Chemical prepattern and reaction-diffusion models for pigmentation (2D)

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System of reacting and diffusing morphogens 3차원 구조 형성물질
could generate a chemical pre-pattern
within the developing integument 외피 via Turing instability

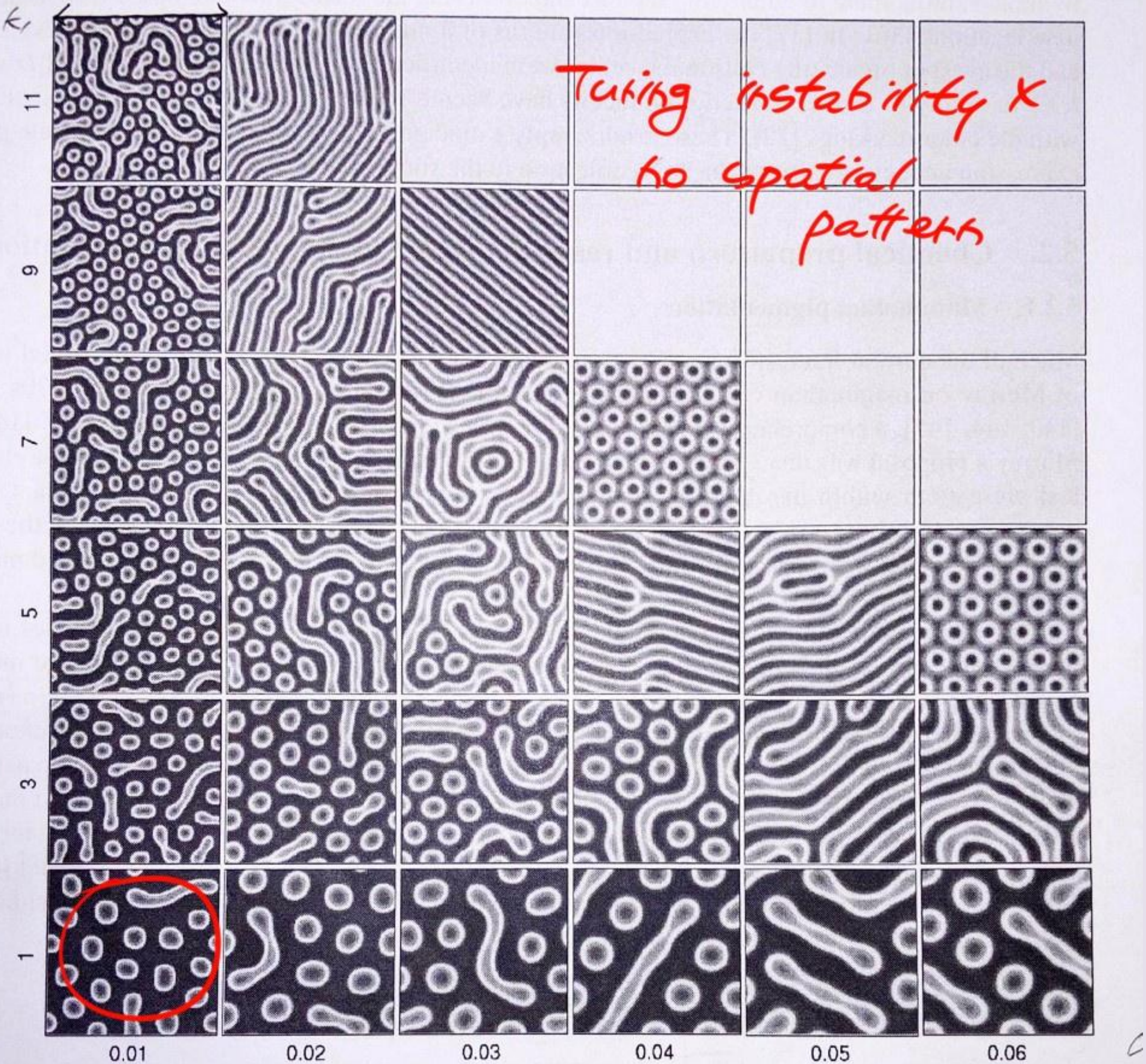
steady state
stable – diffusion X
unstable – diffusion O

Subsequent coat pattern

↑ reflect

Chemical prepattern via differential response
of the pigment cell precursors (= melanoblast) 멜라닌 형성세포

Reaction-diffusion theory of pigmentation
→ Capacity to replicate observed pigment pattern
through a single mechanism



k_1
11
9
7
5
3
1
0.01
0.02
0.03
0.04
0.05
0.06
c
Turing instability ✗
no spatial
pattern

```
% set the parameters (spatial discretization)
xright=10; nx=50;
yright=10; ny=floor(nx*yright/xright);
h=xright/nx;
x=linspace(-0.5*h,xright+0.5*h,nx+2)';
y=linspace(-0.5*h,yright+0.5*h,ny+2);
```

```
% set the parameters (governing equation)
Du=1;
Dv=0.06;
k1=1;
k2=11;
ubar=1+0.04*k2^2;
vbar=0.2*k2;
```

fix

steady state

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + k_1 \left(v - \frac{uv}{1+v^2} \right)$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + k_2 - v - \frac{4uv}{1+v^2}$$

```
% set the parameters (time discretization)
dt=0.1*h^2;
maxit=80000;
nn=1000;

% set the initial condition
u=ubar+0.1*(2*rand(nx+2,ny+2)-1);
v=vbar+0.1*(2*rand(nx+2,ny+2)-1);
nu=u; nv=v;
```

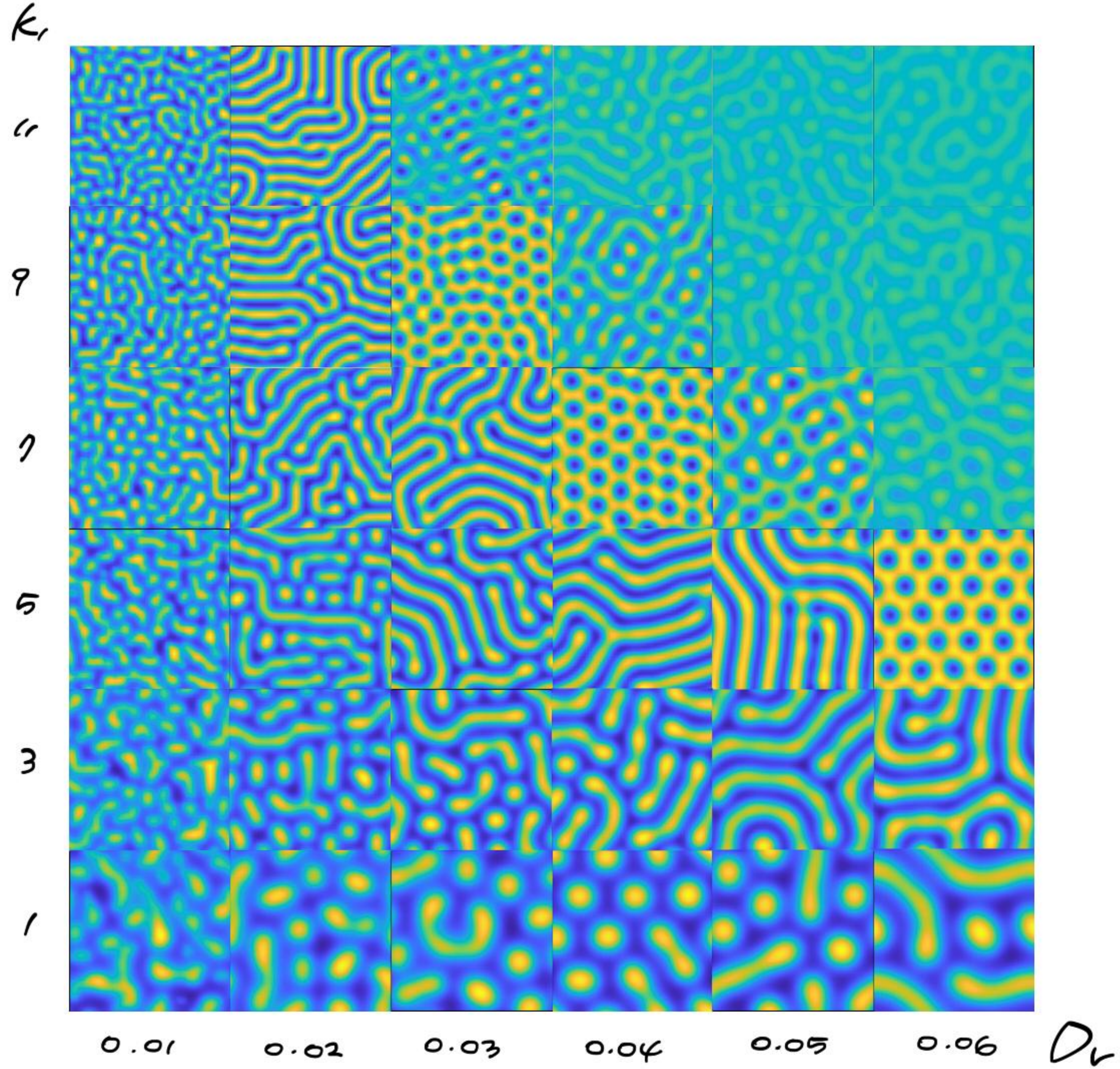
random
perturbation

```
% numerical scheme
for it=1:maxit
    % periodic boundary condition
    u(2:end-1,1)=u(2:end-1,end-1);
    u(2:end-1,end)=u(2:end-1,2);
    u(1,:)=u(end-1,:);
    u(end,:)=u(2,:);
    v(2:end-1,1)=v(2:end-1,end-1);
    v(2:end-1,end)=v(2:end-1,2);
    v(1,:)=v(end-1,:);
    v(end,:)=v(2,:);
```

```
% set the source terms
F=u(2:end-1,2:end-1).*v(2:end-1,2:end-1) ...
    ./(1+v(2:end-1,2:end-1).^2);
f=k1*(v(2:end-1,2:end-1)-F);
g=k2-v(2:end-1,2:end-1)-4*F;

% solve the Eqs. (18) - (19)
nu(2:end-1,2:end-1)=u(2:end-1,2:end-1)+dt*(f+Du*lap(u,h));
nv(2:end-1,2:end-1)=v(2:end-1,2:end-1)+dt*(g+Dv*lap(v,h));


function y=lap(s,h)

y=(s(1:end-2,2:end-1)+s(3:end,2:end-1)+s(2:end-1,1:end-2) ...
        +s(2:end-1,3:end)-4*s(2:end-1,2:end-1))/h^2;
```

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + k_1 \left(v - \frac{uv}{1+v^2} \right)$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + k_2 - v - \frac{4uv}{1+v^2}$$



k1
11
9
7
5
3
1
0.01
0.02
0.03
0.04
0.05
0.06
Dr