Ten lectures on wavelets

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2.7. Parallels with the continuous windowed Fourier transform
2.8. The continuous transforms as tools to build useful operators
2.9. The continuous wavelet transform as a mathematical zoom : The characterization of local regularity

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Basic Definition

Metric

A metric is a way of measuring distance between two points.

Definition 2.1. *A metric space* $(\mathcal{M}, d)$ *is a set* $\mathcal{M}$ *together with a function* $d : \mathcal{M} \times \mathcal{M} \to \mathbb{R}$ *called a metric satisfying four conditions:*

1. $d(x, y) \geq 0$ *for all* $x, y \in \mathcal{M}$.
2. $d(x, y) = 0$ *if and only if* $x = y$.
3. $d(x, y) = d(y, x)$ *for all* $x, y \in \mathcal{M}$.
4. $d(x, y) \leq d(x, z) + d(z, y)$ *for all* $x, y, z \in \mathcal{M}$.

Basic Definition

ball

Definition 2.2. *Let* $(\mathcal{M}, d)$ *be a metric space. The open* r*-ball centered at* x *is the set* $B_r(x) = \{y \in \mathcal{M} : d(x, y) < r\}$ *for any choice of* $x \in \mathcal{M}$ *and* $r > 0$*. A closed* r*-ball centered at* x *is the set* $\overline{B}_r(x) = \{y \in \mathcal{M} : d(x, y) \leq r\}$.

norm

Definition 2.3. *A (complex) normed linear space* $(\mathcal{V}, \|\cdot\|)$ *is a (complex) linear space* $\mathcal{V}$ *together with a function* $\|\cdot\| : \mathcal{V} \to \mathbb{C}$ *called a norm satisfying the following conditions:*

1. $\|v\| \geq 0$ *for all* $v \in \mathcal{V}$.
2. $\|v\| = 0$ *if and only if* $v = 0$.
3. $\|\lambda v\| = |\lambda| \, \|v\|$ *for all* $v \in \mathcal{V}$ *and* $\lambda \in \mathbb{C}$.
4. $\|v + w\| \leq \|v\| + \|w\|$ *for all* $v, w \in \mathcal{V}$.

References

[1] Harris, Terri Joan. "HILBERT SPACES AND FOURIER SERIES." (2015)
[2] Relationship of Fourier series and Hilbert spaces?
https://math.stackexchange.com/questions/184390/relationship-of-fourier-series-and-hilbert-spaces