The PMT positional smearing is calculated by first simulating events along the PMTs with tracking turned on. The distribution of the position of the highest energy Compton scatter is plotted, relative to the position of the PMT in which it originated. The fit is performed for positive $\Delta\rho$ ($\rho = \sqrt{x^2 + y^2}$) and Δz, but the smearing should be performed symmetrically around 0. The fits are shown in Figures 1 and 2.

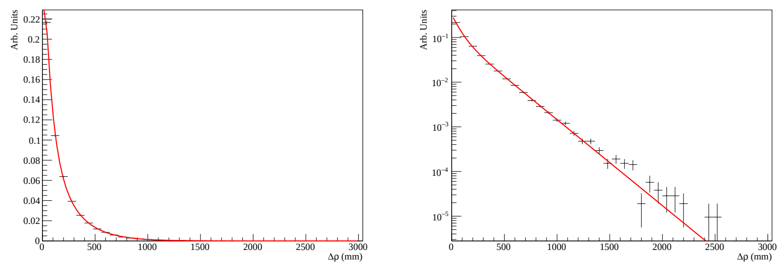

Figure 1: A fit to the $\Delta\rho$ position of the highest energy Compton scattering relative to the PMT it originated in.

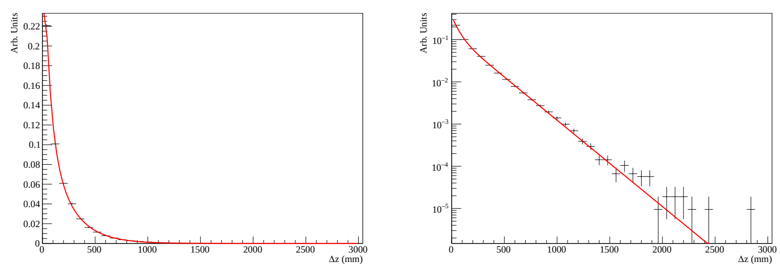

Figure 2: A fit to the Δz position of the highest energy Compton scattering relative to the PMT it originated in.

The fit is performed using the sum of two exponentials:

$$A_1 e^{-r/r_1} + A_2 e^{-r/r_2}. \tag{1}$$

The results of the fit to the $\Delta\rho$ and Δz distributions are shown in Table 1.

	A_1	A_2	r_1	r_2
ρ	0.2033	0.1218	68.85	225.64
z	0.2206	0.1372	54.85	212.29

Table 1: The fitted parameters using Equation 1.

Figure 3 shows the $\Delta\rho$ distribution, extracted from the tracking information, compared to the smeared PMT position using the results from Table 1. Notably the full code for the smearing is given on the next page. The normalization for the first exponential is calculated as:

$$N = (r_1 \times A_1)/(r_1 \times A_1 + R_2 \times A_2) = 0.3374, \tag{2}$$

and the second is:

$$N = (r_2 \times A_2)/(r_1 \times A_1 + R_2 \times A_2) = 0.6626. \tag{3}$$

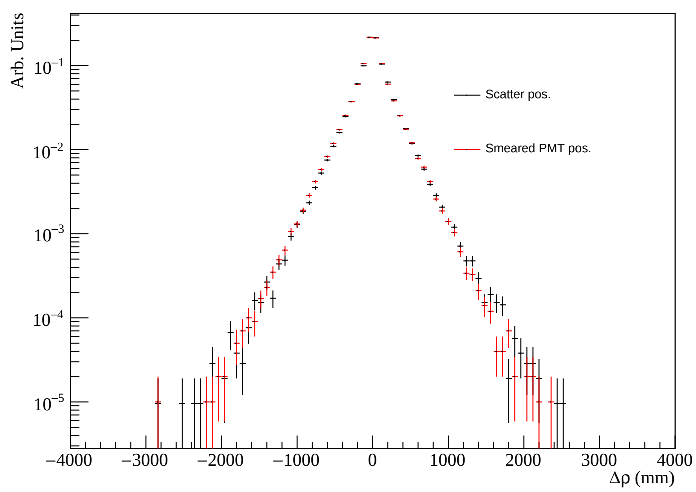

Figure 3: The truth $\Delta\rho$ distribution (black) compared to the $\Delta\rho$ distribution generated from smearing the PMT position according to the fit results in Table 1 (red).

To clarify the procedure for the PMT smearing, I enumerate the steps here:

1. Either choose a random event position (ρ, z) that should correspond to a position of a PMT or use the MC truth information to determine the location of the PMT.

2. Smear the PMT position in ρ according to the procedure outlined below, which yield a value for $\Delta\rho$.

3. Smear the PMT position in ρ according to the procedure outlined below, which yield a value for Δz.

4. The new position is $(\rho + \Delta\rho,\, z + \Delta z)$

```
def pmt_rho_smearing():

    sign = 1
    if np.random.uniform(0, 1) < 0.5:
        sign = -1

    if np.random.uniform(0, 1) < 0.3374:
        r = np.random.exponential(scale=68.85)
        return sign*r

    r = np.random.exponential(scale=225.64)
    return sign*r

def pmt_z_smearing():

    sign = 1
    if np.random.uniform(0, 1) < 0.5:
        sign = -1

    if np.random.uniform(0, 1) < 0.2935:
        r = np.random.exponential(scale=54.85)
        return sign*r

    r = np.random.exponential(scale=212.29)
    return sign*r
```