[IrisToolbox] for Macroeconomic Modeling

Adaptive Random-Walk Metropolis (ARWM) Posterior Simulator

jaromir.benes@iris-toolbox.com

Initialization

Initial vector of parameters θ_0 `poster.InitParam`

Initial proposal covariance matrix Σ_0 `poster.InitProposalCov`

Initial scale σ_0 `poster.InitScale=1/3`

Factorize $\Sigma_0 = P_0 P_0'$

Burn-in

Run a total $N_{\text{burn}} + N$

Return N

Discard N_{burn}

Parameter	Range	IrisT default
N_{burn}	$(0, 1)$	`burnIn=0.10`

n-th proposal

$$\hat{\theta}_n = \theta_{n-1} + w_n = \theta_{n-1} + \sigma_{n-1} P_{n-1} u_n$$

$$w_n \sim N(0, \sigma_{n-1}^2 \Sigma_{n-1})$$

$$u_n \sim N(0, 1)$$

Acceptance or rejection

Accept $\hat{\theta}_n$ with probability α_n

$$\alpha_n = \min\left\{1, \frac{poster(\hat{\theta}_n)}{poster(\theta_{n-1})}\right\}$$

If accepted ($a_n = 1$): $\theta_n = \hat{\theta}_n$

If rejected ($a_n = 0$): $\theta_n = \theta_{n-1}$

Adaptation to target acceptance ratio

Adapt the scale and shape of the proposal covariance matrix to force acceptance ratio towards target α^*

Adaptation

$$\propto \begin{cases} n^{-\gamma}\left(\alpha_n - \alpha^*\right) & \text{if} \quad n \leq \overline{n}_{\text{adapt}} \\ 0 & \text{if} \quad n > \overline{n}_{\text{adapt}} \end{cases}$$

Parameter	Range	IrisT default
γ	$(0.5, 1)$	`gamma=0.8`
$\overline{n}_{\text{adapt}}$	$\{2, 3, 4, \ldots\}$	`lastAdapt=Inf`

Adaptation needs to be vanishing to preserve ergodicity

Scale adaptation

For $n \leq \overline{n}_{\text{adapt}}$:

$$\log \sigma_n = \log \sigma_{n-1} + \kappa_{\text{scale}}\, n^{-\gamma}\, (\alpha_n - \alpha^*)$$

Parameter	Range	IrisT default
κ_{scale}	$(0, \infty)$	`adaptScale=1`

Shape adaptation

For $n \leq \overline{n}_{\text{adapt}}$:

$$\Sigma_n = P_n P_n' = P_{n-1}\left[I + \kappa_{\text{shape}} n^{-\gamma}\left(\alpha_n - \alpha^*\right)\frac{u_n u_n'}{\|u_n\|^2}\right]P_{n-1}'$$

Parameter	Range	IrisT default
κ_{shape}	$(0, \infty)$	`adaptShape=0.5`

Output arguments for diagnostics

Chain of log posteriors $poster(\theta_n)$

Cumulative acceptance ratio $\sum_{k=1}^{n} a_k/n$

Chain of scale factors σ_n

Final proposal covariance matrix Σ_N