Fluid breakthrough experiments

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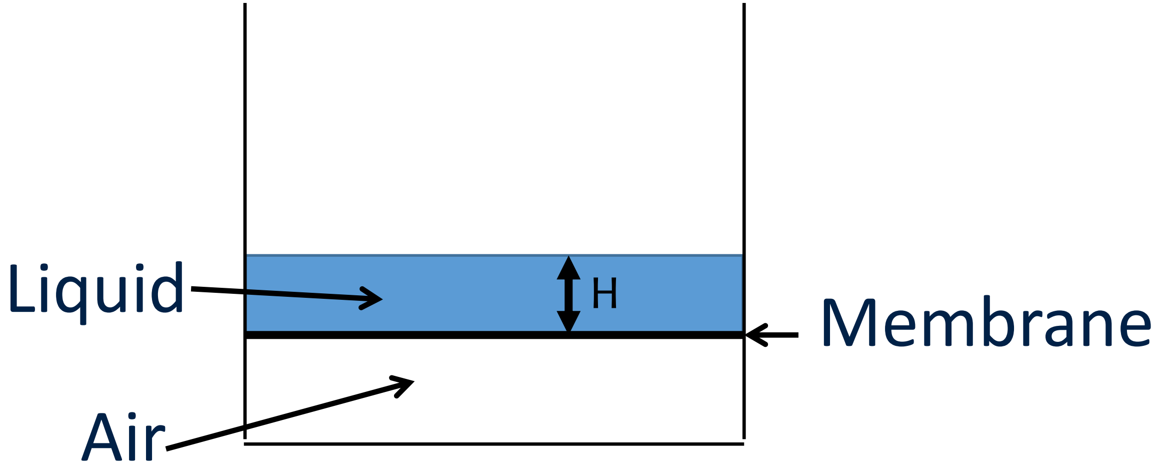
The problem

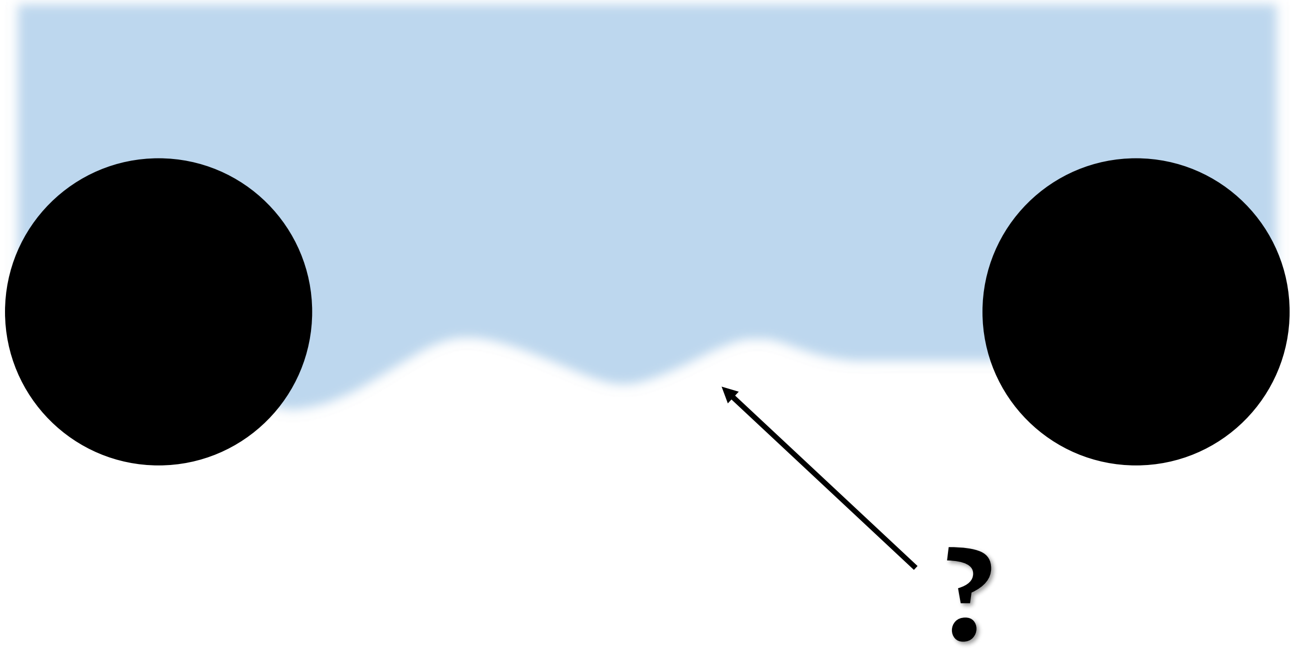
- *Waterproof membranes are used extensively in industry, for example in making waterproof coats.*
- The membranes have a porous structure, comprising a large number of fibres.
- There are various choices for the material used to make the fibres, each with different wettability properties.



The problem

What is the shape of the meniscus between two fibres?

How does this depend on the physical parameters?



Assumptions

- Fluid is **static** and **incompressible**.
- The pressure due to the fluid is **hydrostatic**.
- The upward force is due to the **surface tension** of the fluid.
- The membrane does **not bend**.
- The system is in its **steady state**.

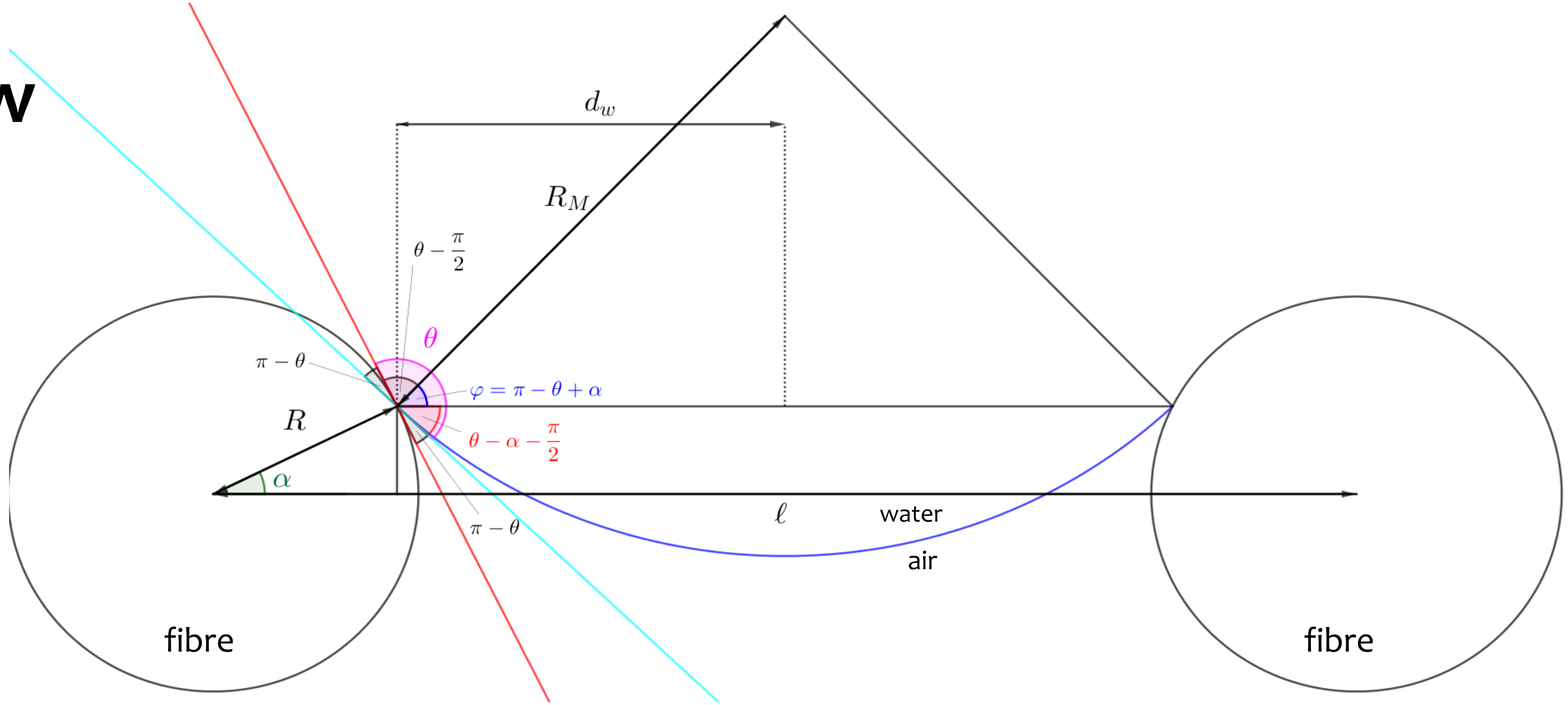
Physical principles

- **Newton's second law**

$$\rho g H = \kappa\gamma$$

- **Curvature**

$$\kappa = \frac{1}{R_M}$$



- **Variables**

α position angle

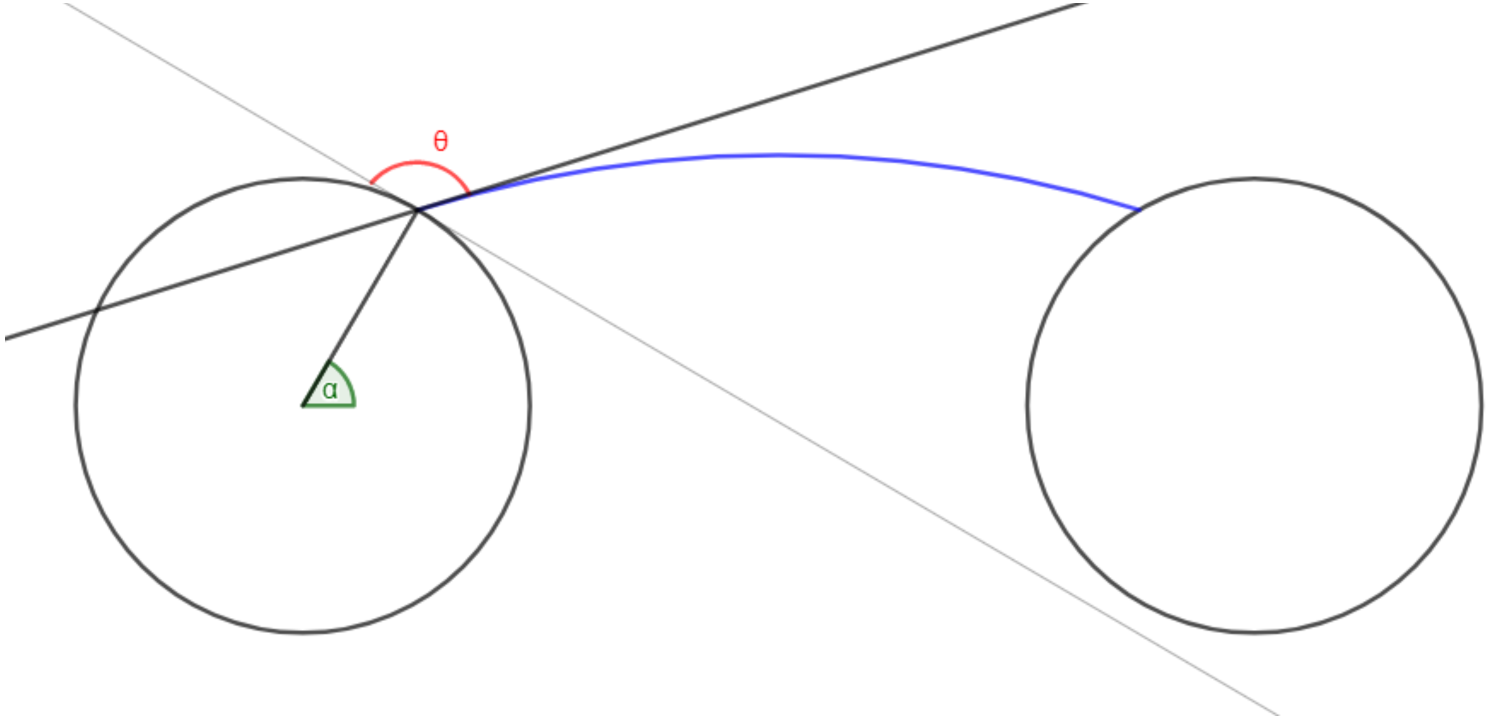
R_M radius of curvature of the meniscus

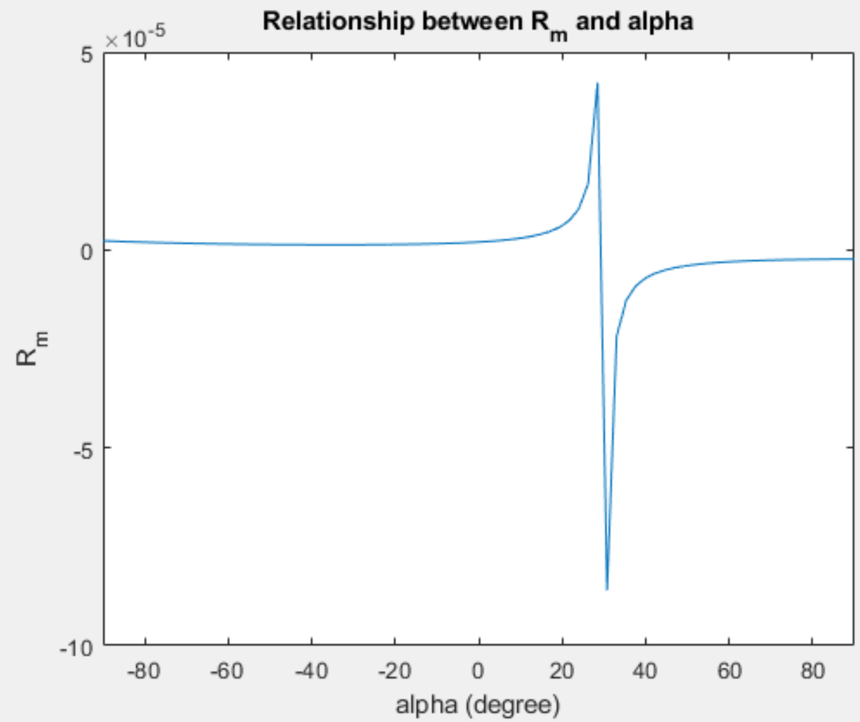
Governing equations

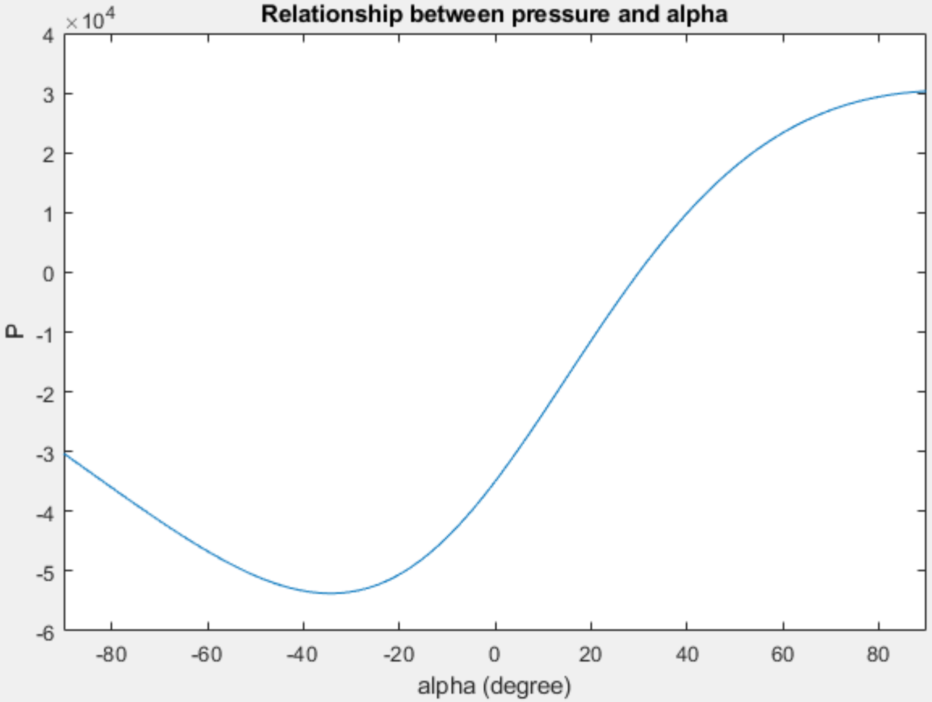
$$R_M(\alpha) = \frac{R\cos(\alpha) - \frac{l}{2}}{\cos(\theta - \alpha)}$$

$$P(\alpha) = \pm\frac{\gamma}{R_M(\alpha)}$$

Hydrophobic $\theta > \frac{\pi}{2}$

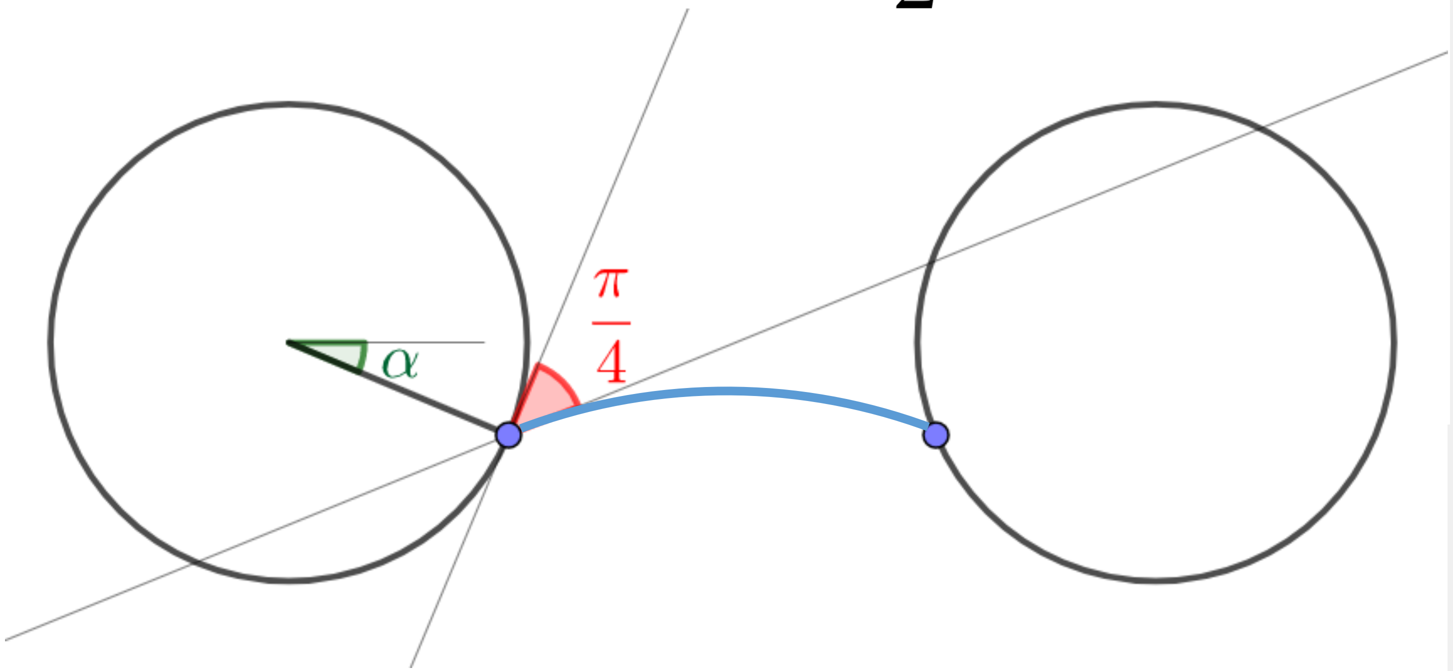


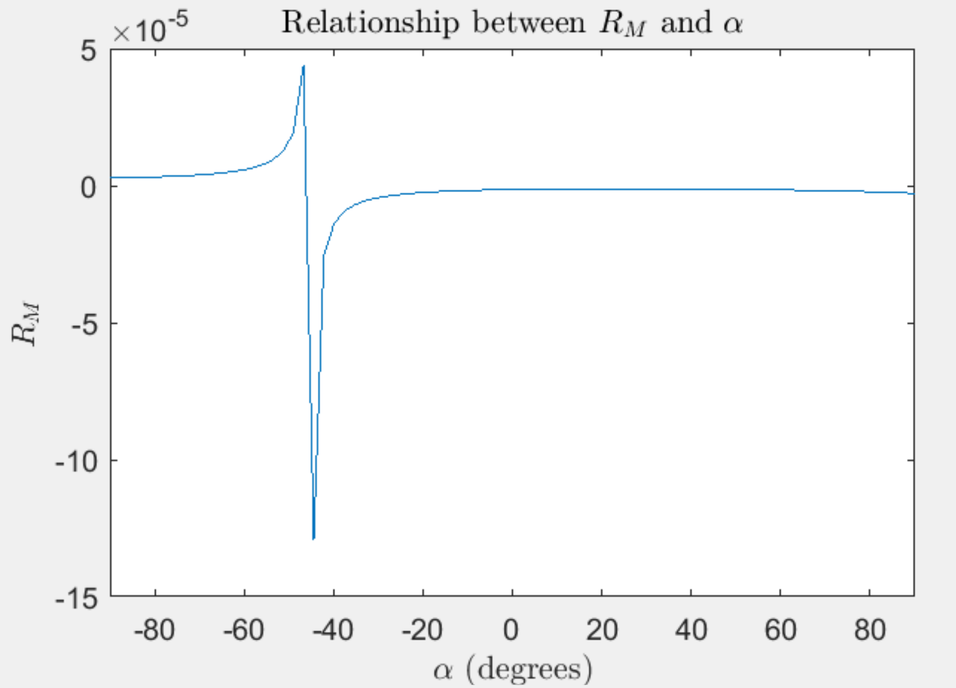


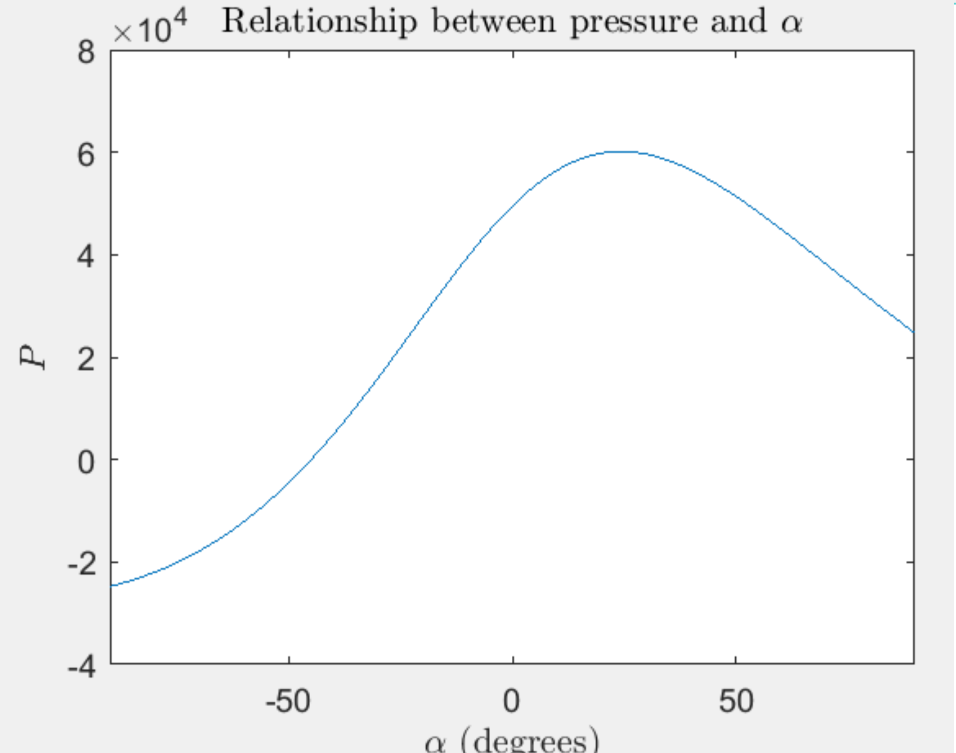


Contact angle: $\theta = \frac{2\pi}{3}$

Hydrophilic $\theta < \frac{\pi}{2}$

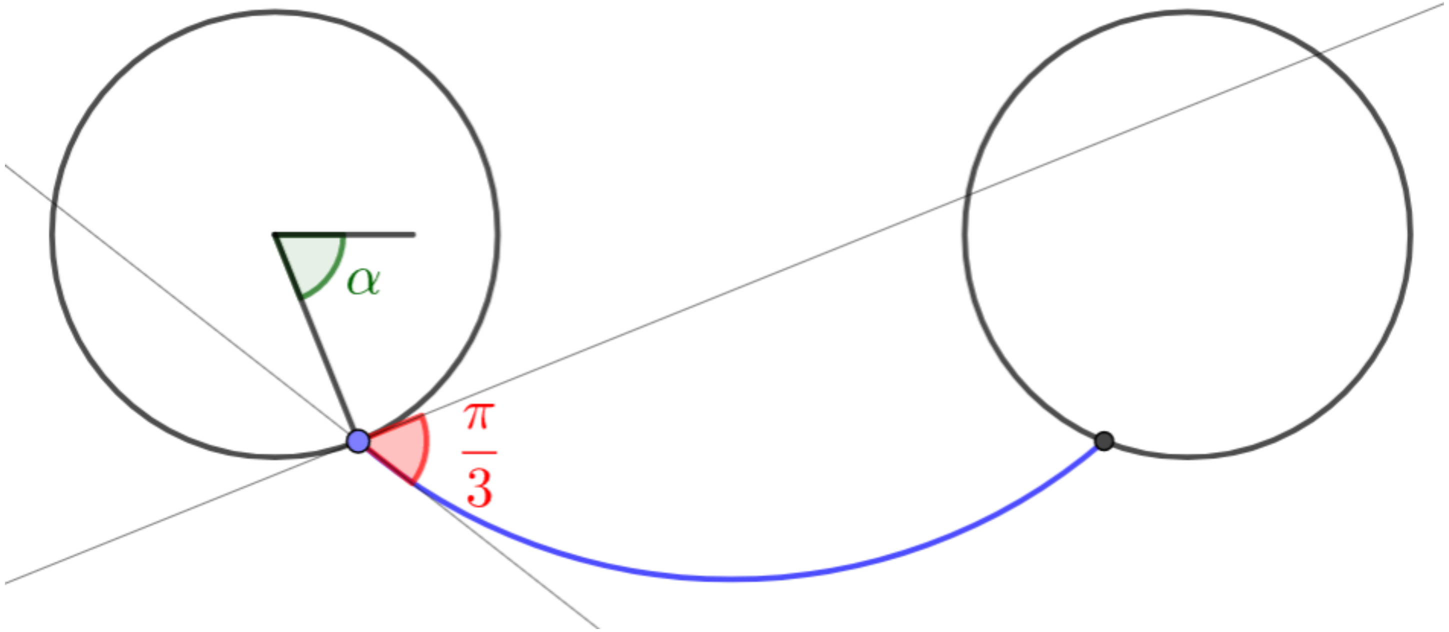


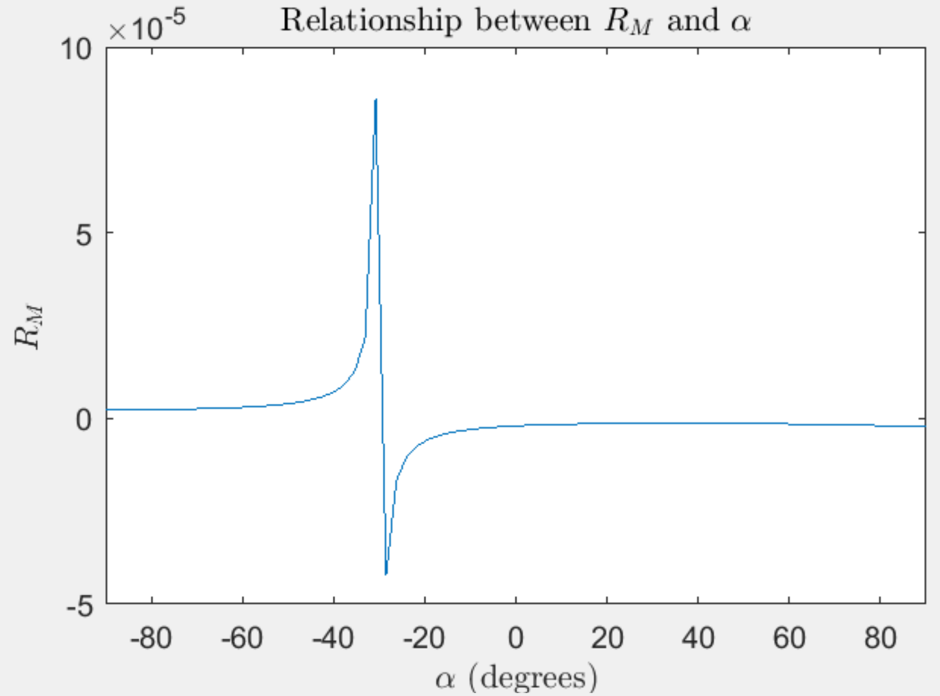


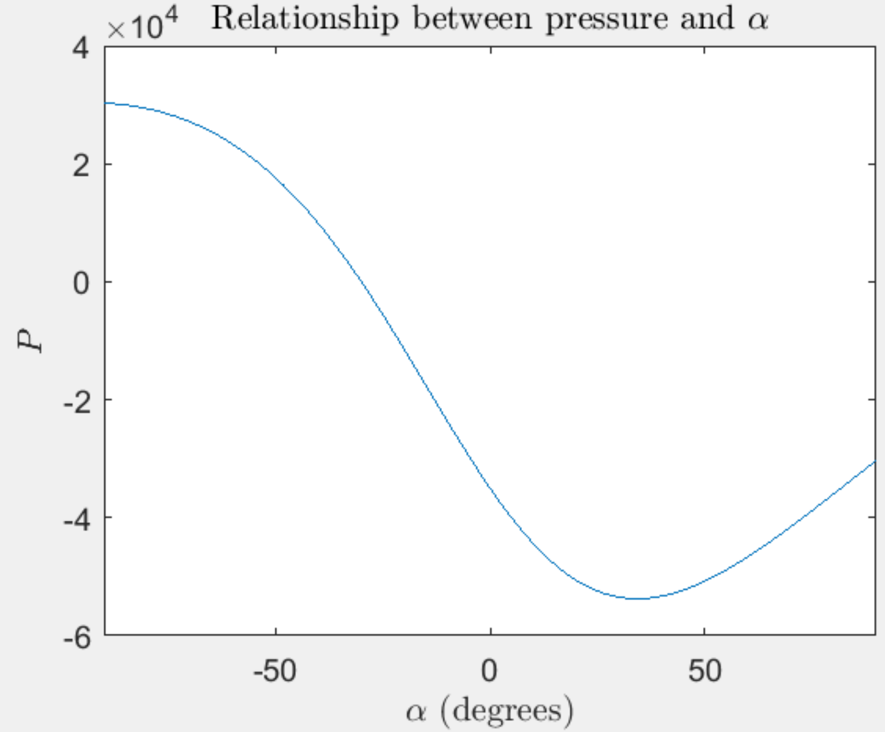


Contact angle: $\theta = \frac{\pi}{4}$

Hydrophilic $\theta < \frac{\pi}{2}$







Contact angle: $\theta = \frac{\pi}{3}$

Conclusions

- **Bursting**

Maximum pressure occurs at $\alpha_{\max} = \theta - \arcsin\left(\frac{2R}{l}\sin(\theta)\right)$

For this maximum pressure, we get critical thickness $H_{crit} = \frac{P(\alpha_{\max})}{\rho g}$

For any $H > H_{crit}$ we have bursting – meniscus cannot support weight of water above it

- **Flooding**

If $H < H_{crit} \ \forall \ \alpha \in \left[-\frac{\pi}{2}\ \frac{\pi}{2}\right]$, then flooding occurs when $H = \frac{P(\frac{\pi}{2})}{\rho g}$

Further work – different radii

$$\begin{cases} 2R_M \ \cos\left(\dfrac{\alpha_1-\alpha_2}{2}\right)\sin\left(\theta+\dfrac{\alpha_1+\alpha_2}{2}\right)+R_1\sin(\alpha_1)+R_2\sin(\alpha_2)-l & =0 \\ 2R_M \ \sin\left(\dfrac{\alpha_1-\alpha_2}{2}\right)\sin\left(\theta+\dfrac{\alpha_1+\alpha_2}{2}\right)+R_1\cos(\alpha_1)-R_2\cos(\alpha_2) & =0 \end{cases}$$