Complex Patterns in a Simple System

2017010698
수학과 오서영

Regular patterns : concentrations of chemically reacting and diffusing systems.

$$U + 2V \rightarrow 3V$$

$$V \rightarrow P$$

Simple **reaction-diffusion model**
~ due to **Gray and Scott**
- Irreversible, P : an inert product.
- A nonequilibrium constraint is represented by a feed term for U.
-Both U and V are removed by the feed process

A variety of spatio-temporal patterns form in response to finite-amplitude perturbations.
- 1D) steady spatial patterns could form even when the diffusion coefficients were **equal**.
- 1D) The response of the is nontrivial and depends both on the control parameters and on the initial perturbation.

Reaction - Diffusion equations

$$\frac{\partial U}{\partial t} = D_u \nabla^2 U - UV^2 + F(1 - U)$$

$$\frac{\partial V}{\partial t} = D_v \nabla^2 V + UV^2 - (F + k)V$$

k : the dimensionless rate constant of the second reaction

F : the dimensionless feed rate

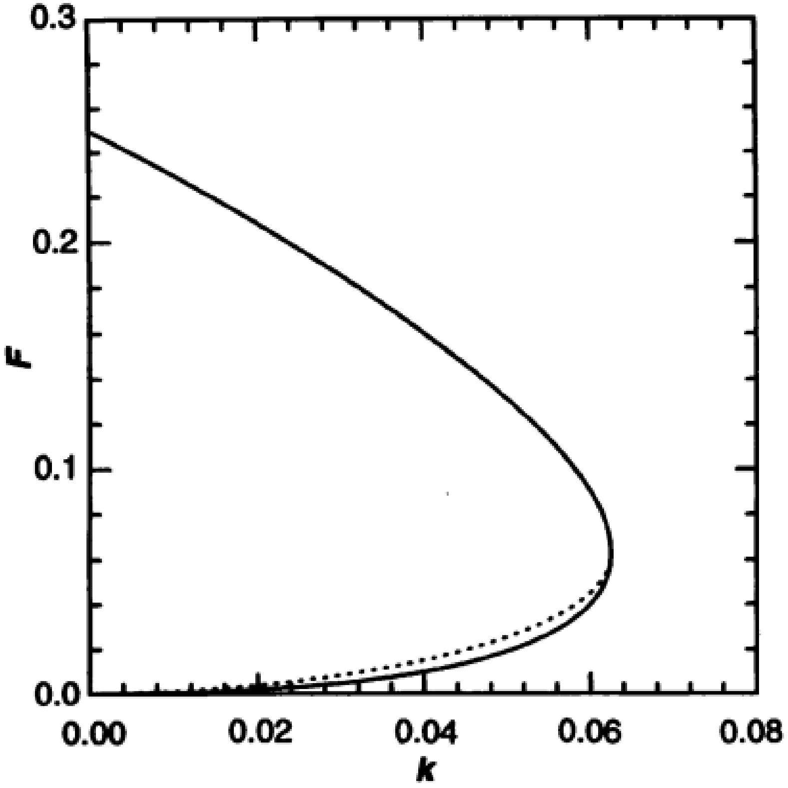
Du = 2 x 10^-5 , Dv = 10^-5

The boundary conditions are periodic.

Trivial steady-state solution :

U = 1, V = 0

- linearly stable for all positive F and k.
- system has two stable steady states.
- For fixed k, the nontrivial stable uniform solution loses stability through saddle-node bifurcation as F is increased through the upper solid line or as F is decreased through the dotted line.
- bifurcating periodic solution is stable for k < 0.035 and unstable for k > 0.035.
- There are no periodic orbits for parameter values outside the region enclosed by the solid line.
- The trivial state is linearly stable and globally attracting.
- Small perturbations decay exponentially but larger perturbations result in a long excursion through phase space before the system returns to the trivial state.

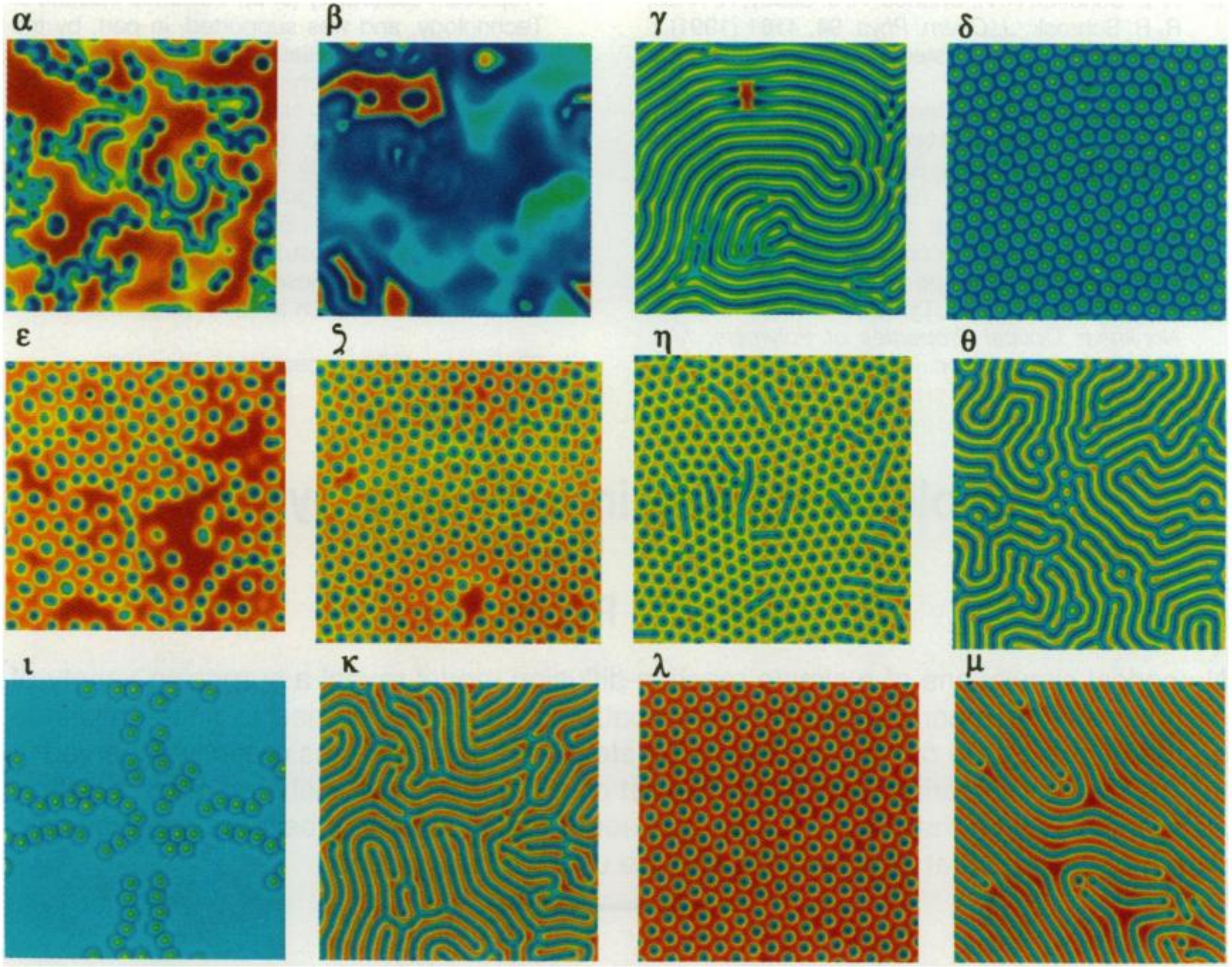


The simulations are forward Euler integrations of the finite-difference equations resulting from discretization of the diffusion operator.

Initially, the entire system was placed in the trivial state ($U = 1, V = 0$).

The center of the grid was then perturbed to ($U = 1/2, V = 1/4$).

The initial disturbance propagated outward from the central square, leaving patterns in its wake, until the entire grid was affected by the initial square perturbation. The propagation was wave-like, with the leading edge of the perturbation moving with an approximately constant velocity.

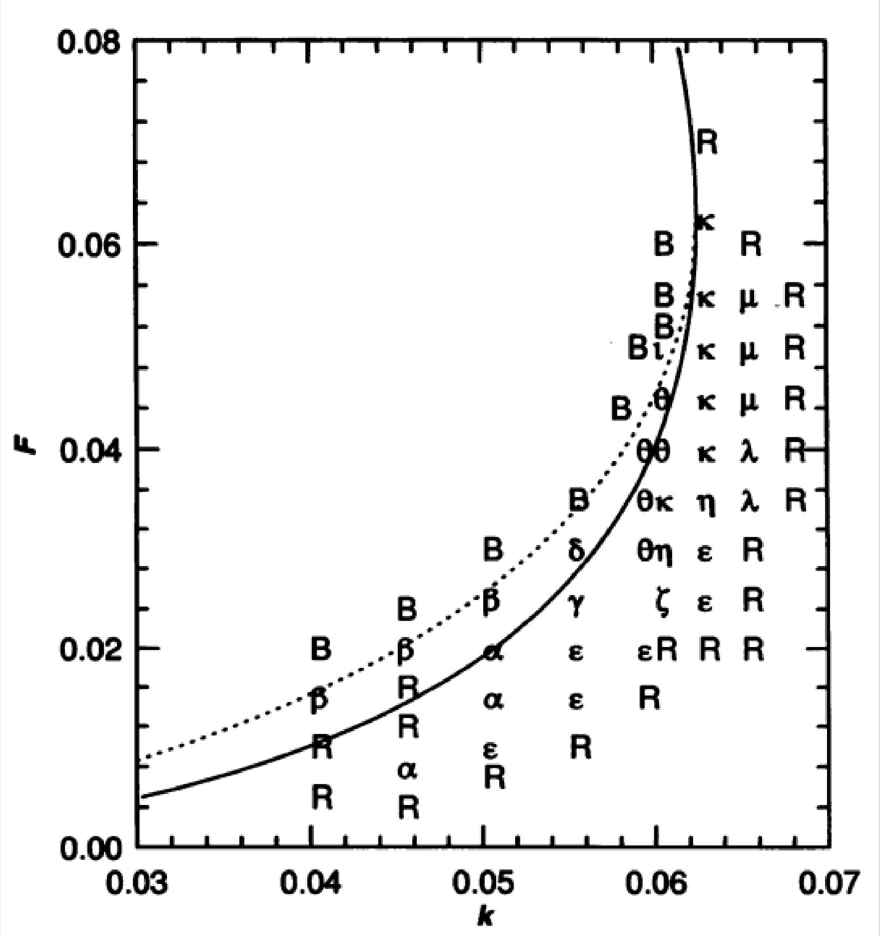


The color : the concentration of U
Red : U = 1 , Blue :U ~ 0.2 , Yellow : intermediate to red and blue

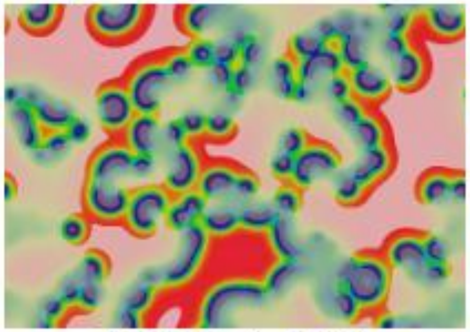
Greek characters indicate the pattern found at that point in parameter space.

R and B : indicating spatially uniform red and blue states, respectively.

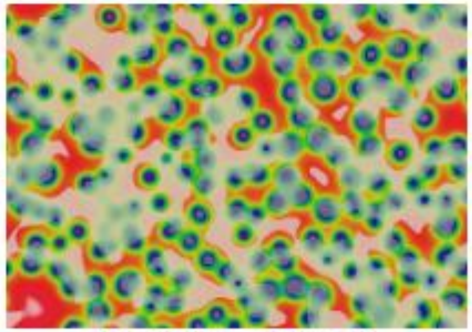
Red state : (U = 1,V = 0)
Blue : (U = 0.3,V = 0.25)



Type alpha (α)

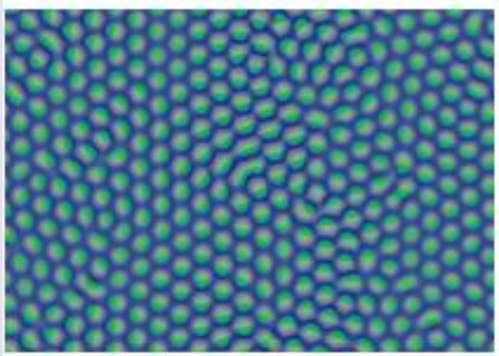


(*F*=0.010, *k*=0.047)

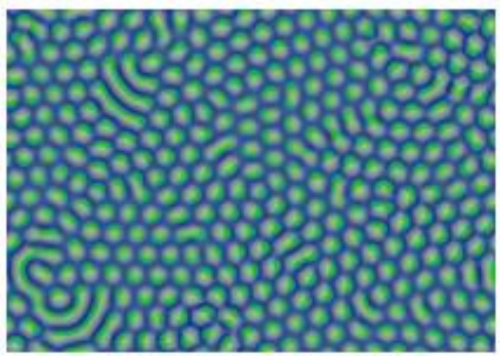


(*F*=0.014, *k*=0.053)

Type delta (δ)



(*F*=0.030, *k*=0.055)



(*F*=0.042, *k*=0.059)

The Two Parameters *F* and *k*
: Describes the parameters *F* and *k* as "feed rate" and "kill rate".

Feed-Rate Spectrum: Stability/Oscillation/Chaos
Varying the parameter *F* while keeping *k* constant causes the system to move vertically
in the parameter map at the top of this page.
Varying *k* in such a way as to follow the "crescent-shaped"
contour will reveal a spectrum of phenomena distinguished
by the amount and type, if any, of oscillation.

Kill-Rate Spectrum: Soliton Type and Shape
Varying just the parameter *k* while keeping *F* constant reveals a second spectrum
distinguished by the presence or absence of large solid regions, stripes, and/or spots.
This spectrum does not appear in the lower *F* values where the system is too chaotic.

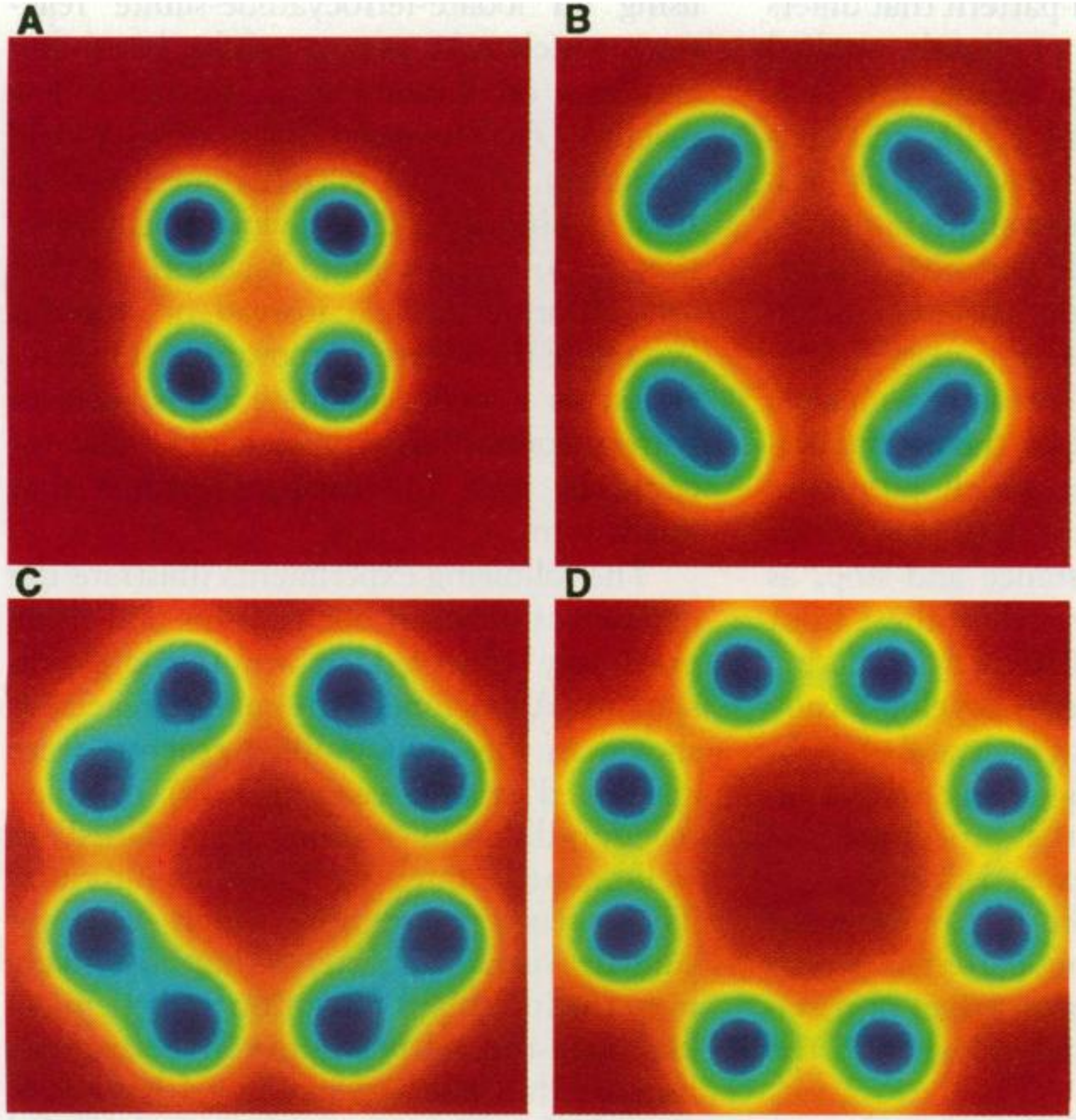


Fig. 4 (left). Time evolution of spot multiplication. This figure was produced in a 256 by 256 simulation with physical dimensions of 0.5 by 0.5 and a time step of 0.01. The times t at which the figures were taken are as follows: (**A**) $t = 0$; (**B**) $t = 350$; (**C**) $t = 510$; and (**D**) $t = 650$.

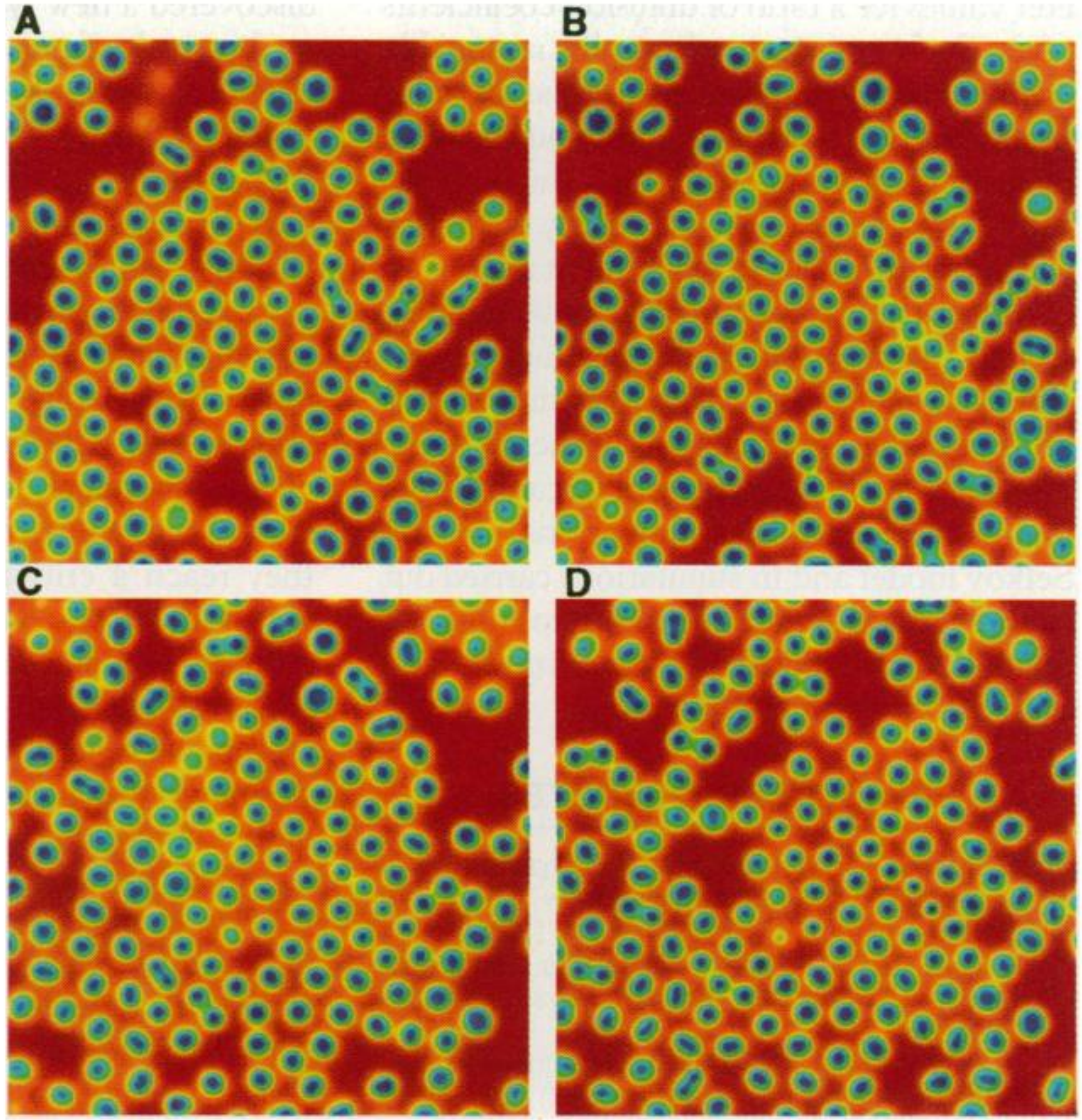


Fig. 5 (right). Time evolution of pattern ε. The images are 250 time units apart. In the corners (which map to the same point in physical space), one can see a yellow region in (**A**) to (**C**). It has decayed to red in (**D**). In (A) and (B), the center of the left border has a red region that is nearly filled in (D).

Turing patterns occur only in a narrow parameter region
in the vicinity of F = k = 0.0625.
In the vicinity of this point,
the branch of small-amplitude Turing patterns is unstable.

With equal diffusion coefficients, no patterns formed in which small
asymmetries in the initial conditions were amplified by the dynamics.
: Nonlinear plane waves in two dimensions cannot be destabilized
by diffusion in the case that all diffusion coefficients are equal.
During the initial stages of the evolution, the corners of the square
perturbation are rounded off.

The perturbation then evolves as a radial wave,
either inward or outward depending on the parameter values.
Such a wave cannot undergo spontaneous symmetry breaking
unless the diffusion coefficients are unequal.
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Reference

John E. Pearson(1993). Complex Patterns in a Simple System