2-sector DSGE model for the Azerbaijan Economy

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1 Households

The economy is populated with a continium of households indexed by $(i) \in [0,1]$ where each household chooses consumption $c_t(i)$, investment $i_t(i)$, money balances $M_t(i)$, hours worked $h_t(i)$, local (foreign) riskless bonds $B_t^d(i)\left(B_t^f(i)\right)$, the supply and the utilisation rate of the capital. The preference of the i^{th} households is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \epsilon_t^P u\left(\tilde{c}_t(i), \frac{M_t(i)}{P_t}, h_t(i)\right) \tag{1.1}$$

where, $\tilde{c}_t = c_t - h_c c_{t-1}$ captures the habit formation and P_t is the aggregate price index in the economy. The functional form is given as follows

$$U_t = \sum_{t=0}^{\infty} \beta^t \epsilon_t^p \left[\frac{(c_t - h_c c_{t-1})^{1-\sigma_c}}{1-\sigma_c} + \frac{\epsilon_t^{mD}}{1-\sigma_m}\left(\frac{M_t}{P_t}\right)^{1-\sigma_m} - \epsilon_t^h \omega \frac{h_t(i)^{1+\kappa}}{1+\kappa}\right] \tag{1.2}$$

For the ease of notation we dropped the index i for all variables except for wages and hours to facilitate the description of the labour market stickiness. Households discount future streams of their utility with a factor β. The preference function involves two 'mean one' stochastic processes; ϵ_t^P is a preference shifter (demand) shock and ϵ_t^h is a labour supply shock. The households' budget constraint is given by

$$P_t c_t + P_t i_t + \frac{B_t^d}{r_t} + e_t \frac{B_t^f}{\phi_t r_t^*} + M_t = W_t(i) h_t(i) + \left(R_t^k u_t - P_t a(u_t)\right) k_{t-1} + B_{t-1}^d + e_t B_{t-1}^f + M_{t-1} - T_t + \Phi_t \tag{1.3}$$

Here $W_t(i)$ and R_t^k are the nominal wage and the nominal rental rate of the capital. We also assume that intensity of the capital utilisation creates an additional cost $(a(u_t))$ for the household which are paid in terms (at a price) of the final consumption bundle (P_t). T_t refers to the lump-sum government taxes (transfers), and Φ_t is the total profit of the households from ownership of firms (Tradable, Non-tradable, Importing) in the economy. The price of domestic bonds is given by $Pb_t^d = \frac{1}{r_t}$. That is, as the interest rate (r_t) increases, domestic bonds become cheaper for households and hence the demand for domestic bonds increases. The price of foreign bonds in national currency $Pb_t^f = \frac{e_t}{\phi_t r_t^*}$ involves an additional term ϕ_t to capture the risk premium of the domestic economy. When the risk premium of the domestic rises foreign bonds become cheaper and households demand more foreign bonds (and hence currency). The functional form for the

$$\phi_t = \exp\left[-\phi_b^1\left(\frac{b_t^f}{gdp_t}\right) + \phi_b^2\left(\left\{\frac{e_{t+1}}{e_t}\frac{e_t}{e_{t-1}}\right\} - 1\right) + \log(\varepsilon_t^{rp})\right] \tag{1.4}$$

Where we assumed that the ϕ_t is negatively related to the country's foreign assets while currency devaluations increases the risk premium. The risk premium also includes a shock term (ϵ_t^P) which aims to capture other factors that drive the risk perception of the counry.

The law of motion for the capital is given by,

$$k_t = (1-\delta)k_{t-1} + \left(1 - s\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{1.5}$$

where we assumed that adjusting the investment involves a cost $s\left(\frac{i_t}{i_{t-1}}\right)$.
First order optimality conditions of households are described as follows[1]:

$$\varepsilon_t^p(c_t - h_c c_{t-1})^{-\sigma_c} - \beta h_c \varepsilon_{t+1}^p (c_{t+1} - h_c c_t)^{-\sigma_c} = \lambda_t \tag{1.6}$$

where λ_t is a Lagrangian multiplier associated with the real budget constraint.

$$1 = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{r_t}{\pi_{t+1}} \tag{1.7}$$

International risk sharing:

$$1 = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{e_{t+1}}{e_t} \frac{\phi_t r_t^*}{\pi_{t+1}} \tag{1.8}$$

Alternatively, this together with the Euler equation gives the UIP condition:

$$1 = \frac{e_{t+1}}{e_t} \frac{\phi_t r_t^*}{r_t} \tag{1.9}$$

w.r.t. money balances

$$U_{m,t} = \lambda_t - \beta \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{1.10}$$

where

$$U_{m,t} = \epsilon_t^P \epsilon_t^{mD} m_t^{-\sigma_m} \tag{1.11}$$

w.r.t. capital:

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[r_{t+1}^k u_{t+1} - a_{t+1}(u_{t+1}) + (1-\delta) q_{t+1} \right] \tag{1.12}$$

w.r.t. investment

$$1 = q_t \left(1 - s\left\{ \frac{i_t}{i_{t-1}} \right\} - s'\left\{ \frac{i_t}{i_{t-1}} \right\} \frac{i_t}{i_{t-1}} \right) + \beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} s'\left\{ \frac{i_{t+1}}{i_t} \right\} \left(\frac{i_{t+1}}{i_t} \right)^2 \tag{1.13}$$

The f.o.c. w.r.t. utilization is given by

$$r_t^k = a'(u_t) \tag{1.14}$$

The stochastic processes are described below. The preference shock is:

$$\log\left(\frac{\epsilon_t^P}{\bar{\epsilon}^P}\right) = \rho^{\epsilon^P} \log\left(\frac{\epsilon_{t-1}^P}{\bar{\epsilon}^P}\right) + \varepsilon_t^{\epsilon^P} \tag{1.15}$$

The labour supply shock is given by:

$$\log\left(\frac{\epsilon_t^h}{\bar{\epsilon}^h}\right) = \rho^{\epsilon^h} \log\left(\frac{\epsilon_{t-1}^h}{\bar{\epsilon}^h}\right) + \varepsilon_t^{\epsilon^h} \tag{1.16}$$

Risk Premium shock:

$$\log\left(\frac{\epsilon_t^{RP}}{\bar{\epsilon}^{RP}}\right) = \rho^{\epsilon^{RP}} \log\left(\frac{\epsilon_{t-1}^{RP}}{\bar{\epsilon}^h}\right) + \varepsilon_t^{\epsilon^{RP}} \tag{1.17}$$

Money Demand Shock:

$$\log\left(\frac{\epsilon_t^{mD}}{\bar{\epsilon}^{mD}}\right) = \rho^{\epsilon^{mD}} \log\left(\frac{\epsilon_{t-1}^{mD}}{\bar{\epsilon}^{mD}}\right) + \varepsilon_t^{\epsilon^{mD}} \tag{1.18}$$

We also assume an exogenous process for the foreign (nominal) interest rate:

$$\log\left(\frac{r_t^*}{\bar{r}^*}\right) = \rho^{r^*} \log\left(\frac{r_{t-1}^*}{\bar{r}^*}\right) + \varepsilon_t^{r^*} \tag{1.19}$$

Households supply labour through the central authority (Labor Unions) and the next section describes labour market decisions.

[1]see Appendix

2 Labour Unions

Households supply differentiated labour types (skills or professions) into the labour market which are packed to homogeneous labour bundle by labor unions. Each labor type has a monopolistically competitive advantage (market power) in the labor market. We assume that the labour unions are owned by households and therefore take into account their preferences when making labour market decisions.

The aggregation technology of the labor packer is given by,

$$h_t^d = \left(\int_0^1 h_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \tag{2.1}$$

The demand function for each type of labor is given by[2]

$$h_t(i) = \left(\frac{w_t(i)}{w_t} \right)^{-\eta} h_t^d \tag{2.2}$$

where $w_t(i)$ is the (real) wage for the labor type i and h_t^d is the total labor input demanded by firms, The aggregate wage index is given by,

$$w_t = \left(\int_0^1 w_t(i)^{1-\eta} di \right)^{\frac{1}{1-\eta}} \tag{2.3}$$

We assume Calvo-Yun arrangement in the labour market. In each period of the time only $1 - \xi_w$ proportion of households (or labour markets) can optimally reset the price of the labour. In all other labour markets the wage is partially indexed to the past inflation by the parameter γ_w. After τ periods in markets which cannot optimise the real wage is given by,

$$w_{t+\tau}(i) = \prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} w_t(i) \tag{2.4}$$

The labor union solves the following problem

$$\max_{w_t(i)} \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left\{ \epsilon_{t+\tau}^p \left[U_{t+\tau}(\cdot, \cdot, h_{t+\tau}(i)) \right] - \lambda_{t+\tau} \left(... - \prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} w_t(i) h_t(i) \right) \right\} \tag{2.5}$$

s.t.

$$h_{t+\tau}(i) = \left(\frac{\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} w_t(i)}{w_{t+\tau}} \right)^{-\eta} h_{t+\tau}^d \tag{2.6}$$

The optimal wage is given by

$$\begin{aligned} \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau & \left\{ \omega \epsilon_{t+\tau}^P \epsilon_{t+\tau}^h \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \frac{w_t^*}{w_{t+\tau}} \right)^{-\eta(1+\kappa)} {h_{t+\tau}^d}^{1+\kappa} \right\} \\ &= \frac{\eta-1}{\eta} w_t^* \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left\{ \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+\tau}} \right)^{-\eta} h_{t+\tau}^d \right\} \end{aligned} \tag{2.7}$$

we can describe the solution recursively,

$$f_t^1 = \frac{\eta-1}{\eta} w_t^* \lambda_t \left(\frac{w_t^*}{w_t} \right)^{-\eta} h_t^d + (\beta \xi_w) \left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}} \right)^{1-\eta} \left(\frac{w_t^*}{w_{t+1}^*} \right)^{1-\eta} f_{t+1}^1 \tag{2.8}$$

$$f_t^2 = \omega \epsilon_t^P \epsilon_t^h \left(\frac{w_t^*}{w_t} \right)^{-\eta(1+\kappa)} {h_t^d}^{1+\kappa} + (\beta \xi_w) \left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}} \right) \left(\frac{w_t^*}{w_{t+1}^*} \right)^{-\eta(1+\kappa)} f_{t+1}^2 \tag{2.9}$$

[2]see the Appendix

And the f.o.c. is given by

$$f_t^1 = f_t^2 \tag{2.10}$$

The law of motion for the aggregate wage is,

$$1 = \xi_w \left(\frac{\pi_{t-1}^{\gamma_w}}{\pi_t}\right)^{1-\eta} \left(\frac{w_{t-1}}{w_t}\right)^{1-\eta} + (1-\xi_w)\left(\frac{w_t^*}{w_t}\right)^{1-\eta} \tag{2.11}$$

The wage dispersion is given by,

$$\vartheta_t^w = \int_0^1 \left(\frac{w_t(i)}{w_t}\right) di = \xi_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\gamma_w}}{\pi_t}\right)^{-\eta} \vartheta_{t-1}^w + (1-\xi_w)\left(\frac{w_t^*}{w_t}\right)^{-\eta} \tag{2.12}$$

which creates a wedge between labor supply and effective demand due to wage stickiness

$$h_t = \vartheta_t^w h_t^d \tag{2.13}$$

3 Firms

The economy is populated by firms which produce non-tradable and tradable goods in a monastically competitive environment. Tradable goods are produced domestically or imported. Domestically produced traded goods are domestically consumed and exported. We assume a perfect capital and labor mobility between sectors, and hence, factor prices are identical across the economy.

3.1 Non-Tradable Firms

Non-tradable firms are indexed by $j \in [0,1]$ and produce goods are imperfectly-substitutes to one-another. The aggregate output in the sector is given by the Dixit-Stiglitz aggregation technology,

$$y_t^N = \left(\int_0^1 y_t^N(j)^{\frac{\zeta^N-1}{\zeta^N}} dj\right)^{\frac{\zeta^N}{\zeta^N-1}} \tag{3.1}$$

where ζ^N is elasticity of the substitution of the differentiated goods. The price index of thee composite non-tradble goods basket is gicen by

$$P_t^N = \left(\int_0^1 P_t^N(j)^{1-\zeta^N} dj\right)^{\frac{1}{1-\zeta^N}} \tag{3.2}$$

where $P_t^N(j)$ is a price of the j^{th} non-tradable variety. The demand for each non-tradable variety is

$$y_t^N(j) = \left(\frac{P_t^N(j)}{P_t^N}\right)^{-\zeta^N} y_t^N \tag{3.3}$$

The production technology of a representative non-tradable firm is given by

$$y_t^N(j) = A_t^N k_t^N(j)^{\alpha^N} \left(A_t h_t^N(j)\right)^{1-\alpha^N} \tag{3.4}$$

the technology is subject to economy-wide and sector specific technology shocks. We assume that the aggregate TFP process is Hicks-neutral. Each non-tradable firm faces a Calvo-Lottery and at each period a firm is allowed to optimally reset its price with a probability $1-\xi^N$.

Nominal profits of a representative non-tradable firm is

$$\psi_t^N(j) = P_t^N(j) y_t^N(j) - R_t^k k_t^N(j) - W_t h_t^N(j) \tag{3.5}$$

Non Tradable firms maximize the net present value of their profits subject to the demand schedule they face and the production technology:

$$\max_{\left[p_t^N(j),h_t(j)^N,k_t(j)\right]} \sum_{\tau=0}^{\infty} \left(\beta\xi^N\right)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}} \psi_{t+\tau}^N(j) \tag{3.6}$$

s.t.:

$$y_t^N(j) = \left(\frac{P_t^N(j)}{P_{t+\tau}^N}\right)^{-\zeta^N} y_{t+\tau}^N \tag{3.7}$$

Where λ_t is the Lagrangian multiplier associated with households real budget constraint and $\left(\beta\xi^N\right)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}}$ is the effective discount rate.

The first-order conditions are given by (see Appendix for detailed derivations)

$$\frac{W_t}{P_t} = \nu_t^N(j)(1-\alpha^N)\frac{Y_t^N}{h_t^N} \tag{3.8}$$

$$\frac{R_t^k}{P_t} = \nu_t^N(j)(\alpha^N)\frac{Y_t^N}{k_t^N} \tag{3.9}$$

$$\tilde{P}_t^N(j) = \frac{\zeta^N}{\zeta^N-1} \frac{\sum_{t=\tau}^{\infty}(\beta\xi^N)^\tau \lambda_{t+\tau} y_{t+\tau}^N \left(P_{t+\tau}^N\right)^{\zeta^N} \nu_{t+\tau}^N(j)}{\sum_{t=\tau}^{\infty}(\beta\xi^N)^\tau \lambda_{t+\tau} \frac{P_{t+\tau}^N}{P_{t+\tau}} y_{t+\tau}^N \left(P_{t+\tau}^N\right)^{\zeta^N-1}} \tag{3.10}$$

Where $\nu_t(j)$ is the real marginal cost of the firm j which is also the Lagrangian multiplier of the associated constraint of the optimization problem. Because firms face same factor prices, the marginal cost, and hence, the optimal reset price are identical within the sector $\left(\nu_{t+\tau}^N(j) = \nu_{t+\tau}^N, \tilde{P}_t^N(j) = \tilde{P}_t^N\right)$. The law of motion for prices can be expressed recursively

$$j_{1,t}^N = \lambda_t y_t^N \nu_t^N + \beta\xi^N j_{1,t+1}^N \left(\pi_{t+1}^N\right)^{\zeta^N} \tag{3.11}$$

$$j_{2,t}^N = \lambda_t \frac{P_t^N}{P_t} y_t^N + \beta\xi^N j_{2,t+1}^N \left(\pi_{t+1}^N\right)^{\zeta^N-1} \tag{3.12}$$

Here $\frac{P_t^N}{P_t} y_t^N$ is the real value of the production (non-tradable GDP) of firms. The price ratio $\left(\frac{P_t^N}{P_t}\right)$ will be defined in section (4). The optimal reset price can be written as,

$$\frac{\tilde{P}_t^N}{P_t^N} = \frac{\zeta^N}{\zeta^N-1} \frac{j_{1,t}^N}{j_{2,t}^N} \tag{3.13}$$

The aggregate price index in the non-tradable market is given by,

$$P_t^N = \left[\int_0^{\xi^N} P_{t-1}^N(j)^{1-\zeta^N} d(j) + (1-\xi^N)\tilde{P}_t^{N\,1-\zeta^N}\right]^{\frac{1}{1-\zeta^N}} \tag{3.14}$$

Owing to Calvo Contracts and because of the "Law of Large Numbers" the law of motion for the non-tradable inflation is given by [3]

$$1 = \xi^N \pi_t^{N\,\xi^N-1} + (1-\xi^N)\left(\frac{\zeta^N}{\zeta^N-1} \frac{j_{1,t}^N}{j_{2,t}^N}\right)^{1-\zeta^N} \tag{3.15}$$

The price dispersion is given as,

$$\vartheta_t^N = \int_0^1 \left(\frac{P_t^N(j)}{P_t^N}\right)^{-\zeta^N} dj = \xi^N \vartheta_{t-1}^N \left(\pi_t^N\right)^{\zeta^N} + (1-\xi^N)\left(\frac{\zeta^N}{\zeta^N-1} \frac{j_{1,t}^N}{j_{2,t}^N}\right)^{-\zeta^N} \tag{3.16}$$

TFP processes are given by,

$$\log\left(\frac{A_t^N}{\bar{A}^N}\right) = \rho^{A^N} \log\left(\frac{A_{t-1}^N}{\bar{A}^N}\right) + \varepsilon_t^{A^N} \tag{3.17}$$

and

$$\log\left(\frac{A_t}{\bar{A}}\right) = \rho^A \log\left(\frac{A_{t-1}}{\bar{A}}\right) + \varepsilon_t^A \tag{3.18}$$

[3] Since optimising firms are randomly chosen and there are large number (continuum) of them the integral over the subset of a unit interval will be proportional to the integral over the entire unit interval.

3.2 Tradable Firms

Tradable firms indexed by $j \in [0,1]$ [4] produce differentiated goods for domestic consumption and foreign markets. Similar to the non-tradable firms the production takes place with the following technology,

$$y_t^T(j) = A_t^T k_t^T(j)^{\alpha^T} \left(A_t h_t^T(j)\right)^{1-\alpha^T} \tag{3.19}$$

Composite domestic and exported tradable goods are produced with Dixit Stiglitz prroduction technologies

$$y_t^{Td} = \left(\int_0^1 y_t^{Td}(j)^{\frac{\zeta^T-1}{\zeta^T}} dj\right)^{\frac{\zeta^T}{\zeta^T-1}} \tag{3.20}$$

and

$$y_t^{Tx} = \left(\int_0^1 y_t^{Tx}(j)^{\frac{\zeta^T-1}{\zeta^T}} dj\right)^{\frac{\zeta^T}{\zeta^T-1}} \tag{3.21}$$

Demand schedules for domestically consumed and exported tradable good varieties are

$$y_t^{Td}(j) = \left(\frac{P_t^{Td}(j)}{P_t^{Td}}\right)^{-\zeta^T} y_t^{Td} \tag{3.22}$$

$$y_t^{Tx}(j) = \left(\frac{P_t^{Tx}(j)}{P_t^{Tx}}\right)^{-\zeta^T} y_t^{Tx} \tag{3.23}$$

The aggregate price indices of domestic and exported composite tradable goods are given by

$$P_t^{Td} = \left(\int_0^1 P_t^{Td}(j)^{1-\zeta^T} dj\right)^{\frac{1}{1-\zeta^T}} \tag{3.24}$$

and

$$P_t^{Tx} = \left(\int_0^1 P_t^{Tx}(j)^{1-\zeta^T} dj\right)^{\frac{1}{1-\zeta^T}} \tag{3.25}$$

Nominal profits of the representative tradable firm is given as

$$\psi_t^T(j) = P_t^{Td}(j) y_t^{Td}(j) + e_t P_t^{Tx}(j) y_t^{Tx}(j) - R_t^k k_t^T(j) - W_t h_t^T(j) \tag{3.26}$$

Tradable firms maximize the net present value of their profits subject to the demand schedule they face and the production technology:

$$\max_{\left[P_t^{Td}(j), P_t^{Tx}(j), h_t(j)^T, k_t(j)^T\right]} \sum_{\tau=0}^{\infty} \left(\beta \xi^N\right)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}} \psi_{t+\tau}^T(j) \tag{3.27}$$

taking into account demand schedules (3.22) and (3.23)

The first-order conditions are

$$\frac{W_t}{P_t} = \nu_t(j)^T (1-\alpha^T) \frac{y_t^T(j)}{h_t^T(j)} \tag{3.28}$$

$$\frac{R_t^K}{P_t} = \nu_t(j)^T (\alpha^T) \frac{y_t^T(j)}{k_t^T(j)} \tag{3.29}$$

$$\tilde{P}_t^{Td} = \frac{\zeta^T}{\zeta^T - 1} \frac{\sum_{\tau=0}^{\infty} (\beta \xi^T)^\tau \lambda_{t+\tau} {P_{t+\tau}^{Td}}^{\zeta^T} y_{t+\tau}^{Td} \nu_{t+\tau}^T}{\sum_{\tau=0}^{\infty} (\beta \xi^T)^\tau \lambda_{t+\tau} {P_{t+\tau}^{Td}}^{\zeta^T - 1} \frac{P_{t+\tau}^{Td}}{P_{t+\tau}} y_{t+\tau}^{Td}} \tag{3.30}$$

[4] for simplicity we use the same index j

$$\tilde{P}_t^{Tx} = \frac{\zeta^T}{\zeta^T - 1} \frac{\sum_{\tau=0}^{\infty} (\beta\xi^T)^\tau \lambda_{t+\tau} {P_{t+\tau}^{Tx}}^{\zeta^T} y_{t+\tau}^{Tx} \nu_{t+\tau}^T}{\sum_{\tau=0}^{\infty} (\beta\xi^T)^\tau \lambda_{t+\tau} {P_{t+\tau}^{Tx}}^{\zeta^T - 1} \frac{e_{t+\tau} P_{t+\tau}^{Tx}}{P_{t+\tau}} y_{t+\tau}^{Tx}} \tag{3.31}$$

Again, price dynamics can be formulated recursively as,

$$j_{1,t}^{Td} = \lambda_t y_t^{Td} \nu_t^T + \beta\xi^T j_{1,t+1}^{Td} \left(\pi_{t+1}^{Td}\right)^{\zeta^T} \tag{3.32}$$

$$j_{2,t}^{Td} = \lambda_t \frac{P_t^{Td}}{P_t} y_t^{Td} + \beta\xi^T j_{2,t+1}^{Td} \left(\pi_{t+1}^{Td}\right)^{\zeta^T - 1} \tag{3.33}$$

Hence the optimal reset price over the market (domestic traded) price for domestically traded goods is given by,

$$\frac{\tilde{P}_t^{Td}}{P_t^{Td}} = \frac{\zeta^T}{\zeta^T - 1} \frac{j_{1,t}^{Td}}{j_{2,t}^{Td}} \tag{3.34}$$

Law of motions for prices are then expressed in terms of this ratio,

$$1 = \zeta^T {\pi_t^{Td}}^{\zeta^T - 1} + (1 - \zeta^T) \left(\frac{\zeta^T}{\zeta^T - 1} \frac{j_{1,t}^{Td}}{j_{2,t}^{Td}} \right)^{1 - \zeta^T} \tag{3.35}$$

And the price dispersion is given by ($\vartheta_t^{Td} = \int_0^1 \left(\frac{P_t^{Td}(j)}{P_t^{Td}} \right)^{-\zeta^T}$),

$$\vartheta_t^{Td} = \zeta^T \vartheta_{t-1}^{Td} \left(\pi_t^{Td}\right)^{\zeta^T} + (1 - \zeta^T) \left(\frac{\zeta^T}{\zeta^T - 1} \frac{j_{1,t}^{Td}}{j_{2,t}^{Td}} \right)^{-\zeta^T} \tag{3.36}$$

Similarly price dynamics in the export market is given as

$$j_{1,t}^{Tx} = \lambda_t y_t^{Tx} \nu_t^T + \beta\xi^T j_{1,t+1}^{Tx} \left(\pi_{t+1}^{Tx}\right)^{\zeta^T} \tag{3.37}$$

$$j_{2,t}^{Tx} = \lambda_t \frac{e_t P_t^{Tx}}{P_t} y_t^{Tx} + \beta\xi^T j_{2,t+1}^{Tx} \left(\pi_{t+1}^{Tx}\right)^{\zeta^T - 1} \tag{3.38}$$

$$\frac{\tilde{P}_t^{Tx}}{P_t^{Tx}} = \frac{\zeta^T}{\zeta^T - 1} \frac{j_{1,t}^{Tx}}{j_{2,t}^{Tx}} \tag{3.39}$$

$$1 = \zeta^T {\pi_t^{Tx}}^{\zeta^T - 1} + (1 - \zeta^T) \left(\frac{\zeta^T}{\zeta^T - 1} \frac{j_{1,t}^{Tx}}{j_{2,t}^{Tx}} \right)^{1 - \zeta^T} \tag{3.40}$$

$$\vartheta_t^{Tx} = \zeta^T \vartheta_{t-1}^{Tx} \left(\pi_t^{Tx}\right)^{\zeta^T} + (1 - \zeta^T) \left(\frac{\zeta^T}{\zeta^T - 1} \frac{j_{1,t}^{Tx}}{j_{2,t}^{Tx}} \right)^{-\zeta^T} \tag{3.41}$$

The demand for exports is analogously given by

$$y_t^{Tx} = \left(\frac{P_t^{Tx}}{P_t^{T*}} \right)^{-\mu_x^*} y_t^* \tag{3.42}$$

where y_t^* is an exogenously given stochastic process and we assume it is captured by the output of the rest of world.

$$\log \left(\frac{y_t^*}{\bar{y}^*} \right) = \rho^{y^*} \log \left(\frac{y_{t-1}^*}{\bar{y}^*} \right) + \varepsilon_t^{y^*} \tag{3.43}$$

The price ratio $\frac{P_t^{Tx}}{P_t^{T*}}$ is defined in equation (4.28). The productivity shock is given as

$$\log \left(\frac{A_t^T}{\bar{A}^T} \right) = \rho^{A^T} \log \left(\frac{A_{t-1}^T}{\bar{A}^T} \right) + \varepsilon_t^{A^T} \tag{3.44}$$

3.3 Importing Firms

Importing firms (indexed by $j \in [0,1]$) buy a composite tradable imports' bundle (actually not a bundle, right?) [5] from the rest of world and produce a differentiated importing varieties. The supply of the composite import bundle takes place in a perfectly competitive environment, while different import varieties are produced in a monopolistic competition. We assume Calvo price stickiness in the import market to allow for an incomplete path-through to import prices.

The aggregation in imports market is given by

$$y_t^{Tm} = \left(\int_0^1 y_t^{Tm}(j)^{\frac{\zeta^{Tm}-1}{\zeta^{Tm}}} dj \right)^{\frac{\zeta^{Tm}}{\zeta^{Tm}-1}} \tag{3.45}$$

Again the aggregate price index of the import bundle is given as

$$P_t^{Tm} = \left(\int_0^1 P_t^{Tm}(j)^{1-\zeta^{Tm}} dj \right)^{\frac{1}{1-\zeta^{Tm}}} \tag{3.46}$$

Profits of Importing Firms are given by

$$\phi_t^m(j) = P_t^{Tm}(j) y_t^{Tm}(j) - e_t P_t^{T*} y_t^{Tm}(j) \tag{3.47}$$

Importing firms maximise their profits as

$$\sum_{\tau=0}^{\infty} \left(\beta \xi^{Tm}\right)^{\tau} \frac{y_{t+\tau}^{Tm} \lambda_{t+\tau}}{P_{t+\tau} \lambda_t} \left[\left(\frac{P_t^{Tm}(j)}{P_{t+\tau}^{Tm}} \right)^{-\zeta^{Tm}} P_t^{Tm}(j) - \left(\frac{P_t^{Tm}(j)}{P_{t+\tau}^{Tm}} \right)^{-\zeta^{Tm}} e_{t+\tau} P_{t+\tau}^{T*} \right] \tag{3.48}$$

The demand The first order conditions of the importing firms is given as

$$P_t^{Tm} = \frac{\zeta^{Tm}}{\zeta^{Tm}-1} \frac{\sum_{\tau=0}^{\infty} \left(\beta \xi^{Tm}\right)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(P_{t+\tau}^{Tm}\right)^{\zeta^{Tm}} \frac{e_{t+\tau} P_{t+\tau}^{T*}}{P_{t+\tau}} y_{t+\tau}^{Tm}}{\sum_{\tau=0}^{\infty} \left(\beta \xi^{Tm}\right)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_{t+\tau}^{Tm} y_{t+\tau}^{Tm}}{P_{t+\tau}}} \tag{3.49}$$

Here $\frac{e_{t+\tau} P_{t+\tau}^{T*}}{P_{t+\tau}}$ is the real exchange rate and acts as a marginal cost for importing firms. Import price dynamics are described recursively

$$j_{1,t}^{Tm} = \lambda_t y_t^{Tm} rer_t + \beta \xi^{Tm} j_{1,t+1}^{Tm} \left(\pi_{t+1}^{Tm}\right)^{\zeta^{Tm}} \tag{3.50}$$

$$j_{2,t}^{Tm} = \lambda_t \{ \frac{P_t^{Tm}}{P_t} \} y_t^{Tm} + \beta \xi^{Tm} j_{2,t+1}^{Tm} \left(\pi_{t+1}^{Tm}\right)^{\zeta^{Tm}-1} \tag{3.51}$$

Here y_t^{Tm} describes the quantities imported and $\frac{P_t^{Tm}}{P_t}$ is the real price of imports and is given by equation (4.12).

$$1 = \xi^{Tm} {\pi_t^{Tm}}^{\zeta^{Tm}-1} + (1-\xi^{Tm}) \left(\frac{\zeta^{Tm}}{\zeta^{Tm}-1} \frac{j_{1,t}^{Tm}}{j_{2,t}^{Tm}} \right)^{1-\zeta^{Tm}} \tag{3.52}$$

$$\vartheta_t^{Tm} = \xi^{Tm} \vartheta_{t-1}^{Tm} \left(\pi_t^{Tm}\right)^{\zeta^{Tm}} + (1-\xi^{Tm}) \left(\frac{\zeta^{Tm}}{\zeta^{Tm}-1} \frac{j_{1,t}^{Tm}}{j_{2,t}^{Tm}} \right)^{-\zeta^{Tm}} \tag{3.53}$$

Since the supply of imported goods is given outside of the model we can ignore the price dispersion in the import market?

[5] There is some inconsistency in the story

4 Final goods producers and price ratios

The final domestically (produced) consumed goods are produced using domestically produced non-tradable goods and domestically produced and consumed tradable products with the CES technology

$$y_t^d = \left[(1-\gamma_d)^{\frac{1}{\mu_d}} {y_t^N}^{\frac{\mu_d-1}{\mu_d}} + \gamma_d^{\frac{1}{\mu_d}} {y_t^{Td}}^{\frac{\mu_d-1}{\mu_d}} \right]^{\frac{\mu_d}{\mu_d-1}} \tag{4.1}$$

Here γ_d is the share of domestic Tradables in the domestic goods in the steady state. μ_d is the elasticity of substitution between non-tradable and domestically absorbed tradable products. The profit maximization implies following demand schedules, Demands are given by,

$$y_t^N = (1-\gamma_d) \left(\frac{P_t^N}{P_t^d} \right)^{-\mu_d} y_t^d \tag{4.2}$$

$$y_t^{Td} = \gamma_d \left(\frac{P_t^{Td}}{P_t^d} \right)^{-\mu_d} y_t^d \tag{4.3}$$

Price index is of the domestically produced (consumables) goods,

$$P_t^d = \left[(1-\gamma_d) {P_t^N}^{1-\mu_d} + \gamma_d {P_t^{Td}}^{1-\mu_d} \right]^{\frac{1}{1-\mu_d}} \tag{4.4}$$

Similarly, the aggregate consumption bundle of the economy is given by CES combination of domestically produced and imported goods,

$$z_t = \left[(1-\gamma_m)^{\frac{1}{\mu_m}} {y_t^d}^{\frac{\mu_m-1}{\mu_m}} + \gamma_m^{\frac{1}{\mu_m}} {y_t^m}^{\frac{\mu_m-1}{\mu_m}} \right]^{\frac{\mu_m}{\mu_m-1}} \tag{4.5}$$

Corresponding demand equations are given by,

$$y_t^d = (1-\gamma_m) \left(\frac{P_t^d}{P_t} \right)^{-\mu_m} z_t \tag{4.6}$$

$$y_t^{Tm} = \gamma_m \left(\frac{P_t^{Tm}}{P_t} \right)^{-\mu_m} z_t \tag{4.7}$$

The CPI index is

$$P_t = \left[(1-\gamma_m) {P_t^d}^{1-\mu_m} + \gamma_m {P_t^{Tm}}^{1-\mu_m} \right]^{\frac{1}{1-\mu_m}} \tag{4.8}$$

Since price levels in this economy is indeterminate we need to express the problem in terms of price ratios. Thus it is a good time to define the price ratios. Divide the equation (4.4) by P_t^N and then P_t^{Td}:

$$\frac{P_t^N}{P_t^d} = \frac{1}{\left[1-\gamma_d + \gamma_d \left(\frac{P_t^{Td}}{P_t^N} \right)^{1-\mu_d} \right]^{\frac{1}{1-\mu_d}}} \tag{4.9}$$

$$\frac{P_t^{Td}}{P_t^d} = \frac{1}{\left[\gamma_d + (1-\gamma_d) \left(\frac{P_t^N}{P_t^{Td}} \right)^{1-\mu_d} \right]^{\frac{1}{1-\mu_d}}} \tag{4.10}$$

Similarly, divide by P_t^d and P_t^{Tm} to get:

$$\frac{P_t^d}{P_t} = \frac{1}{\left[1-\gamma_m + \gamma_m \left(\frac{P_t^{Tm}}{P_t^d} \right)^{1-\mu_m} \right]^{\frac{1}{1-\mu_m}}} \tag{4.11}$$

which is the real price of the domestic consumables. The real price of imports is given by

$$\frac{P_t^{Tm}}{P_t} = \frac{1}{\left[\gamma_m + (1-\gamma_m)\left(\frac{P_t^{Tm}}{P_t^d}\right)^{\mu_m - 1}\right]^{\frac{1}{1-\mu_m}}} \tag{4.12}$$

The terms of trade in the economy is defined as the ratio of import prices over the price of domestic goods.

$$\mathcal{T}_t = \frac{P_t^{Tm}}{P_t^d} \tag{4.13}$$

And also define the sectoral price differential

$$\mathcal{T}_t^{TN} = \frac{P_t^{Td}}{P_t^N} \tag{4.14}$$

Which are linked to respective inflation processes

$$\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\pi_t^m}{\pi_t^d} \tag{4.15}$$

$$\frac{\mathcal{T}_t^{TN}}{\mathcal{T}_{t-1}^{TN}} = \frac{\pi_t^{Td}}{\pi_t^N} \tag{4.16}$$

It is also a good time to define inflation for the domestic price index and the CPI. To do so, divide equaton (4.4) by P_{t-1}^d

$$\pi_t^d = \left[(1-\gamma_d)\left(\frac{P_t^N}{P_{t-1}^d}\frac{P_{t-1}^N}{P_{t-1}^N}\right)^{1-\mu_d} + \gamma_d\left(\frac{P_t^{Td}}{P_{t-1}^d}\frac{P_{t-1}^{Td}}{P_{t-1}^{Td}}\right)^{1-\mu_d}\right]^{\frac{1}{1-\mu_d}} \tag{4.17}$$

which becomes

$$\pi_t^d = \left[(1-\gamma_d)\left(\pi_t^N\frac{P_{t-1}^N}{P_{t-1}^d}\right)^{1-\mu_d} + \gamma^d\left(\pi_t^{Td}\frac{P_{t-1}^{Td}}{P_{t-1}^d}\right)^{1-\mu_d}\right]^{\frac{1}{1-\mu_d}} \tag{4.18}$$

And divide the equation (4.8):

$$\pi_t = \left[(1-\gamma_m)\left(\frac{P_t^d}{P_{t-1}}\frac{P_{t-1}^d}{P_{t-1}^d}\right)^{1-\mu_m} + \gamma_m\left(\frac{P_t^{Tm}}{P_{t-1}}\frac{P_{t-1}^{Tm}}{P_{t-1}^{Tm}}\right)^{1-\mu_m}\right]^{\frac{1}{1-\mu_m}} \tag{4.19}$$

$$\pi_t = \left[(1-\gamma_m)\left(\pi_t^d\frac{P_{t-1}^d}{P_{t-1}}\right)^{1-\mu_m} + \gamma_m\left(\pi_t^m\frac{P_{t-1}^{Tm}}{P_{t-1}}\right)^{1-\mu_m}\right]^{\frac{1}{1-\mu_m}} \tag{4.20}$$

The real prices in sectors, thus easily can be recovered. The real price non-tradable output is

$$\frac{P_t^N}{P_t} = \frac{P_t^N}{P_t^d}\frac{P_t^d}{P_t} \tag{4.21}$$

The real price of domestic tradables is

$$\frac{P_t^{Td}}{P_t} = \frac{P_t^{Td}}{P_t^d}\frac{P_t^d}{P_t} \tag{4.22}$$

For the real price of tradable exports define

$$\Psi_t^{Tx} = \frac{e_t P_t^{Tx}}{P_t^{Td}} \tag{4.23}$$

which we can express as a difference equation

$$\frac{\Psi_t^{Tx}}{\Psi_{t-1}^{Tx}} = \frac{e_t}{e_{t-1}} \frac{\pi_t^{Tx}}{\pi_t^{Td}} \tag{4.24}$$

Now the real price of exports is recovered as

$$\frac{eP_t^{Tx}}{P_t} = \Psi_t^{Tx} \frac{P_t^{Td}}{P_t} \tag{4.25}$$

The real exchange rate $\frac{eP_t^{T*}}{P_t}$ is also defined as a difference equation[6]

$$\frac{rer_t}{rer_{t-1}} = \frac{e_t}{e_{t-1}} \frac{\pi_t^{T*}}{\pi_t} \tag{4.26}$$

Where π_t^{T*} is an exogenously given foreign inflation processes

$$\log\left(\frac{\pi_t^{T*}}{\bar{\pi}^{T*}}\right) = \rho^{\pi^{T*}} \log\left(\frac{\pi_{t-1}^{T*}}{\bar{\pi}^{T*}}\right) + \varepsilon_t^{\pi^{T*}} \tag{4.27}$$

$$\frac{P_t^{Tx}}{P_t^{T*}} = \frac{eP_t^{Tx}}{P_t} \left(\frac{eP_t^{T*}}{P_t}\right)^{-1} = \frac{\frac{eP_t^{Tx}}{P_t}}{rer_t} \tag{4.28}$$

5 Oil sector

We assume exogenously given oil sector in the economy where both the production and oil prices follow stochastic processes. Specifically, quantities are given as

$$\log\left(\frac{y_t^O}{\bar{y}_t^O}\right) = \rho^{y^O} \log\left(\frac{y_{t-1}^O}{\bar{y}_t^O}\right) + \varepsilon_t^{y^O} \tag{5.1}$$

The stochastic process for the oil price is defined in real terms $\frac{P_t^O}{P_t^{T*}}$ to convert the price of oil into the unit of final goods in domestic currency

$$\frac{eP_t^O}{P_t} = \frac{e_t P_t^{T*}}{P_t} \frac{P_t^O}{P_t^{T*}} = rer_t \frac{P_t^O}{P_t^{T*}} \tag{5.2}$$

Define $p_t^O = \frac{P_t^O}{P_t^{T*}}$. Hence the stochastic process for the oil price is

$$\log\left(\frac{p_t^O}{\bar{p}_t^O}\right) = \rho^{p^O} \log\left(\frac{p_{t-1}^O}{\bar{p}_t^O}\right) + \varepsilon_t^{p^O} \tag{5.3}$$

The oil GDP in terms of the final consumption good is given by

$$gdp_t^O = rer_t p_t^O y_t^O \tag{5.4}$$

We assume that the fraction of oil GDP is transferred to the government budget

$$oT_t = \tau_t^{oT} gdp_t^O \tag{5.5}$$

Where τ^{oT} is an exogenously time varying parameter which follows a stochastic process

$$\log\left(\frac{\tau_t^{oT}}{\bar{\tau}^{oT}}\right) = \rho^{oT} \log\left(\frac{\tau_{t-1}^{oT}}{\bar{\tau}^{oT}}\right) + \varepsilon_t^{oT} \tag{5.6}$$

[6]It assumed that all goods are tradable in the foreign economy, or The Non-Tradable price is 1 and constant

6 Government

Government revenues are made of by lump-sum taxes from households and oil Transfers. Government also earns a seniorage

$$g_t = \frac{T_t}{P_t} + oT_t + m_t - \frac{m_{t-1}}{\pi_t} \tag{6.1}$$

And the government consumption shock is given as,

$$\log\left(\frac{g_t}{\bar{g}}\right) = \rho_g \log\left(\frac{g_{t-1}}{\bar{g}}\right) + \epsilon_t^g \tag{6.2}$$

For the monetary policy we consider two alternative regimes. Under the flexible exchange rate the policy is described as

$$\log\left(\frac{r_t}{\bar{r}}\right) = \rho_r \log\left(\frac{r_{t-1}}{\bar{r}}\right) + (1-\rho_r)\left[\theta^{\pi}\log\left(\frac{\pi_t}{\bar{\pi}}\right) + \theta^{gdp}\log\left(\frac{gdp_t}{\bar{gdp}}\right)\right] + \varepsilon_t^M \tag{6.3}$$

where ε_t^M is a monetary policy shock.
Under the peg the policy is given by

$$\frac{e_t}{e_{t-1}} = 1 + \varepsilon_t^M \tag{6.4}$$

7 Aggregation and Consolidated Budget Constraint

The total labour demand across sectors are given by,

$$h_t^d = \int_0^1 h_t^T(j)d(j) + \int_0^1 h_t^N(j)d(j) \tag{7.1}$$

The total capital demand across sectors are given by,

$$u_t k_{t-1} = \int_0^1 k_t^T(j)d(j) + \int_0^1 k_t^N(j)d(j) \tag{7.2}$$

$$k_t^N = \int_0^1 k^N(j)dj \tag{7.3}$$

$$h_t^N = \int_0^1 h^N(j)dj \tag{7.4}$$

$$k_t^T = \int_0^1 k^T(j)dj \tag{7.5}$$

$$h_t^T = \int_0^1 h^T(j)dj \tag{7.6}$$

Integrating over non-tradble firms

$$Y_t^N = \int_0^1 y^N(j)dj = A_t^N k_t^{N\alpha^N}\left(Ah_t^N\right)^{1-\alpha^N} = \vartheta_t^N y_t^N \tag{7.7}$$

$$Y_t^T = \int_0^1 y^T(j)dj = A_t^T k_t^{T\alpha^T}\left(A_t h_t^T\right)^{1-\alpha^T} = \int_0^1 \left(\frac{P_t^{Td}(j)}{P_t^{Td}}\right)^{-\zeta^T} y_t^{Td} + \int_0^1 \left(\frac{P_t^{Tx}(j)}{P_t^{Tx}}\right)^{-\zeta^T} y_t^{Tx} \tag{7.8}$$

Hence,

$$Y_t^T = A_t^T k_t^{T\,\alpha^T} \left(Ah_t^T\right)^{1-\alpha^T} = \vartheta_t^{Td} y_t^{Td} + \vartheta_t^{Tx} y_t^{Tx} \tag{7.9}$$

The real non-oil GDP is defined as the real value of domestically produced goods including export

$$gdp_t = \frac{P_t^N}{P_t} y_t^N + \frac{P_t^{Td}}{P_t} y_t^{Td} + \frac{e_t P_t^{Tx}}{P_t} y_t^{Tx} \tag{7.10}$$

The domestic absorption is comprised of private and government consumption, investment, utilization and investment adjustment costs

$$z_t = c_t + g_t + i_t + a_t(u)k_{t-1} \tag{7.11}$$

The domestic absorption z_t is spent on domestically produced goods (non-tradable and tradable) and imports

$$z_t = \frac{P_t^N}{P_t} y_t^N + \frac{P_t^{Td}}{P_t} y_t^{Td} + \frac{P^{Tm}}{P_t} y^{Tm} \tag{7.12}$$

which implies that the non-oil gdp can be expressed as

$$gdp_t = z_t + nx_t \tag{7.13}$$

where nx_t is the net non-oil exports

$$nx_t = \frac{e_t P_t^{Tx}}{P_t} y_t^{Tx} - \frac{P^{Tm}}{P_t} y^{Tm} \tag{7.14}$$

It can be shown (see the Appendix) the net exports evolves according to

$$nx_t + oT_t = \left(\frac{b_t^f}{\phi_t r_t^*} - \frac{e_t}{e_{t-1}} \frac{b_{t-1}^f}{\pi_t} \right) \tag{7.15}$$

Hence it is also true that

$$gdp_t + oT_t = z_t + \left(\frac{b_t^f}{\phi_t r_t^*} - \frac{e_t}{e_{t-1}} \frac{b_{t-1}^f}{\pi_t} \right) \tag{7.16}$$

Or domestic absorption is equal to the non-oil GDP plus oil transfers less than (or plus) net foreign currency (deposit or lending) holdings (net foreign debt, $d_t^f = -b_t^f$).