

[IrisToolbox] for Macroeconomic Modeling

Stacked time solution method with first-order terminal condition

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Overview

- Creating a stacked time system
- Simulation procedure and terminal condition
- Changes in information sets

Model equations

System of n dynamic conditional-expectations equations

$$\mathbf{E}_t \left[f_1 (x_{t-k}, \dots, x_{t+m}) \right] = 0$$

$$\mathbf{E}_t \left[f_2 (x_{t-k}, \dots, x_{t+m}) \right] = 0$$

$$\vdots$$

$$\mathbf{E}_t \left[f_n (x_{t-k}, \dots, x_{t+m}) \right] = 0$$

where

- n is the number of model equations
- x_t is an $n \times 1$ vector model variables
- $\mathbf{E}_t[\cdot]$ is a conditional expectations operator
- k is the maximum lag
- m is the maximum lead

Stacked time setup

- Simulation range $t = 1, \dots, T$
- Drop the expectations operator
- Stack the n equations for the T simulation periods
- Create a large static system of $T \times n$ equations in $T \times n$ unknowns
- Known initial conditions x_{1-k}, \dots, x_0
- Unknown terminal conditions x_{T+1}, \dots, x_{T+m}

Stacked time system of equations and unknowns

- A total of $n \cdot T$ equations
- A total of $n \cdot T$ unknowns, x_t , $t = 1, \dots, T$

$$f_1(x_{1-k}, \dots, x_{1+m}) = 0$$

$$f_2(x_{1-k}, \dots, x_{1+m}) = 0$$

$$\vdots$$

$$f_n(x_{1-k}, \dots, x_{1+m}) = 0$$

$$\vdots$$
$$\vdots$$

$$f_1(x_{T-k}, \dots, x_{T+m}) = 0$$

$$f_2(x_{T-k}, \dots, x_{T+m}) = 0$$

$$\vdots$$

$$f_n(x_{T-k}, \dots, x_{T+m}) = 0$$

Simulation setup

Initialize

- Create an $n \times (T + k + m)$ matrix
- Fill in initial condition in columns $1, \dots, k$

In each iteration

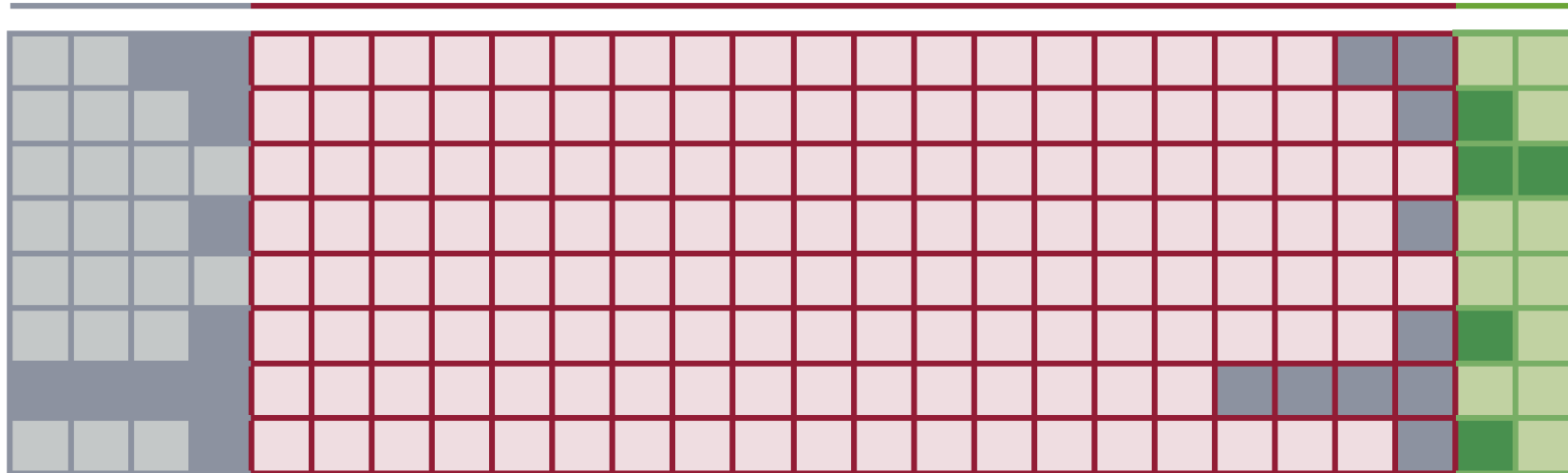
- Fill in the simulation range columns
- Taking the last simulation range columns as initial condition, use the first order simulator to fill in terminal condition
- Evaluate the LHS-RHS discrepancy for all $n \times T$ equations and send this information to the solver

Visualization of the simulation setup

Initial cond

Simulation range

Terminal cond



Terminal condition derived from first-order solution

First-order solution of the model (model-consistent expectations of endogenous variables integrated away)

$$x_t \approx T x_{t-1} + K + R_0 \varepsilon_t + \cdots + R_h \mathbb{E}_t[\varepsilon_{t+h}]$$

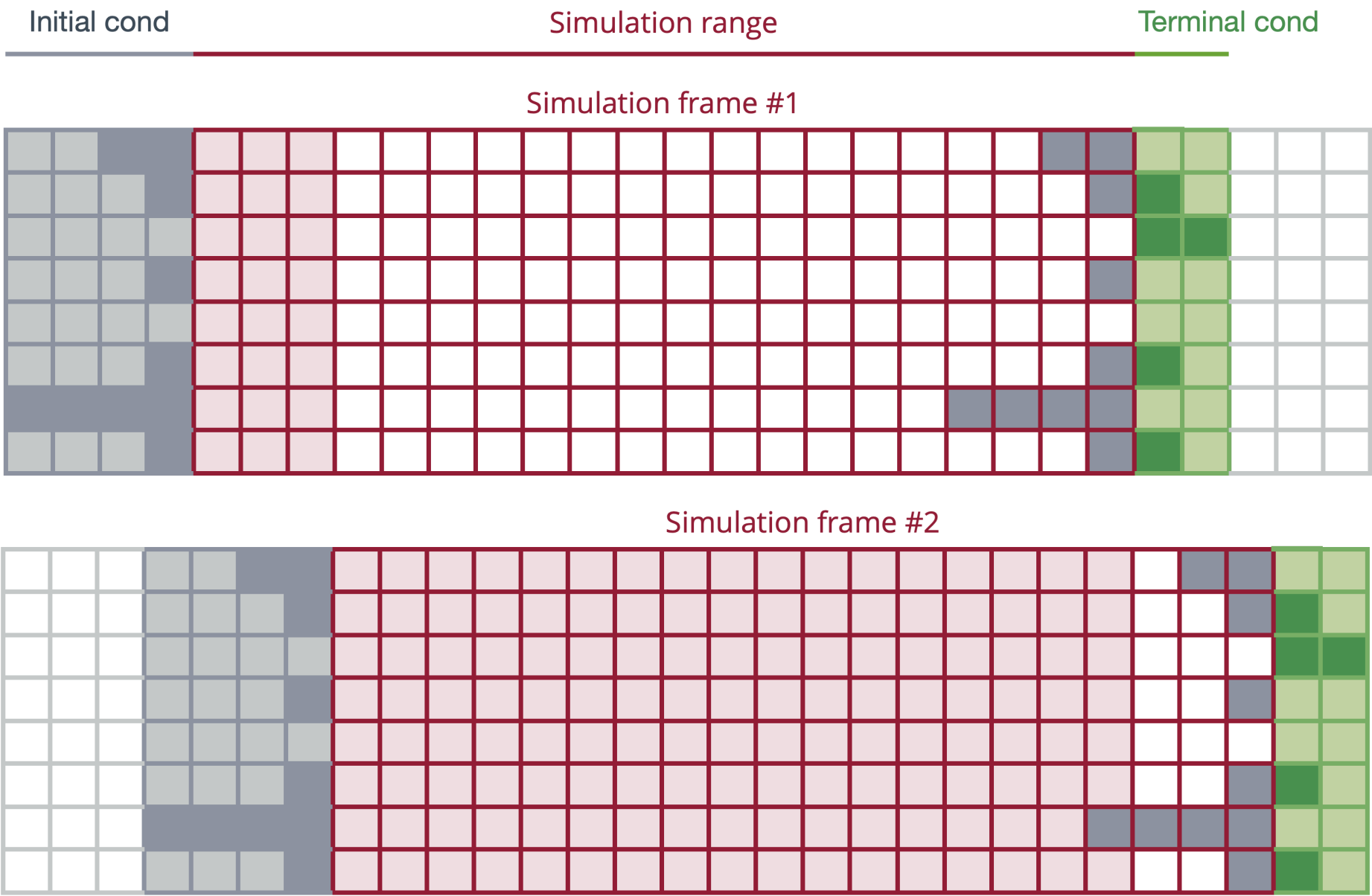
Create a stacked system to calculate the terminal condition points needed

$$\begin{bmatrix} x_{T+1} \\ \vdots \\ x_{T+m} \end{bmatrix} = T^{\text{term}} \begin{bmatrix} x_{T+1-k} \\ \vdots \\ x_T \end{bmatrix} + K^{\text{term}}$$

Changes in information sets within a simulation

- By design, a stacked time simulation is consistent with an assumption of all future events (shocks, swaps) are anticipated
- To simulate a sequence of unanticipated events, the simulation needs to be broken down into a sequence of sub-simulations (simulation frames)

Breakdown of simulation into frames



Implementation in IrisT

Syntax of the `simulate` function for the stacked time method

```
[outputDb, info, frameDb] = simulate( ...  
    model, inputDb, range ...  
    , method="stacked" ...  
);
```

Output structure `info`

Auxiliary output databank with simulation frames `frameDb`

Options to control the setup of the stacked time method

`startIter=`

- `"firstOrder"` (default)
- `"data"`

`terminal=`

- `"firstOrder"` (default)
- `"data"`