Hw 7 Oscar Galindo

Vectors:

SP1:

Find the sums **A+B** and **C+D**, and the differences **A-B** and **C-D** if the vectors **A, B, C,** and **D** are given as follows:

$$A=[7.2 \quad -4.3 \quad 0.6 \quad 1.7] \qquad B=[-11.0 \quad 11.8 \quad 2.4 \quad -1.9]$$

$$C=\begin{bmatrix}1.7\\1.0\\-1.0\\4.3\end{bmatrix} \qquad D=\begin{bmatrix}-2.4\\-0.7\\-6.8\\3.0\end{bmatrix}$$

$$A+B = [(7.2-11.0), (-4.3+11.8), (0.6+2.4), (1.7-1.9)]$$

$$\boxed{A+B = [-3.8, 7.5, 3.0, -0.2]}$$

$$C+D = \begin{bmatrix}1.7+(-2.4)\\1.0+(-0.7)\\-1.0+(-6.8)\\4.3+3.0\end{bmatrix} \Rightarrow \boxed{C+D = \begin{bmatrix}-0.7\\-0.3\\-7.8\\7.3\end{bmatrix}}$$

$$A-B = [(7.2+11.0), (-4.3-11.8), (0.6-2.4), (1.7+1.9)]$$

$$\boxed{A-B = [18.2, -16.1, -1.8, 3.6]}$$

$$C-D = \begin{bmatrix}1.7+2.4\\1.0+0.7\\-1.0+6.8\\4.3-3.0\end{bmatrix} \Rightarrow \boxed{C-D = \begin{bmatrix}4.1\\1.7\\5.8\\1.3\end{bmatrix}}$$

SP2:
Given the vectors specified in SP1, find the following:
$R_1 = 3A - 2B$ $\qquad$ $R_2 = 5C + 2D$

$R_1 = 3A - 2B$

$\Rightarrow 3[7.2 \quad -4.3 \quad 0.6 \quad 1.7] - 2[-11.0 \quad 11.8 \quad 2.4 \quad -1.9]$

$\Rightarrow [21.6 \quad -12.9 \quad 1.8 \quad 5.1] - [-22.0 \quad 23.8 \quad 4.8 \quad -3.8]$

$\boxed{R_1 = [43.6 \quad -36.7 \quad -3 \quad 8.9]}$

$R_2 = 5C + 2D$

$$\Rightarrow 5\begin{bmatrix}1.7\\1.0\\-1.0\\4.3\end{bmatrix} + 2\begin{bmatrix}-2.4\\-0.7\\-6.8\\3.0\end{bmatrix} \Rightarrow \begin{bmatrix}8.5\\5\\-5\\21.5\end{bmatrix} + \begin{bmatrix}-4.8\\-1.4\\-13.6\\6.0\end{bmatrix}$$

$$\boxed{R_2 = \begin{bmatrix}3.7\\3.6\\-18.6\\27.5\end{bmatrix}}$$

SP3:
$E = [7 \quad -1 \quad 4 \quad 2 \quad -8]$ $\qquad$ $F = [1 \quad 2 \quad 9 \quad 0 \quad -4]$

Find $\mathbf{R_3}$ so that
$2E - 3F + R_3 = 0$

$2[7 \quad -1 \quad 4 \quad 2 \quad -8] - 3[1 \quad 2 \quad 9 \quad 0 \quad -4] + R_3 = 0$

$[14 \quad -2 \quad 8 \quad 4 \quad -16] - [3 \quad 6 \quad 18 \quad 0 \quad -12] + R_3 = 0$

$[11 \quad -8 \quad -10 \quad 4 \quad -4] + R_3 = 0$

$\boxed{R_3 = [-11 \quad 8 \quad 10 \quad -4 \quad 4]}$

SP4:

Find the vector **R** from this expression (using vectors in SP1):

$R=(A\cdot B)(2A+B)$

$R = [79.2 \quad -50.74 \quad 1.44 \quad -3.23] \cdot [(14.4 \quad -8.6 \quad 1.2 \quad 3.4) + ((-11) \quad 11.8 \quad 2.4 \quad -1.9)]$

$R = [79.2 \quad -50.74 \quad 1.44 \quad -3.23][3.4 \quad 3.2 \quad 3.6 \quad 1.5]$

$\boxed{R = [269.28 \quad -162.368 \quad 5.184 \quad -4.845]}$

SP5:

Find the component of the vector G in the direction of the vector H and the angle, α, between the two vectors for:

$G=[2 \quad -3 \quad 5] \qquad H=[1 \quad 4 \quad -2]$

$$\cos\theta = \frac{G\cdot H}{|H|} = \frac{2 \quad -12 \quad -10}{\sqrt{21}}$$

$$|G| = \sqrt{2^2 + (-3)^2 + (5)^2} = \sqrt{38}$$

$$|H| = \sqrt{(1)^2 \quad (4)^2 \quad (-2)^2} = \sqrt{21}$$

$$\theta = \cos^{-1}\left(\frac{-15}{\sqrt{38}\cdot\sqrt{21}}\right) = 122.07^{\circ}$$

Matrices

SP6:

Determine the matrix **C** given by:

$C = 3A - 2B$ Note A and B are

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ -4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -3 \\ 1 & 2 \\ -2 & -5 \end{bmatrix}$$

$$3A = \begin{bmatrix} 6 & -3 \\ 0 & 9 \\ -12 & 3 \end{bmatrix} \qquad 2B = \begin{bmatrix} 8 & -6 \\ 2 & 4 \\ -4 & -10 \end{bmatrix}$$

$$C = 3A - 2B$$

$$\boxed{C = \begin{bmatrix} -2 & 3 \\ -2 & 5 \\ -8 & 13 \end{bmatrix}}$$

SP7:

Find the product **CD**:

$$C = \begin{bmatrix} 4 & 0 & -2 & 1 \\ 3 & -2 & 4 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 4 \end{bmatrix}$$

$$CD = \begin{matrix} 4(3) + 0(-2) + (-2)1 + 1(4) \\ 3(3) + (-2)(-2) + (4)(1) + 3(4) \end{matrix} = \begin{bmatrix} 14 \\ 29 \end{bmatrix}$$

SP8:
Find the product **CED**, where **E** is defined as:

$$E=\begin{bmatrix} -2 & 1 & 9 & -2 \\ 3 & -1 & 2 & 7 \\ 0 & -2 & -3 & -9 \\ -5 & 7 & 1 & 6 \end{bmatrix}$$

SP9:
Find the products **FG** and **GF** for the matrices below:

$$F=\begin{bmatrix} -1 & 2 & 2 & 6 \\ 7 & -3 & -4 & 0 \end{bmatrix} \qquad G=\begin{bmatrix} 6 & 3 \\ -1 & 0 \\ 0 & -4 \\ 2 & 1 \end{bmatrix}$$

$$FG = \begin{bmatrix} (-1)(6) + (2)(-1) + (2)(0) + (6)(2) & (-1)(3) + (2)(0) + (2)(-4) + 6(1) \\ (7)(6) + (-3)(-1) + (-4)(0) + (0)(2) & (7)(3) + (-3)(0) + (-4)(-4) + 0(1) \end{bmatrix}$$

$$FG = \begin{bmatrix} 4 & -5 \\ 45 & 37 \end{bmatrix}$$

$$GF = \begin{bmatrix} 6(-1) + 3(7) & 6(2) + 3(-3) & 6(2) + 3(-4) & 6(6) + 3(0) \\ (-1)(-1) + 0(7) & -1(2) + 0(-3) & -1(2) + 0(-4) & -1(6) + 0(0) \\ 0(-1) - 4(7) & 0(2) + -4(-3) & 0(2) + -4(-4) & 0(6) + -4(0) \\ 2(-1) + 1(7) & 2(2) + 1(-3) & 2(4) + 1(-4) & 2(6) + 1(0) \end{bmatrix}$$

$$GF = \begin{bmatrix} 15 & 3 & 0 & 36 \\ 1 & -2 & -2 & -6 \\ -28 & 12 & 16 & 0 \\ 5 & 1 & 0 & 12 \end{bmatrix}$$

SP10:
Find the products HJ and JH for the following:

$$H=\begin{bmatrix}1 & 3 & 0\\ 2 & -1 & 0\\ 3 & 2 & 0\end{bmatrix} \qquad J=\begin{bmatrix}0 & 0 & 0\\ 0 & -0 & 0\\ 2 & 5 & 7\end{bmatrix}$$

$$HJ = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 6 & 10 & 0\end{bmatrix}$$

$$JH = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 33 & 15 & 0\end{bmatrix}$$

SP11:
Use the matrices $\mathbf{B_1}$, $\mathbf{B_2}$, $\mathbf{B_3}$ to show that the following is satisfied:

$$(B_1 B_2 B_3)^T = B_3^T B_2^T B_1^T$$

$$B_1=\begin{bmatrix}-4 & 1\\ 2 & 3\end{bmatrix} \quad B_2=\begin{bmatrix}2 & 1\\ 0 & -3\end{bmatrix} \quad B_3=\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}$$

$$B_{123} = \begin{bmatrix}-17 & -12\\ 14 & -36\end{bmatrix} \qquad B_{123}^T = \begin{bmatrix}-17 & 14\\ -12 & -36\end{bmatrix}$$

$$B_1^T = \begin{bmatrix}-4 & 2\\ 1 & 3\end{bmatrix} \qquad B_2^T = \begin{bmatrix}2 & 0\\ 1 & 3\end{bmatrix} \qquad B_3^T = \begin{bmatrix}3 & -1\\ -2 & 4\end{bmatrix}$$

$$B_1^T B_2^T B_3^T = \begin{bmatrix}-17 & 14\\ -12 & -36\end{bmatrix}$$

SP12:

Solve the following set of equations for *x, y,* and *z* using Cramer's Rule:

$$x+2y+3z=-5$$
$$3x+y-3z=4$$
$$-3x+4y+7z=-7$$

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{bmatrix} = 1\begin{bmatrix} 1 & -3 \\ 4 & 7 \end{bmatrix} - 2\begin{bmatrix} 3 & -3 \\ -3 & 7 \end{bmatrix} + 3\begin{bmatrix} 3 & 1 \\ -3 & 4 \end{bmatrix}$$

$$\Rightarrow 1(7+12) - 2(21-9) + 3(12+3)$$

$$\boxed{M = 40}$$