Chemical prepattern and reaction-diffusion models for pigmentation

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System of reacting and diffusing morphogens could generate a chemical pre-pattern within the developing integument via Turing instability

그 자체로 구조 형성물질

외피

steady state
stable – diffusion X
unstable – diffusion O

Subsequent coat pattern

↑ reflect

Chemical prepattern via differential response of the pigment cell precursors

(= melanoblast)
멜라닌 형성세포

Reaction-diffusion theory of pigmentation
→ Capacity to replicate observed pigment pattern through a single mechanism

Assumption
a) Depth of integument is small
b) initial pigmentation pattern is fixed over relatively short timescale early

Reaction takes place on 2D domain
With periodic boundary condition

equation :

$$\begin{cases} \dfrac{\partial u}{\partial t} = D_u \nabla^2 u + k_1 \left(v - \dfrac{uv}{1+v^2} \right) \\ \dfrac{\partial v}{\partial t} = D_v \nabla^2 v + k_2 - v - \dfrac{uv}{1+v^2} \end{cases}$$

- D_u, D_v, k_1, k_2 : constant

```
nu(ix) = Du*A*u(ix)+u(ix) + dt*k1*(v(ix)-(u(ix).*v(ix))./(1+v(ix).^2));
nv(ix) = Dv*A*v(ix)+v(ix) + dt*(k2-v(ix)-4*(u(ix).*v(ix))./(1+v(ix).^2));
```

Initial distribution

$$(u_0, v_0)$$

$$= (\bar{u} + 0.01\,\mathrm{rand}, \bar{v} + 0.01\,\mathrm{rand})$$

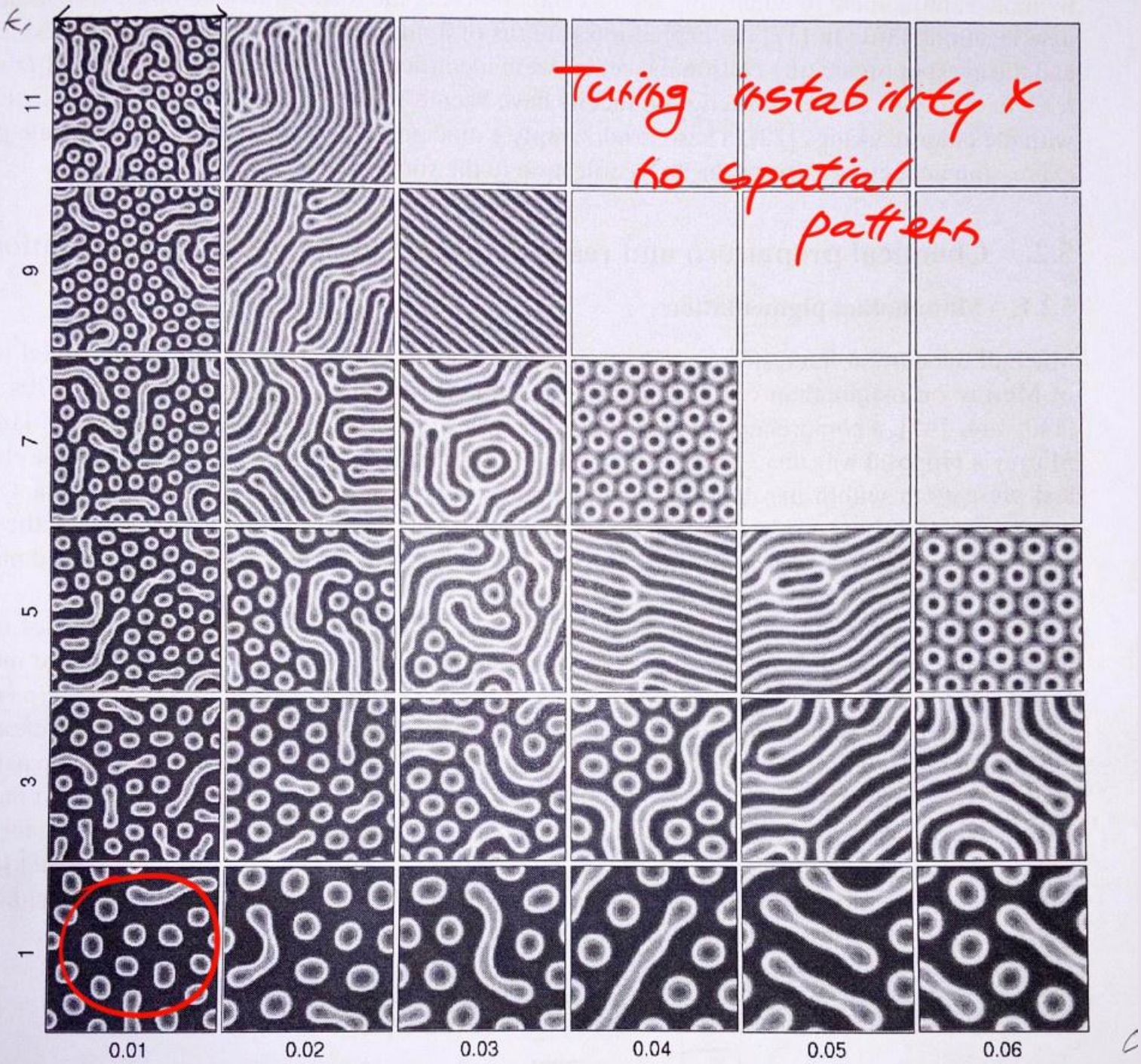
small random perturbation

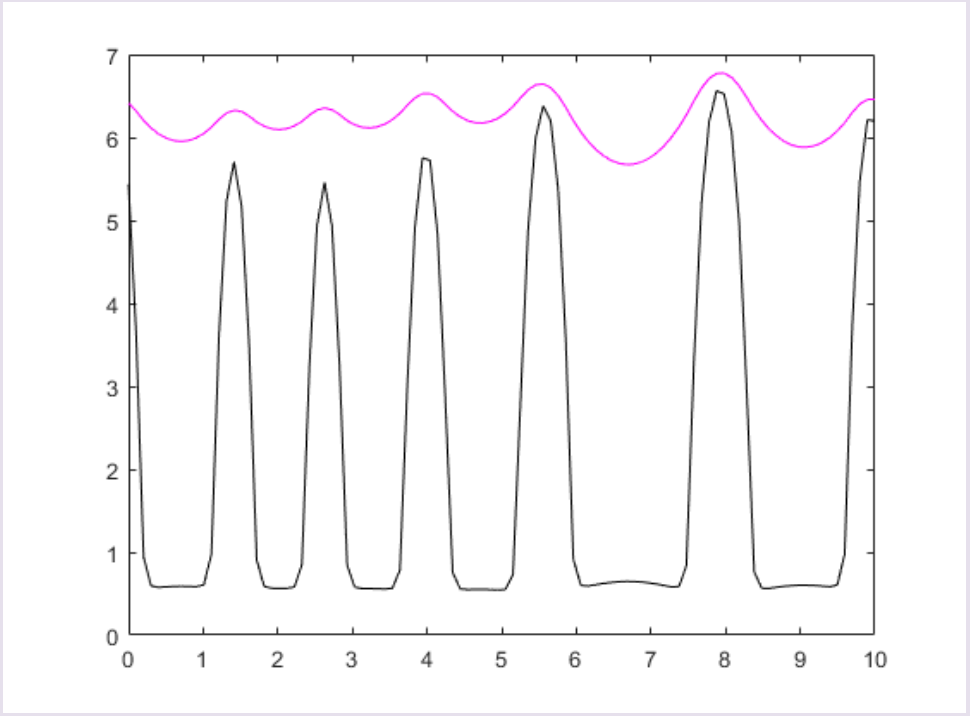
$(\bar{u}, \bar{v}$: uniform steady state$)$

$$D_u = 1.0, \quad k_2 = 11.0$$

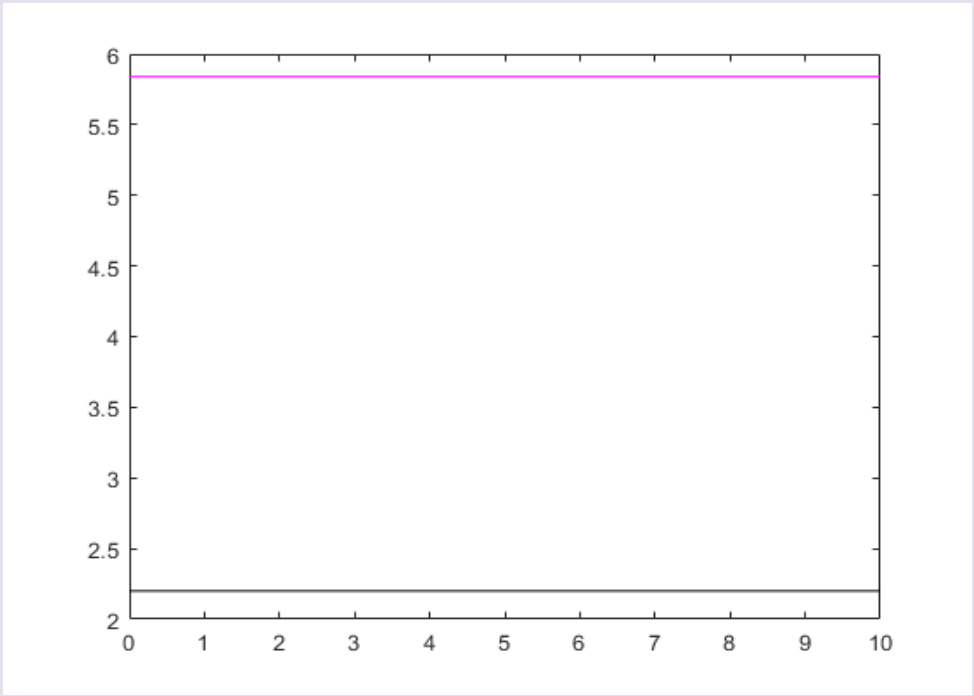
D_v (horizontal axis)

k_1 (vertical axis)


k
11
9
7
5
3
1
0.01
0.02
0.03
0.04
0.05
0.06
Turing instability x
no spatial
pattern



Dv = 0.01
K1 = 1



Dv = 0.06
K1 = 11