Cyclicity :

Important Concepts and Shortcuts of Cyclicity of Numbers

Types of questions based on cyclicity of numbers - There are mainly 3 categories of questions which fall under cyclicity of numbers, which include

1. How to find units digit of a^b
2. How to find units digit of $a^b * c^d * e^f$
3. How to find units digit of a^{bc}

Find units digit of a^b - Given $\mathbf{a^b}$, units place digit of the result depends on units place digit of ***a*** and the divisibility of power ***b***. Consider powers of 2
As we know,
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$.. and so on
What do you observe here? We can see that the units place digit for powers of 2 repeat in an order: 2, 4, 8, 6. So the "cyclicity" of number 2 is 4 (that means the pattern repeats after 4 occurrences) and the cycle pattern is 2, 4, 8, 6. From this you can see that to find the units place digit of powers of 2, you have to divide the exponent by 4.

Example: Find the units place digit of 2^{99}?
Using the above observation of cyclicity of powers of 2, divide the exponent by 4. 99/4 gives reminder as 3. That means, units place digit of 2^{99} is the 3rd item in the cycle which is 8.

Shortcuts to solve problems related to units place digit of a^b

- **Case 1: If b is a multiple of 4**
  - If **a** is an even number, ie: 2, 4, 6 or 8 then the units place digit is 6
  - If **a** is an odd number, ie: 1, 3, 7 or 9 then the units place digit is 1
- **Case 2: If b is not a multiple of 4**
  - Let **r** be the reminder when **b** is divided by 4, then units place of $\mathbf{a^b}$ will be equal to units place of $\mathbf{a^r}$

- **Here we have captured the cyclicity of numbers upto 9 in the below table.**

Number	^1	^2	^3	^4	Cyclicity
2	2	4	8	6	4
3	3	9	7	1	4
4	4	6	4	6	2
5	5	5	5	5	1
6	6	6	6	6	1
7	7	9	3	1	4
8	8	4	2	6	4
9	9	1	9	1	2

Here ^1 means power raised to 1.

Find units digit for numbers of the form $a^b * c^d$

1. First find the unit digit of $\mathbf{a^b}$ and $\mathbf{c^d}$ separately. Let the answers be **x** and **y**
2. Then unit digit of $\mathbf{a^b} * \mathbf{c^d}$ = units digit of **x * y**

Find units digit of a^{bc}

- **Case 1:** If cyclicity of units place digit of a is 4 then we have to divide the exponent of a by 4 and find out the remainder. Depending on the value of remainder we can apply the general rule of cyclicity given above and reach the solution.
- **Case 2:** If cyclicity of units place digit of a is 2, only extra information we need to find is if the exponent will be even or odd. Then we can apply the general rule of cyclicity given above and reach the solution.

Example - Find the units place digit of 2^{4344}

Here cyclicity of units place digit is 4 (Units place digit is 2, from the above table we can see the cyclicity of 2 is 4). Hence case 1 is applicable. Now we have to find the remainder when exponent of 2 is divided by 4, which is the remainder when 4344 is divided by 4. Remainder of 4344/4 = Remainder of (44 – 1)44/4 Using the binomial theorem, (as explained in number system tutorial) we can see that there is only one term in the expansion of (44 – 1)44 which is not divisible by 4. The term is 144/4 Remainder of 144/4 = 1 Now we can apply the general rules of cyclicity, (since reminder is 1, case 2 of general rule of cyclicity is applicable) which says, units place of 24344 = units place of 21 = 2