ If A is 20% more than B, by what percent is B less than A? Do the same problem if A was 37.5% more than B. Why it is important to work with fraction equivalents to save time & calculations.

Hence If A is r % more than B , then B is $\frac{100r}{100+r}$% less than B. Again if A is r % less than B, then B is $\frac{100r}{100-r}$% more than A.

Successive Percent changes

If A's salary increase by 10% in the next year and 20 % in the next to next year, then what is the net percentage increase. (Change it)

- ✓ The net increase of (a%) and a (b%) change is equivalent to (a + b + ab/100%) change. Probably the last example can be more easily solved with multiplying factors. Thus one can use Multiplying factors or a + b + ab/100 interchangeably.

- ✓ Successive percentage change is also useful in any relation of the type C = A * B. If there is a (a%) change in A and a (b%) change in B, then C changes by (a + b + ab/100%). This has also applications in Data Interpretation. Thus if market share grows by 20% and even if the total market size declines by 10%, the sales grows by 1.2 * 0.9 = 1.08 i.e. 8% as Sales = Market size * Market Share.

- ✓ The same relation appears many times in geometry. Thus if any quadrilateral has all its sides increasing by 10%, the area increases by 21% as area is proportional to square of linear dimensions. If sides of a cuboid increase by 20%, volume increases by 72.8% and surface area increases by 44%.

EXAMPLES:

Example 1: The sum of 18% of a number and 6% of the same number is 492. What is 12% of that number?

Sol: Sum of 18% and 6% = 18% + 6% = 24%

24%=492 ; 12% = 492 * 12% / 24%=246

Example 2: In an examination it is required to get 65% of the aggregate marks to pass. A student gets 522 marks and is declared failed by 7% marks. What are the maximum aggregate marks a student can get?

Sol: Pass marks of the examination = 65%

Student failed by 7%, so marks secured by student = 65% – 7% = 58%

58%=522; 100% =900

Example 3: In a test, minimum passing percentage for girls and boys is 30% and 45% respectively. A boy scored 280 marks and failed by 80 marks. How many more marks did a girl require to pass in the test if she scored 108 marks?

Sol: Boy gets 280 marks and fails by 80 marks = 280 + 80 = 360;

45 % = 360 therefore 30 % = 240; so girls got 108 marks and fails by 240 – 108 = 112 marks

Example 4: When 30% of one number is subtracted from another number, the second number reduces to its own four-fifth. What is the ratio between the first and the second numbers respectively?

Sol: Let x and y be the two numbers then

y – 30% x = 4/5 y ; y – 4/5 y = 30% x

y /5 = 30 x/100 => x:y = 2:3

Example 5: In a school there are 800 students out of whom 45 per cent are girls. Monthly fee of each boy is 600 and monthly fee of each girl is 30 per cent less than each boy. What is the total monthly fee of girls and boys together?

Sol: Number of girls = 45 / 100 * 800 = 360

Number of boys = 800 – 360 = 440

Monthly fee of each boy = 600

Monthly fee of each girl = 420

Total fee of boys and girls = 360 × 420 + 440 × 600

= 151200 + 26400

= 415200

Example 6: Ajay spends 25 per cent of his salary on house rent, 5 per cent on food, 15 per cent on travel, 10 per cent on clothes and the remaining amount of ` 27,000 is saved. What is Ajay's income?

Sol: Ajay's total income be 100%

His total expenditure = 25% + 5% + 15% + 10% = 55%

Savings = 100% – 55% = 45%

45% = 27,000 therefore 100% = 27000 * 100/45 = 60,000

Example 7: When the price of eggs is reduced by 20%, it enables a man to buy 20 more eggs for 40. What is the reduced price per egg ?

Sol: Saving due to reduction =20/ 100 * 40 =8

So the sum of Rs 8 enables the man to purchase 20 more eggs at the reduced price (R.P); Reduced price per egg => 8/ 20 =0.4 = 40 paise

Example 8: The price of sugar is increased by 25%. If a family wants to keep its expenses on sugar unaltered, then the family will have to reduce the consumption of sugar by?

Sol: Initial price be 100 and consumption be 100 kg

Total expense = 100 × 100 = 10,000

New price = 100 + 25 = 125

But new expenditure = 10,000
New consumption = 10,000/125 = 80
So consumption of sugar reduced by 100 – 80 = 20%

Example 9: The population of a town increased by 10%, 20% and then decreased by 30%. The new population is what % of the original?

Sol: The overall effect $=1.1\times1.2\times0.7$ (Since 10%, 20% increase and 30% decrease)$=0.924=92.4\%$.

Example 10: Two successive discounts of 10% and 20% are equal to a single discount of?

Sol: Discount is same as decrease of price. So, decrease $=0.9\times0.8=0.72\Rightarrow28\%$ decrease (Since only 72% is remaining)

Example 11: A fruit seller had some apples. He sells 40% apples and still has 420 apples. Originally, he had?

Sol: Suppose originally he had x apples.
Then, (100 - 40)% of x = 420. ; 60/100 * x = 420; x =420 * 100/60 = 700.

Example 12: In a group of students, 70% can speak English and 65% can speak Hindi. If 27% of the students can speak none of the two languages, then what per cent of the group can speak both the languages?

Sol: 27% students speak neither of the languages.

Number of students speaking either of the languages = 100% – 22% = 73% n(E U H) = 73%

$n(E) = 70\%$ $\quad n(H) = 65\%$ $\quad n(E\cap H) = ?$

But $n(E\cup H) = n(E) + n(H) - n(E\cap H)$ $73 = 70 + 65 - n(E\cap H)$

$n(E\cap H) = 135 - 73 = 62$